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PREFACE

The primary objective of these design examples is to provide illustrations of the use of the 2010 AISC Specification for Structural Steel Buildings (ANSI/AISC 360-10) and the 14th Edition of the AISC Steel Construction Manual. The design examples provide coverage of all applicable limit states whether or not a particular limit state controls the design of the member or connection.

In addition to the examples which demonstrate the use of the Manual tables, design examples are provided for connection designs beyond the scope of the tables in the Manual. These design examples are intended to demonstrate an approach to the design, and are not intended to suggest that the approach presented is the only approach. The committee responsible for the development of these design examples recognizes that designers have alternate approaches that work best for them and their projects. Design approaches that differ from those presented in these examples are considered viable as long as the Specification, sound engineering, and project specific requirements are satisfied.

Part I of these examples is organized to correspond with the organization of the Specification. The Chapter titles match the corresponding chapters in the Specification.

Part II is devoted primarily to connection examples that draw on the tables from the Manual, recommended design procedures, and the breadth of the Specification. The chapters of Part II are labeled II-A, II-B, II-C, etc.

Part III addresses aspects of design that are linked to the performance of a building as a whole. This includes coverage of lateral stability and second order analysis, illustrated through a four-story braced-frame and moment-frame building.

The Design Examples are arranged with LRFD and ASD designs presented side by side, for consistency with the AISC Manual. Design with ASD and LRFD are based on the same nominal strength for each element so that the only differences between the approaches are which set of load combinations from ASCE/SEI 7-10 are used for design and whether the resistance factor for LRFD or the safety factor for ASD is used.

CONVENTIONS

The following conventions are used throughout these examples:


2. The source of equations or tabulated values taken from the AISC Specification or AISC Manual is noted along the right-hand edge of the page.

3. When the design process differs between LRFD and ASD, the designs equations are presented side-by-side. This rarely occurs, except when the resistance factor, \( \phi \), and the safety factor, \( \Omega \), are applied.

4. The results of design equations are presented to three significant figures throughout these calculations.

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Part I
Examples Based on the AISC Specification

This part contains design examples demonstrating select provisions of the AISC Specification for Structural Steel Buildings.
Chapter A
General Provisions

A1. SCOPE

These design examples are intended to illustrate the application of the 2010 AISC Specification for Structural Steel Buildings (ANSI/AISC 360-10) (AISC, 2010) and the AISC Steel Construction Manual, 14th Edition (AISC, 2011) in low-seismic applications. For information on design applications requiring seismic detailing, see the AISC Seismic Design Manual.

A2. REFERENCED SPECIFICATIONS, CODES AND STANDARDS

Section A2 includes a detailed list of the specifications, codes and standards referenced throughout the AISC Specification.

A3. MATERIAL

Section A3 includes a list of the steel materials that are approved for use with the AISC Specification. The complete ASTM standards for the most commonly used steel materials can be found in Selected ASTM Standards for Structural Steel Fabrication (ASTM, 2011).

A4. STRUCTURAL DESIGN DRAWINGS AND SPECIFICATIONS

Section A4 requires that structural design drawings and specifications meet the requirements in the AISC Code of Standard Practice for Steel Buildings and Bridges (AISC, 2010b).
CHAPTER A REFERENCES


Chapter B
Design Requirements

B1. GENERAL PROVISIONS

B2. LOADS AND LOAD COMBINATIONS

In the absence of an applicable building code, the default load combinations to be used with this Specification are those from Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10) (ASCE, 2010).

B3. DESIGN BASIS

Chapter B of the AISC Specification and Part 2 of the AISC Manual describe the basis of design, for both LRFD and ASD.

This Section describes three basic types of connections: simple connections, fully restrained (FR) moment connections, and partially restrained (PR) moment connections. Several examples of the design of each of these types of connection are given in Part II of these design examples.

Information on the application of serviceability and ponding provisions may be found in AISC Specification Chapter L and AISC Specification Appendix 2, respectively, and their associated commentaries. Design examples and other useful information on this topic are given in AISC Design Guide 3, Serviceability Design Considerations for Steel Buildings, Second Edition (West et al., 2003).

Information on the application of fire design provisions may be found in AISC Specification Appendix 4 and its associated commentary. Design examples and other useful information on this topic are presented in AISC Design Guide 19, Fire Resistance of Structural Steel Framing (Ruddy et al., 2003).

Corrosion protection and fastener compatibility are discussed in Part 2 of the AISC Manual.

B4. MEMBER PROPERTIES

AISC Specification Tables B4.1a and B4.1b give the complete list of limiting width-to-thickness ratios for all compression and flexural members defined by the AISC Specification.

Except for one section, the W-shapes presented in the compression member selection tables as column sections meet the criteria as nonslender element sections. The W-shapes presented in the flexural member selection tables as beam sections meet the criteria for compact sections, except for 10 specific shapes. When noncompact or slender-element sections are tabulated in the design aids, local buckling criteria are accounted for in the tabulated design values.

The shapes listing and other member design tables in the AISC Manual also include footnoting to highlight sections that exceed local buckling limits in their most commonly available material grades. These footnotes include the following notations:

- Shape is slender for compression.
- Shape exceeds compact limit for flexure.
- The actual size, combination and orientation of fastener components should be compared with the geometry of the cross section to ensure compatibility.
- Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
- Shape does not meet the \( h/t_w \) limit for shear in AISC Specification Section G2.1a.
CHAPTER B REFERENCES

ASCE (2010), Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers, Reston, VA.


Chapter C  
Design for Stability

C1. GENERAL STABILITY REQUIREMENTS

The AISC Specification requires that the designer account for both the stability of the structural system as a whole, and the stability of individual elements. Thus, the lateral analysis used to assess stability must include consideration of the combined effect of gravity and lateral loads, as well as member inelasticity, out-of-plumbness, out-of-straightness and the resulting second-order effects, $P-\Delta$ and $P-\delta$. The effects of “leaning columns” must also be considered, as illustrated in the examples in this chapter and in the four-story building design example in Part III of AISC Design Examples.

$P-\Delta$ and $P-\delta$ effects are illustrated in AISC Specification Commentary Figure C-C2.1. Methods for addressing stability, including $P-\Delta$ and $P-\delta$ effects, are provided in AISC Specification Section C2 and Appendix 7.

C2. CALCULATION OF REQUIRED STRENGTHS

The calculation of required strengths is illustrated in the examples in this chapter and in the four-story building design example in Part III of AISC Design Examples.

C3. CALCULATION OF AVAILABLE STRENGTHS

The calculation of available strengths is illustrated in the four-story building design example in Part III of AISC Design Examples.
EXAMPLE C.1A  DESIGN OF A MOMENT FRAME BY THE DIRECT ANALYSIS METHOD

**Given:**

Determine the required strengths and effective length factors for the columns in the rigid frame shown below for the maximum gravity load combination, using LRFD and ASD. Use the direct analysis method. All members are ASTM A992 material.

Columns are unbraced between the footings and roof in the x- and y-axes and are assumed to have pinned bases.

**Solution:**

From *Manual* Table 1-1, the $W_{12\times65}$ has $A = 19.1$ in.$^2$.

The beams from grid lines A to B, and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-$\Delta$ effects to consider in these members and they may be designed using $K=1.0$.

The moment frame between grid lines B and C is the source of lateral stability and therefore must be designed using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the analysis. For the analysis, the entire frame could be modeled or the model can be simplified as shown in the figure below, in which the stability loads from the three “leaning” columns are combined into a single column.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_u = 1.2D + 1.6L$</td>
<td>$w_u = D + L$</td>
</tr>
<tr>
<td>$= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$</td>
<td>$= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$= 2.40 \text{ kip/ft}$</td>
<td>$= 1.60 \text{ kip/ft}$</td>
</tr>
</tbody>
</table>

Per AISC *Specification* Section C2.1, for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis using 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

*Frame Analysis Gravity Loads*

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:
Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{u}^\prime = 2.40 \text{ kip/ft}$</td>
<td>$w_{u}^\prime = 1.6(1.60 \text{ kip/ft}) = 2.56 \text{ kip/ft}$</td>
</tr>
</tbody>
</table>

Concentrated Gravity Loads on the Pseudo “Leaning” Column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{u}^\prime = (15.0 \text{ ft})(2.40 \text{ kip/ft}) = 36.0 \text{ kips}$</td>
<td>$P_{u}^\prime = 1.6(15.0 \text{ ft})(1.60 \text{ kip/ft}) = 38.4 \text{ kips}$</td>
</tr>
</tbody>
</table>

Frame Analysis Notional Loads

Per AISC Specification Section C2.2, frame out-of-plumbness must be accounted for either by explicit modeling of the assumed out-of-plumbness or by the application of notional loads. Use notional loads.

From AISC Specification Equation C2-1, the notional loads are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1.0$</td>
<td>$\alpha = 1.6$</td>
</tr>
<tr>
<td>$Y_{i} = (120 \text{ ft})(2.40 \text{ kip/ft}) = 288 \text{ kips}$</td>
<td>$Y_{i} = (120 \text{ ft})(1.60 \text{ kip/ft}) = 192 \text{ kips}$</td>
</tr>
<tr>
<td>$N_{i} = 0.002\alpha Y_{i} = 0.002(1.0)(288 \text{ kips}) = 0.576 \text{ kips}$</td>
<td>$N_{i} = 0.002\alpha Y_{i} = 0.002(1.6)(192 \text{ kips}) = 0.614 \text{ kips}$</td>
</tr>
</tbody>
</table>

Summary of Applied Frame Loads
Per AISC Specification Section C2.3, conduct the analysis using 80% of the nominal stiffnesses to account for the effects of inelasticity. Assume, subject to verification, that $\alpha P_y/P_y$ is no greater than 0.5; therefore, no additional stiffness reduction is required.

50% of the gravity load is carried by the columns of the moment resisting frame. Because the gravity load supported by the moment resisting frame columns exceeds one third of the total gravity load tributary to the frame, per AISC Specification Section C2.1, the effects of $P-\Delta$ upon $P-\Delta$ must be included in the frame analysis. If the software used does not account for $P-\Delta$ effects in the frame analysis, this may be accomplished by adding joints to the columns between the footing and beam.

Using analysis software that accounts for both $P-\Delta$ and $P-\Delta$ effects, the following results are obtained:

First-order results

<table>
<thead>
<tr>
<th>$\Delta_{1st}$</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.149 in.</td>
<td>135 kip-ft</td>
<td>146 kip-ft</td>
</tr>
<tr>
<td>6.73 kips</td>
<td>71.6 kips</td>
<td>7.31 kips</td>
</tr>
<tr>
<td>7.31 kips</td>
<td>72.4 kips</td>
<td>4.87 kips</td>
</tr>
</tbody>
</table>

Second-order results

<table>
<thead>
<tr>
<th>$\Delta_{2nd}$</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.217 in.</td>
<td>132 kip-ft</td>
<td>149 kip-ft</td>
</tr>
<tr>
<td>6.66 kips</td>
<td>71.4 kips</td>
<td>7.36 kips</td>
</tr>
<tr>
<td>7.36 kips</td>
<td>72.6 kips</td>
<td>4.91 kips</td>
</tr>
</tbody>
</table>

Check the assumption that $\alpha P_y/P_y \leq 0.5$ and therefore, $\tau_b = 1.0$: 

\[ \frac{\Delta_{2nd}}{\Delta_{1st}} = \frac{0.217}{0.149} = 1.46 \]

\[ \frac{\Delta_{2nd}}{\Delta_{1st}} = \frac{0.239}{0.159} = 1.50 \]


\[ P_y = F_y A_y \]

\[ = 50 \text{ ksi}(19.1 \text{ in.}^2) \]

\[ = 955 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{\alpha P_y}{P_y} = \frac{1.0(72.6\text{kips})}{955\text{kips}} ]</td>
<td>[ \frac{\alpha P_y}{P_y} = \frac{1.6(48.4\text{kips})}{955\text{kips}} ]</td>
</tr>
<tr>
<td>= 0.0760 ≤ 0.5</td>
<td>= 0.0811 ≤ 0.5</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

The stiffness assumption used in the analysis, \( \tau_b = 1.0 \), is verified.

Although the second-order sway multiplier is approximately 1.5, the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Verify the column strengths using the second-order forces shown above, using the following effective lengths (calculations not shown):

Columns:

- Use \( KL_x = 20.0 \text{ ft} \)
- Use \( KL_y = 20.0 \text{ ft} \)
EXAMPLE C.1B  DESIGN OF A MOMENT FRAME BY THE EFFECTIVE LENGTH METHOD

Repeat Example C.1A using the effective length method.

Given:

Determine the required strengths and effective length factors for the columns in the rigid frame shown below for the maximum gravity load combination, using LRFD and ASD. Use the effective length method.

Columns are unbraced between the footings and roof in the x- and y-axes and are assumed to have pinned bases.

Solution:

From Manual Table 1-1, the W12×65 has \( I_x = 533 \) in.\(^4\).

The beams from grid lines A to B, and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no \( P-\Delta \) effects to consider in these members and they may be designed using \( K = 1.0 \).

The moment frame between grid lines B and C is the source of lateral stability and therefore must be designed using the provisions of Appendix 7 of the AISC Specification. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the analysis. For the analysis, the entire frame could be modeled or the model can be simplified as shown in the figure below, in which the stability loads from the three “leaning” columns are combined into a single column.

Check the limitations for the use of the effective length method given in Appendix 7, Section 7.2.1:

1. The structure supports gravity loads through nominally vertical columns.
2. The ratio of maximum second-order drift to the maximum first-order drift will be assumed to be no greater than 1.5, subject to verification following.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u )</td>
<td>( 1.2D + 1.6L )</td>
<td>( D + L )</td>
</tr>
<tr>
<td></td>
<td>( = 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft}) )</td>
<td>( = 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft} )</td>
</tr>
<tr>
<td></td>
<td>( = 2.40 \text{ kip/ft} )</td>
<td>( = 1.60 \text{ kip/ft} )</td>
</tr>
</tbody>
</table>

Per AISC Specification Appendix 7, Section 7.2.1, the analysis must conform to the requirements of AISC Specification Section C2.1, with the exception of the stiffness reduction required by the provisions of Section C2.3.
Per AISC Specification Section C2.1, for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis at 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame Analysis Gravity Loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u' )</td>
<td>2.40 kip/ft</td>
<td>2.56 kip/ft</td>
</tr>
</tbody>
</table>

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u' )</td>
<td>(15.0 ft)(2.40 kip/ft)</td>
<td>36.0 kips</td>
</tr>
<tr>
<td>( P_a' )</td>
<td>1.6(15.0 ft)(1.60 kip/ft)</td>
<td>38.4 kips</td>
</tr>
</tbody>
</table>

Concentrated Gravity Loads on the Pseudo “Leaning” Column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{ul}' )</td>
<td>(60.0 ft)(2.40 kip/ft)</td>
<td>144 kips</td>
</tr>
<tr>
<td>( P_{al}' )</td>
<td>1.6(60.0 ft)(1.60 kip/ft)</td>
<td>154 kips</td>
</tr>
</tbody>
</table>

Frame Analysis Notional Loads

Per AISC Specification Appendix 7, Section 7.2.2, frame out-of-plumbness must be accounted for by the application of notional loads in accordance with AISC Specification Section C2.2b.

From AISC Specification Equation C2-1, the notional loads are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>1.0</td>
<td>1.6</td>
</tr>
<tr>
<td>( Y_i )</td>
<td>(120 ft)(2.40 kip/ft)</td>
<td>(120 ft)(1.60 kip/ft)</td>
</tr>
<tr>
<td></td>
<td>= 288 kips</td>
<td>= 192 kips</td>
</tr>
<tr>
<td>( N_i )</td>
<td>0.002( \alpha Y_i ) (Spec. Eq. C2-1)</td>
<td>0.002( \alpha Y_i ) (Spec. Eq. C2-1)</td>
</tr>
<tr>
<td></td>
<td>= 0.002(1.0)(288 kips)</td>
<td>= 0.002(1.6)(192 kips)</td>
</tr>
<tr>
<td></td>
<td>= 0.576 kips</td>
<td>= 0.614 kips</td>
</tr>
</tbody>
</table>
Summary of Applied Frame Loads

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.0 kips</td>
<td>38.4 kips</td>
</tr>
<tr>
<td>2.40 kip/ft</td>
<td>2.56 kip/ft</td>
</tr>
<tr>
<td>144 kips</td>
<td>154 kips</td>
</tr>
<tr>
<td>0.576 kip</td>
<td>0.614 kip</td>
</tr>
</tbody>
</table>

Per AISC Specification Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

50% of the gravity load is carried by the columns of the moment resisting frame. Because the gravity load supported by the moment resisting frame columns exceeds one third of the total gravity load tributary to the frame, per AISC Specification Section C 2.1, the effects of P-δ must be included in the frame analysis. If the software used does not account for P-δ effects in the frame analysis, this may be accomplished by adding joints to the columns between the footing and beam.

Using analysis software that accounts for both P-Δ and P-δ effects, the following results are obtained:

First-order results

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ₁ = 0.119 in.</td>
<td>Δ₁ = 0.127 in. (prior to dividing by 1.6)</td>
</tr>
<tr>
<td>135 kip-ft</td>
<td>89.8 kip-ft</td>
</tr>
<tr>
<td>146 kip-ft</td>
<td>97.5 kip-ft</td>
</tr>
<tr>
<td>6.73 kips</td>
<td>4.49 kips</td>
</tr>
<tr>
<td>71.6 kips</td>
<td>47.7 kips</td>
</tr>
<tr>
<td>72.4 kips</td>
<td>48.3 kips</td>
</tr>
<tr>
<td>7.31 kips</td>
<td>4.87 kips</td>
</tr>
</tbody>
</table>
Second-order results

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ_{2nd}</td>
<td>0.159 in.</td>
<td>0.174 in. (prior to dividing by 1.6)</td>
</tr>
<tr>
<td>Δ_{2nd} / Δ_{1st}</td>
<td>0.119 in.</td>
<td>0.127 in.</td>
</tr>
<tr>
<td></td>
<td>1.34</td>
<td>1.37</td>
</tr>
</tbody>
</table>

The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is verified; therefore, the effective length method is permitted.

Although the second-order sway multiplier is approximately 1.35, the change in bending moment is small because the only sway moments for this load combination are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Calculate the in-plane effective length factor, \( K_x \), using the “story stiffness method” and Equation C-A-7-5 presented in Commentary Appendix 7, Section 7.2. Take \( K_x = K_2 \)

\[
K_x = K_2 = \sqrt{\frac{\sum P_r}{(0.85 + 0.15R_L)P_r} \left( \frac{\pi^2EI}{L^2} \right)} \geq \sqrt{\frac{\pi^2EI}{L^2} \left( \frac{\Delta_H}{1.7HL} \right)} \quad \text{(Spec. Eq. C-A-7-5)}
\]

Calculate the total load in all columns, \( \sum P_r \)

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum P_r )</td>
<td>2.40 kip/ft (120 ft)</td>
<td>1.60 kip/ft (120 ft)</td>
</tr>
<tr>
<td></td>
<td>= 288 kips</td>
<td>= 192 kips</td>
</tr>
</tbody>
</table>

Calculate the ratio of the leaning column loads to the total load, \( R_L \)

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_L )</td>
<td>( \frac{\sum P_r - \sum P_r \text{ moment frame}}{\sum P_r} )</td>
<td>( \frac{\sum P_r - \sum P_r \text{ moment frame}}{\sum P_r} )</td>
</tr>
<tr>
<td></td>
<td>= 288 kips – (71.5 kips + 72.5 kips)</td>
<td>= 192 kips – (47.7 kips + 48.3 kips)</td>
</tr>
<tr>
<td></td>
<td>288 kips</td>
<td>192 kips</td>
</tr>
<tr>
<td></td>
<td>= 0.500</td>
<td>= 0.500</td>
</tr>
</tbody>
</table>

Calculate the Euler buckling strength of an individual column.
\[
\frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29,000 \text{ ksi})(533 \text{ in.}^4)}{(240 \text{ in.})^2} = 2,650 \text{ kips}
\]

Calculate the drift ratio using the first-order notional loading results.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\Delta_{H}}{L} = \frac{0.119 \text{ in.}}{240 \text{ in.}} = 0.000496 \text{ in./in.}$</td>
<td>$\frac{\Delta_{H}}{L} = \frac{0.127 \text{ in.}}{240 \text{ in.}} = 0.000529 \text{ in./in.}$</td>
</tr>
</tbody>
</table>

For the column at line C:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_x = \sqrt{\frac{288 \text{ kips}}{[0.85 + 0.15(0.500)](72.4 \text{ kips}) \times (2,650 \text{ kips}) \left(\frac{0.000496 \text{ in./in.}}{0.576 \text{ kips}}\right)}} \geq 3.13 \geq 0.324$</td>
<td>$K_x = \sqrt{\frac{1.6(192 \text{ kips})}{<a href="1.6">0.85 + 0.15(0.500)</a>(48.3 \text{ kips}) \times (2,650 \text{ kips}) \left(\frac{0.000529 \text{ in./in.}}{0.614 \text{ kips}}\right)}} \geq 3.13 \geq 0.324$</td>
</tr>
</tbody>
</table>

Use $K_x = 3.13$

Note that it is necessary to multiply the column loads by 1.6 for ASD in the expression above.

Verify the column strengths using the second-order forces shown above, using the following effective lengths (calculations not shown):

Columns:
- Use $K_xL_x = 3.13(20.0 \text{ ft}) = 62.6 \text{ ft}$
- Use $K_xL_y = 20.0 \text{ ft}$
EXAMPLE C.1C  DESIGN OF A MOMENT FRAME BY THE FIRST-ORDER METHOD

Repeat Example C.1A using the first-order analysis method.

Given:

Determine the required strengths and effective length factors for the columns in the rigid frame shown below for the maximum gravity load combination, using LRFD and ASD. Use the first-order analysis method.

Columns are unbraced between the footings and roof in the x- and y-axes and are assumed to have pinned bases.

Solution:

From Manual Table 1-1, the W12x65 has $A = 19.1$ in.$^2$

The beams from grid lines A to B, and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no $P$-$\Delta$ effects to consider in these members and they may be designed using $K=1.0$.

The moment frame between grid lines B and C is the source of lateral stability and therefore must be designed using the provisions of Appendix 7 of the AISC Specification. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the analysis. These members need not be included in the analysis model, except that the forces in the “leaning” columns must be included in the calculation of notional loads.

Check the limitations for the use of the first-order analysis method given in Appendix 7, Section 7.3.1:

1. The structure supports gravity loads through nominally vertical columns.
2. The ratio of maximum second-order drift to the maximum first-order drift will be assumed to be no greater than 1.5, subject to verification.
3. The required axial strength of the members in the moment frame will be assumed to be no more than 50% of the axial yield strength, subject to verification.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_u$</td>
<td>$1.2D + 1.6L$</td>
<td>$D + L$</td>
</tr>
<tr>
<td></td>
<td>$= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$</td>
<td>$= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$</td>
</tr>
<tr>
<td></td>
<td>$= 2.40 \text{ kip/ft}$</td>
<td>$= 1.60 \text{ kip/ft}$</td>
</tr>
</tbody>
</table>
Per AISC Specification Appendix 7, Section 7.3.2, the required strengths are determined from a first-order analysis using notional loads determined below along with a $B_1$ multiplier as determined from Appendix 8.

For ASD, do not multiply loads or divide results by 1.6.

**Frame Analysis Gravity Loads**

The uniform gravity loads to be considered in the first-order analysis on the beam from B to C are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_u' = 2.40$ kip/ft</td>
<td>$w_u' = 1.60$ kip/ft</td>
</tr>
</tbody>
</table>

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u' = (15.0$ ft)$2.40$ kip/ft) = 36.0 kips</td>
<td>$P_u' = (15.0$ ft)$1.60$ kip/ft) = 24.0 kips</td>
</tr>
</tbody>
</table>

**Frame Analysis Notional Loads**

Per AISC Specification Appendix 7, Section 7.3.2, frame out-of-plumbness must be accounted for by the application of notional loads.

From AISC Specification Appendix Equation A-7-2, the required notional loads are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1.0$</td>
<td>$\alpha = 1.6$</td>
</tr>
<tr>
<td>$Y_i = (120$ ft)$2.40$ kip/ft) = 288 kips</td>
<td>$Y_i = (120$ ft)$1.60$ kip/ft) = 192 kips</td>
</tr>
<tr>
<td>$\Delta = 0.0$ in. (no drift in this load combination)</td>
<td>$\Delta = 0.0$ in. (no drift in this load combination)</td>
</tr>
<tr>
<td>$L = 240$ in.</td>
<td>$L = 240$ in.</td>
</tr>
<tr>
<td>$N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_i$</td>
<td>$N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_i$</td>
</tr>
<tr>
<td>$= 2.1(1.0)(0.0$ in./240 in.)(288 kips) $\geq 0.0042(288$ kips)</td>
<td>$= 2.1(1.6)(0.0$ in./240 in.)(192 kips) $\geq 0.0042(192$ kips)</td>
</tr>
<tr>
<td>$= 0.0$ kips $\geq 1.21$ kips</td>
<td>$= 0.0$ kips $\geq 0.806$ kips</td>
</tr>
<tr>
<td>Use $N_i = 1.21$ kips</td>
<td>Use $N_i = 0.806$ kips</td>
</tr>
</tbody>
</table>
Summary of Applied Frame Loads

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="LRFD Diagram" /></td>
<td><img src="image2" alt="ASD Diagram" /></td>
</tr>
</tbody>
</table>

Per AISC Specification Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

Using analysis software, the following first-order results are obtained:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta_{1st} = 0.250) in.</td>
<td>(\Delta_{1st} = 0.167) in.</td>
</tr>
</tbody>
</table>

Check the assumption that the ratio of the second-order drift to the first-order drift does not exceed 1.5. \(B_2\) can be used to check this limit. Calculate \(B_2\) per the provisions of Section 8.2.2 of Appendix 8 using the results of the first-order analysis.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{mf} = 71.2) kips + 72.8 kips (= 144) kips</td>
<td>(P_{mf} = 47.5) kips + 48.5 kips (= 96) kips</td>
</tr>
<tr>
<td>(P_{story} = 144) kips + 4(36 kips) (= 288) kips</td>
<td>(P_{story} = 96) kips + 4(24 kips) (= 192) kips</td>
</tr>
<tr>
<td>(R_M = 1 - 0.15\left(\frac{P_{mf}}{P_{story}}\right)) (\text{Spec. Eq. A-8-8}) (= 1 - 0.15\left(\frac{144\text{kips}}{288\text{kips}}\right)) (= 0.925)</td>
<td>(R_M = 1 - 0.15\left(\frac{P_{mf}}{P_{story}}\right)) (\text{Spec. Eq. A-8-8}) (= 1 - 0.15\left(\frac{96\text{kips}}{192\text{kips}}\right)) (= 0.925)</td>
</tr>
<tr>
<td>(\Delta_H = 0.250) in.</td>
<td>(\Delta_H = 0.167) in.</td>
</tr>
<tr>
<td>(H = 1.21) kips</td>
<td>(H = 0.806) kips</td>
</tr>
<tr>
<td>(L = 240) in.</td>
<td>(L = 240) in.</td>
</tr>
</tbody>
</table>
The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is correct; therefore, the first-order analysis method is permitted.

Check the assumption that $\alpha P_y \leq 0.5 P_y$ and therefore, the first-order analysis method is permitted.

\[
0.5P_y = 0.5 F_y A_g = 0.5(50 \text{ ksi})(19.1 \text{ in.}^2) = 478 \text{ kips}
\]

The assumption that the first-order analysis method can be used is verified.

Although the second-order sway multiplier is approximately 1.4, the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Verify the column strengths using the second-order forces, using the following effective lengths (calculations not shown):

Columns:
- Use $KL_x = 20.0 \text{ ft}$
- Use $KL_y = 20.0 \text{ ft}$
Chapter D
Design of Members for Tension

D1. SLENDERNESS LIMITATIONS

Section D1 does not establish a slenderness limit for tension members, but recommends limiting $L/r$ to a maximum of 300. This is not an absolute requirement. Rods and hangers are specifically excluded from this recommendation.

D2. TENSILE STRENGTH

Both tensile yielding strength and tensile rupture strength must be considered for the design of tension members. It is not unusual for tensile rupture strength to govern the design of a tension member, particularly for small members with holes or heavier sections with multiple rows of holes.

For preliminary design, tables are provided in Part 5 of the AISC Manual for W-shapes, L-shapes, WT-shapes, rectangular HSS, square HSS, round HSS, Pipe and 2L-shapes. The calculations in these tables for available tensile rupture strength assume an effective area, $A_e$, of $0.75A_g$. If the actual effective area is greater than $0.75A_g$, the tabulated values will be conservative and calculations can be performed to obtain higher available strengths. If the actual effective area is less than $0.75A_g$, the tabulated values will be unconservative and calculations are necessary to determine the available strength.

D3. EFFECTIVE NET AREA

The gross area, $A_g$, is the total cross-sectional area of the member.

In computing net area, $A_n$, AISC Specification Section B4.3 requires that an extra $\frac{1}{8}$ in. be added to the bolt hole diameter.

A computation of the effective area for a chain of holes is presented in Example D.9.

Unless all elements of the cross section are connected, $A_e = A_nU$, where $U$ is a reduction factor to account for shear lag. The appropriate values of $U$ can be obtained from Table D3.1 of the AISC Specification.

D4. BUILT-UP MEMBERS

The limitations for connections of built-up members are discussed in Section D4 of the AISC Specification.

D5. PIN-CONNECTED MEMBERS

An example of a pin-connected member is given in Example D.7.

D6. EYEBARS

An example of an eyebar is given in Example D.8. The strength of an eyebar meeting the dimensional requirements of AISC Specification Section D6 is governed by tensile yielding of the body.
EXAMPLE D.1  W-SHAPE TENSION MEMBER

Given:

Select an 8-in. W-shape, ASTM A992, to carry a dead load of 30 kips and a live load of 90 kips in tension. The member is 25 ft long. Verify the member strength by both LRFD and ASD with the bolted end connection shown. Verify that the member satisfies the recommended slenderness limit. Assume that connection limit states do not govern.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

\[
\begin{align*}
P_u & = 1.2(30 \text{ kips}) + 1.6(90 \text{ kips}) \\
& = 180 \text{ kips} \\
P_a & = 30 \text{ kips} + 90 \text{ kips} \\
& = 120 \text{ kips}
\end{align*}
\]

From AISC Manual Table 5-1, try a W8×21.

From AISC Manual Table 2-4, the material properties are as follows:

- W8×21
- ASTM A992
- \( F_y = 50 \text{ ksi} \)
- \( F_u = 65 \text{ ksi} \)

From AISC Manual Tables 1-1 and 1-8, the geometric properties are as follows:

- W8×21
- \( A_g = 6.16 \text{ in.}^2 \)
- \( b_f = 5.27 \text{ in.} \)
- \( t_f = 0.400 \text{ in.} \)
- \( d = 8.28 \text{ in.} \)
- \( r_y = 1.26 \text{ in.} \)

- WT4×10.5
- \( \bar{y} = 0.831 \text{ in.} \)

**Tensile Yielding**

From AISC Manual Table 5-1, the tensile yielding strength is:

\[
\begin{align*}
\text{LRFD} & : 277 \text{ kips} > 180 \text{ kips} \quad \text{o.k.} \\
\text{ASD} & : 184 \text{ kips} > 120 \text{ kips} \quad \text{o.k.}
\end{align*}
\]
**Tensile Rupture**

Verify the table assumption that \( A_e/A_g \geq 0.75 \) for this connection.

Calculate the shear lag factor, \( U \), as the larger of the values from AISC Specification Section D3, Table D3.1 case 2 and case 7.

From AISC Specification Section D3, for open cross sections, \( U \) need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

\[
U = \frac{2b_f t_f}{A_g} = \frac{2(5.27 \text{ in.})(0.400 \text{ in.})}{6.16 \text{ in.}^2} = 0.684
\]

Case 2: Check as two WT-shapes per AISC Specification Commentary Figure C-D3.1, with \( \bar{x} = \bar{y} = 0.831 \text{ in.} \)

\[
U = 1 - \frac{\bar{x}}{l} = 1 - \frac{0.831 \text{ in}}{9.00 \text{ in.}} = 0.908
\]

Case 7:

\[
b_f = 5.27 \text{ in.} \\
d = 8.28 \text{ in.} \\
b_f < \frac{3}{8}d \\
U = 0.85
\]

Use \( U = 0.908 \).

Calculate \( A_n \) using AISC Specification Section B4.3.

\[
A_n = A_g - 4(\frac{1}{16} \text{ in.})t_f = 6.16 \text{ in.}^2 - 4(0.0625 \text{ in.} + \frac{1}{16} \text{ in.})(0.400 \text{ in.}) = 4.76 \text{ in.}^2
\]

Calculate \( A_e \) using AISC Specification Section D3.

\[
A_e = A_n U = 4.76 \text{ in.}^2(0.908) = 4.32 \text{ in.}^2
\]

\[
\frac{A_e}{A_g} = \frac{4.32 \text{ in.}^2}{6.16 \text{ in.}^2} = 0.701 < 0.75; \text{ therefore, table values for rupture are not valid.}
\]

The available tensile rupture strength is,
\[ P_a = F_u A_e \]  
\[ = 65 \text{ ksi}(4.32 \text{ in.}^2) \]  
\[ = 281 \text{ kips} \]  

From AISC *Specification* Section D2, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_t = 0.75 )</td>
<td>( \Omega_t = 2.00 )</td>
</tr>
<tr>
<td>( \phi_t P_a = 0.75(281 \text{ kips}) = 211 \text{ kips} )</td>
<td>( P_a = 281 \text{ kips} )</td>
</tr>
<tr>
<td>211 kips &gt; 180 kips</td>
<td>141 kips &gt; 120 kips</td>
</tr>
</tbody>
</table>

\( \text{O.K.} \)

*Check Recommended Slenderness Limit*

\[ \frac{L}{r} = \left( \frac{25.0 \text{ ft}}{1.26 \text{ in.}} \right) \left( \frac{12.0 \text{ in.}}{\text{ft}} \right) \]

\[ = 238 < 300 \text{ from AISC } \text{*Specification*} \text{ Section D1} \text{ O.K.} \]

The W8\( \times \)21 available tensile strength is governed by the tensile rupture limit state at the end connection.

See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.2 SINGLE ANGLE TENSION MEMBER

Given:
Verify, by both ASD and LRFD, the tensile strength of an L4×4 ½, ASTM A36, with one line of (4) ¾-in.-diameter bolts in standard holes. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Calculate at what length this tension member would cease to satisfy the recommended slenderness limit. Assume that connection limit states do not govern.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

L4×4 ½
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC Manual Table 1-7, the geometric properties are as follows:

L4×4 ½
A_g = 3.75 in.²
r_z = 0.776 in.
\( \overline{y} = 1.18 \text{ in.} = \overline{x} \)

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips}) = 120 \text{ kips} )</td>
<td>( P_a = 20 \text{ kips} + 60 \text{ kips} = 80.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Tensile Yielding

\( P_n = F_y A_g \)
\( = 36 \text{ ksi}(3.75 \text{ in.}²) \)
\( = 135 \text{ kips} \)

From AISC Specification Section D2, the available tensile yielding strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega_t = 1.67 )</td>
</tr>
</tbody>
</table>
| \( \phi P_n = 0.90(135 \text{ kips}) = 122 \text{ kips} \) | \( P_n = 135 \text{ kips} \)
| | \( \Omega_t = 1.67 \) |
| | \( \frac{P_n}{\Omega_t} = 80.8 \text{ kips} \) |
Tensile Rupture

Calculate $U$ as the larger of the values from AISC Specification Section D3, Table D3.1 Case 2 and Case 8.

From AISC Specification Section D3, for open cross sections, $U$ need not be less than the ratio of the gross area of the connected element(s) to the member gross area, therefore,

$$U = 0.500$$

Case 2:

$$U = 1 - \frac{x}{l}$$

$$= 1 - \frac{1.18 \text{ in.}}{9.00 \text{ in.}}$$

$$= 0.869$$

Case 8, with 4 or more fasteners per line in the direction of loading:

$$U = 0.80$$

Use $U = 0.869$.

Calculate $A_n$ using AISC Specification Section B4.3.

$$A_n = A - (d_h + \frac{h}{16})t$$

$$= 3.75 \text{ in.}^2 - (\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{1}{2} \text{ in.})$$

$$= 3.31 \text{ in.}^2$$

Calculate $A_e$ using AISC Specification Section D3.

$$A_e = A_nU$$

$$= 3.31 \text{ in.}^2(0.869)$$

$$= 2.88 \text{ in.}^2$$

$$P_n = F_aA_e$$

$$= 58 \text{ ksi}(2.88 \text{ in.}^2)$$

$$= 167 \text{ kips}$$

From AISC Specification Section D2, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o P_n = 0.75 (167 \text{ kips}) = 125 \text{ kips}$</td>
<td>$\Omega_o = 2.00$</td>
</tr>
<tr>
<td>$\phi_o P_n = 125 \text{ kips}$</td>
<td>$P_n = 167 \text{ kips}$</td>
</tr>
<tr>
<td>$\Omega_o = 2.00$</td>
<td>$83.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

The L4×4×\(\frac{1}{2}\) available tensile strength is governed by the tensile yielding limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o P_n = 122 \text{ kips}$</td>
<td>$P_n = 80.8 \text{ kips}$</td>
</tr>
</tbody>
</table>
| 122 kips > 120 kips | $\Omega_o = 80.8 \text{ kips > 80.0 kips}$ | o.k. | o.k.
Recommended $L_{\text{max}}$

Using AISC Specification Section D1:

$$L_{\text{max}} = 300r_z$$

$$= (300)(0.776\text{in.}) \left(\frac{\text{ft}}{12.0\text{ in.}}\right)$$

$$= 19.4\text{ ft}$$

Note: The $L/r$ limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.3  WT-SHAPE TENSION MEMBER

Given:

A WT6×20, ASTM A992 member has a length of 30 ft and carries a dead load of 40 kips and a live load of 120 kips in tension. The end connection is fillet welded on each side for 16 in. Verify the member tensile strength by both LRFD and ASD. Assume that the gusset plate and the weld are satisfactory.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

WT6×20
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC Manual Table 1-8, the geometric properties are as follows:

WT6×20
A_g = 5.84 in.²
b_f = 8.01 in.
t_f = 0.515 in.
r_x = 1.57 in.
\( \bar{y} = 1.09 \text{ in.} \) = \( \bar{x} \) (in equation for \( U \))

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips}) = 240 \text{ kips} )</td>
<td>( P_a = 40 \text{ kips} + 120 \text{ kips} = 160 \text{ kips} )</td>
</tr>
</tbody>
</table>

Tensile Yielding

Check tensile yielding limit state using AISC Manual Table 5-3.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_a = 263 \text{ kips} &gt; 240 \text{ kips} ) ( \text{ o.k.} )</td>
<td>( \frac{P_a}{\Omega_t} = 175 \text{ kips} &gt; 160 \text{ kips} ) ( \text{ o.k.} )</td>
</tr>
</tbody>
</table>
Tensile Rupture

Check tensile rupture limit state using AISC Manual Table 5-3.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_n = 214 \text{ kips} &lt; 240 \text{ kips}$</td>
<td>n.g.</td>
</tr>
<tr>
<td>$\frac{P_n}{\Omega_t} = 142 \text{ kips} &lt; 160 \text{ kips}$</td>
<td>n.g.</td>
</tr>
</tbody>
</table>

The tabulated available rupture strengths may be conservative for this case; therefore, calculate the exact solution.

Calculate $U$ as the larger of the values from AISC Specification Section D3 and Table D3.1 case 2.

From AISC Specification Section D3, for open cross-sections, $U$ need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$U = \frac{b f t f}{A_g}$$

$$= \frac{8.01 \text{ in.}(0.515 \text{ in.})}{5.84 \text{ in.}^2}$$

$$= 0.706$$

Case 2:

$$U = 1 - \frac{\bar{X}}{l}$$

$$= 1 - \frac{1.09 \text{ in.}}{16.0 \text{ in.}}$$

$$= 0.932$$

Use $U = 0.932$.

Calculate $A_n$ using AISC Specification Section B4.3.

$$A_n = A_g \quad \text{(because there are no reductions due to holes or notches)}$$

$$= 5.84 \text{ in.}^2$$

Calculate $A_e$ using AISC Specification Section D3.

$$A_e = A_n U \quad \text{(Spec. Eq. D3-1)}$$

$$= 5.84 \text{ in.}^2(0.932)$$

$$= 5.44 \text{ in.}^2$$

Calculate $P_n$.

$$P_n = F_{0v} A_e$$

$$= 65 \text{ ksi}(5.44 \text{ in.}^2)$$

$$= 354 \text{ kips}$$
From AISC *Specification* Section D2, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t = 0.75$</td>
<td>$\Omega_t = 2.00$</td>
</tr>
<tr>
<td>$\phi_tP_n = 0.75(354 \text{ kips}) = 266 \text{ kips}$</td>
<td>$P_n = 354 \text{ kips}$</td>
</tr>
<tr>
<td>266 kips $&gt; 240$ kips</td>
<td>$\Omega_t = 2.00$</td>
</tr>
<tr>
<td>$\Omega_t = 177 \text{ kips}$</td>
<td>$= 177 \text{ kips}$</td>
</tr>
<tr>
<td>$177 \text{ kips} &gt; 160 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>o.k.</td>
<td></td>
</tr>
</tbody>
</table>

Alternately, the available tensile rupture strengths can be determined by modifying the tabulated values. The available tensile rupture strengths published in the tension member selection tables are based on the assumption that $A_e = 0.75A_g$. The actual available strengths can be determined by adjusting the values from AISC *Manual* Table 5-3 as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_tP_n = 214 \text{ kips} \left( \frac{A_e}{0.75A_g} \right)$</td>
<td>$P_n = 142 \text{ kips} \left( \frac{A_e}{0.75A_g} \right)$</td>
</tr>
<tr>
<td>$= 214 \text{ kips} \left( \frac{5.44 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)} \right)$</td>
<td>$= 142 \text{ kips} \left( \frac{5.44 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)} \right)$</td>
</tr>
<tr>
<td>$= 266 \text{ kips}$</td>
<td>$= 176 \text{ kips}$</td>
</tr>
<tr>
<td>266 kips $&gt; 240$ kips</td>
<td></td>
</tr>
<tr>
<td>266 kips $&gt; 240$ kips</td>
<td></td>
</tr>
<tr>
<td>176 kips $&gt; 160$ kips</td>
<td></td>
</tr>
<tr>
<td>176 kips $&gt; 160$ kips</td>
<td></td>
</tr>
<tr>
<td>176 kips $&gt; 160$ kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

The WT6×20 available tensile strength is governed by the tensile yielding limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_tP_n = 263 \text{ kips}$</td>
<td>$P_n = 175 \text{ kips}$</td>
</tr>
<tr>
<td>263 kips $&gt; 240$ kips</td>
<td>$\Omega_t = 175 \text{ kips}$</td>
</tr>
<tr>
<td>263 kips $&gt; 240$ kips</td>
<td></td>
</tr>
<tr>
<td>175 kips $&gt; 160$ kips</td>
<td></td>
</tr>
<tr>
<td>175 kips $&gt; 160$ kips</td>
<td></td>
</tr>
<tr>
<td>175 kips $&gt; 160$ kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Recommended Slenderness Limit**

$$\frac{L}{r} = \left( \frac{30.0 \text{ ft}}{1.57 \text{ in.}} \right) \left( \frac{12.0 \text{ in.}}{1.57 \text{ in.}} \right)$$

$$= 229 < 300 \text{ from AISC Specification Section D1} \quad \text{o.k.}$$

See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.4 RECTANGULAR HSS TENSION MEMBER

Given:
Verify the tensile strength of an HSS6×4×3/8, ASTM A500 Grade B, with a length of 30 ft. The member is carrying a dead load of 35 kips and a live load of 105 kips in tension. The end connection is a fillet welded 1/2-in.-thick single concentric gusset plate with a weld length of 16 in. Assume that the gusset plate and weld are satisfactory.

Solution:
From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B
\[ F_y = 46 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-11, the geometric properties are as follows:

HSS6×4×3/8
\[ A_g = 6.18 \text{ in}^2 \]
\[ r_y = 1.55 \text{ in.} \]
\[ t = 0.349 \text{ in.} \]

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
P_u = 1.2(35 \text{ kips}) + 1.6(105 \text{ kips}) & P_a = 35 \text{ kips} + 105 \text{ kips} \\
= 210 \text{ kips} & = 140 \text{ kips} \\
\hline
\end{array}
\]

Tensile Yielding
Check tensile yielding limit state using AISC Manual Table 5-4.

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi P_a = 256 \text{ kips} > 210 \text{ kips} & \text{o.k.} \\
\hline
\end{array}
\]

Tensile Rupture
Check tensile rupture limit state using AISC Manual Table 5-4.
The tabulated available rupture strengths may be conservative in this case; therefore, calculate the exact solution.

Calculate $U$ from AISC Specification Table D3.1 case 6.

\[
\bar{x} = \frac{B^2 + 2BH}{4(B + H)}
\]

\[
= \frac{(4.00 \text{ in.})^2 + 2(4.00 \text{ in.})(6.00 \text{ in.})}{4(4.00 \text{ in.} + 6.00 \text{ in.})}
\]

\[
= 1.60 \text{ in.}
\]

\[
U = 1 - \frac{x}{l}
\]

\[
= 1 - \frac{1.60 \text{ in.}}{16.0 \text{ in.}}
\]

\[
= 0.900
\]

Allowing for a $\frac{1}{16}$-in. gap in fit-up between the HSS and the gusset plate:

\[
A_n = A_e - 2(t_p + \frac{1}{16} \text{ in.})t
\]

\[
= 6.18 \text{ in.}^2 - 2(\frac{1}{2} \text{ in.} + \frac{1}{16} \text{ in.})(0.349 \text{ in.})
\]

\[
= 5.79 \text{ in.}^2
\]

Calculate $A_e$ using AISC Specification Section D3.

\[
A_e = A_n U
\]

\[
= 5.79 \text{ in.}^2(0.900)
\]

\[
= 5.21 \text{ in.}^2
\]

Calculate $P_n$.

\[
P_n = F_u A_e
\]

\[
= 58 \text{ ksi}(5.21 \text{ in.}^2)
\]

\[
= 302 \text{ kips}
\]

From AISC Specification Section D2, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t P_n = 0.75(302 \text{ kips})$</td>
<td>$\frac{P_n}{\Omega_t} = 302 \text{ kips}$</td>
</tr>
<tr>
<td>$\Omega_t = 2.00$</td>
<td>$151 \text{ kips}$</td>
</tr>
<tr>
<td>$227 \text{ kips} &gt; 210 \text{ kips}$</td>
<td>$151 \text{ kips} &gt; 140 \text{ kips}$</td>
</tr>
</tbody>
</table>

The HSS available tensile strength is governed by the tensile rupture limit state.
Recommended Slenderness Limit

\[
\frac{L}{r} = \left( \frac{30.0 \text{ ft}}{1.55 \text{ in.}} \right) \left( \frac{12.0 \text{ in.}}{\text{ft}} \right)
\]

= 232 < 300 from AISC Specification Section D1 \textbf{o.k.}

See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.5  ROUND HSS TENSION MEMBER

Given:

Verify the tensile strength of an HSS6×0.500, ASTM A500 Grade B, with a length of 30 ft. The member carries a dead load of 40 kips and a live load of 120 kips in tension. Assume the end connection is a fillet welded ⅛-in.-thick single concentric gusset plate with a weld length of 16 in. Assume that the gusset plate and weld are satisfactory.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B

\[ F_y = 42 \text{ ksi} \]

\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-13, the geometric properties are as follows:

HSS6×0.500

\[ A_g = 8.09 \text{ in.}^2 \]

\[ r = 1.96 \text{ in.} \]

\[ t = 0.465 \text{ in.} \]

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

\[
\begin{align*}
\text{LRFD} & : & P_u &= 1.2(40 \text{ kips}) + 1.6(120 \text{ kips}) \\
& &= 240 \text{ kips}
\end{align*}
\]

\[
\begin{align*}
\text{ASD} & : & P_a &= 40 \text{ kips} + 120 \text{ kips} \\
& &= 160 \text{ kips}
\end{align*}
\]

**Tensile Yielding**

Check tensile yielding limit state using AISC Manual Table 5-6.

\[
\begin{align*}
\phi P_u & = 306 \text{ kips} > 240 \text{ kips} & \text{o.k.}
\end{align*}
\]

\[
\begin{align*}
\frac{P_a}{\Omega_t} & = 203 \text{ kips} > 160 \text{ kips} & \text{o.k.}
\end{align*}
\]

**Tensile Rupture**

Check tensile rupture limit state using AISC Manual Table 5-6.
Check that $A_e/A_g \geq 0.75$ as assumed in table.

Determine $U$ from AISC Specification Table D3.1 Case 5.

$L = 16.0$ in.
$D = 6.00$ in.
$L/\sqrt{D} = 2.67 > 1.3$, therefore $U = 1.0$

Allowing for a $\sqrt[6]{2}$-in. gap in fit-up between the HSS and the gusset plate,

$$A_n = A_g - 2(t_p + \sqrt{2}\text{in}.)t$$
$$= 8.09 \text{ in.}^2 - 2(0.500 \text{ in.} + \sqrt{2}\text{in})(0.465 \text{ in.})$$
$$= 7.57 \text{ in.}^2$$

Calculate $A_e$ using AISC Specification Section D3.

$$A_e = A_n U$$
$$= 7.57 \text{ in.}^2 (1.0)$$
$$= 7.57 \text{ in.}^2$$

$$\frac{A_e}{A_g} = \frac{7.57 \text{ in.}^2}{8.09 \text{ in.}^2}$$
$$= 0.936 \geq 0.75 \text{ o.k., but conservative}$$

Calculate $P_n$.

$$P_n = F_u A_e$$
$$= (58 \text{ ksi})(7.57 \text{ in.}^2)$$
$$= 439 \text{ kips}$$

From AISC Specification Section D2, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_n = 264 \text{ kips} &gt; 240 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$P_n/\Omega_t = 176 \text{ kips} &gt; 160 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Recommended Slenderness Limit

$$\frac{L}{r} = \left[\frac{30.0 \text{ ft}}{1.96 \text{ in.}}\right] \left[\frac{12.0 \text{ in.}}{\text{ft}}\right]$$
$$= 184 < 300 \text{ from AISC Specification Section D1} \text{ o.k.}$$
See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.6  DOUBLE ANGLE TENSION MEMBER

Given:

A 2L4×4×½ (3⁄8-in. separation), ASTM A36, has one line of (8) ¾-in.-diameter bolts in standard holes and is 25 ft in length. The double angle is carrying a dead load of 40 kips and a live load of 120 kips in tension. Verify the member tensile strength. Assume that the gusset plate and bolts are satisfactory.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36

$F_y = 36$ ksi

$F_u = 58$ ksi

From AISC Manual Tables 1-7 and 1-15, the geometric properties are as follows:

$L4\times4\times\frac{1}{2}$

$A_g = 3.75$ in.$^2$

$x = 1.18$ in.

$2L4\times4\times\frac{1}{2} (s = \frac{3}{8}$ in.)

$r_y = 1.83$ in.

$r_x = 1.21$ in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

\[
\begin{align*}
P_n &= 1.2(40 \text{ kips}) + 1.6(120 \text{ kips}) \\
&= 240 \text{ kips}
\end{align*}
\]

\[
\begin{align*}
P_n &= 40 \text{ kips} + 120 \text{ kips} \\
&= 160 \text{ kips}
\end{align*}
\]

Tensile Yielding

\[
\begin{align*}
P_n &= F_y A_g \\
&= 36 \text{ ksi}(2)(3.75 \text{ in.}^2) \\
&= 270 \text{ kips}
\end{align*}
\]
From AISC *Specification* Section D2, the available tensile yielding strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t$</td>
<td>0.90</td>
<td>$\Omega_t$</td>
</tr>
<tr>
<td>$\phi_tP_n$</td>
<td>0.90(270 kips)</td>
<td>$\frac{P_n}{\Omega_t}$</td>
</tr>
<tr>
<td></td>
<td>= 243 kips</td>
<td>= 270 kips</td>
</tr>
<tr>
<td></td>
<td>$\Omega_t$</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>= 162 kips</td>
<td></td>
</tr>
</tbody>
</table>

**Tensile Rupture**

Calculate $U$ as the larger of the values from AISC *Specification* Section D3, Table D3.1 case 2 and case 8.

From AISC *Specification* Section D3, for open cross-sections, $U$ need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$U = 0.500$$

Case 2:

$$U = 1 - \frac{x}{l}$$

$$= 1 - \frac{1.18}{21.0}$$

$$= 0.944$$

Case 8, with 4 or more fasteners per line in the direction of loading:

$$U = 0.80$$

Use $U = 0.944$.

Calculate $A_n$ using AISC *Specification* Section B4.3.

$$A_n = A_g - 2(d_h + \frac{1}{16} \text{ in.})t$$

$$= 2(3.75 \text{ in.}^2) - 2(\frac{3}{16} \text{ in.} + \frac{1}{16}\text{ in.})(\frac{1}{2} \text{ in.})$$

$$= 6.63 \text{ in.}^2$$

Calculate $A_e$ using AISC *Specification* Section D3.

$$A_e = A_n U$$

$$= 6.63 \text{ in.}^2(0.944)$$

$$= 6.26 \text{ in.}^2$$

Calculate $P_n$.

$$P_n = F_u A_e$$

$$= 58 \text{ ksi}(6.26 \text{ in.}^2)$$

$$= 363 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile rupture strength is:
The double angle available tensile strength is governed by the tensile yielding limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t = 0.75$</td>
<td>$\Omega_t = 2.00$</td>
</tr>
<tr>
<td>$\phi_t P_n = 0.75(363 \text{ kips})$</td>
<td>$P_n = 363 \text{ kips}$</td>
</tr>
<tr>
<td>= 272 kips</td>
<td>$\Omega_t = 2.00$</td>
</tr>
<tr>
<td>$= 182 \text{ kips}$</td>
<td>$= 162 \text{ kips}$</td>
</tr>
</tbody>
</table>

The double angle available tensile strength is governed by the tensile yielding limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>243 kips &gt; 240 kips</td>
<td>162 kips &gt; 160 kips</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Recommended Slenderness Limit**

$$\frac{L}{r_s} = \left( \frac{25.0 \text{ ft}}{1.21 \text{ in. ft}} \right) \left( \frac{12.0 \text{ in.}}{\text{ft}} \right)$$

$$= 248 < 300 \text{ from AISC Specification Section D1 \ o.k.}$$

Note: From AISC Specification Section D4, the longitudinal spacing of connectors between components of built-up members should preferably limit the slenderness ratio in any component between the connectors to a maximum of 300.

See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.7 PIN-CONNECTED TENSION MEMBER

Given:

An ASTM A36 pin-connected tension member with the dimensions shown as follows carries a dead load of 4 kips and a live load of 12 kips in tension. The diameter of the pin is 1 inch, in a ⅛-in. oversized hole. Assume that the pin itself is adequate. Verify the member tensile strength.

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

Plate
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

The geometric properties are as follows:

$w = 4.25$ in.
$t = 0.500$ in.
$d = 1.00$ in.
$a = 2.25$ in.
$c = 2.50$ in.
$d_h = 1.03$ in.

Check dimensional requirements using AISC Specification Section D5.2.

1. $b_e = 2t + 0.63$ in.
   $= 2(0.500\text{ in.}) + 0.63$ in.
   $= 1.63$ in. ≤ 1.61 in.
   $b_e = 1.61$ in. controls

2. $a \geq 1.33b_e$
   2.25 in. $\geq 1.33(1.61\text{ in.})$
   $= 2.14$ in. o.k.
3. \( w \geq 2h + d \)
   
   \[
   4.25 \text{ in.} \geq 2(1.61 \text{ in.}) + 1.00 \text{ in.} \\
   = 4.22 \text{ in.} \\
   \text{o.k.}
   \]

4. \( c \geq a \)
   
   \[
   2.50 \text{ in.} \geq 2.25 \text{ in.} \\
   \text{o.k.}
   \]

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(4 \text{ kips}) + 1.6(12 \text{ kips}) )</td>
<td>( P_u = 4 \text{ kips} + 12 \text{ kips} )</td>
</tr>
<tr>
<td>= 24.0 kips</td>
<td>= 16.0 kips</td>
</tr>
</tbody>
</table>

**Tensile Rupture**

Calculate the available tensile rupture strength on the effective net area.

\[
\phi_t = 0.75 \\
\phi_t P_n = 0.75(93.4 \text{ kips}) \\
= 70.1 \text{ kips}
\]

From AISC Specification Section D5.1, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_t = 0.75 )</td>
<td>( \phi_t P_n = 0.75(93.4 \text{ kips}) )</td>
</tr>
<tr>
<td>= 70.1 kips</td>
<td>= 93.4 kips</td>
</tr>
</tbody>
</table>

\[
\Omega_t = 2.00 \\
\frac{P_n}{\Omega_t} = 46.7 \text{ kips}
\]

**Shear Rupture**

\[
A_{sf} = 2(\frac{a}{2} + \frac{d}{2}) \\
= 2(0.500 \text{ in.})(2.25 \text{ in.} + (1.00 \text{ in.}/2)) \\
= 2.75 \text{ in.}^2
\]

\[
P_s = 0.6F_{u}A_{sf} \\
= 0.6(58 \text{ ksi})(2.75 \text{ in.}^2) \\
= 95.7 \text{ kips}
\]

From AISC Specification Section D5.1, the available shear rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{sf} = 0.75 )</td>
<td>( \phi_{sf} P_n = 0.75(95.7 \text{ kips}) )</td>
</tr>
<tr>
<td>= 71.8 kips</td>
<td>= 95.7 kips</td>
</tr>
</tbody>
</table>

\[
\Omega_{sf} = 2.00 \\
\frac{P_n}{\Omega_{sf}} = 47.9 \text{ kips}
\]

**Bearing**

\[
A_{pb} = 0.500 \text{ in.}(1.00 \text{ in.}) \\
= 0.500 \text{ in.}^2
\]
\[ R_n = 1.8F_y A_{pb} \]  
\[ = 1.8(36 \text{ ksi})(0.500 \text{ in.}^2) \]  
\[ = 32.4 \text{ kips} \]  

(\text{Spec. Eq. J7-1})

From AISC Specification Section J7, the available bearing strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ \phi = 0.75 \]  
\[ \phi P_n = 0.75(32.4 \text{ kips}) \]  
\[ = 24.3 \text{ kips} \] | \[ \Omega = 2.00 \]  
\[ P_n = 32.4 \text{ kips} \]  
\[ \Omega = 2.00 \]  
\[ = 16.2 \text{ kips} \] |

(\text{Spec. Eq. J7-1})

**Tensile Yielding**

\[ A_g = wt \]  
\[ = 4.25 \text{ in.} (0.500 \text{ in.}) \]  
\[ = 2.13 \text{ in.}^2 \]

\[ P_n = F_y A_g \]  
\[ = 36 \text{ ksi} (2.13 \text{ in.}^2) \]  
\[ = 76.7 \text{ kips} \]  

(\text{Spec. Eq. D2-1})

From AISC Specification Section D2, the available tensile yielding strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ \phi_t = 0.90 \]  
\[ \phi_t P_n = 0.90(76.7 \text{ kips}) \]  
\[ = 69.0 \text{ kips} \] | \[ \Omega_t = 1.67 \]  
\[ P_n = 76.7 \text{ kips} \]  
\[ \Omega_t = 1.67 \]  
\[ = 45.9 \text{ kips} \] |

The available tensile strength is governed by the bearing strength limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ \phi P_n = 24.3 \text{ kips} \]  
\[ 24.3 \text{ kips} > 24.0 \text{ kips} \] | \[ \frac{P_n}{\Omega} \]  
\[ = 16.2 \text{ kips} \]  
\[ \frac{P_n}{\Omega} \]  
\[ = 16.2 \text{ kips} > 16.0 \text{ kips} \] |

(\text{o.k.})

See Example J.6 for an illustration of the limit state calculations for a pin in a drilled hole.
EXAMPLE D.8  EYEBAR TENSION MEMBER

Given:

A ½-in.-thick, ASTM A36 eyebar member as shown, carries a dead load of 25 kips and a live load of 15 kips in tension. The pin diameter, \( d \), is 3 in. Verify the member tensile strength.

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

- Plate
  - ASTM A36
  - \( F_y = 36 \text{ ksi} \)
  - \( F_u = 58 \text{ ksi} \)

The geometric properties are as follows:

- \( w = 3.00 \text{ in.} \)
- \( b = 2.23 \text{ in.} \)
- \( t = \frac{3}{8} \text{ in.} \)
- \( d_{head} = 7.50 \text{ in.} \)
- \( d = 3.00 \text{ in.} \)
- \( d_h = 3.03 \text{ in.} \)
- \( R = 8.00 \text{ in.} \)

Check dimensional requirements using AISC Specification Section D6.1 and D6.2.

1. \( t \geq \frac{1}{2} \text{ in.} \)
   \[ \frac{3}{8} \text{ in.} \geq \frac{1}{2} \text{ in.} \]
   **o.k.**

2. \( w \leq 8t \)
   \[ 3.00 \text{ in.} \leq 8(\frac{3}{8} \text{ in.}) \]
   \[ = 5.00 \text{ in.} \]
   **o.k.**
3. \( d \geq \frac{3}{8} w \)
   
   \[
   \begin{align*}
   3.00 \text{ in.} & \geq \frac{3}{8}(3.00 \text{ in.}) \\
   & = 2.63 \text{ in.} \quad \text{o.k.}
   \end{align*}
   \]

4. \( d_h \leq d + \frac{1}{8}w \)
   
   \[
   \begin{align*}
   3.03 \text{ in.} & \leq 3.00 \text{ in.} + (\frac{1}{8} \text{ in.}) \\
   & = 3.03 \text{ in.} \quad \text{o.k.}
   \end{align*}
   \]

5. \( R \geq d_{load} \)
   
   \[
   \begin{align*}
   8.00 \text{ in.} & \geq 7.50 \text{ in.} \quad \text{o.k.}
   \end{align*}
   \]

6. \( \frac{3}{8} w < b \leq \frac{3}{4} w \)
   
   \[
   \begin{align*}
   \frac{3}{8}(3.00 \text{ in.}) & < 2.23 \text{ in.} \leq \frac{3}{4}(3.00 \text{ in.}) \\
   2.00 \text{ in.} & < 2.23 \text{ in.} \leq 2.25 \text{ in.} \quad \text{o.k.}
   \end{align*}
   \]

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(25.0 \text{ kips}) + 1.6(15.0 \text{ kips}) )</td>
<td>( P_u = 25.0 \text{ kips} + 15.0 \text{ kips} )</td>
</tr>
<tr>
<td>= 54.0 \text{ kips}</td>
<td>= 40.0 \text{ kips}</td>
</tr>
</tbody>
</table>

**Tensile Yielding**

Calculate the available tensile yielding strength at the eyebar body (at \( w \)).

\[
A_g = w t \\
= 3.00 \text{ in.} (\frac{3}{8} \text{ in.}) \\
= 1.88 \text{ in.}^2
\]

\[
P_n = F_y A_g \\
= 36 \text{ ksi}(1.88 \text{ in.}^2) \\
= 67.7 \text{ kips}
\]

From AISC *Specification* Section D2, the available tensile yielding strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_t = 0.90 )</td>
<td>( \Omega_t = 1.67 )</td>
</tr>
<tr>
<td>( \phi_t P_n = 0.90(67.7 \text{ kips}) )</td>
<td>( \frac{P_n}{\Omega_t} = 67.7 \text{ kips} )</td>
</tr>
<tr>
<td>= 60.9 \text{ kips}</td>
<td>= 40.5 \text{ kips}</td>
</tr>
</tbody>
</table>

60.9 \text{ kips} > 54.0 \text{ kips} \quad \text{o.k.}

The eyebar tension member available strength is governed by the tensile yielding limit state.

Note: The eyebar detailing limitations ensure that the tensile yielding limit state at the eyebar body will control the strength of the eyebar itself. The pin should also be checked for shear yielding, and, if the material strength is less than that of the eyebar, bearing.

See Example J.6 for an illustration of the limit state calculations for a pin in a drilled hole.
EXAMPLE D.9  PLATE WITH STAGGERED BOLTS

Given:

Compute $A_s$ and $A_e$ for a 14-in.-wide and $\frac{1}{2}$-in.-thick plate subject to tensile loading with staggered holes as shown.

Solution:

Calculate net hole diameter using AISC Specification Section B4.3.

$$d_{net} = d_h + \frac{1}{16} \text{ in.}$$
$$= \frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}$$
$$= 0.875 \text{ in.}$$

Compute the net width for all possible paths across the plate. Because of symmetry, many of the net widths are identical and need not be calculated.

$$w = 14.0 - \sum d_{net} + \sum \frac{s^2}{4g}$$  from AISC Specification Section B4.3.

Line A-B-E-F:  $w = 14.0 \text{ in.} - 2(0.875 \text{ in.})$
$$= 12.3 \text{ in.}$$

Line A-B-C-D-E-F:  $w = 14.0 \text{ in.} - 4(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})}$
$$= 11.5 \text{ in.}$$  controls

Line A-B-C-D-G:  $w = 14.0 \text{ in.} - 3(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})}$
$$= 11.9 \text{ in.}$$

Line A-B-D-E-F:  $w = 14.0 \text{ in.} - 3(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(7.00 \text{ in.})} + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})}$
$$= 12.1 \text{ in.}$$

Therefore, $A_e = 11.5 \text{ in.}(0.500 \text{ in.})$
= 5.75 in.²

Calculate $U$.

From AISC Specification Table D3.1 case 1, because tension load is transmitted to all elements by the fasteners,

$U = 1.0$

$$A_e = A_n U$$

$$= 5.75 \text{ in.}^2 (1.0)$$

$$= 5.75 \text{ in.}^2$$
Chapter E
Design of Members for Compression

This chapter covers the design of compression members, the most common of which are columns. The AISC Manual includes design tables for the following compression member types in their most commonly available grades.

- wide-flange column shapes
- HSS
- double angles
- single angles

LRFD and ASD information is presented side-by-side for quick selection, design or verification. All of the tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD designs is similar to that of previous AISC Specifications, and will provide similar designs. In this AISC Specification, ASD and LRFD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used, when possible.

E1. GENERAL PROVISIONS

The design compressive strength, $\phi P_n$, and the allowable compressive strength, $P_n/\Omega_c$, are determined as follows:

$$P_n = \text{nominal compressive strength based on the controlling buckling mode}$$

$$\phi_c = 0.90 \text{ (LRFD)}$$

$$\Omega_c = 1.67 \text{ (ASD)}$$

Because $F_{cr}$ is used extensively in calculations for compression members, it has been tabulated in AISC Manual Table 4-22 for all of the common steel yield strengths.

E2. EFFECTIVE LENGTH

In the AISC Specification, there is no limit on slenderness, $KL/r$. Per the AISC Specification Commentary, it is recommended that $KL/r$ not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, the tables of the AISC Manual are stopped at common or practical lengths for ordinary usage. For example, a double L3×3×1/4, with a 3/16-in. separation has an $r_y$ of 1.38 in. At a $KL/r$ of 200, this strut would be 23'-0" long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the AISC Manual, shapes that contain slender elements when supplied in their most common material grade are footnoted with the letter “c”. For example, see a W14×22c.

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Nonslender sections, including nonslender built-up I-shaped columns and nonslender HSS columns, are governed by these provisions. The general design curve for critical stress versus $KL/r$ is shown in Figure E-1.
The term $L$ is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.

### E3-3
Inelastic buckling

### E3-2
Elastic buckling

---

![Fig. E-1. Standard column curve.](image)

**Table:**

<table>
<thead>
<tr>
<th>$F_y$, ksi</th>
<th>Limiting $KL/r$</th>
<th>$0.44F_y$, ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>134</td>
<td>15.8</td>
</tr>
<tr>
<td>50</td>
<td>113</td>
<td>22.0</td>
</tr>
<tr>
<td>60</td>
<td>104</td>
<td>26.4</td>
</tr>
<tr>
<td>70</td>
<td>96</td>
<td>30.8</td>
</tr>
</tbody>
</table>

### E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and WT sections, which are singly-symmetric shapes subject to torsional and flexural-torsional buckling. The available strengths in axial compression of these shapes are tabulated in Part 4 of the AISC *Manual* and examples on the use of these tables have been included in this chapter for the shapes with $KL_z = KL_y$.

### E5. SINGLE ANGLE COMPRESSION MEMBERS

The available strength of single angle compression members is tabulated in Part 4 of the AISC *Manual*.

### E6. BUILT-UP MEMBERS

There are no tables for built-up shapes in the AISC *Manual*, due to the number of possible geometries. This section suggests the selection of built-up members without slender elements, thereby making the analysis relatively straightforward.

### E7. MEMBERS WITH SLENDER ELEMENTS

The design of these members is similar to members without slender elements except that the formulas are modified by a reduction factor for slender elements, $Q$. Note the similarity of Equation E7-2 with Equation E3-2, and the similarity of Equation E7-3 with Equation E3-3.
The tables of Part 4 of the AISC *Manual* incorporate the appropriate reductions in available strength to account for slender elements.

Design examples have been included in this Chapter for built-up I-shaped members with slender webs and slender flanges. Examples have also been included for a double angle, WT and an HSS shape with slender elements.
EXAMPLE E.1A  W-SHAPE COLUMN DESIGN WITH PINNED ENDS

Given:
Select an ASTM A992 ($F_y = 50$ ksi) W-shape column to carry an axial dead load of 140 kips and live load of 420 kips. The column is 30 ft long and is pinned top and bottom in both axes. Limit the column size to a nominal 14-in. shape.

Solution:
From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>1.2(140 kips) + 1.6(420 kips) = 840 kips</td>
<td>$P_a = 140$ kips + 420 kips = 560 kips</td>
</tr>
</tbody>
</table>

Column Selection
From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, $K = 1.0$.

Because the unbraced length is the same in both the $x$-$x$ and $y$-$y$ directions and $r_x$ exceeds $r_y$ for all W-shapes, $y$-$y$ axis bucking will govern.

Enter the table with an effective length, $KL_y$, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a $W_{14}$×132.
Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>W14&lt;</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>lb/ft</td>
<td>145</td>
<td>132</td>
</tr>
<tr>
<td>Design</td>
<td>P₀/ℓ₀</td>
<td>0</td>
</tr>
<tr>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>0</td>
<td>1290</td>
<td>1590</td>
</tr>
<tr>
<td>6</td>
<td>1250</td>
<td>1550</td>
</tr>
<tr>
<td>7</td>
<td>1240</td>
<td>1530</td>
</tr>
<tr>
<td>8</td>
<td>1230</td>
<td>1510</td>
</tr>
<tr>
<td>9</td>
<td>1210</td>
<td>1490</td>
</tr>
<tr>
<td>10</td>
<td>1200</td>
<td>1480</td>
</tr>
<tr>
<td>11</td>
<td>1190</td>
<td>1470</td>
</tr>
<tr>
<td>12</td>
<td>1180</td>
<td>1460</td>
</tr>
<tr>
<td>13</td>
<td>1170</td>
<td>1450</td>
</tr>
<tr>
<td>14</td>
<td>1160</td>
<td>1440</td>
</tr>
<tr>
<td>15</td>
<td>1150</td>
<td>1430</td>
</tr>
<tr>
<td>16</td>
<td>1140</td>
<td>1420</td>
</tr>
<tr>
<td>17</td>
<td>1130</td>
<td>1410</td>
</tr>
<tr>
<td>18</td>
<td>1120</td>
<td>1400</td>
</tr>
<tr>
<td>19</td>
<td>1110</td>
<td>1390</td>
</tr>
<tr>
<td>20</td>
<td>1100</td>
<td>1380</td>
</tr>
<tr>
<td>21</td>
<td>1090</td>
<td>1370</td>
</tr>
<tr>
<td>22</td>
<td>1080</td>
<td>1360</td>
</tr>
<tr>
<td>23</td>
<td>1070</td>
<td>1350</td>
</tr>
<tr>
<td>24</td>
<td>1060</td>
<td>1340</td>
</tr>
<tr>
<td>25</td>
<td>1050</td>
<td>1330</td>
</tr>
<tr>
<td>26</td>
<td>1040</td>
<td>1320</td>
</tr>
<tr>
<td>27</td>
<td>1030</td>
<td>1310</td>
</tr>
<tr>
<td>28</td>
<td>1020</td>
<td>1300</td>
</tr>
<tr>
<td>29</td>
<td>1010</td>
<td>1290</td>
</tr>
<tr>
<td>30</td>
<td>1000</td>
<td>1280</td>
</tr>
</tbody>
</table>

From AISC Manual Table 4-1, the available strength for a y-y axis effective length of 30 ft is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_0 ) = 893 kips &gt; 840 kips</td>
<td>( P_\phi ) = 594 kips &gt; 560 kips</td>
</tr>
</tbody>
</table>

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
EXAMPLE E.1B  W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING

Given:

Redesign the column from Example E.1A assuming the column is laterally braced about the y-y axis and torsionally braced at the midpoint.

Solution:

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

\[ P_u = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips}) = 840 \text{ kips} \]

\[ P_a = 140 \text{ kips} + 420 \text{ kips} = 560 \text{ kips} \]

Column Selection

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \).

Because the unbraced lengths differ in the two axes, select the member using the y-y axis then verify the strength in the x-x axis.

Enter AISC Manual Table 4-1 with a y-y axis effective length, \( K L_y \), of 15 ft and proceed across the table until reaching a shape with an available strength that equals or exceeds the required strength. Try a \( W_{14} \times 90 \). A 15 ft long \( W_{14} \times 90 \) provides an available strength in the y-y direction of:

\[ \phi c P_n = 1,000 \text{ kips} \]

\[ P = 667 \text{ kips} \]

The \( r_y / r_x \) ratio for this column, shown at the bottom of AISC Manual Table 4-1, is 1.66. The equivalent y-y axis effective length for strong axis buckling is computed as:

\[ KL = \frac{30.0 \text{ ft}}{1.66} = 18.1 \text{ ft} \]
From AISC Manual Table 4-1, the available strength of a W14×90 with an effective length of 18 ft is:

<table>
<thead>
<tr>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
<tr>
<td>ASD</td>
</tr>
</tbody>
</table>

The available compressive strength is governed by the x-x axis flexural buckling limit state.

The available strengths of the columns described in Examples E.1A and E.1B are easily selected directly from the AISC Manual Tables. The available strengths can also be verified by hand calculations, as shown in the following Examples E.1C and E.1D.
EXAMPLE E.1C  W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×132 column with unbraced lengths of 30 ft in both axes. The material properties and loads are as given in Example E.1A.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

- W14×132
  - $A_g = 38.8$ in.$^2$
  - $r_x = 6.28$ in.
  - $r_y = 3.76$ in.

Slenderness Check

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, $K = 1.0$.

Because the unbraced length is the same for both axes, the $y$-y axis will govern.

$$
\frac{K_y L_y}{r_y} = \left( \frac{1.0(30.0 \text{ ft})}{3.76 \text{ in.}} \right) \left( \frac{12.0 \text{ in.}}{\text{ft}} \right) = 95.7
$$

For $F_y = 50$ ksi, the available critical stresses, $\phi c F_{cr}$ and $F_{cr}/\Omega_c$ for $K L/r = 95.7$ are interpolated from AISC Manual Table 4-22 as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi c F_{cr}$</td>
<td>23.0 ksi</td>
<td>$F_{cr} = 15.4$ ksi</td>
</tr>
<tr>
<td>$\phi c P_{cr} = 38.8 \text{ in.}^2 (23.0 \text{ ksi})$</td>
<td>892 kips &gt; 840 kips</td>
<td>$P_{cr} = 38.8 \text{ in.}^2 (15.4 \text{ ksi})$</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
<td>= 598 kips &gt; 560 kips</td>
</tr>
</tbody>
</table>

Note that the calculated values are approximately equal to the tabulated values.
EXAMPLE E.1D  W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a \( W_{14} \times 90 \) with a strong axis unbraced length of 30.0 ft and weak axis and torsional unbraced lengths of 15.0 ft. The material properties and loads are as given in Example E.1A.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

- \( W_{14} \times 90 \)
  - \( A_g = 26.5 \text{ in}^2 \)
  - \( r_x = 6.14 \text{ in.} \)
  - \( r_y = 3.70 \text{ in.} \)

Slenderness Check

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \).

\[
\frac{KL}{r_x} = 1.0 \left( \frac{30.0 \text{ ft}}{6.14 \text{ in.}} \right) \left( \frac{12 \text{ in.}}{\text{ft}} \right)
= 58.6 \text{ governs}
\]

\[
\frac{KL}{r_y} = 1.0 \left( \frac{15.0 \text{ ft}}{3.70 \text{ in.}} \right) \left( \frac{12 \text{ in.}}{\text{ft}} \right)
= 48.6
\]

Critical Stresses

The available critical stresses may be interpolated from AISC Manual Table 4-22 or calculated directly as follows:

Calculate the elastic critical buckling stress, \( F_e \).

\[
F_e = \frac{\pi^2 E}{(KL/r_x)^2} \quad \text{(Spec. Eq. E3-4)}
\]

\[
= \frac{\pi^2 \left( 29,000 \text{ksi} \right)}{(58.6)^2}
= 83.3 \text{ ksi}
\]

Calculate the flexural buckling stress, \( F_{cr} \).

\[
4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ksi}}{50 \text{ksi}}}
= 113
\]

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Because \( \frac{KL}{r} = 58.6 \leq 113 \),

\[
F_{cr} = \begin{bmatrix} 0.658 \frac{F_y}{F_y} \\ \frac{50.0}{50.0} \end{bmatrix} \begin{bmatrix} F_y \\ 50.0 \text{ksi} \end{bmatrix}
\]

\[
= 0.658 \times \frac{50.0}{50.0} \times 50.0 \text{ksi}
= 38.9 \text{ ksi}
\]

**Nominal Compressive Strength**

\[
P_a = F_{cr}A_g
\]

\[
= 38.9 \text{ ksi} \left(26.5 \text{ in}^2\right)
= 1,030 \text{ kips}
\]

From AISC *Specification* Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
<td>( P_a = 1,030 \text{ kips} )</td>
</tr>
<tr>
<td>( \Phi_c P_a = 0.90(1,030 \text{ kips}) )</td>
<td>( \Omega_c = 1.67 )</td>
<td>( \frac{P_a}{\Omega_c} = 1,030 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( = 927 \text{ kips &gt; 840 kips} ) o.k.</td>
<td>( = 617 \text{ kips &gt; 560 kips} ) o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE E.2  BUILT-UP COLUMN WITH A SLENDER WEB

Given:

Verify that a built-up, ASTM A572 Grade 50 column with PL1 in. × 8 in. flanges and a PL4 in. × 15 in. web is sufficient to carry a dead load of 70 kips and live load of 210 kips in axial compression. The column length is 15 ft and the ends are pinned in both axes.

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

Built-Up Column
ASTM A572 Grade 50
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

The geometric properties are as follows:

Built-Up Column
\( d = 17.0 \text{ in.} \)
\( b_f = 8.00 \text{ in.} \)
\( t_f = 1.00 \text{ in.} \)
\( h = 15.0 \text{ in.} \)
\( t_w = \frac{3}{4} \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(70 \text{ kips}) + 1.6(210 \text{ kips}) = 420 \text{ kips} )</td>
<td>( P_u = 70 \text{ kips} + 210 \text{ kips} = 280 \text{ kips} )</td>
</tr>
</tbody>
</table>

Built-Up Section Properties (ignoring fillet welds)

\[
A = 2(8.00 \text{ in.})(1.00 \text{ in.}) + 15.0 \text{ in.}(\frac{3}{4} \text{ in.}) \\
= 19.8 \text{ in.}^2
\]

\[
I_z = \frac{2(1.00 \text{ in.})(8.00 \text{ in.})^3 + 15.0 \text{ in.}(\frac{3}{4} \text{ in.})^3}{12}
\]
\[ r_y = \sqrt{\frac{I_y}{A}} \]
\[ = \sqrt{\frac{85.4 \text{ in.}^4}{19.8 \text{ in.}^2}} \]
\[ = 2.08 \text{ in.} \]

\[ I_x = \sum Ad^2 + \sum \frac{bh^3}{12} \]
\[ = 2 \left( 8.00 \text{ in.}^2 \right) (8.00 \text{ in.})^2 + \frac{(1/4 \text{ in.})(15.00 \text{ in.})^3}{12} + \frac{2(8.00 \text{ in.})(1.00 \text{ in.})^3}{12} \]
\[ = 1,100 \text{ in.}^4 \]

**Elastic Flexural Buckling Stress**

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \).

Because the unbraced length is the same for both axes, the \( y-y \) axis will govern by inspection.

\[ \frac{K L_y}{r_y} = \frac{1.0(15.0 \text{ ft})(12.0 \text{ in.})}{2.08 \text{ in.}} \]
\[ = 86.5 \]

\[ F_x = \frac{\pi^2 E}{(\frac{K L_y}{r_y})^2} \quad \text{(Spec. Eq. E3-4)} \]
\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(86.5)^2} \]
\[ = 38.3 \text{ ksi} \]

**Elastic Critical Torsional Buckling Stress**

Note: Torsional buckling generally will not govern if \( K L_y \geq K L_z \); however, the check is included here to illustrate the calculation.

From the User Note in AISC Specification Section E4,

\[ C_w = \frac{I_x h_y}{4} \]
\[ = \frac{85.4 \text{ in.}^4 (16.0 \text{ in.})}{4} \]
\[ = 5,470 \text{ in.}^6 \]
From AISC Design Guide 9, Equation 3.4,

\[
J = \frac{\sum b t^3}{3} = \frac{2(8.00 \text{ in.})(1.00 \text{ in.})^3 + (15.0 \text{ in.})(\frac{1}{4} \text{ in.})^3}{3} = 5.41 \text{ in}^4
\]

\[
F_\varepsilon = \left[ \frac{\pi^2 E C_w}{(K_c L)^2} + GJ \right] \frac{1}{I_e + I_y} = \left[ \frac{\pi^2 (29,000 \text{ ksi})(5.470 \text{ in}^6)}{[(1.0)(15 \text{ ft})(12)]^2} + (11,200 \text{ ksi})(5.41 \text{ in}^4) \right] \left( \frac{1}{1,100 \text{ in}^4 + 85.4 \text{ in}^4} \right) = 91.9 \text{ ksi} > 38.3 \text{ ksi}
\]

Therefore, the flexural buckling limit state controls.

Use \( F_\varepsilon = 38.3 \text{ ksi} \).

**Slenderness**

Check for slender flanges using AISC Specification Table B4.1a, then determine \( Q_s \), the unstiffened element (flange) reduction factor using AISC Specification Section E7.1.

Calculate \( k_c \) using AISC Specification Table B4.1b note [a].

\[
k_c = \frac{4}{\sqrt{b/t}} = \frac{4}{\sqrt{15.0 \text{ in.}/\frac{1}{4} \text{ in.}}} = 0.516, \text{ which is between 0.35 and 0.76}
\]

For the flanges,

\[
\lambda = \frac{b}{t} = \frac{4.00 \text{ in.}}{1.00 \text{ in.}} = 4.00
\]

Determine the flange limiting slenderness ratio, \( \lambda_r \), from AISC Specification Table B4.1a case 2

\[
\lambda_r = 0.64 \frac{k_c E}{F_\varepsilon} = 0.64 \frac{0.516(29,000 \text{ ksi})}{50 \text{ ksi}} = 11.1
\]

\( \lambda < \lambda_r \); therefore, the flange is not slender and \( Q_s = 1.0 \).

Check for a slender web, then determine \( Q_a \), the stiffened element (web) reduction factor using AISC Specification Section E7.2.
\[ \lambda = \frac{h}{t} \]
\[ = \frac{15.0 \text{ in.}}{\frac{1}{4} \text{ in.}} \]
\[ = 60.0 \]

Determine the slender web limit from AISC Specification Table B4.1a case 5

\[ \lambda_e = 1.49 \sqrt{\frac{E}{F_y}} \]
\[ = 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]
\[ = 35.9 \]

\[ \lambda > \lambda_e \]; therefore, the web is slender

\[ Q_a = \frac{A_e}{A_y} \]  \hspace{1cm} (Spec. Eq. E7-16)

where \( A_e \) = effective area based on the reduced effective width, \( b_e \)

For AISC Specification Equation E7-17, take \( f \) as \( F_{cr} \) with \( F_{cr} \) calculated based on \( Q = 1.0 \).

Select between AISC Specification Equations E7-2 and E7-3 based on \( KL/r_y \).  

\[ KL/r = 86.5 \] as previously calculated

\[ 4.71 \sqrt{\frac{E}{QF_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{1.0(50 \text{ ksi})}} \]
\[ = 113 > 86.5 \]

Because \( \frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{QF_y}} \),

\[ F_{cr} = Q \left[ 0.658 \frac{QF_y}{F_y} \right] \]
\[ = 1.0 \left[ 0.658 \left( \frac{1.0(50 \text{ ksi})}{50 \text{ ksi}} \right) \right] (50 \text{ ksi}) \]
\[ = 29.0 \text{ ksi} \]

\[ b_e = 1.92 \sqrt{\frac{E}{F} \left[ 1 - \frac{0.34}{(b/t)} \right]} \leq b, \text{ where } b = h \]  \hspace{1cm} (Spec. Eq. E7-17)

\[ = 1.92 \left( \frac{\frac{1}{4} \text{ in.}}{29,000 \text{ ksi}} \right) \left[ 29,000 \text{ ksi} \right] \left[ 1 - \frac{0.34}{(15.0 \text{ in.}/\frac{1}{4} \text{ in.})} \right] \leq 15.0 \text{ in.} \]
\[ = 12.5 \text{ in.} \leq 15.0 \text{ in.} \]; therefore, compute \( A_e \) with reduced effective web width

\[ A_e = b_e t_e + 2b_e t_f \]
\[ = 12.5 \text{ in.} \left( \frac{1}{4} \text{ in.} \right) + 2 \left( 8.00 \text{ in.} \right) \left( 1.00 \text{ in.} \right) \]
\[ = 19.1 \text{ in.}^2 \]
\[ Q_a = \frac{A_e}{A} \]
\[ = \frac{19.1 \text{ in.}^2}{19.8 \text{ in.}^2} \]
\[ = 0.965 \]

\( Q = Q_a Q_s \) from AISC Specification Section E7
\[ = 1.00(0.965) \]
\[ = 0.965 \]

**Flexural Buckling Stress**

Determine whether AISC Specification Equation E7-2 or E7-3 applies.

\[ KL/r = 86.5 \] as previously calculated

\[ 4.71 \sqrt{\frac{E}{QF_e}} = 4.71 \sqrt{\frac{29,000 \text{ksi}}{0.965(50 \text{ksi})}} \]
\[ = 115 > 86.5 \]

Therefore, AISC Specification Equation E7-2 applies.

\[ F_{cr} = Q \left[ 0.658 \frac{QF_e}{F_y} \right] F_y \]
\[ = 0.965 \left[ 0.658 \frac{0.965(50 \text{ksi})}{38.3 \text{ksi}} \right] (50 \text{ksi}) \]
\[ = 28.5 \text{ ksi} \]

**Nominal Compressive Strength**

\[ P_n = F_{cr} A_g \]
\[ = 28.5 \text{ ksi}(19.8 \text{ in.}^2) \]
\[ = 564 \text{ kips} \]

From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
</tbody>
</table>
| \( \phi_c P_n = 0.90(564 \text{ kips}) \) | \( P_n = \frac{564 \text{ kips}}{1.67} \)
| = 508 kips > 420 kips | \( = 338 \text{ kips} > 280 \text{ kips} \) | o.k. | o.k. |
EXAMPLE E.3  BUILT-UP COLUMN WITH SLENDER FLANGES

Given:

Determine if a built-up, ASTM A572 Grade 50 column with PL\(\frac{3}{8}\) in. \(\times 10\frac{1}{2}\) in. flanges and a PL\(\frac{1}{4}\) in. \(\times 7\frac{1}{4}\) in. web has sufficient available strength to carry a dead load of 40 kips and a live load of 120 kips in axial compression. The column’s unbraced length is 15.0 ft in both axes and the ends are pinned.

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

- Built-Up Column
  - ASTM A572 Grade 50
  - \(F_y = 50\) ksi
  - \(F_u = 65\) ksi

The geometric properties are as follows:

- Built-Up Column
  - \(d = 8.00\) in.
  - \(b_f = 10\frac{1}{2}\) in.
  - \(t_f = \frac{3}{8}\) in.
  - \(h = 7\frac{1}{4}\) in.
  - \(t_w = \frac{1}{4}\) in.

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_u)</td>
<td>(1.2(40) kips (+ 1.6(120) kips)</td>
<td>(P_u = 40) kips (+ 120) kips)</td>
</tr>
<tr>
<td>(= 240) kips</td>
<td></td>
<td>(= 160) kips</td>
</tr>
</tbody>
</table>

Built-Up Section Properties (ignoring fillet welds)

- \(A_g = 2(10\frac{1}{2}\) in.)\(\frac{3}{8}\) in. \(+ 7\frac{1}{4}\) in.\(\frac{1}{4}\) in. \(= 9.69\) in.²

Because the unbraced length is the same for both axes, the weak axis will govern.
\[ I_y = 2 \left[ \frac{\left( \frac{3}{8} \text{ in.} \right) \left( \frac{10}{2} \text{ in.} \right)^3}{12} \right] + \frac{\left( \frac{7}{4} \text{ in.} \right) \left( \frac{1}{4} \text{ in.} \right)^3}{12} \]
\[ = 72.4 \text{ in.}^4 \]

\[ r_y = \frac{I_y}{A} \]
\[ = \sqrt{\frac{72.4 \text{ in.}^4}{9.69 \text{ in.}^2}} \]
\[ = 2.73 \text{ in.} \]

\[ I_x = 2 \left( \frac{10}{2} \text{ in.} \right) \left( \frac{3}{8} \text{ in.} \right) \left( \frac{3.81 \text{ in.}}{2} \right)^2 + \frac{\left( \frac{1}{4} \text{ in.} \right) \left( \frac{7}{4} \text{ in.} \right)^3}{12} + \frac{2 \left( \frac{10}{2} \text{ in.} \right) \left( \frac{1}{8} \text{ in.} \right)^3}{12} \]
\[ = 122 \text{ in.}^4 \]

**Web Slenderness**

Determine the limiting slenderness ratio, \( \lambda_w \), from AISC Specification Table B4.1a case 5:

\[ \lambda_y = 1.49 \sqrt{\frac{E}{F_y}} \]
\[ = 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]
\[ = 35.9 \]

\[ \lambda = \frac{h}{t_w} \]
\[ = \frac{7/4 \text{ in.}}{1/4 \text{ in.}} \]
\[ = 29.0 \]

\( \lambda < \lambda_y \); therefore, the web is not slender.

Note that the fillet welds are ignored in the calculation of \( h \) for built up sections.

**Flange Slenderness**

Calculate \( k_c \).

\[ k_c = \frac{4}{\sqrt{h/t_w}} \text{ from AISC Specification Table B4.1b note [a]} \]
\[ = \frac{4}{\sqrt{7/4 \text{ in.}/1/4 \text{ in.}}} \]
\[ = 0.743, \text{ where } 0.35 \leq k_c \leq 0.76 \text{ o.k.} \]

Use \( k_c = 0.743 \)
Determine the limiting slenderness ratio, $\lambda_r$, from AISC Specification Table B4.1a case 2.

$$\lambda_r = 0.64 \sqrt{\frac{k_k E}{F_y}}$$

$$= 0.64 \sqrt{\frac{29,000 \text{ksi}(0.743)}{50 \text{ksi}}} = 13.3$$

$$\lambda = \frac{b}{t}$$

$$= \frac{5.25}{0.64} \text{ in.} = 8.25 \text{ in.}$$

$$= 14.0$$

$\lambda > \lambda_r$; therefore, the flanges are slender.

For compression members with slender elements, Section E7 of the AISC Specification applies. The nominal compressive strength, $P_n$, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling. Depending on the slenderness of the column, AISC Specification Equation E7-2 or E7-3 applies. $F_e$ is used in both equations and is calculated as the lesser of AISC Specification Equations E3-4 and E4-4.

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, $K = 1.0$.

Because the unbraced length is the same for both axes, the weak axis will govern.

$$\frac{K L_y}{r_y} = \frac{1.0(15.0 \text{ ft})(12 \text{ in.})}{2.73 \text{ in.}} = 65.9$$

Elastic Critical Stress, $F_e$, for Flexural Buckling

$$F_e = \frac{\pi^2 E}{(KL)^2} \left( \frac{r}{r_y} \right)$$

$$= \frac{\pi^2 (29,000 \text{ ksi})}{(65.9)^2} = 65.9 \text{ ksi}$$

Elastic Critical Stress, $F_e$, for Torsional Buckling

Note: This limit state is not likely to govern, but the check is included here for completeness.

From the User Note in AISC Specification Section E4,

$$C_w = \frac{I_k b_y^2}{4} = 72.4 \text{ in.}^4 (7.63 \text{ in.})^2 = 1,050 \text{ in.}^6$$

From AISC Design Guide 9, Equation 3.4,
\[ J = \sum \frac{bt^3}{3} \]
\[ = \frac{2(10\frac{1}{2} \text{ in.})(\frac{3}{8} \text{ in.})^3}{3} + 7\frac{1}{4} \text{ in.}(\frac{1}{4} \text{ in.})^3 \]
\[ = 0.407 \text{ in.}^3 \]

\[ F_e = \left[ \frac{\pi^2 EC_w}{(K_c L)^2} + \frac{1}{I_x + I_y} \right] \]
\[ = \left[ \frac{\pi^2 (29,000 \text{ ksi})(1,050 \text{ in}^6)}{(180 \text{ in.})^2} + (11,200 \text{ ksi})(0.407 \text{ in}^4) \right] \left( \frac{1}{122 \text{ in}^4 + 72.4 \text{ in}^4} \right) \]
\[ = 71.2 \text{ ksi} > 65.9 \text{ ksi} \]

Therefore, use \( F_e = 65.9 \text{ ksi} \).

**Slenderness Reduction Factor, Q**

\[ Q = Q_s Q_a \quad \text{from AISC Specification Section E7, where } Q_a = 1.0 \text{ because the web is not slender.} \]

Calculate \( Q_s \), the unstiffened element (flange) reduction factor from AISC Specification Section E7.1(b).

Determine the proper equation for \( Q_s \) by checking limits for AISC Specification Equations E7-7 to E7-9.

\[ b \]

\[ t = 14.0 \quad \text{as previously calculated} \]

\[ \frac{0.64}{E_k} \sqrt{F_y} = 0.64 \sqrt{\frac{29,000 \text{ ksi}(0.743)}{50 \text{ ksi}}} \]

\[ = 13.3 \]

\[ \frac{1.17}{E_k} \sqrt{F_y} = 1.17 \sqrt{\frac{29,000 \text{ ksi}(0.743)}{50 \text{ ksi}}} \]

\[ = 24.3 \]

\[ 0.64 \frac{E_k}{F_y} < \frac{b}{t} \leq 1.17 \frac{E_k}{F_y} \quad \text{therefore, AISC Specification Equation E7-8 applies} \]

\[ Q_s = 1.415 - 0.65 \left( \frac{b}{t} \right) \sqrt{\frac{F_y}{E_k}} \quad (\text{Spec. Eq. E7-8}) \]

\[ = 1.415 - 0.65(14.0) \sqrt{\frac{50 \text{ ksi}}{(29,000 \text{ ksi})(0.743)}} \]

\[ = 0.977 \]

\[ Q = Q_s Q_a \]

\[ = 0.977(1.0) \]

\[ = 0.977 \]

**Nominal Compressive Strength**
\[ 4.71 \sqrt{\frac{E}{QF_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{0.977 \times 50 \text{ ksi}}} \]
\[ = 115 > 65.9, \text{ therefore, AISC Specification Equation E7-2 applies} \]

\[ F_{cr} = Q \left[ \frac{0.658}{0.977} \right] F_y \]
\[ = 9.69 \text{ ksi} \]
\[ = 35.8 \text{ ksi} \]

\[ P_n = F_{cr} A_y \]
\[ = 35.8 \text{ ksi} \times 9.69 \text{ in.}^2 \]
\[ = 347 \text{ kips} \]

From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c = 0.90$</td>
<td>$\Omega_c = 1.67$</td>
</tr>
<tr>
<td>$\phi_c P_n = 0.90 \times 347 \text{ kips}$</td>
<td>$P_n = 347 \text{ kips} \Rightarrow \Omega_c = 1.67$</td>
</tr>
<tr>
<td>$= 312 \text{ kips} &gt; 240 \text{ kips}$</td>
<td>$= 208 \text{ kips} &gt; 160 \text{ kips}$</td>
</tr>
</tbody>
</table>

Note: Built-up sections are generally more expensive than standard rolled shapes; therefore, a standard compact shape, such as a W8×35 might be a better choice even if the weight is somewhat higher. This selection could be taken directly from AISC Manual Table 4-1.
EXAMPLE E.4A  W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns in conjunction with the effective length method.

**Given:**

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the \( x \)-axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the \( y \)-axis of the column).

**Solution:**

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

- **W18×50**
  - \( I_x = 800 \text{ in.}^4 \)

- **W24×55**
  - \( I_x = 1350 \text{ in.}^4 \)

- **W14×82**
  - \( A_g = 24.0 \text{ in.}^2 \)
  - \( I_x = 881 \text{ in.}^4 \)

**Column B-C**

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the roof and floor is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_e = 1.2(41.5 \text{ kips}) + 1.6(125 \text{ kips}) )</td>
<td>( P_a = 41.5 + 125 )</td>
</tr>
<tr>
<td>( = 250 \text{ kips} )</td>
<td>( = 167 \text{ kips} )</td>
</tr>
</tbody>
</table>
Effective Length Factor

Calculate the stiffness reduction parameter, τₙ, using AISC *Manual* Table 4-21.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pₙ</td>
<td>250 kips</td>
<td>167 kips</td>
</tr>
<tr>
<td>Aₙ</td>
<td>24.0 in.²</td>
<td>24.0 in.²</td>
</tr>
<tr>
<td>τₙ</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Therefore, no reduction in stiffness for inelastic buckling will be required.

Determine \( G_{\text{top}} \) and \( G_{\text{bottom}} \).

\[
G_{\text{top}} = \tau \sum \frac{(E I_c / L_c)}{\sum (E I_g / L_g)}
\]


\[
= (1.00) \left( \frac{29,000 \text{ ksi}}{2(29,000 \text{ ksi})} \right) \left( \frac{881 \text{ in.}^4}{14.0 \text{ ft}} \right)
\]

\[
= 1.38
\]

\[
G_{\text{bottom}} = \tau \sum \frac{(E I_c / L_c)}{\sum (E I_g / L_g)}
\]


\[
= (1.00) \left( \frac{2(29,000 \text{ ksi})}{2(29,000 \text{ ksi})} \right) \left( \frac{1,350 \text{ in.}^4}{35.0 \text{ ft}} \right)
\]

\[
= 1.63
\]

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, \( K \) is slightly less than 1.5; therefore use \( K = 1.5 \). Because the column available strength tables are based on the \( KL \) about the \( y-y \) axis, the equivalent effective column length of the upper segment for use in the table is:

\[
KL = \frac{(KL)}{r_y}
\]

\[
= \frac{1.5(14.0 \text{ ft})}{2.44}
\]

\[
= 8.61 \text{ ft}
\]

Take the available strength of the W14×82 from AISC *Manual* Table 4-1.

At \( KL = 9 \text{ ft} \), the available strength in axial compression is:
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_c = 940 \text{ kips} &gt; 250 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{P_c}{\Omega_c} = 626 \text{ kips} &gt; 167 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Column A-B**

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the floor and the foundation is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 1.2(100 \text{ kips}) + 1.6(300 \text{ kips})$</td>
<td>$P_u = 100 \text{ kips} + 300 \text{ kips}$</td>
</tr>
<tr>
<td>= 600 kips</td>
<td>= 400 kips</td>
</tr>
</tbody>
</table>

**Effective Length Factor**

Calculate the stiffness reduction parameter, $\tau_b$, using AISC *Manual* Table 4-21.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{P_u}{A_e} = \frac{600 \text{ kips}}{24.0 \text{ in.}^2}$</td>
<td>$\frac{P_u}{A_e} = \frac{400 \text{ kips}}{24.0 \text{ in.}^2}$</td>
</tr>
<tr>
<td>= 25.0 ksi</td>
<td>= 16.7 ksi</td>
</tr>
<tr>
<td>$\tau_b = 1.00$</td>
<td>$\tau_b = 0.994$</td>
</tr>
</tbody>
</table>

Determine $G_{\text{top}}$ and $G_{\text{bottom}}$ accounting for column inelasticity by replacing $E_iL_i$ with $\tau_b(E_cL_c)$. Use $\tau_b = 0.994$.

\[
G_{\text{top}} = \tau \left( \frac{\sum (E_iL_i)}{\sum (E_cL_c)} \right) = (0.994)\left( \frac{29,000 \text{ ksi}(881 \text{ in.}^4)}{14.0 \text{ ft}} \right) = 1.62
\]

\[
G_{\text{bottom}} = 1.0 \ (\text{fixed}) \text{ from AISC Specification Commentary Appendix 7, Section 7.2}
\]

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, $K$ is approximately 1.40. Because the column available strength tables are based on the $KL$ about the $y$-$y'$ axis, the effective column length of the lower segment for use in the table is:

\[
KL = \left( \frac{(KL)_x}{r_x} \right) = \left( \frac{(14.0 \text{ ft})}{2.44} \right) = 8.03 \text{ ft}
\]

Take the available strength of the W14×82 from AISC *Manual* Table 4-1.
At $KL = 9$ ft, (conservative) the available strength in axial compression is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_e = 940$ kips &gt; 600 kips</td>
<td>$P_e/\Omega_e = 626$ kips &gt; 400 kips</td>
</tr>
</tbody>
</table>

A more accurate strength could be determined by interpolation from AISC Manual Table 4-1.
EXAMPLE E.4B  W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

Given:

Using the effective length method, determine the available strength of the column shown subject to the same gravity loads shown in Example E.4A with the column pinned at the base about the y-y axis. All other assumptions remain the same.

Solution:

As determined in Example E.4A, for the column segment B-C between the roof and the floor, the column strength is adequate.

As determined in Example E.4A, for the column segment A-B between the floor and the foundation, 

\[ G_{top} = 1.62 \]

At the base, 

\[ G_{bottom} = 10 \text{ (pinned)} \] from AISC Specification Commentary Appendix 7, Section 7.2

Note: this is the only change in the analysis.

From the alignment chart, AISC Specification Commentary Figure C-A-7.2, \( K \) is approximately equal to 2.00. Because the column available strength tables are based on the effective length, \( KL \), about the y-y axis, the effective column length of the segment A-B for use in the table is:

\[ KL = \frac{(KL)}{r_r} \]

\[ = 2.00(14.0 \text{ ft}) \]

\[ = 2.44 \text{ ft} \]

\[ = 11.5 \text{ ft} \]

Interpolate the available strength of the W14x82 from AISC Manual Table 4-1.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_u = 861 \text{ kips} &gt; 600 \text{ kips} )</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE E.5  DOUBLE ANGLE COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:

Verify the strength of a 2L4×3⅜×¾ LLBB (⅜-in. separation) strut, ASTM A36, with a length of 8 ft and pinned ends carrying an axial dead load of 20 kips and live load of 60 kips. Also, calculate the required number of pretensioned bolted or welded intermediate connectors required.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36

\[ F_y = 36 \text{ ksi} \]

\[ F_u = 58 \text{ ksi} \]

From AISC Manual Tables 1-7 and 1-15, the geometric properties are as follows:

L4×3⅜×¾ LLBB

\[ r_x = 0.719 \text{ in.} \]

2L4×3⅜×¾ LLBB

\[ r_x = 1.25 \text{ in.} \]

\[ r_y = 1.55 \text{ in. for } ⅜\text{-in. separation} \]

\[ r_y = 1.69 \text{ in. for } ¼\text{-in. separation} \]

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

\[
\begin{align*}
LRFD & \\
& P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips}) \\
& = 120 \text{ kips} \\
& \text{ASD} & \\
& P_u = 20 \text{ kips} + 60 \text{ kips} \\
& = 80.0 \text{ kips}
\end{align*}
\]

Table Solution

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \).

For \((KL)_x = 8 \text{ ft}\), the available strength in axial compression is taken from the upper (X-X) portion of AISC Manual Table 4-9 as:

\[
\begin{align*}
\text{LRFD} & \\
\phi P_u &= 127 \text{ kips} > 120 \text{ kips} & \text{o.k.} \\
& \text{ASD} & \\
\frac{P_u}{\Omega_x} &= 84.7 \text{ kips} > 80.0 \text{ kips} & \text{o.k.}
\end{align*}
\]

For buckling about the \(y\)-\(y\) axis, the values are tabulated for a separation of ¾ in.

To adjust to a spacing of ¼ in., \((KL)_y\) is multiplied by the ratio of the \(r_y\) for a ¾-in. separation to the \(r_y\) for a ¼-in. separation. Thus,

\[
\begin{align*}
(KL)_y &= 1.0(8.00 \text{ ft}) \left( \frac{1.55 \text{ in.}}{1.69 \text{ in.}} \right) \\
& = 7.34 \text{ ft}
\end{align*}
\]
The calculation of the equivalent \((KL)_y\) in the preceding text is a simplified approximation of AISC Specification Section E6.1. To ensure a conservative adjustment for a \(\frac{3}{4}\)-in. separation, take \((KL)_y = 8\) ft.

The available strength in axial compression is taken from the lower \((Y-Y)\) portion of AISC Manual Table 4-9 as:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi P_u = 130) kips (&gt; 120) kips</td>
<td>(P_u = 86.3) kips (&gt; 80.0) kips</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Therefore, \(x-x\) axis flexural buckling governs.

Intermediate Connectors

From AISC Manual Table 4-9, at least two welded or pretensioned bolted intermediate connectors are required. This can be verified as follows:

\[
a = \text{distance between connectors} = \frac{8.00\ \text{ft}(12\ \text{in./ft})}{3 \text{ spaces}} = 32.0\ \text{in.}
\]

From AISC Specification Section E6.2, the effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, \(a\), must not exceed three-fourths of the governing slenderness ratio of the built-up member.

Therefore,

\[
\frac{Ka}{n} \leq \frac{3}{4} \left( \frac{KL}{r} \right)_{\text{max}}
\]

Solving for \(a\) gives,

\[
a \leq \frac{3n \left( \frac{KL}{r} \right)_{\text{max}}}{4K}
\]

\[
\frac{KL}{r_x} = \frac{1.0(8.00\ \text{ft})(12\ \text{in./ft})}{1.25\ \text{in.}} = 76.8 \ \text{controls}
\]

\[
\frac{KL}{r_y} = \frac{1.0(8.00\ \text{ft})(12\ \text{in./ft})}{1.69\ \text{in.}} = 56.8
\]
Thus,  
\[ a \leq \frac{3z \left( \frac{KL}{r} \right)_{\text{max}}}{4K} \]
\[ = \frac{3(0.719 \text{ in.})(76.8)}{4(1.0)} \]
\[ = 41.4 \text{ in.} > 32.0 \text{ in.} \quad \text{o.k.} \]

Note that one connector would not be adequate as 48.0 in. > 41.4 in. The available strength can be easily determined by using the tables of the AISC Manual. Available strength values can be verified by hand calculations, as follows:

**Calculation Solution**

From AISC Manual Tables 1-7 and 1-15, the geometric properties are as follows:

- \( L4 \times 37/8 \times 37/8 \)
  - \( J = 0.132 \text{ in.}^4 \)
  - \( r_y = 1.05 \text{ in.} \)
  - \( \bar{x} = 0.947 \text{ in.} \)

- \( 2L4 \times 37/8 \times 37/8 \text{ LLBB (3/8 in. separation)} \)
  - \( A_g = 5.36 \text{ in.}^2 \)
  - \( r_y = 1.69 \text{ in.} \)
  - \( \bar{r}_y = 2.33 \text{ in.} \)
  - \( H = 0.813 \)

**Slenderness Check**

\[ \lambda = \frac{b}{t} \]
\[ = \frac{4.00 \text{ in.}}{\frac{3}{8} \text{ in.}} \]
\[ = 10.7 \]

Determine the limiting slenderness ratio, \( \lambda_r \), from AISC Specification Table B4.1a Case 3

\[ \lambda_r = 0.45 \sqrt{E/F_y} \]
\[ = 0.45 \sqrt{29,000 \text{ ksi}/36 \text{ ksi}} \]
\[ = 12.8 \]

\( \lambda < \lambda_r \); therefore, there are no slender elements.

For compression members without slender elements, AISC Specification Sections E3 and E4 apply.

The nominal compressive strength, \( P_n \), shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling.

**Flexural Buckling about the x-x Axis**

\[ \frac{KL}{r_x} = 1.0(8.00 \text{ ft})(12 \text{ in./ft}) \]
\[ = 1.25 \text{ in.} \]
$F_y = \pi^2 \frac{E}{KL}^2$ 

(Spec. Eq. E3-4)

$= \pi^2 (29,000 \text{ ksi}) \frac{(76.8)^2}{(29,000 \text{ ksi}) \frac{36 \text{ ksi}}{48.5 \text{ ksi}}} \approx 48.5 \text{ ksi}$

$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 134 > 76.8$, therefore

$F_{cr} = F_y = \left[ 0.658 \frac{E}{F_y} \right] F_y$ 

(Spec. Eq. E3-2)

$= \left[ 0.658 \frac{36 \text{ ksi}}{48.5 \text{ ksi}} \right] (36 \text{ ksi}) = 26.4 \text{ ksi}$

**Torsional and Flexural-Torsional Buckling**

For nonslender double angle compression members, AISC *Specification* Equation E4-2 applies.

$F_{cr y}$ is taken as $F_{cr}$, for flexural buckling about the $y$-$y$ axis from AISC *Specification* Equation E3-2 or E3-3 as applicable.

Using AISC *Specification* Section E6, compute the modified $KL/r_i$ for built up members with pretensioned bolted or welded connectors. Assume two connectors are required.

$a = 96.0 \text{ in.}/3 = 32.0 \text{ in.}$

$r_i = r_z$ (single angle) = 0.719 in.

$a = \frac{32 \text{ in.}}{0.719 \text{ in.}} = 44.5 > 40$, therefore

$$\left( \frac{KL}{r_i} \right)_u \leq \sqrt{\left( \frac{KL}{r} \right)_u^2 + \left( \frac{K_i a}{r_i} \right)^2}$$

where $K_i = 0.50$ for angles back-to-back 

(Spec. Eq. E6-2b)

$$= \sqrt{(56.8)^2 + \left( \frac{0.50(32.0 \text{ in.})}{0.719 \text{ in.}} \right)^2} = 61.0 \leq 134$$
\[ F_y = \frac{\pi^2E}{(KL)^2} \]  
\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(61.0)^2} \]  
\[ = 76.9 \text{ ksi} \] 

\[ F_{cry} = \left[ 0.658 \frac{E}{F_y} \right] F_y \]  
\[ = \left[ 0.658 \left( \frac{36 \text{ ksi}}{76.9 \text{ ksi}} \right) \right] (36 \text{ ksi}) \]  
\[ = 29.6 \text{ ksi} \] 

\[ F_{crz} = \frac{GJ}{A_e \bar{t}_o^2} \]  
\[ = \frac{(11,200 \text{ ksi})(2 \text{ angles})(0.132 \text{ in.}^4)}{(5.36 \text{ in.}^2)(2.33 \text{ in.})^2} \]  
\[ = 102 \text{ ksi} \] 

\[ F_{cr} = \left( \frac{F_{cry} + F_{crz}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{cry} F_{crz} H}{(F_{cry} + F_{crz})^2}} \right] \]  
\[ = \left( \frac{29.6 \text{ ksi} + 102 \text{ ksi}}{2(0.813)} \right) \left[ 1 - \sqrt{1 - \frac{4(29.6 \text{ ksi})(102 \text{ ksi})(0.813)}{(29.6 \text{ ksi} + 102 \text{ ksi})^2}} \right] \]  
\[ = 27.7 \text{ ksi} \quad \text{does not control} \] 

**Nominal Compressive Strength**

\[ P_e = F_{cr} A_g \]  
\[ = 26.4 \text{ ksi}(5.36 \text{ in.}^2) \]  
\[ = 142 \text{ kips} \] 

From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>Design Example</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \phi_c P_a = 0.90(142 \text{ kips}) )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>= 128 kips ( &gt; 120 \text{ kips} )</td>
<td>( \frac{\Omega_c}{P_a} = \frac{1.67}{142 \text{ kips}} )</td>
<td>( = 85.0 \text{ kips} &gt; 80.0 \text{ kips} )</td>
</tr>
</tbody>
</table>
EXAMPLE E.6  DOUBLE ANGLE COMPRESSION MEMBER WITH SLENDER ELEMENTS

Given:

Determine if a 2L5×3×¾ LLBB (¼-in. separation) strut, ASTM A36, with a length of 8 ft and pinned ends has sufficient available strength to support a dead load of 10 kips and live load of 30 kips in axial compression. Also, calculate the required number of pretensioned bolted or welded intermediate connectors.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36

\[F_y = 36 \text{ ksi}\]

\[F_u = 58 \text{ ksi}\]

From AISC Manual Tables 1-7 and 1-15, the geometric properties are as follows:

L5×3×¾

\[r_z = 0.652 \text{ in.}\]

2L5×3×¾ LLBB

\[r_x = 1.62 \text{ in.}\]

\[r_y = 1.19 \text{ in. for ⅛-in. separation}\]

\[r_y = 1.33 \text{ in. for ⅛-in. separation}\]

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

\[P_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips})\]

\[= 60.0 \text{ kips}\]

\[P_u = 10 \text{ kips} + 30 \text{ kips}\]

\[= 40.0 \text{ kips}\]

Table Solution

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \(K = 1.0\).

From the upper portion of AISC Manual Table 4-9, the available strength for buckling about the \(x-x\) axis, with \((KL)_x = 8\) ft is:

\[\phi_P = 87.1 \text{ kips} > 60.0 \text{ kips}\]

\[\frac{P_u}{\Omega_c} = 58.0 \text{ kips} > 40.0 \text{ kips}\]
For buckling about the \( y \)-axis, the tabulated values are based on a separation of \( \frac{3}{8} \) in. To adjust for a spacing of \( \frac{1}{4} \) in., \((KL)_y\) is multiplied by the ratio of \( r_x \) for a \( \frac{3}{8} \)-in. separation to \( r_y \) for a \( \frac{1}{4} \)-in. separation.

\[
(KL)_y = 1.0(8.0 \text{ ft})(1.19 \text{ in.})
\]

\[
= 7.16 \text{ ft}
\]

This calculation of the equivalent \((KL)_y\) does not completely take into account the effect of AISC Specification Section E6.1 and is slightly unconservative.

From the lower portion of AISC Manual Table 4-9, interpolate for a value at \((KL)_y = 7.16 \text{ ft}\).

The available strength in compression is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_L P_{ey} )</td>
<td>65.2 kips &gt; 60.0 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>( \frac{P_{ew}}{\Omega_L} )</td>
<td>43.3 kips &gt; 40.0 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

These strengths are approximate due to the linear interpolation from the table and the approximate value of the equivalent \((KL)_y\) noted in the preceding text. These can be compared to the more accurate values calculated in detail as follows:

Intermediate Connectors

From AISC Manual Table 4-9, it is determined that at least two welded or pretensioned bolted intermediate connectors are required. This can be confirmed by calculation, as follows:

\[
a = \text{distance between connectors}
\]

\[
= \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{3 \text{ spaces}}
\]

\[
= 32.0 \text{ in.}
\]

From AISC Specification Section E6.2, the effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, \( a \), must not exceed three-fourths of the governing slenderness ratio of the built-up member.

Therefore,

\[
\frac{K_a}{r_i} \leq \frac{3}{4} \left( \frac{K_l}{r} \right)_{\text{max}}
\]

Solving for \( a \) gives,

\[
a \leq \frac{3r_i \left( \frac{KL}{r} \right)_{\text{max}}}{4K}
\]

\[
r_i = r_i = 0.652 \text{ in.}
\]

\[
\frac{KL_y}{r_y} = \frac{1.0(8.0 \text{ ft})(12.0 \text{ in./ft})}{1.33 \text{ in.}}
\]

\[
= 72.2 \text{ controls}
\]
Thus, 
\[
3r_\frac{KL}{r_{max}} \leq 4K
\]
\[
= \frac{3(0.652 \text{ in.})(72.2)}{4(1.0)}
\]
\[
= 35.3 \text{ in.} > 32.0 \text{ in.} \quad \text{o.k.}
\]

The governing slenderness ratio used in the calculations of the AISC Manual tables includes the effects of the provisions of Section E6.1 and is slightly higher as a result. See the following for these calculations. As a result, the maximum connector spacing calculated here is slightly conservative.

Available strength values can be verified by hand calculations, as follows.

**Calculation Solution**

From AISC Manual Tables 1-7 and 1-15, the geometric properties are as follows.

- \(L5 \times 3 \times \frac{3}{4}\)
  - \(J = 0.0438 \text{ in.}^4\)
  - \(r_y = 0.853 \text{ in.}\)
  - \(\bar{x} = 0.648 \text{ in.}\)

- \(2L5 \times 3 \times \frac{3}{4} \text{ LLBB}\)
  - \(A_g = 3.88 \text{ in.}^2\)
  - \(r_y = 1.33 \text{ in.}\)
  - \(\bar{r} = 2.59 \text{ in.}\)
  - \(H = 0.657\)

**Slenderness Check**

\[
\lambda = \frac{b}{t}
\]

- \(= \frac{5.00 \text{ in.}}{\frac{3}{4} \text{ in.}}\)
- \(= 20.0\)

Calculate the limiting slenderness ratio, \(\lambda_r\), from AISC Specification Table B4.1a Case 3.

\[
\lambda_r = 0.45 \sqrt{\frac{E}{F_y}}
\]

- \(= 0.45 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}}\)
- \(= 12.8\)

\(\lambda > \lambda_r\); therefore, the angle has a slender element

For a double angle compression member with slender elements, AISC Specification Section E7 applies. The nominal compressive strength, \(P_n\), shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling. \(F_{cr}\) will be determined by AISC Specification Equation E7-2 or E7-3.

Calculate the slenderness reduction factor, \(Q\).
\[ Q = Q_s Q_a \] from AISC Specification Section E7.

Calculate \( Q_s \) for the angles individually using AISC Specification Section E7.1c.

\[ 0.45 \sqrt{\frac{E}{F_y}} = 12.8 < 20.0 \]
\[ 0.91 \sqrt{\frac{E}{F_y}} = 0.91 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 25.8 \geq 20.0 \]

Therefore, AISC Specification Equation E7-11 applies.

\[ Q_s = 1.34 - 0.76 \left( \frac{b}{t} \right) \sqrt{\frac{F_y}{E}} \]
\[ = 1.34 - 0.76(20.0) \sqrt{\frac{36 \text{ ksi}}{29,000 \text{ ksi}}} = 0.804 \]

\( Q_s = 1.0 \) (no stiffened elements)

Therefore, \( Q = Q_a Q_s \)
\[ = 0.804(1.0) = 0.804 \]

Critical Stress, \( F_{cr} \)

From the preceding text, \( K = 1.0 \).

AISC Specification Equations E7-2 and E7-3 require the computation of \( F_c \). For singly symmetric members, AISC Specification Equations E3-4 and E4-5 apply.

Flexural Buckling about the \( x-x \) Axis

\[ \frac{K_s L}{r_e} = \frac{1.0(8.0 \text{ ft})(12.0 \text{ in./ft})}{1.62 \text{ in.}} = 59.3 \]
\[ F_c = \frac{\pi^2 E}{(K_s L)^2} \left( \frac{r_e}{K_s L} \right) \]
\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(59.3)^2} = 81.4 \text{ ksi} \text{ does not control} \]

Torsional and Flexural-Torsional Buckling

\[ \frac{K_t L}{r_y} = \frac{1.0(8.0 \text{ ft})(12.0 \text{ in./ft})}{1.33 \text{ in.}} \]
Using AISC Specification Section E6, compute the modified $KL/r_y$ for built-up members with pretensioned bolted or welded connectors.

\[ a = 96.0 \text{ in./3} \]
\[ = 32.0 \text{ in.} \]
\[ r_i = r_z \text{ (single angle)} \]
\[ = 0.652 \text{ in.} \]
\[ \frac{a}{r_i} = \frac{32}{0.652} \text{ in.} \]
\[ = 49.1 > 40, \text{ therefore,} \]

\[
\left( \frac{KL}{r} \right)_m = \sqrt{\left( \frac{KL}{r} \right)_o + \left( \frac{K_i a}{r_i} \right)^2} \quad \text{where } K_i = 0.50 \text{ for angles back-to-back (Spec. Eq. E6-2b)}
\]
\[
= \sqrt{(72.2)^2 + \left(\frac{0.50(32.0 \text{ in.})}{0.652 \text{ in.}}\right)^2}
\]
\[ = 76.3 \]

\[
F_{xy} = \frac{\pi^2 E}{\left( \frac{K_i L}{r_i} \right)_m^2}
\]
\[
= \frac{\pi^2 (29,000 \text{ ksi})}{(76.3)^2}
\]
\[ = 49.2 \text{ ksi} \]

\[
F_{ae} = \frac{\pi^2 E C_w}{(K_i L)^2 + GJ} \frac{1}{A_{gF_o}^2} \quad \text{(Spec. Eq. E4-9)}
\]

For double angles, omit term with $C_w$ per the User Note at the end of AISC Specification Section E4.

\[
F_{ae} = \frac{GJ}{A_{gF_o}^2}
\]
\[
= \frac{(11,200 \text{ ksi})(2 \text{ angles})(0.0438 \text{in.}^4)}{(3.88 \text{in.}^2)(2.59 \text{in.})^2}
\]
\[ = 37.7 \text{ ksi} \]

\[
F_a = \frac{F_{xy} + F_{ae}}{2H} \left[ 1 - \sqrt{1 - \frac{4F_{ae}F_{ae}H}{(F_{xy} + F_{ae})}} \right] \quad \text{(Spec. Eq. E4-5)}
\]
\[
= \left( \frac{49.2 \text{ ksi} + 37.7 \text{ ksi}}{2(0.657)} \right) \left[ 1 - \sqrt{1 - \frac{4(49.2 \text{ ksi})(37.7 \text{ ksi})(0.657)}{(49.2 \text{ ksi} + 37.7 \text{ ksi})^2}} \right]
\]
Use the limits based on $F_e$ to determine whether to apply Specification Equation E7-2 or E7-3.

$$QF_e = \frac{0.804(36 \text{ ksi})}{2.25}$$

$$= 12.9 \text{ ksi} \leq 26.8 \text{ ksi}, \text{ therefore AISC Specification Equation E7-2 applies}$$

$$F_{y, cr} = 0.804 \cdot 36 \text{ ksi}$$

$$= 26.8 \text{ ksi}$$

$$y_{QF} = 2.25$$

$$\frac{QF_e}{y_{QF}} = 12.9 \text{ ksi} < 26.8 \text{ ksi}, \text{ therefore AISC Specification Equation E7-2 applies}$$

$$F_{y, cr} = \frac{QF_e}{y_{QF}}$$

$$= 0.804 \cdot 36 \text{ ksi}$$

$$= 18.4 \text{ ksi}$$

**Nominal Compressive Strength**

$$P_n = F_{y, cr} A_g$$

$$= 18.4 \text{ ksi} \cdot 3.88 \text{ in.}^2$$

$$= 71.4 \text{ kips}$$

From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c$</td>
<td>0.90</td>
<td>$\Omega_c = 1.67$</td>
</tr>
<tr>
<td>$\phi_c P_n$</td>
<td>0.90(71.4 kips)</td>
<td>$P_n = 71.4 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>= 64.3 kips $&gt; 60.0 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

$$\Omega_c = \frac{P_n}{71.4 \text{ kips}}$$

$$= \frac{42.8 \text{ kips}}{40.0 \text{ kips}}$$

o.k.
EXAMPLE E.7  WT COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:

Select an ASTM A992 nonslender WT-shape compression member with a length of 20 ft to support a dead load of 20 kips and live load of 60 kips in axial compression. The ends are pinned.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
  $F_y = 50$ ksi
  $F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

$$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips}) = 120 \text{ kips}$$

Table Solution

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, $K = 1.0$.

Therefore, $(KL)_x = (KL)_y = 20.0$ ft.

Select the lightest nonslender member from AISC Manual Table 4-7 with sufficient available strength about both the $x$-$x$ axis (upper portion of the table) and the $y$-$y$ axis (lower portion of the table) to support the required strength.

Try a WT7×34.

The available strength in compression is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_x = 120$ kips</td>
<td>$P_x = 85.5$ kips &gt; $80.0$ kips controls o.k.</td>
</tr>
<tr>
<td>$P_y = 147$ kips &gt; $80.0$ kips controls o.k.</td>
<td></td>
</tr>
</tbody>
</table>

The available strength can be easily determined by using the tables of the AISC Manual. Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC Manual Table 1-8, the geometric properties are as follows.

<table>
<thead>
<tr>
<th>WT7×34</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_g = 10.0$ in.$^2$</td>
</tr>
<tr>
<td>$r_x = 1.81$ in.</td>
</tr>
<tr>
<td>$r_y = 2.46$ in.</td>
</tr>
</tbody>
</table>
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\[ J = 1.50 \text{ in.}^4 \]
\[ \overline{y} = 1.29 \text{ in.} \]
\[ I_x = 32.6 \text{ in.}^4 \]
\[ I_y = 60.7 \text{ in.}^4 \]
\[ d = 7.02 \text{ in.} \]
\[ t_w = 0.415 \text{ in.} \]
\[ b_f = 10.0 \text{ in.} \]
\[ t_f = 0.720 \text{ in.} \]

**Stem Slenderness Check**

\[ \lambda = \frac{d}{t_w} \]
\[ = \frac{7.02 \text{ in.}}{0.415 \text{ in.}} \]
\[ = 16.9 \]

Determine the stem limiting slenderness ratio, \( \lambda_s \), from AISC Specification Table B4.1a Case 4

\[ \lambda_s = 0.75 \sqrt{\frac{E}{F_y}} \]
\[ = 0.75 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]
\[ = 18.1 \]

\( \lambda < \lambda_s \); therefore, the stem is not slender

**Flange Slenderness Check**

\[ \lambda = \frac{b_f}{2t_f} \]
\[ = \frac{10 \text{ in.}}{2(0.720 \text{ in.})} \]
\[ = 6.94 \]

Determine the flange limiting slenderness ratio, \( \lambda_f \), from AISC Specification Table B4.1a Case 1

\[ \lambda_f = 0.56 \sqrt{\frac{E}{F_y}} \]
\[ = 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]
\[ = 13.5 \]

\( \lambda < \lambda_f \); therefore, the flange is not slender

There are no slender elements.

For compression members without slender elements, AISC Specification Sections E3 and E4 apply. The nominal compressive strength, \( P_n \), shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling.

**Flexural Buckling About the x-x Axis**
Design Examples V14.1

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\[ KL = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{1.81 \text{ in.}} = 133 \]

\[ \sqrt[4.71]{\frac{E}{F_y}} = 4.71 \sqrt[50]{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 113 < 133, \text{ therefore, AISC Specification Equation E3-3 applies} \]

\[ F_e = \frac{\pi^2 E}{KL^2 r} \]
\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(133)^2} = 16.2 \text{ ksi} \]
\[ F_{cr} = 0.877 F_e \]
\[ = 0.877(16.2 \text{ ksi}) = 14.2 \text{ ksi controls} \]

Torsional and Flexural-Torsional Buckling

Because the WT7×34 section does not have any slender elements, AISC Specification Section E4 will be applicable for torsional and flexural-torsional buckling. \( F_{cr} \) will be calculated using AISC Specification Equation E4-2.

Calculate \( F_{cry} \).

\( F_{cry} \) is taken as \( F_{cr} \) from AISC Specification Section E3, where \( KL/r = KL/r_y \).

\[ KL = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{2.46 \text{ in.}} = 97.6 < 113, \text{ therefore, AISC Specification Equation E3-2 applies} \]

\[ F_e = \frac{\pi^2 E}{KL^2 r_y} \]
\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(97.6)^2} = 30.0 \text{ ksi} \]
\[ F_{cry} = F_{cr} = \left[ 0.658 \frac{F_y}{F_{cr}} \right] F_y \]
\[ = \left[ 0.658 \frac{50.0 \text{ ksi}}{30.0 \text{ ksi}} \right] 50.0 \text{ ksi} = 24.9 \text{ ksi} \]
The shear center for a T-shaped section is located on the axis of symmetry at the mid-depth of the flange.

\[ x_0 = 0.0 \text{ in.} \]

\[ y_0 = \bar{y} - \frac{I_f}{2} \]
\[ = 1.29 \text{ in.} - \frac{0.720 \text{ in.}}{2} \]
\[ = 0.930 \text{ in.} \]

\[ \bar{r}_o^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_y} \]  
(Spec. Eq. E4-11)
\[ = (0.0 \text{ in.})^2 + (0.930 \text{ in.})^2 + \frac{32.6 \text{ in.}^4 + 60.7 \text{ in.}^4}{10.0 \text{ in.}^2} \]
\[ = 10.2 \text{ in.}^2 \]

\[ \bar{r}_o = \sqrt{\bar{r}_o^2} \]
\[ = \sqrt{10.2 \text{ in.}^2} \]
\[ = 3.19 \text{ in.} \]

\[ H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_o^2} \]  
(Spec. Eq. E4-10)
\[ = 1 - \frac{(0.0 \text{ in.})^2 + (0.930 \text{ in.})^2}{10.2 \text{ in.}^2} \]
\[ = 0.915 \]

\[ F_{crx} = \frac{GJ}{A_y \bar{r}_o^2} \]  
(Spec. Eq. E4-3)
\[ = \frac{(11,200 \text{ ksi})(1.50 \text{ in.}^4)}{(10.0 \text{ in.}^2)(10.2 \text{ in.}^2)} \]
\[ = 165 \text{ ksi} \]

\[ F_{crx} = \left( \frac{F_{crx} + F_{crz}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{crx}F_{crz}H}{(F_{crx} + F_{crz})^2}} \right] \]  
(Spec. Eq. E4-2)
\[ = \left( \frac{24.9 \text{ ksi} + 165 \text{ ksi}}{2(0.915)} \right) \left[ 1 - \sqrt{1 - \frac{4(24.9 \text{ ksi})(165 \text{ ksi})(0.915)}{(24.9 \text{ ksi} + 165 \text{ ksi})^2}} \right] \]
\[ = 24.5 \text{ ksi} \quad \text{does not control} \]

\[ x-x \text{ axis flexural buckling governs, therefore,} \]

\[ P_x = F_{crx}A_y \]  
(Spec. Eq. E3-1)
\[ = 14.2 \text{ ksi}(10.0 \text{ in.}^2) \]
\[ = 142 \text{ kips} \]
From AISC *Specification* Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_P = 0.90(142 \text{kips})$</td>
<td>$P_c = 142 \text{kips}$</td>
<td>$\Omega_c = 1.67$</td>
</tr>
<tr>
<td></td>
<td>$= 128 \text{kips} &gt; 120 \text{kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE E.8  WT COMPRESSION MEMBER WITH SLENDER ELEMENTS

Given:

Select an ASTM A992 WT-shape compression member with a length of 20 ft to support a dead load of 6 kips and live load of 18 kips in axial compression. The ends are pinned.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\( F_y = 50 \) ksi

\( F_u = 65 \) ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

\[
\begin{align*}
\text{LRFD} & : P_e = 1.2(6 \text{kips}) + 1.6(18 \text{kips}) = 36.0 \text{kips} \\
\text{ASD} & : P_u = 6 \text{kips} + 18 \text{kips} = 24.0 \text{kips}
\end{align*}
\]

Table Solution

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \).

Therefore, \((KL)_x = (KL)_y = 20.0 \text{ ft.}\)

Select the lightest member from AISC Manual Table 4-7 with sufficient available strength about the both the \(x-x\) axis (upper portion of the table) and the \(y-y\) axis (lower portion of the table) to support the required strength.

Try a WT7×15.

The available strength in axial compression from AISC Manual Table 4-7 is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \phi_x P_{xx} ) = 66.7 kips &gt; 36.0 kips</td>
<td>( \phi_x P_{xx} ) o.k.</td>
</tr>
<tr>
<td></td>
<td>( \phi_y P_{yy} ) = 36.6 kips &gt; 36.0 kips</td>
<td>( \phi_y P_{yy} ) controls o.k.</td>
</tr>
<tr>
<td></td>
<td>( \frac{P_w}{\Omega_c} ) &gt; 24.4 kips &gt; 24.0 kips</td>
<td>( \frac{P_w}{\Omega_c} ) o.k. controls o.k.</td>
</tr>
</tbody>
</table>

The available strength can be easily determined by using the tables of the AISC Manual. Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC Manual Table 1-8, the geometric properties are as follows:

WT7×15

\( A_g = 4.42 \text{ in.}^2 \)

\( r_x = 2.07 \text{ in.} \)

\( r_y = 1.49 \text{ in.} \)

\( J = 0.190 \text{ in.}^4 \)

\( Q_c = 0.611 \)
\[ \bar{y} = 1.58 \text{ in.} \]
\[ I_x = 19.0 \text{ in.}^4 \]
\[ I_y = 9.79 \text{ in.}^4 \]
\[ d = 6.92 \text{ in.} \]
\[ t_w = 0.270 \text{ in.} \]
\[ b_f = 6.73 \text{ in.} \]
\[ t_f = 0.385 \text{ in.} \]

Stem Slenderness Check
\[ \lambda = \frac{d}{t_w} = \frac{6.92 \text{ in.}}{0.270 \text{ in.}} = 25.6 \]

Determine stem limiting slenderness ratio, \( \lambda_r \), from AISC Specification Table B4.1a case 4
\[ \lambda_r = 0.75 \frac{E}{F_y} = 0.75 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 18.1 \]

\( \lambda > \lambda_r \); therefore, the web is slender

Flange Slenderness Check
\[ \lambda = \frac{b_f}{2t_f} = \frac{6.73 \text{ in.}}{2(0.385 \text{ in.})} = 8.74 \]

Determine flange limiting slenderness ratio, \( \lambda_r \), from AISC Specification Table B4.1a Case 1
\[ \lambda_r = 0.56 \frac{E}{F_y} = 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 13.5 \]

\( \lambda < \lambda_r \); therefore, the flange is not slender

Because this WT7×15 has a slender web, AISC Specification Section E7 is applicable. The nominal compressive strength, \( P_n \), shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling.

\( x-x \) Axis Critical Elastic Flexural Buckling Stress
\[ \frac{K_{cr}L}{r_e} = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{2.07 \text{ in.}} = 116 \]
\[ F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \]  
\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(116)^2} \]  
\[ = 21.3 \text{ ksi} \]  

**Critical Elastic Torsional and Flexural-Torsional Buckling Stress**

\[ K_r L = \frac{1.0 (20.0 \text{ ft})(12 \text{ in./ft})}{1.49 \text{ in.}} \]  
\[ = 161 \]  

\[ F_{ey} = \frac{\pi^2 E}{\left(\frac{K_r L}{r_e}\right)^2} \]  
\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(161)^2} \]  
\[ = 11.0 \text{ ksi} \]  

**Torsional Parameters**

The shear center for a T-shaped section is located on the axis of symmetry at the mid-depth of the flange.

\[ x_0 = 0.0 \text{ in.} \]  

\[ y_0 = y - \frac{t_f}{2} \]  
\[ = 1.58 \text{ in.} - \frac{0.385 \text{ in.}}{2} \]  
\[ = 1.39 \text{ in.} \]  

\[ \bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_y} \]  
\[ = (0.0 \text{ in.})^2 + (1.39 \text{ in.})^2 + \frac{19.0 \text{ in.}^4 + 9.79 \text{ in.}^4}{4.42 \text{ in.}^2} \]  
\[ = 8.45 \text{ in.}^2 \]  

\[ \bar{r}_0 = \sqrt{\bar{r}_0^2} \]  
\[ = \sqrt{8.45 \text{ in.}^2} \]  
\[ = 2.91 \text{ in.} \]  

\[ H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} \]  
\[ = 1 - \frac{(0.0 \text{ in.})^2 + (1.39 \text{ in.})^2}{8.45 \text{ in.}^2} \]  
\[ = 0.771 \]
\[
F_{\omega} = \left( \frac{\pi^2 E C_w}{(K_e L)^2} + GJ \right) \frac{1}{A_p r_c^2}
\]

(Spec. Eq. E4-9)

Omit term with \( C_w \) per User Note at end of AISC Specification Section E4.

\[
F_{\omega} = \frac{GJ}{A_p r_c^2} = \frac{11,200 \text{ ksi}(0.190 \text{ in.}^2)}{4.42 \text{ in.}^2(0.190 \text{ in.}^2)} = 57.0 \text{ ksi}
\]

\[
F_e = \left( \frac{F_{\omega} + F_{\omega}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{\omega}F_e H}{(F_{\omega} + F_e)^2}} \right] \quad \text{(Spec. Eq. E4-5)}
\]

\[
= \frac{11.0 \text{ ksi} + 57.0 \text{ ksi}}{2(0.771)} \left[ 1 - \sqrt{1 - \frac{4(11.0 \text{ ksi})(57.0 \text{ ksi})(0.771)}{(11.0 \text{ ksi} + 57.0 \text{ ksi})^2}} \right] = 10.5 \text{ ksi}
\]

Check limit for the applicable equation.

\[
QF_x = \frac{(0.611)(50 \text{ ksi})}{2.25} = 13.6 \text{ ksi} > 10.5 \text{ ksi}, \text{ therefore, AISC Specification Equation E7-3 applies}
\]

\[
F_{\omega} = 0.877F_e \quad \text{(Spec. Eq. E7-3)}
\]

\[
= 0.877(10.5 \text{ ksi}) = 9.21 \text{ ksi}
\]

\[
P_h = F_{\omega} A_p \quad \text{(Spec. Eq. E7-1)}
\]

\[
= 9.21 \text{ ksi} \left( 4.42 \text{ in.}^2 \right) = 40.7 \text{ kips}
\]
From AISC *Specification* Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c = 0.90$</td>
<td>$\Omega_c = 1.67$</td>
</tr>
<tr>
<td>$\phi_c P_n = 0.90 \times 40.7 \text{ kips}$</td>
<td>$\frac{P_n}{\Omega_c} = \frac{40.7 \text{ kips}}{1.67}$</td>
</tr>
<tr>
<td>$= 36.6 \text{ kips} &gt; 36.0 \text{ kips}$</td>
<td>$= 24.4 \text{ kips} &gt; 24.0 \text{ kips}$</td>
</tr>
</tbody>
</table>
EXAMPLE E.9  RECTANGULAR HSS COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:
Select an ASTM A500 Grade B rectangular HSS compression member, with a length of 20 ft, to support a dead load of 85 kips and live load of 255 kips in axial compression. The base is fixed and the top is pinned.

Solution:
From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B  
\( F_y = 46 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

\[
\left(1.2 \times 85 \text{ kips} + 1.6 \times 255 \text{ kips}\right) = 510 \text{ kips}
\]

\[
\left(85 \text{ kips} + 255 \text{ kips}\right) = 340 \text{ kips}
\]

Table Solution
From AISC Specification Commentary Table C-A-7.1, for a fixed-pinned condition, \( K = 0.8 \).

\[
(KL)_x = (KL)_y = 0.8(20.0 \text{ ft}) = 16.0 \text{ ft}
\]

Enter AISC Manual Table 4-3 for rectangular sections or AISC Manual Table 4-4 for square sections.

Try an HSS12\times10\times3\%.

From AISC Manual Table 4-3, the available strength in axial compression is:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
P_r = 1.2(85 \text{ kips}) + 1.6(255 \text{ kips}) & P_a = 85 \text{ kips} + 255 \text{ kips} \\
= 510 \text{ kips} & = 340 \text{ kips} \\
\hline
\end{array}
\]

The available strength can be easily determined by using the tables of the AISC Manual. Available strength values can be verified by hand calculations, as follows.

Calculation Solution
From AISC Manual Table 1-11, the geometric properties are as follows:

HSS12\times10\times3\%
\( A_g = 14.6 \text{ in.}^2 \)
\( r_x = 4.61 \text{ in.} \)
\( r_y = 4.01 \text{ in.} \)
\( t_{des} = 0.349 \text{ in.} \)

Slenderness Check
Note: According to AISC Specification Section B4.1b, if the corner radius is not known, \( b \) and \( h \) shall be taken as the outside dimension minus three times the design wall thickness. This is generally a conservative assumption.

Calculate \( \lambda b/t \) of the most slender wall.

\[
\lambda = \frac{h}{t} = \frac{12.0\text{ in.} - 3(0.349\text{ in.})}{0.349\text{ in.}} = 31.4
\]

Determine the wall limiting slenderness ratio, \( \lambda_r \), from AISC Specification Table B4.1a Case 6

\[
\lambda_r = 1.40 \sqrt{\frac{E}{F_y}} = 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 35.2
\]

\( \lambda < \lambda_r \); therefore, the section does not contain slender elements.

Because \( r_y < r_x \) and \( (KL)_x = (KL)_y \), \( r_y \) will govern the available strength.

Determine the applicable equation.

\[
\frac{K_rL}{r_y} = \frac{0.8(20.0 \text{ ft})(12 \text{ in./ft})}{4.01 \text{ in.}} = 47.9
\]

\[
4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 118 \geq 47.9, \text{ therefore, use AISC Specification Equation E3-2}
\]

\[
F_r = \frac{\pi^2 E}{(KL/r)^2} 
\]

\[
= \frac{\pi^2(29,000 \text{ ksi})}{(47.9)^2} = 125 \text{ ksi}
\]

\[
F_{yr} = \left(0.658 \frac{E}{F_y}\right)F_y 
\]

\[
= \left(0.658 \frac{29,000 \text{ ksi}}{46 \text{ ksi}}\right)(46 \text{ ksi}) = 39.4 \text{ ksi}
\]

\[
P_b = F_{yr} A_b
\]

\[
= 39.4 \text{ ksi}(14.6 \text{ in.}^2)
\]
From AISC *Specification* Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi_c P_x = 0.90(575 \text{ kips})$</td>
<td>$P_x = \frac{575 \text{ kips}}{1.67}$</td>
</tr>
<tr>
<td>$= 518 \text{ kips} &gt; 510 \text{ kips}$</td>
<td>$= 344 \text{ kips} &gt; 340 \text{ kips}$</td>
</tr>
</tbody>
</table>
EXAMPLE E.10  RECTANGULAR HSS COMPRESSION MEMBER WITH SLENDER ELEMENTS

Given:
Select an ASTM A500 Grade B rectangular HSS12×8 compression member with a length of 30 ft, to support an axial dead load of 26 kips and live load of 77 kips. The base is fixed and the top is pinned.

A column with slender elements has been selected to demonstrate the design of such a member.

Solution:
From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B
\( F_y = 46 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:
\[
\left( P_a \right)_{LRFD} = 1.2(26 \text{ kips}) + 1.6(77 \text{ kips}) = 154 \text{ kips}
\]
\[
\left( P_a \right)_{ASD} = 26 \text{ kips} + 77 \text{ kips} = 103 \text{ kips}
\]

Table Solution
From AISC Specification Commentary Table C-A-7.1, for a fixed-pinned condition, \( K = 0.8 \).

\[
(KL)_x = (KL)_y = 0.8(30.0 \text{ ft}) = 24.0 \text{ ft}
\]

Enter AISC Manual Table 4-3, for the HSS12×8 section and proceed to the lightest section with an available strength that is equal to or greater than the required strength, in this case an HSS 12

From AISC Manual Table 4-3, the available strength in axial compression is:
\[
\left( P_a \right)_{LRFD} = 156 \text{ kips} \quad \text{o.k.}
\]
\[
\left( P_a \right)_{ASD} = \frac{P_a}{\Omega} = 103 \text{ kips} \geq 103 \text{ kips} \quad \text{o.k.}
\]

The available strength can be easily determined by using the tables of the AISC Manual. Available strength values can be verified by hand calculations, as follows, including adjustments for slender elements.

Calculation Solution
From AISC Manual Table 1-11, the geometric properties are as follows:
HSS12×8×3/8

$A_g = 6.76 \text{ in}^2$

$r_x = 4.56 \text{ in.}$

$r_y = 3.35 \text{ in.}$

$\frac{b}{t} = 43.0$

$h = 66.0$

$t_{des} = 0.174 \text{ in.}$

**Slenderness Check**

Calculate the limiting slenderness ratio, $\lambda_r$, from AISC Specification Table B4.1a case 6 for walls of HSS.

$$
\lambda_r = 1.40 \sqrt{\frac{E}{F_y}}
$$

$$
= 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}}
$$

$= 35.2 < 43.0$ and $35.2 < 66.0$, therefore both the 8-in. and 12-in. walls are slender elements

**Note** that for determining the width-to-thickness ratio, $b$ is taken as the outside dimension minus three times the design wall thickness per AISC Specification Section B4.1b(d).

For the selected shape,

$\frac{b}{t} = 8.0 \text{ in.} - 3(0.174 \text{ in.})$

$= 7.48 \text{ in.}$

$\frac{h}{t} = 12.0 \text{ in.} - 3(0.174 \text{ in.})$

$= 11.5 \text{ in.}$

AISC Specification Section E7 is used for an HSS member with slender elements. The nominal compressive strength, $P_n$, is determined based upon the limit states of flexural buckling. Torsional buckling will not govern for HSS unless the torsional unbraced length greatly exceeds the controlling flexural unbraced length.

**Effective Area, $A_e$**

$$Q_x = \frac{A_e}{A_{ig}}$$  
\text{(Spec. Eq. E7-16)}

where $A_e$ = summation of the effective areas of the cross section based on the reduced effective widths, $b_e$

For flanges of square and rectangular slender-element sections of uniform thickness,

$$b_e = 1.92t \sqrt{\frac{E}{f}} \left[1 - \frac{0.38}{(b/t) \sqrt{\frac{E}{f}}}\right] \leq b$$  
\text{(Spec. Eq. E7-18)}

where $f = P_n/A_{ig}$, but can conservatively be taken as $F_y$, according to the User Note in Specification Section E7.2.
For the 8-in. walls,
\[
b_e = 1.92t \left[ \frac{E}{F_y} \left[ 1 - \frac{0.38}{b/t} \sqrt{\frac{E}{F_y}} \right] \right] \quad \text{(Spec. Eq. E7-18)}
\]
\[
= 1.92(0.174 \text{ in.}) \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \left[ 1 - \frac{0.38}{(43.0)} \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \right]
\]
\[
= 6.53 \text{ in.} \leq 7.48 \text{ in.}
\]
Length that is ineffective = \( b - b_e \)
\[
= 7.48 \text{ in.} - 6.53 \text{ in.}
\]
\[
= 0.950 \text{ in.}
\]

For the 12-in. walls,
\[
b_e = 1.92t \left[ \frac{E}{F_y} \left[ 1 - \frac{0.38}{b/t} \sqrt{\frac{E}{F_y}} \right] \right] \quad \text{(Spec. Eq. E7-18)}
\]
\[
= 1.92(0.174 \text{ in.}) \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \left[ 1 - \frac{0.38}{(66.0)} \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \right]
\]
\[
= 7.18 \text{ in.} \leq 11.5 \text{ in.}
\]
Length that is ineffective = \( b - b_e \)
\[
= 11.5 \text{ in.} - 7.18 \text{ in.}
\]
\[
= 4.32 \text{ in.}
\]
\[
A_e = 6.76 \text{ in.}^2 - 2(0.174 \text{ in.})(0.950 \text{ in.}) - 2(0.174 \text{ in.})(4.32 \text{ in.})
\]
\[
= 4.93 \text{ in.}^2
\]

For cross sections composed of only stiffened slender elements, \( Q = Q_a (Q_s = 1.0) \).
\[
Q = \frac{A_e}{A_g}
\]
\[
= \frac{4.93 \text{ in.}^2}{6.76 \text{ in.}^2}
\]
\[
= 0.729
\]

Critical Stress, \( F_{cr} \)
\[
\frac{K_rL}{r_y} = 0.8(30.0 \text{ ft})(12 \text{ in.}/ \text{ ft})
\]
\[
= 86.0
\]
\[
4.71 \sqrt{\frac{E}{QF_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{0.729(46 \text{ ksi})}}
\]
\[
= 139 \geq 86.0, \text{ therefore AISC Specification Equation E7-2 applies}
\]

For the limit state of flexural buckling.
\[ F_e = \frac{\pi^2 E}{KL^2} \left( \frac{r}{r} \right) \]  
= \frac{\pi^2 (29,000 \text{ ksi})}{(86.0)^2}  
= 38.7 \text{ ksi} \]  

\[ F_{cr} = Q \left[ \frac{0.658}{F_y} \right] F_y \]  
= 0.729 \left[ \frac{0.658}{38.7 \text{ ksi}} \right] 46 \text{ ksi}  
= 23.3 \text{ ksi} \]  

**Nominal Compressive Strength**

\[ P_n = F_{cr}A_g \]  
= 23.3 ksi(6.76 in.²)  
= 158 kips

From AISC *Specification* Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>( \phi_c P_n = 0.90(158 \text{ kips}) )</td>
<td>( P_{cr} = 158 \text{ kips} )</td>
</tr>
<tr>
<td>= 142 kips &lt; 154 kips</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td><strong>See following note.</strong></td>
<td>( = 94.6 \text{ kips} &lt; 103 \text{ kips} )</td>
</tr>
</tbody>
</table>

Note: A smaller available strength is calculated here because a conservative initial assumption \( f = F_y \) was made in applying AISC *Specification* Equation E7-18. A more exact solution is obtained by iterating from the effective area, \( A_e \), step using \( f = P_{cr}/A_e \) until the value of \( f \) converges. The HSS column strength tables in the AISC *Manual* were calculated using this iterative procedure.
EXAMPLE E.11 PIPE COMPRESSION MEMBER

Given:

Select an ASTM A53 Grade B Pipe compression member with a length of 30 ft to support a dead load of 35 kips and live load of 105 kips in axial compression. The column is pin-connected at the ends in both axes and braced at the midpoint in the y-y direction.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A53 Grade B

\( F_y = 35 \text{ ksi} \)

\( F_u = 60 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

\[
\begin{align*}
P_u & = 1.2 \times 35 \text{ kips} + 1.6 \times 105 \text{ kips} \\
& = 210 \text{ kips}
\end{align*}
\]

\[
\begin{align*}
P_a & = 35 \text{ kips} + 105 \text{ kips} \\
& = 140 \text{ kips}
\end{align*}
\]

Table Solution

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \).

Therefore, \( (KL)_x = 30.0 \text{ ft} \) and \( (KL)_y = 15.0 \text{ ft} \). Buckling about the \( x-x \) axis controls.

Enter AISC Manual Table 4-6 with a \( KL \) of 30 ft and proceed across the table until reaching the lightest section with sufficient available strength to support the required strength.

Try a 10-in. Standard Pipe.

From AISC Manual Table 4-6, the available strength in axial compression is:

\[
\begin{align*}
\phi, P_a & = 222 \text{ kips} > 210 \text{ kips} \quad \text{O.K.}
\end{align*}
\]

\[
\begin{align*}
P_a = 148 \text{ kips} > 140 \text{ kips} \quad \text{O.K.}
\end{align*}
\]

The available strength can be easily determined by using the tables of the AISC Manual. Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC Manual Table 1-14, the geometric properties are as follows:
Pipe 10 Std.

$A_g = 11.5 \text{ in.}^2$

$r = 3.68 \text{ in.}$

$\lambda = \frac{D}{t} = 31.6$

No Pipes shown in AISC Manual Table 4-6 are slender at 35 ksi, so no local buckling check is required; however, some round HSS are slender at higher steel strengths. The following calculations illustrate the required check.

### Limiting Width-to-Thickness Ratio

$\lambda_s = 0.11 \frac{E}{F_y}$ from AISC Specification Table B4.1a case 9

$= 0.11 \left( \frac{29,000 \text{ ksi}}{35 \text{ ksi}} \right)$

$= 91.1$

$\lambda < \lambda_s$; therefore, the pipe is not slender

### Critical Stress, $F_{cr}$

$$KL = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{3.68 \text{ in.}} = 97.8$$

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{35 \text{ ksi}}}$$

$= 136 \geq 97.8$, therefore AISC Specification Equation E3-2 applies

$$F_y = \frac{\pi^2 E}{KL}$$

$$= \frac{\pi^2 (29,000 \text{ ksi})}{(97.8)^2}$$

$= 29.9 \text{ ksi}$

$$F_{cr} = \left( 0.658 \frac{F_y}{F_y} \right) F_y$$

$$= \left( 0.658 \frac{35 \text{ ksi}}{29.9 \text{ ksi}} \right) (35 \text{ ksi})$$

$= 21.4 \text{ ksi}$

### Nominal Compressive Strength

$P_n = F_{cr} A_g$

$= 21.4 \text{ ksi} (11.5 \text{ in.}^2)$

$= 246 \text{ kips}$

*Design Examples V14.1*

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0 = 0.90$</td>
<td>$\Omega_c = 1.67$</td>
<td>$\Omega_c = 1.67$</td>
</tr>
<tr>
<td>$\phi_0 P_e = 0.90(246 \text{ kips})$</td>
<td>$P_e = \frac{246 \text{ kips}}{1.67}$</td>
<td>$P_e = \frac{246 \text{ kips}}{1.67}$</td>
</tr>
<tr>
<td></td>
<td>$= 221 \text{ kips} &gt; 210 \text{ kips}$</td>
<td>$= 147 \text{ kips} &gt; 140 \text{ kips}$</td>
</tr>
</tbody>
</table>

Note that the design procedure would be similar for a round HSS column.
EXAMPLE E.12  BUILT-UP I-SHAPED MEMBER WITH DIFFERENT FLANGE SIZES

Given:

Compute the available strength of a built-up compression member with a length of 14 ft. The ends are pinned. The outside flange is PL3/8-in.×5-in., the inside flange is PL3/8-in.×8-in., and the web is PL3/8-in.×10½-in. Material is ASTM A572 Grade 50.

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

ASTM A572 Grade 50

\(F_y = 50 \text{ ksi}\)

\(F_u = 65 \text{ ksi}\)

There are no tables for special built-up shapes; therefore the available strength is calculated as follows.

Slenderness Check

Check outside flange slenderness.

Calculate \(k_c\).

\[
k_c = \frac{4}{\sqrt{h/w}} \quad \text{from AISC Specification Table B4.1b note [a]}
\]

\[
= \frac{4}{\sqrt{10\frac{1}{2} \text{ in.}/\frac{3}{8} \text{ in.}}}
\]

\[
= 0.756, \quad 0.35 \leq k_c \leq 0.76 \quad \text{o.k.}
\]

For the outside flange, the slenderness ratio is,

\[
\lambda = \frac{b}{t}
\]

\[
= \frac{2.50 \text{ in.}}{\frac{3}{8} \text{ in.}}
\]

\[
= 3.33
\]
Determine the limiting slenderness ratio, $\lambda$, from AISC Specification Table B4.1a case 2

$$\lambda = 0.64 \frac{k_n E}{F_y}$$

$$= 0.64 \sqrt{\frac{0.756(29,000 \text{ ksi})}{50 \text{ ksi}}}$$

$$= 13.4$$

$\lambda \leq \lambda_c$; therefore, the outside flange is not slender

Check inside flange slenderness.

$$\lambda = \frac{b}{t}$$

$$= \frac{4.0 \text{ in.}}{\frac{3}{4} \text{ in.}}$$

$$= 5.33$$

$\lambda \leq \lambda_c$; therefore, the inside flange is not slender

Check web slenderness.

$$\lambda = \frac{h}{t}$$

$$= \frac{10.5 \text{ in.}}{\frac{3}{8} \text{ in.}}$$

$$= 28.0$$

Determine the limiting slenderness ratio, $\lambda$, for the web from AISC Specification Table B4.1a Case 8

$$\lambda = 1.49 \frac{E}{F_y}$$

$$= 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 35.9$$

$\lambda \leq \lambda_c$; therefore, the web is not slender

Section Properties (ignoring welds)

$$A_k = b_1 t_{f1} + h t_w + b_2 t_{f2}$$

$$= (8.00 \text{ in.})(\frac{3}{4} \text{ in.}) + (10.5 \text{ in.})(\frac{3}{8} \text{ in.}) + (5.00 \text{ in.})(\frac{3}{4} \text{ in.})$$

$$= 13.7 \text{ in.}^2$$

$$y = \frac{\Sigma A_i y_i}{\Sigma A_i}$$

$$= \left( \frac{6.00 \text{ in.}^2)(11.6 \text{ in.}) + (3.94 \text{ in.}^2)(6.00 \text{ in.}) + (3.75 \text{ in.}^2)(0.375 \text{ in.})}{6.00 \text{ in.}^2 + (3.94 \text{ in.}^2) + (3.75 \text{ in.}^2)} \right)$$

$$= 6.91 \text{ in.}$$
Note that the center of gravity about the x-axis is measured from the bottom of the outside flange.

\[
I_x = \left[ \frac{(8.00 \text{ in.})(\frac{3}{4} \text{ in.})^3}{12} \right] + \left[ \frac{(\frac{3}{8} \text{ in.})(10\frac{1}{2} \text{ in.})^3}{12} \right] + \left[ \frac{(\frac{5}{8} \text{ in.})(5.00 \text{ in.})^3}{12} \right] + \left[ \frac{(\frac{1}{4} \text{ in.})(12 \text{ in.})^3}{12} \right]
\]

\[
= 334 \text{ in.}^4
\]

\[
r_x = \frac{I_x}{A}
\]

\[
= \frac{334 \text{ in.}^4}{13.7 \text{ in.}^2}
\]

\[
= 24.6 \text{ in.}
\]

\[
I_y = \left[ \frac{(\frac{3}{4} \text{ in.})(8.00 \text{ in.})^3}{12} \right] + \left[ \frac{(10\frac{1}{2} \text{ in.})(\frac{3}{8} \text{ in.})^3}{12} \right] + \left[ \frac{(\frac{5}{8} \text{ in.})(5.00 \text{ in.})^3}{12} \right] + \left[ \frac{(\frac{1}{4} \text{ in.})(12 \text{ in.})^3}{12} \right]
\]

\[
= 39.9 \text{ in.}^4
\]

\[
r_y = \frac{I_y}{A}
\]

\[
= \frac{39.9 \text{ in.}^4}{13.7 \text{ in.}^2}
\]

\[
= 2.93 \text{ in.}
\]

**x-x Axis Flexural Elastic Critical Buckling Stress, } F_e **

\[
K, L = \frac{1.0(14.0 \text{ ft})(12 \text{ in./ft})}{4.94 \text{ in.}}
\]

\[
= 34.0
\]

\[
F_e = \frac{\pi^2 E}{\left( \frac{KL}{r} \right)^2}
\]

\[
= \frac{\pi^2 (29,000 \text{ ksi})}{(34.0)^2}
\]

\[
= 248 \text{ ksi does not control}
\]

**Flexural-Torsional Critical Elastic Buckling Stress**

Calculate torsional constant, J.
$J = \sum \left( \frac{bh^3}{3} \right)$ from AISC Design Guide 9

$= \left( \frac{8.00 \text{ in.}}{3} \right)^3 + \left( \frac{10\frac{1}{2} \text{ in.}}{3} \right)^3 + \left( \frac{5.00 \text{ in.}}{3} \right)^3$

$= 2.01 \text{ in.}^4$

Distance between flange centroids:

$$h_o = d - \frac{t_{\alpha}}{2} - \frac{t_{\beta}}{2}$$

$= 12.0 \text{ in.} - \frac{\frac{3}{4} \text{ in.}}{2} - \frac{\frac{3}{4} \text{ in.}}{2}$

$= 11.3 \text{ in.}$

Warping constant:

$$C_w = \frac{t}{12} \left( \frac{h_1 b_2^3}{b_1^3 + b_2^3} \right)$$

$= \frac{\frac{3}{4} \text{ in.}}{12} \left( \frac{11.3 \text{ in.}^3}{(8.00 \text{ in.})^3 + (5.00 \text{ in.})^3} \right)$

$= 802 \text{ in.}^6$

Due to symmetry, both the centroid and the shear center lie on the $y$-axis. Therefore $x_c = 0$. The distance from the center of the outside flange to the shear center is:

$$e = h_o \left( \frac{b_1^3}{b_1^3 + b_2^3} \right)$$

$= 11.3 \text{ in.} \left( \frac{(8.00 \text{ in.})^3}{(8.00 \text{ in.})^3 + (5.00 \text{ in.})^3} \right)$

$= 9.08 \text{ in.}$

Add one-half the flange thickness to determine the shear center location measured from the bottom of the outside flange.

$$e + \frac{t_f}{2} = 9.08 \text{ in.} + \frac{\frac{3}{4} \text{ in.}}{2}$$

$= 9.46 \text{ in.}$

$$y_o = \left( e + \frac{t_f}{2} \right) - \bar{y}$$

$= 9.46 \text{ in.} - 6.91 \text{ in.}$

$= 2.55 \text{ in.}$

$$\bar{y}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_k}$$

$(Spec. \text{ Eq. E4-11})$

$= 0.0 + (2.55 \text{ in.})^2 + \frac{334 \text{ in.}^4 + 39.9 \text{ in.}^4}{13.7 \text{ in.}^2}$
From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, $K = 1.0$.

$$KL = \frac{1.0(14.0 \text{ ft})(12.0 \text{ in./ft})}{1.71 \text{ in.}} = 98.2$$

$$F_{\nu y} = \frac{\pi^2 E}{\left( \frac{K, L}{r_y} \right)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(98.2)^2} = 29.7 \text{ ksi}$$

$$F_{\nu z} = \left[ \frac{\pi^2 E_{\nu z}}{(K, L)^2} + GJ \left( \frac{1}{A_y r_o^2} \right) \right]$$

$$= \left[ \frac{\pi^2 (29,000 \text{ ksi})(802 \text{ in}.)}{\left( (1.0)(14.0 \text{ ft})(12 \text{ in./ft}) \right)^2} + (11,200 \text{ ksi})(2.01 \text{ in}.) \right] \left( \frac{1}{13.7 \text{ in}^2(33.8 \text{ in}^2)} \right)$$

$$= 66.2 \text{ ksi}$$

$$F_\nu = \left( \frac{F_{\nu y} + F_{\nu z}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{\nu y}F_{\nu z}H}{\left( F_{\nu y} + F_{\nu z} \right)^2}} \right]$$

$$= \left( \frac{29.7 \text{ ksi} + 66.2 \text{ ksi}}{2(0.808)} \right) \left[ 1 - \sqrt{1 - \frac{4(29.7 \text{ ksi})(66.2 \text{ ksi})(0.808)}{(29.7 \text{ ksi} + 66.2 \text{ ksi})^2}} \right]$$

$$= 26.4 \text{ ksi} \quad \text{controls}$$

Torsional and flexural-torsional buckling governs.

$$F_{\nu z} = \frac{50 \text{ ksi}}{2.25} = 22.2 \text{ ksi} \leq 26.4 \text{ ksi}, \text{ therefore, AISC Specification Equation E3-2 applies}$$

$$F_{\nu y} = \left[ 0.658 \frac{50}{76.4 \text{ ksi}} \right] F_\nu$$

$$= \left[ 0.658 \frac{50}{76.4 \text{ ksi}} \right] (50 \text{ ksi})$$
= 22.6 ksi

\[ P_e = F_{ce} A_g \]

\[ = 22.6 \text{ ksi} (13.7 \text{ in.}^2) \]

\[ = 310 \text{ kips} \]

From AISC *Specification* Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>( \phi_c P_e = 0.90(310 \text{ kips}) )</td>
<td>( P_e = 310 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( \Omega_e = \frac{1.67}{1.67} )</td>
</tr>
<tr>
<td></td>
<td>= 186 kips</td>
</tr>
</tbody>
</table>
EXAMPLE E.13 DOUBLE-WT COMPRESSION MEMBER

Given:

Determine the available compressive strength for the double-WT9×20 compression member shown below. Assume that ½-in.-thick connectors are welded in position at the ends and at equal intervals "a" along the length. Use the minimum number of intermediate connectors needed to force the two WT-shapes to act as a single built-up compressive section.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Tee
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

From AISC Manual Table 1-8 the geometric properties for a single WT9×20 are as follows:

\[ A = 5.88 \text{ in.}^2 \]
\[ I_x = 44.8 \text{ in.}^4 \]
\[ I_y = 9.55 \text{ in.}^4 \]
\[ r_x = 2.76 \text{ in.} \]
\[ r_y = 1.27 \text{ in.} \]
\[ \bar{y} = 2.29 \text{ in.} \]
\[ J = 0.404 \text{ in.}^4 \]
\[ C_w = 0.788 \text{ in.}^6 \]
\[ Q_3 = 0.496 \]
From mechanics of materials, the combined section properties for two WT9×20's, flange-to-flange, spaced 0.50 in. apart, are as follows:

\[
A = \sum A_{\text{single tee}} \\
= 2(5.88 \text{ in.}^2) \\
= 11.8 \text{ in.}^2 \\
I_x = \sum (I_x + A\bar{y}^2) \\
= 2\left[44.8 \text{ in.}^4 + (5.88 \text{ in.}^2)(2.29 \text{ in.} + \frac{1}{4} \text{ in.})^2\right] \\
= 165 \text{ in.}^4 \\
r_x = \sqrt{\frac{I_x}{A}} \\
= \sqrt{\frac{165 \text{ in.}^4}{11.8 \text{ in.}^2}} \\
= 3.74 \text{ in.} \\
I_y = \sum I_y \text{ single tee} \\
= 2(9.55 \text{ in.}^4) \\
= 19.1 \text{ in.}^4 \\
r_y = \sqrt{\frac{I_y}{A}} \\
= \sqrt{\frac{19.1 \text{ in.}^4}{11.8 \text{ in.}^2}} \\
= 1.27 \text{ in.} \\
J = \sum J \text{ single tee} \\
= 2(0.404 \text{ in.}^4) \\
= 0.808 \text{ in.}^4
\]

For the double-WT (cruciform) shape it is reasonable to take C_w = 0 and ignore any warping contribution to column strength.

The y-axis of the combined section is the same as the y-axis of the single section. When buckling occurs about the y-axis, there is no relative slip between the two WTs. For buckling about the x-axis of the combined section, the WT's will slip relative to each other unless restrained by welded or slip-critical end connections.

Intermediate Connectors

Determine the minimum adequate number of intermediate connectors.

From AISC Specification Section E6.2, the maximum slenderness ratio of each tee may not exceed three-quarters of the maximum slenderness ratio of the double-tee built-up section. For a WT9×20, the minimum radius of gyration, r_i = r_y = 1.27 in.

Use K = 1.0 for both the single tee and the double tee:
\[
\left( \frac{Ka}{n} \right)_{\text{single tee}} \leq 0.75 \left( \frac{KL}{r_{\text{min}}/\text{double tee}} \right)
\]

\[
a \leq 0.75 \left( \frac{r_{y}}{r_{\text{double tee}}} \right) \left( \frac{K_{\text{double tee}}L}{K_{\text{single tee}}} \right)
\]

\[
= 0.75 \left( \frac{1.27 \text{ in.}}{1.27 \text{ in.}} \right) \left( \frac{1.0(9.00 \text{ ft})(12 \text{ in./ft})}{1.0} \right)
\]

\[
= 81.0 \text{ in.}
\]

Thus, one intermediate connector at mid-length \((a = 4.5 \text{ ft} = 54 \text{ in.})\) satisfies AISC Specification Section E6.2.

Flexural Buckling and Torsional Buckling Strength

The nominal compressive strength, \(P_n\), is computed using

\[P_n = F_{ce} A_g\]  

(Spec. Eq. E3-1 or Eq. E7-1)

where the critical stress is determined using AISC Specification equations in Sections E3, E4 or E7, as appropriate.

For the WT9\times20, the stem is slender because \(d/t_w = 28.4 > 0.75 \sqrt{29,000 \text{ ksi}/50 \text{ ksi}} = 18.1\). Therefore, the member is a slender element member and the provisions of Section E7 must be followed. Determine the elastic buckling stress for flexural buckling about the \(y\)- and \(x\)-axes, and torsional buckling. Then, using \(Q_s\), determine the critical buckling stress and the nominal strength.

Flexural buckling about the \(y\)-axis:

\[
r_y = 1.27 \text{ in.}
\]

\[
KL = 1.0(9.0 \text{ ft})(12 \text{ in./ft})
\]

\[
r_y = 1.27 \text{ in.}
\]

\[
= 85.0
\]

\[
F_{ce} = \frac{\pi^2 E}{(KL/r)^2}
\]  

(Spec. Eq. E3-4)
Flexural buckling about the x-axis:

Flexural buckling about the x-axis is determined using the modified slenderness ratio to account for shear deformation of the intermediate connectors.

Note that the provisions of AISC Specification Section E6.1, which require that KL/r be replaced with (KL/r)_m, apply if “the buckling mode involves relative deformations that produce shear forces in the connectors between individual shapes...”. Relative slip between the two sections occurs for buckling about the x-axis so the provisions of the section apply only to buckling about the x-axis.

The connectors are welded at the ends and the intermediate point. The modified slenderness is calculated using the spacing between intermediate connectors:

\[
a = 4.5 \text{ ft}(12.0 \text{ in./ft}) = 54.0 \text{ in.}
\]

\[
r_{lb} = r_y = 1.27 \text{ in.}
\]

\[
\frac{a}{r_{lb}} = \frac{54.0 \text{ in.}}{1.27 \text{ in.}} = 42.5
\]

Because \(a/r_{lb} > 40\), use AISC Specification Equation E6-2b.

\[
\left( \frac{KL}{r} \right)_m = \sqrt{\left( \frac{KL}{r} \right)_o^2 + \left( \frac{K_i a}{r_i} \right)^2}
\]

(Spec. Eq. E6-2b)

where

\[
\left( \frac{KL}{r} \right)_o = \left( \frac{1.0(9.00 \text{ ft})(12.0 \text{ in./ft})}{3.74 \text{ in.}} \right) = 28.9
\]

\[
\left( \frac{K_i a}{r_i} \right) = \left( \frac{0.86(4.50 \text{ ft})(12.0 \text{ in./ft})}{1.27 \text{ in.}} \right) = 36.6
\]

Thus,

\[
\left( \frac{KL}{r} \right)_m = \sqrt{(28.9)^2 + (36.6)^2} = 46.6
\]

and

\[
F_e = \frac{\pi^2 E}{\left( \frac{KL}{r_x} \right)_m^2}
\]

(from Spec. Eq. E3-4)


\[
\frac{\pi^2 (29,000 \text{ ksi})}{(46.6)^2} = 132 \text{ ksi}
\]

Torsional buckling:

\[
F_e = \frac{\pi^2 E C_w}{(K_z L)^2} + \frac{GJ}{I_x + I_y} \frac{1}{(K_z L)^2} \quad \quad \text{(Spec. Eq. E4-4)}
\]

The cruciform section made up of two back-to-back WT's has virtually no warping resistance, thus the warping contribution is ignored and Equation E4-4 becomes

\[
F_e = \frac{GJ}{I_x + I_y}
\]

\[
= \frac{(11,200 \text{ ksi})(0.808 \text{ in.}^4)}{(165 \text{ in.}^4 + 19.1 \text{ in.}^4)} = 49.2 \text{ ksi}
\]

Use the smallest elastic buckling stress, \( F_e \), from the limit states considered above to determine \( F_{cr} \) by AISC Specification Equation E7-2 or Equation E7-3, as follows:

\[
Q_s = 0.496
\]

\[
F_e = F_{e\text{,smallest}}
\]

\[
= 39.6 \text{ ksi (y-axis flexural buckling)}
\]

\[
\frac{Q F_y}{F_e} = \frac{0.496(50 \text{ ksi})}{39.6 \text{ ksi}} = 0.626 < 2.25
\]

Therefore use Equation E7-2,

\[
F_{cr} = Q \left[ 0.658 F_e \right] F_y
\]

\[
= 0.496 \left[ 0.658 \times 0.626 \right] (50 \text{ ksi})
\]

\[
= 19.1 \text{ ksi}
\]

Determine the nominal compressive strength, \( P_n \):

\[
P_n = F_{cr} A_g \quad \quad \text{(Spec. Eq. E7-1)}
\]

\[
= (19.1 \text{ ksi})(11.8 \text{ in.}^2) = 225 \text{ kips}
\]

Determine the available compressive strength:
\[ \phi_c = 0.90 \]
\[ \phi_c P_a = 0.90(225 \text{ kips}) \]
\[ = 203 \text{ kips} \]

\[ \Omega_c = 1.67 \]
\[ \frac{P_a}{\Omega_c} = \frac{225 \text{ kips}}{1.67} \]
\[ = 135 \text{ kips} \]
EXAMPLE E.14  ECCENTRICALLY LOADED SINGLE-ANGLE COMPRESSION MEMBER
(LONG LEG ATTACHED)

Given:

Determine the available strength of an eccentrically loaded ASTM A36 L8×4×½ single angle, as shown, with an effective length of 5 ft. The long leg of the angle is the attached leg, and the eccentric load is applied at 0.75t as shown. Use the provisions of the AISC Specification and compare the results to the available strength found in AISC Manual Table 4-12.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36

\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-7:

L8×4×½

\( b = 8.00 \text{ in.} \)
\( d = 4.00 \text{ in.} \)
\( t = 3/8 \text{ in.} \)
\( \bar{x} = 0.829 \text{ in.} \)
\( \bar{y} = 2.81 \text{ in.} \)
\( A = 5.11 \text{ in.}^2 \)
\( I_x = 34.2 \text{ in.}^4 \)
\( I_y = 6.03 \text{ in.}^4 \)
\( I_z = 3.84 \text{ in.}^4 \)
\( r_i = 0.867 \text{ in.} \)
\( \tan \alpha = 0.268 \)
From AISC Shapes Database V14.0:

\[ I_w = 36.4 \text{ in.}^4 \]
\[ S_{wA} = 11.0 \text{ in.}^3 \]
\[ S_{wB} = 14.6 \text{ in.}^3 \]
\[ S_{wC} = 7.04 \text{ in.}^3 \]
\[ S_{zA} = 1.61 \text{ in.}^3 \]
\[ S_{zB} = 2.51 \text{ in.}^3 \]
\[ S_{zC} = 5.09 \text{ in.}^3 \]

From geometry, distances between points A, B, C and the principal \( w-w \) and \( z-z \) axes, as shown in Figure E.14-1, are determined as follows:

\[ \alpha = \tan^{-1}(0.268) \]
\[ = 15.0^\circ \]
\[ w_A = (d - \overline{x}) \cos \alpha - (\overline{y} - \frac{t}{2}) \sin \alpha \]
\[ = (4.00 \text{ in.} - 0.829 \text{ in.}) \cos 15.0^\circ - \left(2.81 \text{ in.} - \frac{\gamma_0 \text{ in.}}{2}\right) \sin 15.0^\circ \]
\[ = 2.39 \text{ in.} \]
\[ w_B = \overline{x} \cos \alpha + \overline{y} \sin \alpha \]
\[ = 0.829 \text{ in.}(\cos 15.0^\circ) + 2.81 \text{ in.}(\sin 15.0^\circ) \]
\[ = 1.53 \text{ in.} \]
\[ w_C = (b - \overline{y}) \sin \alpha - (\overline{x} - \frac{t}{2}) \cos \alpha \]
\[ = (8.00 \text{ in.} - 2.81 \text{ in.}) \sin 15.0^\circ - \left(0.829 \text{ in.} - \frac{\gamma_0 \text{ in.}}{2}\right) \cos 15.0^\circ \]
\[ = 0.754 \text{ in.} \]
\[ z_A = \left[(d - \overline{x}) - (\overline{y} - \frac{t}{2}) \tan \alpha\right] \sin \alpha + \frac{\overline{y} - \frac{t}{2}}{\cos \alpha} \]
\[ = \left[(4.00 \text{ in.} - 0.829 \text{ in.}) - \left(2.81 \text{ in.} - \frac{\gamma_0 \text{ in.}}{2}\right)(0.268)\right] \sin 15.0^\circ + \frac{2.81 \text{ in.} - \frac{\gamma_0 \text{ in.}}{2}}{\cos 15.0^\circ} \]
\[ = 3.32 \text{ in.} \]
\[ z_B = \overline{y} \cos \alpha - \overline{x} \sin \alpha \]
\[ = 2.81 \text{ in.}(\cos 15.0^\circ) - 0.829 \text{ in.}(\sin 15.0^\circ) \]
\[ = 2.50 \text{ in.} \]
\[ z_C = (b - \overline{y}) \cos \alpha + \left(\overline{x} - \frac{t}{2}\right) \sin \alpha \]
\[ = (8.00 \text{ in.} - 2.81 \text{ in.}) \cos 15.0^\circ + \left(0.829 \text{ in.} - \frac{\gamma_0 \text{ in.}}{2}\right) \sin 15.0^\circ \]
\[ = 5.17 \text{ in.} \]
The load is applied at the location shown in Figure E.14-1. Determine the eccentricities about the major (w-w axis) and minor (z-z axis) principal axes for the load, $P$. From AISC Manual Table 1-7, the angle of the principal axes is found to be $\alpha = \tan^{-1}(0.268) = 15.0^\circ$.

Using the geometry shown in Figures E.14-1 and E.14-2:

$$e_w = \left[ (\bar{x} + 0.75r) - (0.5b - \bar{y}) \tan \alpha \right] \sin \alpha + \left( \frac{0.5b - \bar{y}}{\cos \alpha} \right)$$

$$= \left[ (0.829 \text{ in.} + 0.75(\gamma\text{ in.})) - (4.00 \text{ in.} - 2.81 \text{ in.})(0.268) \right] (0.259) + \left( \frac{4.00 \text{ in.} - 2.81 \text{ in.}}{0.966} \right)$$

$$= 1.45 \text{ in.}$$

$$e_z = (\bar{x} + 0.75r) \cos \alpha - (0.5b - \bar{y}) \sin \alpha$$

$$= \left[ (0.829 \text{ in.} + 0.75(\gamma \text{ in.}))(0.966) - (4.00 \text{ in.} - 2.81 \text{ in.})(0.259) \right]$$

$$= 0.810 \text{ in.}$$

Because of these eccentricities, the moment resultant has components about both principal axes; therefore, the combined stress provisions of AISC Specification Section H2 must be followed.

$$\left| \frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}} \right| \leq 1.0$$
Due to the load and the given eccentricities, moments about the w-w and z-z axes will have different effects on points A, B and C. The axial force will produce a compressive stress and the moments, where positive moments are in the direction shown in Figure E.14-2, will produce stresses with a sign indicated by the sense given in the following. In this example, compressive stresses will be taken as positive and tensile stresses will be taken as negative.

<table>
<thead>
<tr>
<th>Point</th>
<th>Caused by $M_w$</th>
<th>Caused by $M_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>tension</td>
<td>tension</td>
</tr>
<tr>
<td>B</td>
<td>tension</td>
<td>compression</td>
</tr>
<tr>
<td>C</td>
<td>compression</td>
<td>tension</td>
</tr>
</tbody>
</table>

Available Compressive Strength

Check the slenderness of the longest leg for uniform compression.

$$\lambda = \frac{b}{t}$$

$$= \frac{8.00 \text{ in.}}{0.45 \text{ in.}}$$

$$= 18.3$$

From AISC Specification Table B4.1a, the limiting width-to-thickness ratio is:

$$\lambda_r = 0.45 \sqrt{\frac{E}{F_y}}$$

$$= 0.45 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}}$$

$$= 12.8$$

Since $b/t = 18.3 > 12.8$, this angle must be treated as a slender element compression member according to AISC Specification Section E7.1(c). To determine the appropriate equation for determination of $Q_s$, compare $b/t$ to 0.91
Thus, \( Q \) is computed for a slender unstiffened element in compression from AISC Specification Equation E7-11.

\[
Q = Q_s
\]

\[
= 1.34 - 0.76 \left( \frac{b}{t} \right) \sqrt{\frac{F_y}{E}}
\]

\[
= 1.34 - 0.76 (18.3) \sqrt{\frac{36 \text{ ksi}}{29,000 \text{ ksi}}}
\]

\[
= 0.850
\]

Determine the critical stress, \( F_{cr} \), with \( KL = 60.0 \text{ in.} \) for buckling about the z-z axis.

\[
KL = 60.0 \text{ in.}
\]

\[
r_z = 0.867 \text{ in.}
\]

\[
= 69.2
\]

\[
< 4.71 \sqrt{\frac{E}{QF_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{(0.850)(36 \text{ ksi})}}
\]

\[
< 145
\]

Therefore, use Equation E7-2:

\[
F_{cr} = Q \left[ 0.658 \frac{QF}{F_y} \right] F_y
\]

where

\[
F_e = \frac{\pi^2 E}{(KL)^2}
\]

\[
= \frac{\pi^2 (29,000 \text{ ksi})}{(69.2)^2}
\]

\[
= 59.8 \text{ ksi}
\]

Therefore:

\[
F_{cr} = Q \left[ 0.658 \frac{QF}{F_y} \right] F_y
\]

\[
= 0.850 \left[ 0.658 \frac{0.850(36 \text{ ksi})}{59.8 \text{ ksi}} \right] (36 \text{ ksi})
\]

\[
= 24.7 \text{ ksi}
\]

The nominal strength, \( P_n \), is:
\( P_n = F_{cr} A_g \)  
\[ = 24.7 \text{ ksi}(5.11 \text{ in.}) \]
\[ = 126 \text{ kips} \]

From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>( \phi_c P_c = 0.90(126 \text{ kips}) )</td>
<td>( P_c = \frac{126 \text{ kips}}{1.67} = 75.4 \text{ kips} )</td>
</tr>
</tbody>
</table>

Determine the available flexural strengths, \( M_{bcw} \) and \( M_{bcz} \), and the available flexural stresses at each point on the cross section.

Limit State of Yielding

Consider the limit state of yielding for bending about the \( w-w \) and \( z-z \) axes at points A, B and C, according to AISC Specification Section F10.1.

**w-w axis:**

\( S_{wA} = 11.0 \text{ in.}^3 \)
\( M_{ywA} = F_y S_{wA} \)
\[ = (36 \text{ ksi})(11.0 \text{ in.}^3) \]
\[ = 396 \text{ kip-in.} \]
\( M_{awA} = 1.5 M_{ywA} \)
\[ = 1.5(396 \text{ kip-in.}) \]
\[ = 594 \text{ kip-in.} \]

\( S_{wB} = 14.6 \text{ in.}^3 \)
\( M_{ywB} = F_y S_{wB} \)
\[ = (36 \text{ ksi})(14.6 \text{ in.}^3) \]
\[ = 526 \text{ kip-in.} \]
\( M_{awB} = 1.5 M_{ywB} \)
\[ = 1.5(526 \text{ kip-in.}) \]
\[ = 789 \text{ kip-in.} \]

\( S_{wC} = 7.04 \text{ in.}^3 \)
\( M_{ywC} = F_y S_{wC} \)
\[ = (36 \text{ ksi})(7.04 \text{ in.}^3) \]
\[ = 253 \text{ kip-in.} \]
\( M_{awC} = 1.5 M_{ywC} \)
\[ = 1.5(253 \text{ kip-in.}) \]
\[ = 380 \text{ kip-in.} \]

**z-z axis:**
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\[ S_{ZA} = 1.61 \text{ in.}^3 \]
\[ M_{yzA} = F_y S_{ZA} \]
\[ = (36 \text{ ksi})(1.61 \text{ in.}^3) \]
\[ = 58.0 \text{ kip-in.} \]
\[ M_{nZA} = 1.5M_{yzA} \]
\[ = 1.5(58.0 \text{ kip-in.}) \]
\[ = 87.0 \text{ kip-in.} \]
\[ S_{zB} = 2.51 \text{ in.}^3 \]
\[ M_{yzB} = F_y S_{zB} \]
\[ = (36 \text{ ksi})(2.51 \text{ in.}^3) \]
\[ = 90.4 \text{ kip-in.} \]
\[ M_{nZB} = 1.5M_{yzB} \]
\[ = 1.5(90.4 \text{ kip-in.}) \]
\[ = 136 \text{ kip-in.} \]
\[ S_{zC} = 5.09 \text{ in.}^3 \]
\[ M_{yzC} = F_y S_{zC} \]
\[ = (36 \text{ ksi})(5.09 \text{ in.}^3) \]
\[ = 183 \text{ kip-in.} \]
\[ M_{nZC} = 1.5M_{yzC} \]
\[ = 1.5(183 \text{ kip-in.}) \]
\[ = 275 \text{ kip-in.} \]

Select the least \( M_n \) for each axis:

For the limit state of yielding about the \( w-w \) axis,
\[ M_{nw} = 380 \text{ kip-in.} \]

For the limit state of yielding about the \( z-z \) axis,
\[ M_{nZ} = 87.0 \text{ kip-in.} \]

Limit State of Lateral-Torsional Buckling

From AISC Specification Section F10.2, the limit state of lateral-torsional buckling of a single angle without continuous restraint along its length is a function of the elastic lateral-torsional buckling moment about the major principal axis. For bending about the major principal axis for an unequal leg angle:

\[
M_e = \frac{4.9EIZ_b}{L_b^2} \left[ \sqrt{\beta_w^2 + 0.052 \left( \frac{L_b t}{r_z} \right)^2} + \beta_w \right] \quad (Spec. \ Eq. F10-5)
\]

From AISC Specification Section F1, for uniform moment along the member length, \( C_b = 1.0 \). From AISC Specification Commentary Table C-F10.1, an L8×4×7/8 has \( \beta_w = 5.48 \text{ in.} \). From AISC Specification Commentary
Figure C-F10.4b, with the tip of the long leg (point C) in compression for bending about the \(w\)-axis, \(\beta_w\) is taken as negative. Thus:

\[
M_c = \frac{4.9(29,000 \text{ ksi})(3.84 \text{ in.}^4)(1.0)}{(60.0 \text{ in.})^2} \left[ \sqrt{(-5.48 \text{ in.})^2 + (0.052) \left( \frac{(60.0 \text{ in.})(0.052 \text{ in.})}{0.867 \text{ in.}} \right)^2} + (-5.48 \text{ in.}) \right]
\]

\[
= 505 \text{ kip-in.}
\]

Because \(M_{ywC} = 253 \text{ kip-in.} < M_c = 505 \text{ kip-in.}\), determine \(M_n\) from AISC Specification Equation F10-3:

\[
M_{nwC} = \left( 1.92 - 1.17 \sqrt{\frac{M_{ywC}}{M_c}} \right) M_{ywC} \leq 1.5 M_{ywC}
\]

\[
= \left( 1.92 - 1.17 \sqrt{\frac{253 \text{ kip-in.}}{505 \text{ kip-in.}}} \right) (253 \text{ kip-in.}) \leq 1.5 (253 \text{ kip-in.})
\]

\[
= 276 \text{ kip-in.} \leq 380 \text{ kip-in.}
\]

Limit State of Leg Local Buckling

From AISC Specification Section F10.3, the limit state of leg local buckling applies when the toe of the leg is in compression. As discussed previously and indicated in Table E.14-1, the only case in which a toe is in compression is point C for bending about the \(w\)-\(w\) axis. Thus, determine the slenderness of the long leg. From AISC Specification Table B4.1b:

\[
\lambda_p = 0.54 \sqrt{\frac{E}{F_y}}
\]

\[
= 0.54 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}}
\]

\[
= 15.3
\]

\[
\lambda_r = 0.91 \sqrt{\frac{E}{F_y}}
\]

\[
= 0.91 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}}
\]

\[
= 25.8
\]

\[
\lambda = \frac{b}{t} = \frac{8.0 \text{ in.}}{\gamma_0 \text{ in.}} = 18.3
\]

Because \(\lambda_p < \lambda < \lambda_r\), the angle is noncompact for flexure for this loading. From AISC Specification Equation F10-7:

\[
M_{nwC} = F_y S_{wc} \left( 2.43 - 1.72 \left( \frac{b}{t} \right) \sqrt{\frac{F_y}{E}} \right)
\]

\[
(Spec. \ Eq. \ F10-7)
\]
\[
\frac{\phi_b}{\phi_{bw}} = \phi_b = 0.90 \\
M_{bw} = \phi_b M_{bw} \\
= 0.90(276 \text{ kip-in.}) \\
= 248 \text{ kip-in.}
\]

\[
\phi_b = 0.90 \\
M_{bw} = \phi_b M_{bw} \\
= 0.90(276 \text{ kip-in.}) \\
= 248 \text{ kip-in.}
\]

\[
M_{ew} = \frac{276 \text{ kip-in.}}{1.67} = 165 \text{ kip-in.}
\]

\[
M_{zc} = \frac{87.0 \text{ kip-in.}}{1.67} = 52.1 \text{ kip-in.}
\]

**Available Flexural Strength**

Select the controlling nominal flexural strength for the \( w-w \) and \( z-z \) axes.

For the \( w-w \) axis:

\[
M_{nw} = 276 \text{ kip-in.}
\]

For the \( z-z \) axis:

\[
M_{nz} = 87.0 \text{ kip-in.}
\]

From AISC *Specification* Section F1, determine the available flexural strength for each axis, \( w-w \) and \( z-z \) as follows:

**Required Flexural Strength**

The load on the column is applied at eccentricities about the \( w-w \) and \( z-z \) axes resulting in the following moments:

\[
M_w = P_r e_w \\
= P_r (1.45 \text{ in.})
\]
The combination of axial load and moment will produce second-order effects in the column which must be accounted for.

Using AISC Specification Appendix 8.2, an approximate second-order analysis can be performed. The required second-order flexural strengths will be $B_{1w}M_w$ and $B_{1z}M_z$, respectively, where

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{el}}}$$  \hspace{1cm} (Spec. Eq. A-8-3)

and

$\alpha = 1.0$ (LRFD)

$\alpha = 1.6$ (ASD)

$C_m = 1.0$ for a column with uniform moment along its length

For each axis, parameters $P_{elw}$ and $P_{elz}$, as used in the moment magnification terms, $B_{1w}$ and $B_{1z}$, are:

$$P_{elw} = \frac{\pi^2 EI_w}{(KL)^2}$$  \hspace{1cm} (Spec. Eq. A-8-5)

$$= \frac{\pi^2 (29,000 \text{ ksi})(36.4 \text{ in.}^4)}{(60.0 \text{ in.})^2}$$

$$= 2,890 \text{ kips}$$

$$P_{elz} = \frac{\pi^2 EI_z}{(KL)^2}$$

$$= \frac{\pi^2 (29,000 \text{ ksi})(3.84 \text{ in.}^4)}{(60.0 \text{ in.})^2}$$

$$= 305 \text{ kips}$$

and

$$B_{1w} = \frac{C_m}{1 - \frac{\alpha P_r}{P_{elw}}}$$  \hspace{1cm} (from Spec. Eq. A-8-3)

$$= \frac{1.0}{1 - \frac{\alpha P_r}{2,890 \text{ kips}}}$$

$$B_{1z} = \frac{C_m}{1 - \frac{\alpha P_r}{P_{elz}}}$$  \hspace{1cm} (from Spec. Eq. A-8-3)

Thus, the required second-order flexural strengths are:
Interaction of Axial and Flexural Strength

Evaluate the interaction of axial and flexural stresses according to the provisions of AISC Specification Section H2.

The interaction equation is given as:

$$\left(\frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}}\right) \leq 1.0$$

(Spec. Eq. H2-1)

where the stresses are to be considered at each point on the cross section with the appropriate sign representing the sense of the stress. Because the required stress and available stress at any point are both functions of the same section property, $A$ or $S$, it is possible to convert Equation H2-1 from a stress based equation to a force based equation where the section properties will cancel.

Substituting the available strengths and the expressions for the required second-order flexural strengths into AISC Specification Equation H2-1 yields:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[
\left| \frac{P_u}{113 \text{ kips}} + \frac{P_u (1.45 \text{ in.})}{248 \text{ kip-in.}} \left(1 - \frac{1.00P_u}{2,890 \text{ kips}}\right) \right| \leq 1.0
\] | \[
\left| \frac{P_u}{75.4 \text{ kips}} + \frac{P_u (1.45 \text{ in.})}{165 \text{ kip-in.}} \left(1 - \frac{1.60P_u}{2,890 \text{ kips}}\right) \right| \leq 1.0
\] |

| \[
\left| \frac{P_u (0.810 \text{ in.})}{78.3 \text{ kip-in.}} \left(1 - \frac{1.00P_u}{305 \text{ kips}}\right) \right| \leq 1.0
\] | \[
\left| P_u (0.810 \text{ in.}) \left(1 - \frac{1.60P_u}{305 \text{ kips}}\right) \right| \leq 1.0
\] |

These interaction equations must now be applied at each critical point on the section, points A, B and C using the appropriate sign for the sense of the resulting stress, with compression taken as positive.

For point A, the $w$ term is negative and the $z$ term is negative. Thus:
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<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{P_u}{113 \text{ kips}} - \frac{P_t (1.45 \text{ in.})}{248 \text{ kip-in.}} \left( \frac{1.0}{1 - \frac{1.00P_u}{2,890 \text{ kips}}} \right) \leq 1.0 ]</td>
<td>[ \frac{P_u}{75.4 \text{ kips}} - \frac{P_t (1.45 \text{ in.})}{165 \text{ kip-in.}} \left( \frac{1.0}{1 - \frac{1.60P_u}{2,890 \text{ kips}}} \right) \leq 1.0 ]</td>
</tr>
</tbody>
</table>

By iteration, \( P_u = 86.2 \text{ kips} \). By iteration, \( P_u = 56.1 \text{ kips} \).

For point B, the \( w \) term is negative and the \( z \) term is positive. Thus:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{P_u}{113 \text{ kips}} + \frac{P_t (0.810 \text{ in.})}{78.3 \text{ kip-in.}} \left( \frac{1.0}{1 - \frac{1.00P_u}{305 \text{ kips}}} \right) \leq 1.0 ]</td>
<td>[ \frac{P_u}{75.4 \text{ kips}} + \frac{P_t (0.810 \text{ in.})}{52.1 \text{ kip-in.}} \left( \frac{1.0}{1 - \frac{1.60P_u}{305 \text{ kips}}} \right) \leq 1.0 ]</td>
</tr>
</tbody>
</table>

By iteration, \( P_u = 62.9 \text{ kips} \). By iteration, \( P_u = 41.1 \text{ kips} \).

For point C, the \( w \) term is positive and the \( z \) term is negative. Thus:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{P_u}{113 \text{ kips}} + \frac{P_t (1.45 \text{ in.})}{248 \text{ kip-in.}} \left( \frac{1.0}{1 - \frac{1.00P_u}{2,890 \text{ kips}}} \right) \leq 1.0 ]</td>
<td>[ \frac{P_u}{75.4 \text{ kips}} + \frac{P_t (1.45 \text{ in.})}{165 \text{ kip-in.}} \left( \frac{1.0}{1 - \frac{1.60P_u}{2,890 \text{ kips}}} \right) \leq 1.0 ]</td>
</tr>
</tbody>
</table>

By iteration, \( P_u = 158 \text{ kips} \). By iteration, \( P_u = 99.7 \text{ kips} \).

**Governing Available Strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the above iterations, ( P_u = 62.9 \text{ kips} ). From AISC Manual Table 4-12, ( \phi P_u = 62.8 \text{ kips} ).</td>
<td>From the above iterations, ( P_u = 41.1 \text{ kips} ). From AISC Manual Table 4-12, ( P_u/\Omega = 41.4 \text{ kips} ).</td>
</tr>
</tbody>
</table>

Thus, the calculations demonstrate how the values for this member in AISC Manual Table 4-12 can be confirmed. The slight variations between the calculated solutions and those from AISC Manual Table 4-12 are due to rounding.
Chapter F
Design of Members for Flexure

INTRODUCTION

This Specification chapter contains provisions for calculating the flexural strength of members subject to simple bending about one principal axis. Included are specific provisions for I-shaped members, channels, HSS, tees, double angles, single angles, rectangular bars, rounds and unsymmetrical shapes. Also included is a section with proportioning requirements for beams and girders.

There are selection tables in the AISC Manual for standard beams in the commonly available yield strengths. The section property tables for most cross sections provide information that can be used to conveniently identify noncompact and slender element sections. LRFD and ASD information is presented side-by-side.

Most of the formulas from this chapter are illustrated by the following examples. The design and selection techniques illustrated in the examples for both LRFD and ASD will result in similar designs.

F1. GENERAL PROVISIONS

Selection and evaluation of all members is based on deflection requirements and strength, which is determined as the design flexural strength, \( \phi_b M_n \), or the allowable flexural strength, \( M_n/\Omega_b \), where

\[ M_n = \text{the lowest nominal flexural strength based on the limit states of yielding, lateral torsional-buckling, and local buckling, where applicable} \]

\[ \phi_b = 0.90 \text{ (LRFD)} \quad \Omega_b = 1.67 \text{ (ASD)} \]

This design approach is followed in all examples.

The term \( L_b \) is used throughout this chapter to describe the length between points which are either braced against lateral displacement of the compression flange or braced against twist of the cross section. Requirements for bracing systems and the required strength and stiffness at brace points are given in AISC Specification Appendix 6.

The use of \( C_b \) is illustrated in several of the following examples. AISC Manual Table 3-1 provides tabulated \( C_b \) values for some common situations.

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

AISC Specification Section F2 applies to the design of compact beams and channels. As indicated in the User Note in Section F2 of the AISC Specification, the vast majority of rolled I-shaped beams and channels fall into this category. The curve presented as a solid line in Figure F-1 is a generic plot of the nominal flexural strength, \( M_n \), as a function of the unbraced length, \( L_b \). The horizontal segment of the curve at the far left, between \( L_b = 0 \) ft and \( L_p \), is the range where the strength is limited by flexural yielding. In this region, the nominal strength is taken as the full plastic moment strength of the section as given by AISC Specification Equation F2-1. In the range of the curve at the far right, starting at \( L_r \), the strength is limited by elastic buckling. The strength in this region is given by AISC Specification Equation F2-3. Between these regions, within the linear region of the curve between \( M_n = M_p \) at \( L_p \) on the left, and \( M_n = 0.7M_f = 0.7F_y S_x \) at \( L_r \) on the right, the strength is limited by inelastic buckling. The strength in this region is provided in AISC Specification Equation F2-2.
The curve plotted as a heavy solid line represents the case where \( C_b = 1.0 \), while the heavy dashed line represents the case where \( C_b \) exceeds 1.0. The nominal strengths calculated in both AISC Specification Equations F2-2 and F2-3 are linearly proportional to \( C_b \), but are limited to \( M_p \) as shown in the figure.

\[
M_n = M_p = F_y Z_x
\]  
(Spec. Eq. F2-1)

\[
M_n = C_b \left[ M_p - \left( M_p - 0.7F_y S_x \right) \left( \frac{L_0 - L_p}{L_r} \right) \right] \leq M_p
\]  
(Spec. Eq. F2-2)

\[
M_n = F_{cr} S_x \leq M_p
\]  
(Spec. Eq. F2-3)

where

\[
F_{cr} = \frac{C_b \pi^2 E}{\left( \frac{L_0}{r_c} \right)^2} \sqrt{1 + 0.078 \frac{J_c}{h_n \left( \frac{L_0}{r_c} \right)^2}}
\]  
(Spec. Eq. F2-4)

The provisions of this section are illustrated in Example F.1 (W-shape beam) and Example F.2 (channel).

Plastic design provisions are given in AISC Specification Appendix 1. \( L_{pd} \), the maximum unbraced length for prismatic member segments containing plastic hinges is less than \( L_p \).
F3. DOUBLY SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES BENT ABOUT THEIR MAJOR AXIS

The strength of shapes designed according to this section is limited by local buckling of the compression flange. Only a few standard wide flange shapes have noncompact flanges. For these sections, the strength reduction for $F_y = 50$ ksi steel varies. The approximate percentages of $M_p$ about the strong axis that can be developed by noncompact members when braced such that $L_b \leq L_p$ are shown as follows:

- $W21\times48 = 99\%$
- $W14\times99 = 99\%$
- $W14\times90 = 97\%$
- $W12\times65 = 98\%$
- $W10\times12 = 99\%$
- $W8\times31 = 99\%$
- $W8\times10 = 99\%$
- $W6\times15 = 94\%$
- $W6\times8.5 = 97\%$

The strength curve for the flange local buckling limit state, shown in Figure F-2, is similar in nature to that of the lateral-torsional buckling curve. The horizontal axis parameter is $\lambda = b_t/2t_f$. The flat portion of the curve to the left of $\lambda_{pf}$ is the plastic yielding strength, $M_p$. The curved portion to the right of $\lambda_{rf}$ is the strength limited by elastic buckling of the flange. The linear transition between these two regions is the strength limited by inelastic flange buckling.

![Diagram](image)

$M_n = M_p = F_y Z_x$  
(Spec. Eq. F2-1)

$M_n = \left[ M_p - (M_p - 0.7F_y S_x) \left( \frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right) \right]$  
(Spec. Eq. F3-1)

$M_n = \frac{0.9E k_c S_x}{\lambda^2}$  
(Spec. Eq. F3-2)

where

$k_c = \frac{4}{\sqrt{b_t/t_f}}$ from AISC Specification Table B4.1b footnote [a], where $0.35 \leq k_c \leq 0.76$

The strength reductions due to flange local buckling of the few standard rolled shapes with noncompact flanges are incorporated into the design tables in Chapter 3 of the AISC Manual.
There are no standard I-shaped members with slender flanges. The noncompact flange provisions of this section are illustrated in Example F.3.

F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS BENT ABOUT THEIR MAJOR AXIS

This section of the AISC Specification applies to doubly symmetric I-shaped members with noncompact webs and singly symmetric I-shaped members (those having different flanges) with compact or noncompact webs.

F5. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to I-shaped members with slender webs, formerly designated as “plate girders”.

F6. I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MINOR AXIS

I-shaped members and channels bent about their minor axis are not subject to lateral-torsional buckling. Rolled or built-up shapes with noncompact or slender flanges, as determined by AISC Specification Tables B4.1a and B4.1b, must be checked for strength based on the limit state of flange local buckling using Equations F6-2 or F6-3 as applicable.

The vast majority of W, M, C and MC shapes have compact flanges, and can therefore develop the full plastic moment, $M_p$, about the minor axis. The provisions of this section are illustrated in Example F.5.

F7. SQUARE AND RECTANGULAR HSS AND BOX-SHAPED MEMBERS

Square and rectangular HSS need only be checked for the limit states of yielding and local buckling. Although lateral-torsional buckling is theoretically possible for very long rectangular HSS bent about the strong axis, deflection will control the design as a practical matter.

The design and section property tables in the AISC Manual were calculated using a design wall thickness of 93% of the nominal wall thickness. Strength reductions due to local buckling have been accounted for in the AISC Manual design tables. The selection of rectangular or square HSS with compact flanges is illustrated in Example F.6. The provisions for rectangular or square HSS with noncompact flanges are illustrated in Example F.7. The provisions for HSS with slender flanges are illustrated in Example F.8. Available flexural strengths of rectangular and square HSS are listed in Tables 3-12 and 3-13, respectively.

F8. ROUND HSS

The definition of HSS encompasses both tube and pipe products. The lateral-torsional buckling limit state does not apply, but round HSS are subject to strength reductions from local buckling. Available strengths of round HSS and Pipes are listed in AISC Manual Tables 3-14 and 3-15, respectively. The tabulated properties and available flexural strengths of these shapes in the AISC Manual are calculated using a design wall thickness of 93% of the nominal wall thickness. The design of a Pipe is illustrated in Example F.9.

F9. TEES AND DOUBLE ANGLES LOADED IN THE PLANE OF SYMMETRY

The AISC Specification provides a check for flange local buckling, which applies only when the flange is in compression due to flexure. This limit state will seldom govern. A check for local buckling of the web was added in the 2010 edition of the Specification. Attention should be given to end conditions of tees to avoid inadvertent fixed end moments which induce compression in the web unless this limit state is checked. The design of a WT-shape in bending is illustrated in Example F.10.
F10. SINGLE ANGLES

Section F10 permits the flexural design of single angles using either the principal axes or geometric axes (x-x and y-y axes). When designing single angles without continuous bracing using the geometric axis design provisions, $M_y$ must be multiplied by 0.80 for use in Equations F10-1, F10-2 and F10-3. The design of a single angle in bending is illustrated in Example F.11.

F11. RECTANGULAR BARS AND-rounds

There are no design tables in the AISC Manual for these shapes. The local buckling limit state does not apply to any bars. With the exception of rectangular bars bent about the strong axis, solid square, rectangular and round bars are not subject to lateral-torsional buckling and are governed by the yielding limit state only. Rectangular bars bent about the strong axis are subject to lateral-torsional buckling and are checked for this limit state with Equations F11-2 and F11-3, as applicable.

These provisions can be used to check plates and webs of tees in connections. A design example of a rectangular bar in bending is illustrated in Example F.12. A design example of a round bar in bending is illustrated in Example F.13.

F12. UNSYMMETRICAL SHAPES

Due to the wide range of possible unsymmetrical cross sections, specific lateral-torsional and local buckling provisions are not provided in this Specification section. A general template is provided, but appropriate literature investigation and engineering judgment are required for the application of this section. A Z-shaped section is designed in Example F.14.

F13. PROPORTIONS OF BEAMS AND GIRDERs

This section of the Specification includes a limit state check for tensile rupture due to holes in the tension flange of beams, proportioning limits for I-shaped members, detail requirements for cover plates and connection requirements for built-up beams connected side-to-side. Also included are unbraced length requirements for beams designed using the moment redistribution provisions of AISC Specification Section B3.7.
EXAMPLE F.1-1A  W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, CONTINUOUSLY BRACED

Given:

Select an ASTM A992 W-shape beam with a simple span of 35 ft. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to \( L/360 \). The nominal loads are a uniform dead load of 0.45 kip/ft and a uniform live load of 0.75 kip/ft. Assume the beam is continuously braced.

\[
\begin{align*}
  w_D &= 0.45 \text{ kip/ft} \\
  w_L &= 0.75 \text{ kip/ft}
\end{align*}
\]

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\( F_y = 50 \text{ ksi} \)

\( F_u = 65 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
\begin{align*}
  w_u &= 1.2(0.45 \text{ kip/ft}) + 1.6(0.75 \text{ kip/ft}) \\
  &= 1.74 \text{ kip/ft} \\
  M_u &= \frac{1.74 \text{ kip/ft} (35.0 \text{ ft})^2}{8} \\
  &= 266 \text{ kip-ft}
\end{align*}
\]

\[
\begin{align*}
  w_0 &= 0.45 \text{ kip/ft} + 0.75 \text{ kip/ft} \\
  &= 1.20 \text{ kip/ft} \\
  M_0 &= \frac{1.20 \text{ kip/ft} (35.0 \text{ ft})^2}{8} \\
  &= 184 \text{ kip-ft}
\end{align*}
\]

Required Moment of Inertia for Live-Load Deflection Criterion of \( L/360 \)

\[
\begin{align*}
  \Delta_{\text{max}} &= \frac{L}{360} \\
  &= \frac{35.0 \text{ ft}(12 \text{ in./ft})}{360} \\
  &= 1.17 \text{ in.}
\end{align*}
\]

\[
\begin{align*}
  I_{\text{reqd}} &= \frac{5w_lI^4}{384E\Delta_{\text{max}}} \text{ from AISC Manual Table 3-23 Case 1} \\
  &= \frac{5(0.750 \text{ kip/ft})(35.0 \text{ ft})^4}{384 (29,000 \text{ ksi})(1.17 \text{ in.})} \\
  &= 746 \text{ in.}^4
\end{align*}
\]
**Beam Selection**

Select a W18×50 from AISC Manual Table 3-2.

Per the User Note in AISC Specification Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

From AISC Manual Table 3-2, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_a M_a = \phi_p M_{pu} )</td>
<td>( = 379 \text{ kip-ft} &gt; 266 \text{ kip-ft} )</td>
<td>o.k.</td>
</tr>
<tr>
<td></td>
<td>( M_a = M_{pu} )</td>
<td>( \Omega_\alpha = \Omega_{\alpha} )</td>
</tr>
<tr>
<td></td>
<td>( \Omega_\alpha = 252 \text{ kip-ft} &gt; 184 \text{ kip-ft} )</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

From AISC Manual Table 3-2, \( I_x = 800 \text{ in.}^4 > 746 \text{ in.}^4 \) o.k.
EXAMPLE F.1-1B  W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, CONTINUOUSLY BRACED

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A by applying the requirements of the AISC Specification directly.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

W18×50
ASTM A992
Fy = 50 ksi
Fu = 65 ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

W18×50
Zx = 101 in.³

The required flexural strength from Example F.1-1A is:

LRFD ASD
Mu = 266 kip-ft  Mₗ = 184 kip-ft

Nominal Flexural Strength, Mₙ

Per the User Note in AISC Specification Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

Mₙ = Mp
= Fy Zx  (Spec. Eq. F2-1)
= 50 ksi(101 in.³)
= 5,050 kip-in. or 421 kip-ft

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>φₙMₙ</td>
<td>0.90(421 kip-ft)</td>
<td>0.90(252 kip-ft)</td>
</tr>
<tr>
<td>φₙMₙ</td>
<td>= 379 kip-ft &gt; 266 kip-ft</td>
<td>o.k. = 252 kip-ft &gt; 184 kip-ft</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-2A  W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and third points. Use the AISC Manual tables.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u = 266$ kip-ft</td>
<td>$M_u = 184$ kip-ft</td>
</tr>
</tbody>
</table>

Unbraced Length

\[ L_u = \frac{35.0}{3} \]
\[ = 11.7 \text{ ft} \]

By inspection, the middle segment will govern. From AISC Manual Table 3-1, for a uniformly loaded beam braced at the ends and third points, $C_b = 1.01$ in the middle segment. Conservatively neglect this small adjustment in this case.

Available Strength

Enter AISC Manual Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of $11.7$ ft. Obtain the available strength from the appropriate vertical scale to the left.

From AISC Manual Table 3-10, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_n \approx 302$ kip-ft &gt; $266$ kip-ft</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-2B  W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and third points. Apply the requirements of the AISC Specification directly.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

W18×50
\( S_x = 88.9 \text{ in.}^3 \)

The required flexural strength from Example F.1-1A is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 266 \text{ kip-ft} )</td>
<td>( M_u = 184 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Nominal Flexural Strength, \( M_n \)

Calculate \( C_b \).

For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using AISC Specification Equation F1-1.

\[
C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C}
\]  
(Spec. Eq. F1-1)

For the center segment of the beam, the required moments for AISC Specification Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: \( M_{\text{max}} = 1.00, M_A = 0.972, M_B = 1.00, \text{ and } M_C = 0.972. \)

\[
C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.972) + 4(1.00) + 3(0.972)} = 1.01
\]

For the end-span beam segments, the required moments for AISC Specification Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: \( M_{\text{max}} = 0.889, M_A = 0.306, M_B = 0.556, \text{ and } M_C = 0.750. \)

\[
C_b = \frac{12.5(0.889)}{2.5(0.889) + 3(0.306) + 4(0.556) + 3(0.750)} = 1.46
\]

Thus, the center span, with the higher required strength and lower \( C_b \), will govern.

From AISC Manual Table 3-2:
For a compact beam with an unbraced length of \( L_p < L_b \leq L_r \), the lesser of either the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the nominal strength.

\[ M_p = 5,050 \text{ kip-in. (from Example F.1-1B)} \]

\[
M_a = C_b \left[ M_p - (M_p - 0.7F_S) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p
\]

\[
= 1.01 \left[ 5,050 \text{ kip-in.} - \left[ 5,050 \text{ kip-in.} - 0.7(50 \text{ ksi})(88.9 \text{ in.}^3) \right] \left( \frac{11.7 \text{ ft} - 5.83 \text{ ft}}{16.9 \text{ ft} - 5.83 \text{ ft}} \right) \right] \leq 5,050 \text{ kip-in.}
\]

\[
= 4,060 \text{ kip-in. or } 339 \text{ kip-ft}
\]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_a = 0.90(339 \text{ kip-ft}) )</td>
<td>( M_a = 339 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( = 305 \text{ kip-ft} &gt; 266 \text{ kip-ft} )</td>
<td>( \Omega_b ) o.k.</td>
</tr>
<tr>
<td></td>
<td>( = 203 \text{ kip-ft} &gt; 184 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-3A  W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and center point. Use the AISC Manual tables.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

\[ \omega_0 = 0.45 \text{ kip/ft} \]
\[ \omega_1 = 0.75 \text{ kip/ft} \]

\[ 35' - 0" \]

**Beam Loading & Bracing Diagram**
*(bracing at ends & midpoint)*

Unbraced Length

\[ L_o = \frac{35.0 \text{ ft}}{2} = 17.5 \text{ ft} \]

From AISC Manual Table 3-1, for a uniformly loaded beam braced at the ends and at the center point, \( C_b = 1.30 \). There are several ways to make adjustments to AISC Manual Table 3-10 to account for \( C_b \) greater than 1.0.

Procedure A

Available moments from the sloped and curved portions of the plots from AISC Manual Table 3-10 may be multiplied by \( C_b \), but may not exceed the value of the horizontal portion (\( \phi M_p \) for LRFD, \( M_p/\Omega \) for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from AISC Manual Table 3-10.

Enter AISC Manual Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 17.5 ft. Obtain the available strength from the appropriate vertical scale to the left.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_o = 266 \text{ kip-ft} )</td>
<td>( M_o = 184 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( \phi ) ( M_o ) ≈ 222 kip-ft</td>
<td>( M_o/\Omega_0 \approx 148 \text{ kip-ft} )</td>
</tr>
<tr>
<td>From Manual Table 3-2, ( \phi ) ( M_p \approx 379 \text{ kip-ft} ) (upper limit on ( C_p M_o ))</td>
<td>From Manual Table 3-2, ( M_p/\Omega_0 = 252 \text{ kip-ft} ) (upper limit on ( C_p M_o ))</td>
</tr>
</tbody>
</table>
Adjust for $C_b$.

$1.30(222 \text{ kip-ft}) = 289 \text{ kip-ft}$

Check Limit.

$289 \text{ kip-ft} \leq \phi_b M_p = 379 \text{ kip-ft}$  \( \text{o.k.} \)

Check available versus required strength.

$289 \text{ kip-ft} > 266 \text{ kip-ft}$  \( \text{o.k.} \)

Adjust for $C_b$.  

$1.30(147 \text{ kip-ft}) = 191 \text{ kip-ft}$

Check Limit.

$191 \text{ kip-ft} \leq \frac{M_p}{\Omega_b} = 252 \text{ kip-ft}$  \( \text{o.k.} \)

Check available versus required strength.

$191 \text{ kip-ft} > 184 \text{ kip-ft}$  \( \text{o.k.} \)

**Procedure B**

For preliminary selection, the required strength can be divided by $C_b$ and directly compared to the strengths in *AISC Manual* Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

Calculate the adjusted required strength.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u' = 266 \text{ kip-ft}/1.30 = 205 \text{ kip-ft}$</td>
<td>$M_u' = 184 \text{ kip-ft}/1.30 = 142 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from *AISC Manual* Table 3-10.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_u \approx 222 \text{ kip-ft} &gt; 205 \text{ kip-ft}$  ( \text{o.k.} )</td>
<td>$\frac{M_u}{\Omega_b} \approx 148 \text{ kip-ft} &gt; 142 \text{ kip-ft}$  ( \text{o.k.} )</td>
</tr>
<tr>
<td>$\phi_b M_p = 379 \text{ kip-ft} &gt; 266 \text{ kip-ft}$  ( \text{o.k.} )</td>
<td>$\frac{M_p}{\Omega_b} = 252 \text{ kip-ft} &gt; 184 \text{ kip-ft}$  ( \text{o.k.} )</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-3B  W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and center point. Apply the requirements of the AISC Specification directly.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\( F_y = 50 \text{ ksi} \)

\( F_u = 65 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

W18×50

\( r_{ts} = 1.98 \text{ in.} \)

\( S_x = 88.9 \text{ in.}^3 \)

\( J = 1.24 \text{ in.}^4 \)

\( h_o = 17.4 \text{ in.} \)

The required flexural strength from Example F.1-1A is:

\[ M_u = 266 \text{ kip-ft} \]

\[ M_a = 184 \text{ kip-ft} \]

Nominal Flexural Strength, \( M_n \)

Calculate \( C_b \),

\[ C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C} \quad (\text{Spec. Eq. F1-1}) \]

The required moments for AISC Specification Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: \( M_{max} = 1.00 \), \( M_A = 0.438 \), \( M_B = 0.751 \), and \( M_C = 0.938 \).

\[ C_b = \frac{12.5 (1.00)}{2.5 (1.00) + 3 (0.438) + 4 (0.751) + 3 (0.938)} = 1.30 \]

From AISC Manual Table 3-2:

\( L_p = 5.83 \text{ ft} \)

\( L_r = 16.9 \text{ ft} \)

For a compact beam with an unbraced length \( L_b > L_r \), the limit state of elastic lateral-torsional buckling applies.
Calculate $F_{cr}$ with $L_b = 17.5$ ft.

$$
F_{cr} = \frac{C_s \pi^2 E}{\left(\frac{L_b}{r_s}\right)^2} \sqrt{1 + 0.078 \frac{J_c}{S_h} \left(\frac{L_b}{r_s}\right)^3}
$$

where $c = 1.0$ for doubly symmetric I-shapes

(Spec. Eq. F2-4)

$$
= \frac{1.30 \pi^2 (29,000 \text{ ksi})}{\left(\frac{17.5 \text{ ft}(12 \text{ in./ft})}{1.98 \text{ in.}}\right)^2} \sqrt{1 + 0.078 \frac{1.24 \text{ in.}^4 (1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})(1.98 \text{ in.})} \left(\frac{17.5 \text{ ft}(12 \text{ in./ft})}{1.98 \text{ in.}}\right)^2
$$

= 43.2 \text{ ksi}

$M_n = F_{cr} S_i \leq M_p$

= 43.2 ksi (88.9 in.³)

= 3,840 kip-in. ≤ 5,050 kip-in. (from Example F.1-1B)

$M_n = 3,840 \text{ kip-in. or 320 kip-ft}$

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_b M_n = 0.90(320 \text{ kip-ft})$</td>
<td>$M_n = \frac{320 \text{ kip-ft}}{1.67}$</td>
</tr>
<tr>
<td>= 288 kip-ft</td>
<td>= 192 kip-ft</td>
</tr>
</tbody>
</table>

288 kip-ft > 266 kip-ft o.k. 192 kip-ft > 184 kip-ft o.k.
EXAMPLE F.2-1A  COMPACT CHANNEL FLEXURAL MEMBER, CONTINUOUSLY BRACED

Given:

Select an ASTM A36 channel to serve as a roof edge beam with a simple span of 25 ft. Limit the live load deflection to $L/360$. The nominal loads are a uniform dead load of 0.23 kip/ft and a uniform live load of 0.69 kip/ft. The beam is continuously braced.

\[ w_d = 0.23 \text{ kip/ft} \]
\[ w_l = 0.69 \text{ kip/ft} \]

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A36
  - $F_y = 36 \text{ ksi}$
  - $F_u = 58 \text{ ksi}$

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[ w_u = 1.2(0.23 \text{ kip/ft}) + 1.6(0.69 \text{ kip/ft}) = 1.38 \text{ kip/ft} \]
\[ w_a = 0.23 \text{ kip/ft} + 0.69 \text{ kip/ft} = 0.920 \text{ kip/ft} \]

\[ M_u = \frac{1.38 \text{ kip/ft}(25.0 \text{ ft})^2}{8} = 108 \text{ kip-ft} \]
\[ M_a = \frac{0.920 \text{ kip/ft}(25.0 \text{ ft})^2}{8} = 71.9 \text{ kip-ft} \]

Beam Selection

Per the User Note in AISC Specification Section F2, all ASTM A36 channels are compact. Because the beam is compact and continuously braced, the yielding limit state governs and $M_n = M_p$. Try C15×33.9 from AISC Manual Table 3-8.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_n M_n = \phi_b M_p$</td>
<td>$M_a = M_p$</td>
</tr>
<tr>
<td>$= 137 \text{ kip-ft} &gt; 108 \text{ kip-ft}$</td>
<td>$\frac{\Omega_a}{\Omega_b} = 91.3 \text{ kip-ft} &gt; 71.9 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>


**Live Load Deflection**

Assume the live load deflection at the center of the beam is limited to $L/360$.

$$\Delta_{\text{max}} = \frac{L}{360}$$

$$= \frac{25.0 \text{ ft}(12 \text{ in./ft})}{360}$$

$$= 0.833 \text{ in.}$$

For C15×33.9, $I_x = 315 \text{ in.}^4$ from AISC *Manual* Table 1-5.

The maximum calculated deflection is:

$$\Delta_{\text{max}} = \frac{5wL^4}{384EI} \text{ from AISC Manual Table 3-23 Case 1}$$

$$= \frac{5(0.69 \text{ kip/ft})(25.0 \text{ ft})^4(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(315 \text{ in.}^4)}$$

$$= 0.664 \text{ in.} < 0.833 \text{ in.} \quad \text{o.k.}$$
EXAMPLE F.2-1B  COMPACT CHANNEL FLEXURAL MEMBER, CONTINUOUSLY BRACED

Given:

Example F.2-1A can be easily solved by utilizing the tables of the AISC Manual. Verify the results by applying the requirements of the AISC Specification directly.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A36
  - \( F_y = 36 \) ksi
  - \( F_u = 58 \) ksi

From AISC Manual Table 1-5, the geometric properties are as follows:

- \( C_{15 \times 33.9} \)
  - \( Z_x = 50.8 \text{ in.}^3 \)

The required flexural strength from Example F.2-1A is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 108 \text{ kip-ft} )</td>
<td>( M_a = 71.9 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

**Nominal Flexural Strength, \( M_n \)**

Per the User Note in AISC Specification Section F2, all ASTM A36 C- and MC-shapes are compact.

A channel that is continuously braced and compact is governed by the yielding limit state.

\[
M_n = M_p = F_y Z_x = 36 \text{ ksi}(50.8 \text{ in.}^3) = 1,830 \text{ kip-in.} \text{ or} 152 \text{ kip-ft}
\]  

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_n = 0.90(152 \text{ kip-ft}) )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>= 137 kip-ft &gt; 108 kip-ft</td>
<td>( M_a = \frac{152 \text{ kip-ft}}{1.67} )</td>
</tr>
</tbody>
</table>

\( \Omega_b = 1.67 \) \( M_a = 91.0 \text{ kip-ft} > 71.9 \text{ kip-ft} \) \( \text{o.k.} \)
EXAMPLE F.2-2A  COMPACT CHANNEL FLEXURAL MEMBER WITH BRACING AT ENDS AND FIFTH POINTS

Given:

Check the C15×33.9 beam selected in Example F.2-1A, assuming it is braced at the ends and the fifth points rather than continuously braced.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36

$F_y = 36$ ksi

$F_u = 58$ ksi

The center segment will govern by inspection.

The required flexural strength at midspan from Example F.2-1A is:

\[
\begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
M_a = 108 \text{ kip-ft} & M_a = 71.9 \text{ kip-ft} \\
\end{array}
\]

From AISC Manual Table 3-1, with an almost uniform moment across the center segment, $C_b = 1.00$; therefore, no adjustment is required.

Unbraced Length

\[
L_b = \frac{25.0 \text{ ft}}{5} = 5.00 \text{ ft}
\]

Obtain the strength of the C15×33.9 with an unbraced length of 5.00 ft from AISC Manual Table 3-11.

Enter AISC Manual Table 3-11 and find the intersection of the curve for the C15×33.9 with an unbraced length of 5.00 ft. Obtain the available strength from the appropriate vertical scale to the left.
EXAMPLE F.2-2B  COMPACT CHANNEL FLEXURAL MEMBER WITH BRACING AT ENDS AND FIFTH POINTS

Given:
Verify the results from Example F.2-2A by calculation using the provisions of the AISC *Specification*.

Solution:
From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC *Manual* Table 1-5, the geometric properties are as follows:

\[ C_{15\times33.9} \]
\[ S_y = 42.0 \text{ in.}^3 \]

The required flexural strength from Example F.2-1A is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 108 \text{ kip-ft} )</td>
<td>( M_u = 71.9 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Nominal Flexural Strength, \( M_n \)

Per the User Note in AISC *Specification* Section F2, all ASTM A36 C- and MC-shapes are compact.

From AISC *Manual* Table 3-1, for the center segment of a uniformly loaded beam braced at the ends and the fifth points:

\[ C_b = 1.00 \]

From AISC *Manual* Table 3-8, for a \( C_{15\times33.9} \):

\[ L_p = 3.75 \text{ ft} \]
\[ L_r = 14.5 \text{ ft} \]

For a compact channel with \( L_p < L_b \leq L_r \), the lesser of the flexural yielding limit state or the inelastic lateral-torsional buckling limit-state controls the available flexural strength.

The nominal flexural strength based on the flexural yielding limit state, from Example F.2-1B, is:

\[ M_n = M_p = 1,830 \text{ kip-in.} \]

The nominal flexural strength based on the lateral-torsional buckling limit state is:
\[ M_n = C_b \left[ M_p - (M_p - 0.7F_yS_y) \left( \frac{L_a - L_p}{L_p - L_a} \right) \right] \leq M_p \]  
(Spec. Eq. F2-2)

\[ = 1.0 \left[ 1,830 \text{ kip-in.} - \left[ 1,830 \text{ kip-in.} - 0.7(36 \text{ ksi})(42.0 \text{ in.}^3) \right] \left( \frac{5.00 \text{ ft} - 3.75 \text{ ft}}{14.5 \text{ ft} - 3.75 \text{ ft}} \right) \right] \leq 1,830 \text{ kip-in.} \]

\[ = 1,740 \text{ kip-in.} \leq 1,830 \text{ kip-in.} \]  \text{o.k.}

\[ M_n = 1,740 \text{ kip-in. or 145 kip-ft} \]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_n = 0.90(145 \text{ kip-ft}) )</td>
<td>( \frac{M_n}{\Omega_b} = 145 \text{ kip-ft} )</td>
</tr>
<tr>
<td>= 131 kip-ft</td>
<td>= 86.8 kip-ft</td>
</tr>
<tr>
<td>131 kip-ft &gt; 108 kip-ft \text{ o.k.}</td>
<td>86.8 kip-ft &gt; 71.9 kip-ft \text{ o.k.}</td>
</tr>
</tbody>
</table>
EXAMPLE F.3A  W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN STRONG-AXIS BENDING

Given:

Select an ASTM A992 W-shape beam with a simple span of 40 ft. The nominal loads are a uniform dead load of 0.05 kip/ft and two equal 18 kip concentrated live loads acting at the third points of the beam. The beam is continuously braced. Also calculate the deflection.

Note: A beam with noncompact flanges will be selected to demonstrate that the tabulated values of the AISC Manual account for flange compactness.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\( F_y = 50 \text{ ksi} \)

\( F_u = 65 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required flexural strength at midspan is:

\[
\begin{align*}
\text{LRFD} & : & P_u & = 1.6(18 \text{ kips}) = 28.8 \text{ kips} \\
& & M_u & = \left( \frac{0.0600 \text{ kip/ft}(40.0 \text{ ft})^2}{8} + (28.8 \text{ kips}) \frac{40.0 \text{ ft}}{3} \right) \\
& & & = 396 \text{ kip-ft} \\
\text{ASD} & : & P_u & = 18 \text{ kips} \\
& & M_u & = \left( \frac{0.0500 \text{ kip/ft}(40.0 \text{ ft})^2}{8} + (18.0 \text{ kips}) \frac{40.0 \text{ ft}}{3} \right) \\
& & & = 250 \text{ kip-ft}
\end{align*}
\]

Beam Selection

For a continuously braced W-shape, the available flexural strength equals the available plastic flexural strength.

Select the lightest section providing the required strength from the bold entries in AISC Manual Table 3-2.

Try a W21×48.

This beam has a noncompact compression flange at \( F_y = 50 \text{ ksi} \) as indicated by footnote “f” in AISC Manual Table 3-2. This shape is also footnoted in AISC Manual Table 1-1.
From AISC Manual Table 3-2, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_a = \phi_b M_{px}$</td>
<td>$M_a = M_{px}$</td>
</tr>
<tr>
<td>$= 398 \text{ kip-ft} &gt; 396 \text{ kip-ft}$</td>
<td>$\frac{M_a}{\Omega_b} = \frac{M_{px}}{\Omega_b}$</td>
</tr>
<tr>
<td></td>
<td>$= 265 \text{ kip-ft} &gt; 250 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Note: The value $M_{px}$ in AISC Manual Table 3-2 includes the strength reductions due to the noncompact nature of the shape.

**Deflection**

$I_x = 959 \text{ in.}^4$ from AISC Manual Table 1-1

The maximum deflection occurs at the center of the beam.

$$\Delta_{max} = \frac{5w_pl^2}{384EI} + \frac{Pl^3}{28EI} \text{ from AISC Manual Table 3-23 cases 1 and 9}$$

$$= \frac{5(0.0500 \text{ kip/ft})(40.0 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(959 \text{ in.}^4)} + \frac{18.0 \text{ kips}(40.0 \text{ ft})^3 (12 \text{ in./ft})^3}{28(29,000 \text{ ksi})(959 \text{ in.}^4)}$$

$$= 2.66 \text{ in.}$$

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than strength in beam design.
EXAMPLE F.3B  W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN STRONG-AXIS BENDING

Given:

Verify the results from Example F.3A by calculation using the provisions of the AISC Specification.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\( F_y = 50 \text{ ksi} \)

\( F_u = 65 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

\( W_{21 \times 48} \)

\( S_x = 93.0 \text{ in.}^3 \)

\( Z_x = 107 \text{ in.}^3 \)

\( \frac{b_f}{2t_f} = 9.47 \)

The required flexural strength from Example F.3A is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 396 \text{ kip-ft} )</td>
<td>( M_u = 250 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Flange Slenderness

\( \lambda = \frac{b_f}{2t_f} = 9.47 \)

The limiting width-to-thickness ratios for the compression flange are:

\( \lambda_{pf} = 0.38 \sqrt{\frac{E}{F_y}} \) from AISC Specification Table B4.1b Case 10

\[ = 0.38 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]

\[ = 9.15 \]

\( \lambda_{rf} = 1.0 \sqrt{\frac{E}{F_y}} \) from AISC Specification Table B4.1b Case 10

\[ = 1.0 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]

\[ = 24.1 \]

\( \lambda_{rf} > \lambda > \lambda_{pf} \), therefore, the compression flange is noncompact. This could also be determined from the footnote “f” in AISC Manual Table 1-1.
Nominal Flexural Strength, $M_n$

Because the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the available strength is governed by AISC Specification Section F3.2, Compression Flange Local Buckling.

$$M_p = F_y Z_e = 50 \text{ ksi}(107 \text{ in.}^3) = 5,350 \text{ kip-in. or 446 kip-ft}$$

$$M_n = M_p - \left( M_p - 0.7 F_y S_e \right) \left( \frac{\lambda_f - \lambda_{ref}}{\lambda_{ref} - \lambda_{ref}} \right)$$  \hspace{1cm} (Spec. Eq. F3-1)

$$= \left[ 5,350 \text{ kip-in.} - \left[ 5,350 \text{ kip-in.} - 0.7(50 \text{ ksi})(93.0 \text{ in.}^3) \right] \left( \frac{9.47 - 9.15}{24.1 - 9.15} \right) \right]$$

$$= 5,310 \text{ kip-in. or 442 kip-ft}$$

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_b M_n = 0.90(442 \text{ kip-ft})$</td>
<td>$M_n = 442 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$= 398 \text{ kip-ft} &gt; 396 \text{ kip-ft}$</td>
<td>$\Omega_b M_n = 265 \text{ kip-ft} &gt; 250 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Note that these available strengths are identical to the tabulated values in AISC Manual Table 3-2, which account for the noncompact flange.
EXAMPLE F.4 W-SHAPE FLEXURAL MEMBER, SELECTION BY MOMENT OF INERTIA FOR STRONG-AXIS BENDING

Given:

Select an ASTM A992 W-shape flexural member by the moment of inertia, to limit the live load deflection to 1 in. The span length is 30 ft. The loads are a uniform dead load of 0.80 kip/ft and a uniform live load of 2 kip/ft. The beam is continuously braced.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{u}$</td>
<td>$1.2(0.800 \text{ kip/ft}) + 1.6(2 \text{ kip/ft})$</td>
<td>$w_{u} = 0.80 \text{ kip/ft + 2 kip/ft}$</td>
</tr>
<tr>
<td></td>
<td>$= 4.16 \text{ kip/ft}$</td>
<td>$= 2.80 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$M_{u}$</td>
<td>$\frac{4.16 \text{ kip/ft}(30.0 \text{ ft})^2}{8}$</td>
<td>$M_{u} = \frac{2.80 \text{ kip/ft}(30.0 \text{ ft})^2}{8}$</td>
</tr>
<tr>
<td></td>
<td>$= 468 \text{ kip-ft}$</td>
<td>$= 315 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Minimum Required Moment of Inertia

The maximum live load deflection, $\Delta_{\text{max}}$, occurs at midspan and is calculated as:

$$\Delta_{\text{max}} = \frac{5w_{u} I^{4}}{384EI}$$

from AISC Manual Table 3-23 case 1

Rearranging and substituting $\Delta_{\text{max}} = 1.00$ in.,

$$I_{\text{min}} = \frac{5(2 \text{ kips/ft})(30.0 \text{ ft})^4(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(1.00 \text{ in.})}$$

$$= 1,260 \text{ in.}^4$$

Beam Selection

Select the lightest section with the required moment of inertia from the bold entries in AISC Manual Table 3-3.
Try a W24×55.

\[ I_x = 1,350 \text{ in.}^4 > 1,260 \text{ in.}^4 \quad \text{o.k.} \]

Because the W24×55 is continuously braced and compact, its strength is governed by the yielding limit state and AISC Specification Section F2.1

From AISC Manual Table 3-2, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_a = \phi_b M_{pl} ) = 503 kip-ft</td>
<td>( \frac{M_a}{\Omega_a} = \frac{M_{pl}}{\Omega_{pl}} ) = 334 kip-ft</td>
</tr>
<tr>
<td>503 kip-ft &gt; 468 kip-ft o.k.</td>
<td>334 kip-ft &gt; 315 kip-ft o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE F.5  I-SHAPED FLEXURAL MEMBER IN MINOR-AXIS BENDING

Given:

Select an ASTM A992 W-shape beam loaded in its minor axis with a simple span of 15 ft. The loads are a total uniform dead load of 0.667 kip/ft and a uniform live load of 2 kip/ft. Limit the live load deflection to $L/240$. The beam is braced at the ends only.

Note: Although not a common design case, this example is being used to illustrate AISC Specification Section F6 (I-shaped members and channels bent about their minor axis).

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
- $F_y = 50$ ksi
- $F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

$$w_u = 1.2(0.667 \text{ kip/ft}) + 1.6(2 \text{ kip/ft})$$
$$= 4.00 \text{ kip/ft}$$

$$M_u = \frac{4.00 \text{ kip/ft}(15.0 \text{ ft})^2}{8}$$
$$= 113 \text{ kip-ft}$$

$$w_l = 0.667 \text{ kip/ft} + 2 \text{ kip/ft}$$
$$= 2.67 \text{ kip/ft}$$

$$M_l = \frac{2.67 \text{ kip/ft}(15.0 \text{ ft})^2}{8}$$
$$= 75.1 \text{ kip-ft}$$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{15.0 \text{ ft}(12 \text{ in./ft})}{240}$$
$$= 0.750 \text{ in.}$$

$$I_{req} = \frac{5w_lL^3}{384E\Delta_{max}} \text{ from AISC Manual Table 3-23 case 1}$$
$$= \frac{5(2.00 \text{ kip/ft})(15.0 \text{ ft})^3(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(0.750 \text{ in.})}$$
Beam Selection

Select the lightest section from the bold entries in AISC Manual Table 3-5, due to the likelihood that deflection will govern this design.

Try a W12×58.

From AISC Manual Table 1-1, the geometric properties are as follows:

\[
\begin{align*}
W12\times58 & \\
S_y &= 21.4 \text{ in.}^3 \\
Z_y &= 32.5 \text{ in.}^3 \\
I_y &= 107 \text{ in.}^4 > 105 \text{ in.}^4 \quad \text{o.k.}
\end{align*}
\]

AISC Specification Section F6 applies. Because the W12×58 has compact flanges per the User Note in this Section, the yielding limit state governs the design.

\[
M_y = M_p = F_y Z_y \leq 1.6 F_y S_y \quad (\text{Spec. Eq. F6-1})
\]

\[
= 50 \text{ ksi}(32.5 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(21.4 \text{ in.}^3)
\]

\[
= 1,630 \text{ kip-in.} \leq 1,710 \text{ kip-in.} \quad \text{o.k.}
\]

\[
M_y = 1,630 \text{ kip-in.} \text{ or 136 kip-ft}
\]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_b M_y = 0.90(136 \text{ kip-ft})$</td>
<td>$M_y = \frac{136 \text{ kip-ft}}{1.67}$</td>
</tr>
<tr>
<td>$= 122 \text{ kip-ft}$</td>
<td>$= 81.4 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>
| $122 \text{ kip-ft} > 113 \text{ kip-ft}$ | $81.4 \text{ kip-ft} > 75.1 \text{ kip-ft}$ | o.k. | o.k.
EXAMPLE F.6  HSS FLEXURAL MEMBER WITH COMPACT FLANGES

Given:
Select a square ASTM A500 Grade B HSS beam to span 7.5 ft. The loads are a uniform dead load of 0.145 kip/ft and a uniform live load of 0.435 kip/ft. Limit the live load deflection to $L/240$. The beam is continuously braced.

Solution:
From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A500 Grade B
- $F_y = 46$ ksi
- $F_u = 58$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_a = 1.2(0.145 \text{ kip/ft}) + 1.6(0.435 \text{ kip/ft})$</td>
<td>$w_a = 0.145 \text{ kip/ft} + 0.435 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$= 0.870 \text{ kip/ft}$</td>
<td>$= 0.580 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$M_u = \frac{(0.870 \text{ kip/ft})(7.50 \text{ ft})^2}{8}$</td>
<td>$M_u = \frac{(0.580 \text{ kip/ft})(7.50 \text{ ft})^2}{8}$</td>
</tr>
<tr>
<td>$= 6.12 \text{ kip-ft}$</td>
<td>$= 4.08 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240} = \frac{7.50 \text{ ft}(12 \text{ in./ft})}{240} = 0.375 \text{ in.}$$

Determine the minimum required $I$ as follows.

$$I_{req} = \frac{5w_f I^4}{384E \Delta_{max}} \text{ from AISC Manual Table 3-23 Case 1}$$

$$= \frac{5(0.435 \text{ kip/ft})(7.50 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(0.375 \text{ in.})}$$

$$= 2.85 \text{ in.}^4$$
**Beam Selection**

Select an HSS with a minimum $I_x$ of 2.85 in.⁴, using AISC *Manual* Table 1-12, and having adequate available strength, using AISC *Manual* Table 3-13.

Try an HSS3½×3½×r₆.

From AISC *Manual* Table 1-12, $I_x = 2.90$ in.⁴ > 2.85 in.⁴ **o.k.**

From AISC *Manual* Table 3-13, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_u = 6.67$ kip-ft &gt; $6.12$ kip-ft</td>
<td><strong>o.k.</strong></td>
</tr>
</tbody>
</table>
EXAMPLE F.7A  HSS FLEXURAL MEMBER WITH NONCOMPACT FLANGES

Given:

Select a rectangular ASTM A500 Grade B HSS beam with a span of 21 ft. The loads include a uniform dead load of 0.15 kip/ft and a uniform live load of 0.4 kip/ft. Limit the live load deflection to $L/240$. The beam is braced at the end points only. A noncompact member was selected here to illustrate the relative ease of selecting noncompact shapes from the AISC Manual, as compared to designing a similar shape by applying the AISC Specification requirements directly, as shown in Example F.7B.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

<table>
<thead>
<tr>
<th>Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A500 Grade B</td>
</tr>
<tr>
<td>$F_y = 46$ ksi</td>
</tr>
<tr>
<td>$F_u = 58$ ksi</td>
</tr>
</tbody>
</table>

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

$$ w_u = 1.2(0.15 \text{ kip/ft}) + 1.6(0.4 \text{ kip/ft}) $$
$$ M_u = \frac{0.820 \text{ kip/ft}(21.0 \text{ ft})^2}{8} $$
$$ = 45.2 \text{ kip-ft} $$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$ \Delta_{max} = \frac{L}{240} $$
$$ = \frac{21.0 \text{ ft}(12 \text{ in./ft})}{240} $$
$$ = 1.05 \text{ in.} $$

The maximum calculated deflection is:

$$ \Delta_{max} = \frac{5w_l l^4}{384EI} \text{ from AISC Manual Table 3-23 case 1} $$

Rearranging and substituting $\Delta_{max} = 1.05$ in.,
Beam Selection

Select a rectangular HSS with a minimum $I_x$ of 57.5 in.$^4$, using AISC Manual Table 1-11, and having adequate available strength, using AISC Manual Table 3-12.

Try an HSS10×6×3/8 oriented in the strong direction. This rectangular HSS section was purposely selected for illustration purposes because it has a noncompact flange. See AISC Manual Table 1-12A for compactness criteria.

$I_x = 74.6$ in.$^4 > 57.5$ in.$^4$  o.k.

From AISC Manual Table 3-12, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_u = 57.0$ kip-ft $&gt; 45.2$ kip-ft</td>
<td>o.k.</td>
<td>$\frac{M_u}{\Omega_b} = 37.9$ kip-ft $&gt; 30.3$ kip-ft</td>
</tr>
</tbody>
</table>
EXAMPLE F.7B  HSS FLEXURAL MEMBER WITH NONCOMPACT FLANGES

Given:

Notice that in Example F.7A the required information was easily determined by consulting the tables of the AISC Manual. The purpose of the following calculation is to demonstrate the use of the AISC Specification equations to calculate the flexural strength of an HSS member with a noncompact compression flange.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B

\[ F_y = 46 \text{ ksi} \]

\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-11, the geometric properties are as follows:

HSS10×6×3/16

\[ Z_x = 18.0 \text{ in.}^3 \]

\[ S_x = 14.9 \text{ in.}^3 \]

**Flange Compactness**

\[ \lambda = \frac{b}{t} \]

\[ = 31.5 \text{ from AISC Manual Table 1-11} \]

Determine the limiting ratio for a compact HSS flange in flexure from AISC Specification Table B4.1b Case 17.

\[ \lambda_p = 1.12 \sqrt{\frac{E}{F_y}} \]

\[ = 1.12 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \]

\[ = 28.1 \]

**Flange Slenderness**

Determine the limiting ratio for a slender HSS flange in flexure from AISC Specification Table B4.1b Case 17.

\[ \lambda_r = 1.40 \sqrt{\frac{E}{F_y}} \]

\[ = 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \]

\[ = 35.2 \]

\( \lambda_p < \lambda < \lambda_r; \) therefore, the flange is noncompact. For this situation, AISC Specification Equation F7-2 applies.
Web Slenderness

\[ \lambda = \frac{h}{t} \]

= 54.5 from AISC Manual Table 1-11

Determine the limiting ratio for a compact HSS web in flexure from AISC Specification Table B4.1b Case 19.

\[ \lambda_p = 2.42 \sqrt{\frac{E}{F_y}} \]

= 2.42 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}}

= 60.8

\( \lambda < \lambda_p \); therefore, the web is compact

For HSS with noncompact flanges and compact webs, AISC Specification Section F7.2(b) applies.

\[ M_p = F_y Z_x \]

= 46 ksi(18.0 in. \(^3\))

= 828 kip-in.

\[ M_e = M_p - (M_p - F_y S) \left( 3.57 \frac{b}{t_f} \sqrt{\frac{F_y}{E}} - 4.0 \right) \leq M_p \]  

(Spec. Eq. F7-2)

= 828 kip-in. - \( \left[ 828 \text{ kip-in.} - 46 \text{ ksi} (14.9 \text{ in.}^3) \right] \left( 3.57 (31.5) \sqrt{29,000 \text{ ksi}} - 4.0 \right) \)

= 760 kip-in. or 63.3 kip-ft

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_w = 0.90 (63.3 \text{ kip-ft}) )</td>
<td>( M_w = 63.3 \text{ kip-ft} )</td>
</tr>
<tr>
<td>= 57.0 kip-ft</td>
<td>( \Omega_w = 1.67 )</td>
</tr>
<tr>
<td></td>
<td>( = 37.9 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>
EXAMPLE F.8A  HSS FLEXURAL MEMBER WITH SLENDER FLANGES

Given:

Verify the strength of an ASTM A500 Grade B HSS8×8×3/16 with a span of 21 ft. The loads are a dead load of 0.125 kip/ft and a live load of 0.375 kip/ft. Limit the live load deflection to L/240. The beam is continuously braced.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B (rectangular HSS)

\[ F_y = 46 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
\begin{align*}
   w_u &= 1.2(0.125 \text{ kip/ft}) + 1.6(0.375 \text{ kip/ft}) \\
   &= 0.750 \text{ kip/ft} \\
   M_u &= \frac{0.750 \text{ kip/ft} (21.0 \text{ ft})^2}{8} \\
   &= 41.3 \text{ kip-ft} \\
\end{align*}
\]

\[
\begin{align*}
   w_a &= 0.125 \text{ kip/ft} + 0.375 \text{ kip/ft} \\
   &= 0.500 \text{ kip/ft} \\
   M_a &= \frac{0.500 \text{ kip/ft} (21.0 \text{ ft})^2}{8} \\
   &= 27.6 \text{ kip-ft} \\
\end{align*}
\]

Obtain the available flexural strength of the HSS8×8×3/16 from AISC Manual Table 3-13.

\[
\begin{align*}
   \phi_bM_a &= 43.3 \text{ kip-ft} > 41.3 \text{ kip-ft} \quad \text{o.k.} \\
   \frac{M_a}{\Omega_b} &= 28.8 \text{ kip-ft} > 27.6 \text{ kip-ft} \quad \text{o.k.} \\
\end{align*}
\]

Note that the strengths given in AISC Manual Table 3-13 incorporate the effects of noncompact and slender elements.
Deflection

The maximum live load deflection permitted is:

\[ \Delta_{\text{max}} = \frac{L}{240} \]

\[ = \frac{21.0 \text{ ft}(12 \text{ in./ft})}{240} \]

\[ = 1.05 \text{ in.} \]

\[ I_x = 54.4 \text{ in.}^4 \text{ from AISC Manual Table 1-12} \]

The maximum calculated deflection is:

\[ \Delta_{\text{max}} = \frac{5wL^4}{384EI} \text{ from AISC Manual Table 3-23 Case 1} \]

\[ = \frac{5(0.375 \text{ kip/ft})(21.0 \text{ ft})^4(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(54.4 \text{ in.}^4)} \]

\[ = 1.04 \text{ in.} < 1.05 \text{ in.} \text{ o.k.} \]
EXAMPLE F.8B  HSS FLEXURAL MEMBER WITH SLENDER FLANGES

Given:

In Example F.8A the available strengths were easily determined from the tables of the AISC Manual. The purpose of the following calculation is to demonstrate the use of the AISC Specification equations to calculate the flexural strength of an HSS member with slender flanges.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B (rectangular HSS)

\( F_y = 46 \text{ ksi} \)

\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-12, the geometric properties are as follows:

<table>
<thead>
<tr>
<th>HSS8×8×( \frac{3}{8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_x = 54.4 \text{ in.}^4 )</td>
</tr>
<tr>
<td>( Z_x = 15.7 \text{ in.}^3 )</td>
</tr>
<tr>
<td>( S_x = 13.6 \text{ in.}^3 )</td>
</tr>
<tr>
<td>( B = 8.00 \text{ in.} )</td>
</tr>
<tr>
<td>( H = 8.00 \text{ in.} )</td>
</tr>
<tr>
<td>( t = 0.174 \text{ in.} )</td>
</tr>
<tr>
<td>( b/t = 43.0 )</td>
</tr>
<tr>
<td>( h/t = 43.0 )</td>
</tr>
</tbody>
</table>

The required flexural strength from Example F.8A is:

\( M_u = 41.3 \text{ kip-ft} \)

\( M_a = 27.6 \text{ kip-ft} \)

Flange Slenderness

The assumed outside radius of the corners of HSS shapes is \( 1.5t \). The design thickness is used to check compactness.

Determine the limiting ratio for a slender HSS flange in flexure from AISC Specification Table B4.1b Case 17.

\[
\lambda_c = 1.40 \sqrt{\frac{E}{F_y}}
\]

\[
= 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}}
\]

\[
= 35.2
\]

\[
\lambda = \frac{b}{t}
\]

\[
= 43.0 > \lambda_c; \text{ therefore, the flange is slender}
\]
Web Slenderness

Determine the limiting ratio for a compact web in flexure from AISC Specification Table B4.1b Case 19.

\[ \lambda_p = 2.42 \sqrt{\frac{E}{F_y}} \]

\[ = 2.42 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \]

\[ = 60.8 \]

\[ \lambda = \frac{h}{t} \]

\[ = 43.0 < \lambda_p, \text{ therefore the web is compact} \]

Nominal Flexural Strength, \( M_n \)

For HSS sections with slender flanges and compact webs, AISC Specification Section F7.2(c) applies.

\[ M_n = F_y S_c \]  \hspace{1cm} (Spec. Eq. F7-3)

Where \( S_c \) is the effective section modulus determined with the effective width of the compression flange taken as:

\[ b_c = 1.92 t_c \sqrt{F_y} \left[ 1 - \frac{0.38}{b_c / t_c} \sqrt{\frac{E}{F_y}} \right] \leq b \]  \hspace{1cm} (Spec. Eq. F7-4)

\[ = 1.92 \left( 0.174 \text{ in.} \right) \sqrt{29,000 \text{ ksi}} \left[ 1 - \frac{0.38}{43.0} \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \right] \]

\[ = 6.53 \text{ in.} \]

\[ b = 8.00 \text{ in.} - 3 \left( 0.174 \text{ in.} \right) \text{ from AISC Specification Section B4.1b(d)} \]

\[ = 7.48 \text{ in.} > 6.53 \text{ in.} \text{ o.k.} \]

The ineffective width of the compression flange is:

\[ b - b_c = 7.48 \text{ in.} - 6.53 \text{ in.} \]

\[ = 0.950 \text{ in.} \]

An exact calculation of the effective moment of inertia and section modulus could be performed taking into account the ineffective width of the compression flange and the resulting neutral axis shift. Alternatively, a simpler but slightly conservative calculation can be performed by removing the ineffective width symmetrically from both the top and bottom flanges.

\[ I_{eff} \approx 54.4 \text{ in.}^4 - 2 \left[ \frac{(0.950 \text{ in.})(0.174 \text{ in.})^3}{12} + (0.950 \text{ in.})(0.174 \text{ in.})(3.91)^2 \right] \]

\[ = 49.3 \text{ in.}^4 \]
The effective section modulus can now be calculated as follows:

\[
S_e = \frac{I_{ef}}{d / 2}
\]

\[
= \frac{49.3 \text{ in.}^4}{8.00 \text{ in.} / 2}
\]

\[
= 12.3 \text{ in.}^3
\]

\[M_n = F_y S_e \quad \text{(Spec. Eq. F7-3)}\]

\[
= 46 \text{ ksi}(12.3 \text{ in.}^3)
\]

\[
= 566 \text{ kip-in. or 47.2 kip-ft}
\]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_b)</td>
<td>0.90</td>
<td>(\Omega_b = 1.67)</td>
</tr>
<tr>
<td>(\phi_b M_n)</td>
<td>0.90(47.2 kip-ft)</td>
<td>(M_n = 47.2 \text{ kip-ft} \quad \Omega_b = 1.67)</td>
</tr>
<tr>
<td></td>
<td>= 42.5 kip-ft &gt; 41.3 kip-ft o.k.</td>
<td>= 28.3 kip-ft &gt; 27.6 kip-ft o.k.</td>
</tr>
</tbody>
</table>

Note that the calculated available strengths are somewhat lower than those in AISC Manual Table 3-13 due to the use of the conservative calculation of the effective section modulus.
EXAMPLE F.9A  PIPE FLEXURAL MEMBER

Given:

Select an ASTM A53 Grade B Pipe shape with an 8-in. nominal depth and a simple span of 16 ft. The loads are a total uniform dead load of 0.32 kip/ft and a uniform live load of 0.96 kip/ft. There is no deflection limit for this beam. The beam is braced only at the ends.

\[ w_D = 0.32 \text{ kip/ft} \]
\[ w_L = 0.96 \text{ kip/ft} \]

\[ 16'-0" \]

Beam Loading & Bracing Diagram
(braced at end points only)

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A53 Grade B
\[ F_y = 35 \text{ ksi} \]
\[ F_u = 60 \text{ ksi} \]

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[ w_u = 1.2(0.32 \text{ kip/ft}) + 1.6(0.96 \text{ kip/ft}) = 1.92 \text{ kip/ft} \]
\[ M_u = \frac{(1.92 \text{ kip/ft})(16.0 \text{ ft})^2}{8} = 61.4 \text{ kip-ft} \]

\[ w_a = 0.32 \text{ kip/ft} + 0.96 \text{ kip/ft} = 1.28 \text{ kip/ft} \]
\[ M_a = \frac{(1.28 \text{ kip/ft})(16.0 \text{ ft})^2}{8} = 41.0 \text{ kip-ft} \]

Pipe Selection

Select a member from AISC Manual Table 3-15 having the required strength.

Select Pipe 8 x-Strong.

From AISC Manual Table 3-15, the available flexural strength is:

\[ \phi M_u = 81.4 \text{ kip-ft} > 61.4 \text{ kip-ft} \text{ o.k.} \]
\[ \frac{M_a}{\Omega_b} = 54.1 \text{ kip-ft} > 41.0 \text{ kip-ft} \text{ o.k.} \]
EXAMPLE F.9B  PIPE FLEXURAL MEMBER

Given:

The available strength in Example F.9A was easily determined using AISC Manual Table 3-15. The following example demonstrates the calculation of the available strength by directly applying the requirements of the AISC Specification.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A53 Grade B
  - \( F_y = 35 \text{ ksi} \)
  - \( F_u = 60 \text{ ksi} \)

From AISC Manual Table 1-14, the geometric properties are as follows:

- Pipe 8 x-Strong
  - \( Z = 31.0 \text{ in.}^3 \)
  - \( D = 8.63 \text{ in.} \)
  - \( t = 0.465 \text{ in.} \)
  - \( D/t = 18.5 \)

The required flexural strength from Example F.9A is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u )</td>
<td>61.4 kip-ft</td>
<td>41.0 kip-ft</td>
</tr>
</tbody>
</table>

Slenderness Check

Determine the limiting diameter-to-thickness ratio for a compact section from AISC Specification Table B4.1b Case 20.

\[
\lambda_p = \frac{0.07E}{F_y} = \frac{0.07(29,000 \text{ ksi})}{35 \text{ ksi}} = 58.0
\]

\[
\lambda = \frac{D}{t} = 18.5 < \lambda_p ; \text{ therefore, the section is compact and the limit state of flange local buckling does not apply}
\]

\[
\frac{D}{t} < \frac{0.45E}{F_y} = 373, \text{ therefore AISC Specification Section F8 applies}
\]
Nominal Flexural Strength Based on Flexural Yielding

\[ M_a = M_p \]
\[ = F_y Z_s \]
\[ = 35 \text{ ksi} \left( 31.0 \text{ in.}^3 \right) \]
\[ = 1,090 \text{ kip-in. or 90.4 kip-ft} \]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td></td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_a = 0.90(90.4 \text{ kip-ft}) )</td>
<td>( M_a = \frac{90.4 \text{ kip-ft}}{1.67} )</td>
<td>= 54.1 kip-ft &gt; 41.0 kip-ft o.k.</td>
</tr>
<tr>
<td></td>
<td>= 81.4 kip-ft &gt; 61.4 kip-ft o.k.</td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE F.10  WT-SHAPE FLEXURAL MEMBER

Given:

Select an ASTM A992 WT beam with a 5-in. nominal depth and a simple span of 6 ft. The toe of the stem of the WT is in tension. The loads are a uniform dead load of 0.08 kip/ft and a uniform live load of 0.24 kip/ft. There is no deflection limit for this member. The beam is continuously braced.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
\begin{align*}
\text{LRFD} & : w_u = 1.2(0.08 \text{ kip/ft}) + 1.6(0.24 \text{ kip/ft}) \\
& = 0.480 \text{ kip/ft} \\
M_u & = \frac{0.480 \text{ kip/ft}(6.00 \text{ ft})^2}{8} \\
& = 2.16 \text{ kip-ft} \\
\text{ASD} & : w_u = 0.08 \text{ kip/ft} + 0.24 \text{ kip/ft} \\
& = 0.320 \text{ kip/ft} \\
M_u & = \frac{0.320 \text{ kip/ft}(6.00 \text{ ft})^2}{8} \\
& = 1.44 \text{ kip-ft}
\end{align*}
\]

Try a WT5×6.

From AISC Manual Table 1-8, the geometric properties are as follows:

WT5×6
\[
\begin{align*}
d & = 4.94 \text{ in.} \\
I_x & = 4.35 \text{ in.}^4 \\
Z_x & = 2.20 \text{ in.}^2 \\
S_x & = 1.22 \text{ in.}^3 \\
b_f & = 3.96 \text{ in.} \\
t_f & = 0.210 \text{ in.} \\
\bar{y} & = 1.36 \text{ in.} \\
b_f/2t_f & = 9.43
\end{align*}
\]
\[ S_{wc} = \frac{I_y}{y} \]
\[ = 4.35 \text{ in.}^4 \]
\[ = 1.36 \text{ in.} \]
\[ = 3.20 \text{ in.}^3 \]

**Flexural Yielding**

\[ M_n = M_p \]  \hspace{1cm} (Spec. Eq. F9-1)

\[ M_p = F_y Z_s \leq 1.6 M_y \]  \hspace{1cm} for stems in tension  \hspace{1cm} (Spec. Eq. F9-2)

\[ 1.6 M_y = 1.6 F_y S_s \]
\[ = 1.6 \times 50 \text{ ksi} \times 1.22 \text{ in.}^3 \]
\[ = 97.6 \text{ kip-in.} \]

\[ M_p = F_y Z_s \]
\[ = 50 \text{ ksi} \times 2.20 \text{ in.}^3 \]
\[ = 110 \text{ kip-in.} > 97.6 \text{ kip-in.}, \text{ therefore, use} \]

\[ M_p = 97.6 \text{ kip-in. or } 8.13 \text{ kip-ft} \]

**Lateral-Torsional Buckling (AISC Specification Section F9.2)**

Because the WT is continuously braced, no check of the lateral-torsional buckling limit state is required.

**Flange Local Buckling (AISC Specification Section F9.3)**

Check flange compactness.

Determine the limiting slenderness ratio for a compact flange from AISC Specification Table B4.1b Case 10.

\[ \lambda_{sf} = 0.38 \sqrt{\frac{E}{F_y}} \]
\[ = 0.38 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]
\[ = 9.15 \]

\[ \lambda = \frac{b_f}{2 t_f} \]
\[ = 9.43 > \lambda_{sf}; \text{ therefore, the flange is not compact} \]
Check flange slenderness.

\[ \lambda_{sf} = 1.0 \sqrt{\frac{E}{F_y}} \text{ from AISC Specification Table B4.1b Case 10} \]

\[ = 1.0 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]

\[ = 24.1 \]

\[ \lambda = \frac{b_f}{2t_f} \]

\[ = 9.43 < \lambda_{sf} ; \text{ therefore, the flange is not slender} \]

For a WT with a noncompact flange, the nominal flexural strength due to flange local buckling is:

\[ M_n = \left[ M_p - \left( M_p - 0.7 F_y S_w \right) \left( \frac{\lambda_s - \lambda_{sf}}{\lambda_s - \lambda_{sf}} \right) \right] \leq 1.6 M_p \]  

(Spec. Eq. F9-6)

\[ = \left\{ 110 \text{ kip-in.} - \left[ 110 \text{ kip-in.} - 0.7 (50 \text{ ksi})(3.20 \text{ in.}^3) \right] \left( \frac{9.43 - 9.15}{24.1 - 9.15} \right) \right\} \leq 97.6 \text{ kip-in.} \]

\[ = 110 \text{ kip-in.} > 97.6 \text{ kip-in.} \]

Therefore use:

\[ M_n = 97.6 \text{ kip-in. or 8.13 kip-ft} \]

\[ M_n = M_p = 8.13 \text{ kip-ft} \text{ yielding limit state controls} \]  

(Spec. Eq. F9-1)

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_s = 0.90 )</td>
<td></td>
<td>( \Omega_s = 1.67 )</td>
</tr>
<tr>
<td>( \phi_s M_n = 0.90(8.13 \text{ kip-ft}) )</td>
<td>= 7.32 kip-ft &gt; 2.16 kip-ft</td>
<td>( M_n = 8.13 \text{ kip-ft} )</td>
</tr>
<tr>
<td></td>
<td>( \Omega_s = 1.67 )</td>
<td>= 4.87 kip-ft &gt; 1.44 kip-ft</td>
</tr>
</tbody>
</table>
EXAMPLE F.11A SINGLE ANGLE FLEXURAL MEMBER

Given:

Select an ASTM A36 single angle with a simple span of 6 ft. The vertical leg of the single angle is up and the toe is in compression. The vertical loads are a uniform dead load of 0.05 kip/ft and a uniform live load of 0.15 kip/ft. There are no horizontal loads. There is no deflection limit for this angle. The angle is braced at the end points only. Assume bending about the geometric x-x axis and that there is no lateral-torsional restraint.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A36
  - \( F_y = 36 \text{ ksi} \)
  - \( F_u = 58 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
\begin{align*}
\text{LRFD} & \quad \text{ASD} \\
\text{w}_{\text{ud}} & = 1.2(0.05 \text{ kip/ft}) + 1.6(0.15 \text{ kip/ft}) & \text{w}_{\text{ue}} & = 0.05 \text{ kip/ft} + 0.15 \text{ kip/ft} \\
& = 0.300 \text{ kip/ft} & & = 0.200 \text{ kip/ft} \\
M_{\text{ud}} & = 0.300 \text{ kip/ft} \left( \frac{6 \text{ ft}}{8} \right)^2 & M_{\text{ue}} & = 0.200 \text{ kip/ft} \left( \frac{6 \text{ ft}}{8} \right)^2 \\
& = 1.35 \text{ kip-ft} & & = 0.900 \text{ kip-ft}
\end{align*}
\]

Try a L4×4×¼.

From AISC Manual Table 1-7, the geometric properties are as follows:

- L4×4×¼
  - \( S_x = 1.03 \text{ in.}^3 \)

Nominal Flexural Strength, \( M_n \)

Flexural Yielding

From AISC Specification Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

\[
M_n = 1.5M_y = 1.5F_yS_x = 1.5(36 \text{ ksi})(1.03\text{ in.}^3) = 55.6 \text{ kip-in.}
\]
Lateral-Torsional Buckling

From AISC Specification Section F10.2, for single angles bending about a geometric axis with no lateral-torsional restraint, $M_y$ is taken as 0.80 times the yield moment calculated using the geometric section modulus.

$$M_y = 0.80F_yS_e$$

= $0.80(36\text{ ksi})(1.03\text{ in.}^2) = 29.7 \text{ kip-in.}$

Determine $M_e$.

For bending moment about one of the geometric axes of an equal-leg angle with no axial compression, with no lateral-torsional restraint, and with maximum compression at the toe, use AISC Specification Section F10.2(b)(iii)(a)(i), Equation F10-6a.

$$C_b = 1.14 \text{ from AISC Manual Table 3-1}$$

$$M_e = \frac{0.66Eb^4C_b}{L_o^2}\left[\sqrt{1+0.78\left(\frac{L_0t}{b^2}\right)^2} - 1\right]$$

= $0.66(29,000 \text{ ksi})(4.00 \text{ in.})^4(0.25 \text{ in.})(1.14)\left[\sqrt{1+0.78\left(\frac{72.0 \text{ in.}}{4.00 \text{ in.}}\right)^2} - 1\right]$

= 110 kip-in. > 29.7 kip-in.; therefore, AISC Specification Equation F10-3 is applicable

$$M_n = \left(1.92 - 1.17\frac{M_y}{M_e}\right)M_y \leq 1.5M_y$$

= $1.92 - 1.17\frac{29.7 \text{ kip-in.}}{110 \text{ kip-in.}} \leq 29.7 \text{ kip-in.} \leq 1.5(29.7 \text{ kip-in.})$

= 39.0 kip-in. ≤ 44.6 kip-in.; therefore, $M_n = 39.0 \text{ kip-in.}$

Leg Local Buckling

AISC Specification Section F10.3 applies when the toe of the leg is in compression.

Check slenderness of the leg in compression.

$$\lambda = \frac{b}{t}$$

= $\frac{4.00 \text{ in.}}{\frac{3}{16} \text{ in.}} = 16.0$

Determine the limiting compact slenderness ratios from AISC Specification Table B4.1b Case 12.

$$\lambda_y = 0.54\sqrt{\frac{E}{F_y}}$$
\[
= 0.54 \sqrt{\frac{29,000 \text{ksi}}{36 \text{ksi}}} \\
= 15.3
\]

Determine the limiting noncompact slenderness ratios from AISC Specification Table B4.1b Case 12.

\[
\lambda_y = 0.91 \sqrt{\frac{E}{F_y}} \\
= 0.91 \sqrt{\frac{29,000 \text{ksi}}{36 \text{ksi}}} \\
= 25.8
\]

\[\lambda_y < \lambda < \lambda_r, \text{ therefore, the leg is noncompact in flexure}\]

\[
M_n = F_c S_c \left[ 2.43 - 1.72 \left( \frac{b}{t} \right) \sqrt{\frac{F_c}{E}} \right] \\
= 0.80 S_c \\
= 0.80 (1.03 \text{in.}^3) \\
= 0.824 \text{in.}^3
\]

\[M_n = 36 \text{ksi} \left( 0.824 \text{ in.}^3 \right) \left[ 2.43 - 1.72(16.0) \sqrt{\frac{36 \text{ ksi}}{29,000 \text{ ksi}}} \right] \\
= 43.3 \text{ kip-in.}
\]

The lateral-torsional buckling limit state controls.

\[M_e = 39.0 \text{ kip-in. or 3.25 kip-ft}\]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_b = 0.90)</td>
<td></td>
<td>(\Omega_b = 1.67)</td>
</tr>
<tr>
<td>(\phi_b M_n = 0.90 \times 3.25 \text{ kip-ft})</td>
<td>(M_n = 3.25 \text{ kip-ft})</td>
<td>(\Omega_b = \frac{3.25 \text{ kip-ft}}{1.67})</td>
</tr>
<tr>
<td></td>
<td>= 2.93 kip-ft &gt; 1.35 kip-ft</td>
<td>= 1.95 kip-ft &gt; 0.900 kip-ft</td>
</tr>
</tbody>
</table>

o.k.
EXAMPLE F.11B SINGLE ANGLE FLEXURAL MEMBER

Given:

Select an ASTM A36 single angle with a simple span of 6 ft. The vertical leg of the single angle is up and the toe is in compression. The vertical loads are a uniform dead load of 0.05 kip/ft and a uniform live load of 0.15 kip/ft. There are no horizontal loads. There is no deflection limit for this angle. The angle is braced at the end points and at the midspan. Assume bending about the geometric x-x axis and that there is lateral-torsional restraint at the midspan and ends only.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
LRFD: \quad w_{ud} = 1.2(0.05 \text{ kip/ft}) + 1.6(0.15 \text{ kip/ft}) = 0.300 \text{ kip/ft} \\
M_{ud} = \frac{0.300 \text{ kip/ft} (6 \text{ ft})^2}{8} = 1.35 \text{ kip-ft} \\
\]

\[
ASD: \quad w_{ue} = 0.05 \text{ kip/ft} + 0.15 \text{ kip/ft} = 0.200 \text{ kip/ft} \\
M_{ue} = \frac{0.200 \text{ kip/ft} (6 \text{ ft})^2}{8} = 0.900 \text{ kip-ft} \\
\]

Try a L4\( \times \)4\( \times \)4.

From AISC Manual Table 1-7, the geometric properties are as follows:

\[
L4\times4\times4 \\
S_x = 1.03 \text{ in.}^3 \\
\]

Nominal Flexural Strength, \( M_n \)

Flexural Yielding

From AISC Specification Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

\[
M_n = 1.5M_y \\
= 1.5F_yS_x \\
\]

\( \text{(Spec. Eq. F10-1)} \)
Lateral-Torsional Buckling

From AISC Specification Section F10.2(b)(iii)(b), for single angles with lateral-torsional restraint at the point of maximum moment, $M_y$ is taken as the yield moment calculated using the geometric section modulus.

$$M_y = F_s S_z$$

$$= 36 \text{ ksi} (1.03 \text{ in.}^2)$$

$$= 37.1 \text{ kip-in.}$$

Determine $M_a$.

For bending moment about one of the geometric axes of an equal-leg angle with no axial compression, with lateral-torsional restraint at the point of maximum moment only (at midspan in this case), and with maximum compression at the toe, $M_a$ shall be taken as 1.25 times $M_e$ computed using AISC Specification Equation F10-6a.

$$C_b = 1.30 \text{ from AISC Manual Table } 3-1$$

$$M_e = 1.25 \left( \frac{0.66E_b t C_b}{L_a^2} \right) \left( 1 + 0.78 \left( \frac{L_t}{b^2} \right)^{2} - 1 \right)$$  (Spec. Eq. F10-6a)

$$= 1.25 \left[ \frac{0.66(29,000 \text{ ksi})(4.00 \text{ in.})^{4} (\frac{1}{4} \text{ in.})(1.30)}{(36.0 \text{ in.})^{2}} \right] \sqrt{1 + 0.78 \left( \frac{36.0 \text{ in.}}{4.00 \text{ in.}} \right)^{2} - 1}$$

$$= 179 \text{ kip-in.} > 37.1 \text{ kip-in.}, \text{ therefore, AISC Specification Equation F10-3 is applicable}$$

$$M_a = \begin{cases} 1.92 - 1.17 \left( \frac{M_e}{M_y} \right) & M_y \leq 1.5M_y \\ 1.92 - 1.17 \left( \frac{37.1 \text{ kip-in.}}{179 \text{ kip-in.}} \right) & 37.1 \text{ kip-in.} \leq 1.5(37.1 \text{ kip-in.}) \\ \end{cases}$$  (Spec. Eq. F10-3)

$$= 51.5 \text{ kip-in.} \leq 55.7 \text{ kip-in.}, \text{ therefore, } M_a = 51.5 \text{ kip-in.}$$

Leg Local Buckling

$$M_a = 43.3 \text{ kip-in.} \text{ from Example F.11A.}$$

The leg local buckling limit state controls.

$$M_e = 43.3 \text{ kip-in. or 3.61 kip-ft}$$

From AISC Specification Section F1, the available flexural strength is:
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
</tbody>
</table>
| \( \phi_b M_s = 0.90(3.61 \text{ kip-ft}) \)  
  \( = 3.25 \text{ kip-ft} > 1.35 \text{ kip-ft} \) | \( M_s \)  
  \( = \frac{3.61 \text{ kip-ft}}{1.67} \)  
  \( = 2.16 \text{ kip-ft} > 0.900 \text{ kip-ft} \) | o.k.  
  o.k. |
EXAMPLE F.11C SINGLE ANGLE FLEXURAL MEMBER

**Given:**

Select an ASTM A36 single angle with a simple span of 6 ft. The vertical loads are a uniform dead load of 0.05 kip/ft and a uniform live load of 0.15 kip/ft. The horizontal load is a uniform wind load of 0.12 kip/ft. There is no deflection limit for this angle. The angle is braced at the end points only and there is no lateral-torsional restraint. Use load combination 4 from Section 2.3.2 of ASCE/SEI 7 for LRFD and load combination 6a from Section 2.4.1 of ASCE/SEI 7 for ASD.

![Beam Bracing Diagram](https://via.placeholder.com/150)

**Solution:**

From AISC *Manual* Table 2-4, the material properties are as follows:

- **ASTM A36**
  - $F_y = 36$ ksi
  - $F_u = 58$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{ux}$</td>
<td>$1.2(0.05 \text{ kip/ft} + 0.15 \text{ kip/ft}) = 0.210 \text{ kip/ft}$</td>
<td>$w_{ux} = 0.05 \text{ kip/ft} + 0.75(0.15 \text{ kip/ft}) = 0.163 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$w_{uy}$</td>
<td>$1.0(0.12 \text{ kip/ft}) = 0.12 \text{ kip/ft}$</td>
<td>$w_{uy} = 0.75[(0.6)(0.12 \text{ kip/ft})] = 0.054 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$M_{ux}$</td>
<td>$0.210 \text{ kip/ft} (6 \text{ ft})^2 \div 8 = 0.945 \text{ kip-ft}$</td>
<td>$M_{ux} = 0.163 \text{ kip/ft} (6 \text{ ft})^2 \div 8 = 0.734 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$M_{uy}$</td>
<td>$0.12 \text{ kip/ft} (6 \text{ ft})^2 \div 8 = 0.540 \text{ kip-ft}$</td>
<td>$M_{uy} = 0.054 \text{ kip/ft} (6 \text{ ft})^2 \div 8 = 0.243 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Try a L4×4×¾.
Fig. F.11C-1. Example F.11C single angle geometric and principal axes moments.

Positive Geometric and Principal Axes

For an equal leg angle, tan\(\alpha\) = 1.00 and \(\alpha = 45^\circ\)

Principal Axis Moments

From AISC Manual Table 1-7, the geometric properties are as follows:

- \(L_{4\times4\times4}\)
- \(S_y = S_y = 1.03 \text{ in.}^3\)
- \(I_x = I_y = 3.00 \text{ in.}^4\)
- \(I_z = 1.18 \text{ in.}^4\)

Additional properties from the angle geometry are as follows:

- \(w_B = 1.53 \text{ in.}\)
- \(w_C = 1.39 \text{ in.}\)
- \(z_C = 2.74 \text{ in.}\)
Additional principal axes properties from the AISC *Shapes Database* are as follows:

\[
I_w = 4.82 \text{ in.}^4 \\
S_{zb} = 0.779 \text{ in.}^3 \\
S_{zc} = 0.857 \text{ in.}^3 \\
S_{wc} = 1.76 \text{ in.}^3
\]

**Z-Axis Nominal Flexural Strength, \(M_{nz}\)**

Note that \(M_{nz}\) and \(M_{nc}\) are positive; therefore, the toes of the angle are in compression.

**Flexural Yielding**

From AISC *Specification* Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

\[
M_{nz} = 1.5M_z \\
= 1.5F_yS_{zb} \\
= 1.5(36 \text{ ksi})(0.779 \text{ in.}^3) \\
= 42.1 \text{ kip-in.}
\]

**Lateral-Torsional Buckling**

From the User Note in AISC *Specification* Section F10, the limit state of lateral-torsional buckling does not apply for bending about the minor axis.

**Leg Local Buckling**

Check slenderness of outstanding leg in compression.

\[
\lambda = \frac{b}{t} \\
= \frac{4.00 \text{ in.}}{\frac{1}{4} \text{ in.}} \\
= 16.0
\]

The limiting width-to-thickness ratios are:

\[
\lambda_p = 0.54 \sqrt{\frac{E}{F_y}} \text{ from AISC Specification Table B4.1b Case 12} \\
\lambda_p = 0.54 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} \\
\lambda_p = 15.3
\]

\[
\lambda_r = 0.91 \sqrt{\frac{E}{F_y}} \text{ from AISC Specification Table B4.1b Case 12} \\
\lambda_r = 0.91 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} \\
\lambda_r = 25.8
\]
\( \lambda_p < \lambda < \lambda_c \), therefore, the leg is noncompact in flexure

\[
M_{ac} = F_rS_c \left( 2.43 - 1.72 \left( \frac{b}{t} \right) \frac{F_s}{E} \right)
\]

(Spec. Eq. F10-7)

\( S_c = S_{cc} \) (to toe in compression)

\[
= 0.857 \text{ in.}^3
\]

\[
M_{ac} = 36 \text{ ksi} \left( 0.857 \text{ in.}^3 \right) \left[ 2.43 - 1.72(16.0) \frac{36 \text{ ksi}}{29,000 \text{ ksi}} \right]
\]

\[
= 45.1 \text{ kip-in.}
\]

The flexural yielding limit state controls.

\( M_{ac} = 42.1 \text{ kip-in.} \)

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_y = 0.90 )</td>
<td>( \Omega_y = 1.67 )</td>
</tr>
<tr>
<td>( \phi_y M_{ac} = 0.90(42.1 \text{ kip-in.}) )</td>
<td>( M_{ac} = 42.1 \text{ kip-in.} )</td>
</tr>
<tr>
<td>= 37.9 kip-in.</td>
<td>( \Omega_y = 1.67 )</td>
</tr>
<tr>
<td></td>
<td>( = 25.2 \text{ kip-in.} )</td>
</tr>
</tbody>
</table>

**W-Axis Nominal Flexural Strength, \( M_{nw} \)**

Flexural Yielding

\[
M_{nw} = 1.5M_y
\]

(Spec. Eq. F10-1)

\[
= 1.5F_rS_{nc}
\]

\[
= 1.5(36 \text{ ksi})(1.76 \text{ in.}^3)
\]

= 95.0 kip-in.

Lateral-Torsional Buckling

Determine \( M_c \).

For bending about the major principal axis of an equal-leg angle without continuous lateral-torsional restraint, use AISC Specification Equation F10-4.

\( C_b = 1.14 \) from AISC Manual Table 3-1

\[
M_c = \frac{0.46E b^2 \tau^2 C_b}{L_b}
\]

(Spec. Eq. F10-4)

\[
= \frac{0.46(29,000 \text{ ksi})(4.00 \text{ in.})^2(\frac{1}{2} \text{ in.})^2(1.14)}{72.0 \text{ in.}}
\]

\[
= 211 \text{ kip-in.}
\]
\[ M_e = F_y S_w C \]
\[ = 36 \text{ ksi}(1.76 \text{ in.}^3) \]
\[ = 63.4 \text{ kip-in.} \]

\[ M_e > M_y, \text{ therefore, AISC Specification Equation F10-3 is applicable} \]

\[ M_{nw} = \left( 1.92 - 1.17 \frac{M_e}{M_y} \right) M_y \leq 1.5 M_y \]
\[ = \left( 1.92 - 1.17 \frac{63.4 \text{ kip-in.}}{211 \text{ kip-in.}} \right) 63.4 \text{ kip-in.} \leq 1.5(63.4 \text{ kip-in.}) \]
\[ = 81.1 \text{ kip-in.} \leq 95.1 \text{ kip-in.}, \text{ therefore, } M_{nw} = 81.1 \text{ kip-in.} \]

**Leg Local Buckling**

From the preceding calculations, the leg is noncompact in flexure.

\[ M_{nw} = F_y S_c \left( 2.43 - 1.72 \left( \frac{b}{t} \right) \sqrt{\frac{F_y}{E}} \right) \]
\[ = 36 \text{ ksi}(1.76 \text{ in.}^3) \left( 2.43 - 1.72(16.0) \sqrt{\frac{36 \text{ ksi}}{29,000 \text{ ksi}}} \right) \]
\[ = 92.5 \text{ kip-in.} \]

The lateral-torsional buckling limit state controls.

\[ M_{nw} = 81.1 \text{ kip-in.} \]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_{yw} = 0.90(81.1 \text{ kip-in.}) )</td>
<td>( M_{nw} = 81.1 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( = 73.0 \text{ kip-in.} )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td></td>
<td>( = 48.6 \text{ kip-in.} )</td>
</tr>
</tbody>
</table>

The moment resultant has components about both principal axes; therefore, the combined stress ratio must be checked using the provisions of AISC Specification Section H2.

\[ \left| \frac{f_{ru} + f_{rdu}}{F_{ru} + F_{rdu}} \right| \leq 1.0 \]
\[ (Spec. \text{ Eq. H2-1}) \]

Note: Rather than convert moments into stresses, it is acceptable to simply use the moments in the interaction equation because the section properties that would be used to convert the moments to stresses are the same in the numerator and denominator of each term. It is also important for the designer to keep track of the signs of the stresses at each point so that the proper sign is applied when the terms are combined. The sign of the moments...
used to convert geometric axis moments to principal axis moments will indicate which points are in tension and which are in compression but those signs will not be used in the interaction equations directly.

Based on Figure F.11C-1, the required flexural strength and available flexural strength for this beam can be summarized as:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{uw}$ = 0.286 kip-ft</td>
<td>$M_{uw}$ = 0.347 kip-ft</td>
</tr>
<tr>
<td>$\phi \frac{M_{uw}}{M_{uw}} = \frac{73.0 \text{ kip-in.}}{12 \text{ in./ft}} = 6.08 \text{ kip-ft}$</td>
<td>$M_{uw}$ = 48.6 kip-in. $\Omega_b = \frac{12 \text{ in.}}{12 \text{ in./ft}} = 4.05 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$M_{oz}$ = 1.05 kip-ft</td>
<td>$M_{oz}$ = 0.691 kip-ft</td>
</tr>
<tr>
<td>$\phi \frac{M_{oz}}{M_{oz}} = \frac{37.9 \text{ kip-in.}}{12 \text{ in./ft}} = 3.16 \text{ kip-ft}$</td>
<td>$M_{oz}$ = 25.2 kip-in. $\Omega_b = \frac{12 \text{ in.}}{12 \text{ in./ft}} = 2.10 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

At point B:

$M_w$ causes no stress at point B; therefore, the stress ratio is set to zero. $M_z$ causes tension at point B; therefore it will be taken as negative.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{0-1.05 \text{ kip-ft}}{3.16 \text{ kip-ft}} = 0.332 \leq 1.0$</td>
<td>o.k. $\frac{0-0.691 \text{ kip-ft}}{2.10 \text{ kip-ft}} = 0.329 \leq 1.0$ o.k.</td>
</tr>
</tbody>
</table>

At point C:

$M_w$ causes tension at point C; therefore, it will be taken as negative. $M_z$ causes compression at point C; therefore, it will be taken as positive.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{0.286 \text{ kip-ft} + 1.05 \text{ kip-ft}}{6.08 \text{ kip-ft} + 3.16 \text{ kip-ft}} = 0.285 \leq 1.0$</td>
<td>o.k. $\frac{0.347 \text{ kip-ft} + 0.691 \text{ kip-ft}}{4.05 \text{ kip-ft} + 2.10 \text{ kip-ft}} = 0.243 \leq 1.0$ o.k.</td>
</tr>
</tbody>
</table>

At point A:

$M_w$ and $M_z$ cause compression at point A; therefore, both will be taken as positive.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{0.286 \text{ kip-ft} + 1.05 \text{ kip-ft}}{6.08 \text{ kip-ft} + 3.16 \text{ kip-ft}} = 0.379 \leq 1.0$</td>
<td>o.k. $\frac{0.347 \text{ kip-ft} + 0.691 \text{ kip-ft}}{4.05 \text{ kip-ft} + 2.10 \text{ kip-ft}} = 0.415 \leq 1.0$ o.k.</td>
</tr>
</tbody>
</table>

Thus, the interaction of stresses at each point is seen to be less than 1.0 and this member is adequate to carry the required load. Although all three points were checked, it was expected that point A would be the controlling point because compressive stresses add at this point.
EXAMPLE F.12  RECTANGULAR BAR IN STRONG-AXIS BENDING

Given:
Select an ASTM A36 rectangular bar with a span of 12 ft. The bar is braced at the ends and at the midpoint. Conservatively use \( C_b = 1.0 \). Limit the depth of the member to 5 in. The loads are a total uniform dead load of 0.44 kip/ft and a uniform live load of 1.32 kip/ft.

\[
\begin{align*}
\text{Given:} \\
& w_d = 0.44 \text{ kip/ft} \\
& w_l = 1.32 \text{ kip/ft} \\
\end{align*}
\]

Beam Loading & Bracing Diagram
(braced at end points and midspan)

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
\begin{align*}
\text{LRFD} & \\
\text{LRFD} & \\
\text{ASD} & \\
\text{ASD} &
\end{align*}
\]

Try a BAR 5 in.\( \times \)3 in.

From AISC Manual Table 17-27, the geometric properties are as follows:

\[
S_y = \frac{bd^2}{6} = \frac{(3.00 \text{ in.})(5.00 \text{ in.})^2}{6} = 12.5 \text{ in.}^3
\]
\[ Z_s = \frac{bd^2}{4} \]
\[ = \frac{(3.00 \text{ in.})(5.00 \text{ in.})^2}{4} \]
\[ = 18.8 \text{ in.}^3 \]

**Nominal Flexural Strength, \( M_n \)**

**Flexural Yielding**

Check limit from AISC *Specification* Section F11.1.

\[
\frac{L_o d}{t^2} \leq \frac{0.08E}{F_y} \]
\[
\frac{72.0 \text{ in.}(5.00 \text{ in.})}{(3.00 \text{ in.})^2} \leq \frac{0.08(29,000 \text{ ksi})}{36 \text{ ksi}} \]

\[ 40.0 < 64.4, \text{ therefore, the yielding limit state applies} \]

\[ M_n = M_p \]
\[ = F_y Z \leq 1.6 M_y \]

\[ 1.6 M_y = 1.6 F_y S_s \]
\[ = 1.6(36 \text{ ksi})(12.5 \text{ in.}^3) \]
\[ = 720 \text{ kip-in.} \]

\[ M_p = F_y Z_s \]
\[ = 36 \text{ ksi}(18.8 \text{ in.}^3) \]
\[ = 677 \text{ kip-in.} \leq 720 \text{ kip-in.} \]

Use \( M_n = M_p \)
\[ = 677 \text{ kip-in. or 56.4 kip-ft} \]

**Lateral-Torsional Buckling (AISC *Specification* Section F11.2)**

As previously calculated, \( L_o d/t^2 \leq 0.08E/F_y \), therefore, the lateral-torsional buckling limit state does not apply.

From AISC *Specification* Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td></td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_n = 0.90(56.4 \text{ kip-ft}) )</td>
<td>( M_s = 56.4 \text{ kip-ft} )</td>
<td>( \Omega_s = \frac{56.4 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>[ = 50.8 \text{ kip-ft} &gt; 47.5 \text{ kip-ft} ]</td>
<td>o.k.</td>
<td>[ = 33.8 \text{ kip-ft} &gt; 31.7 \text{ kip-ft} ]</td>
</tr>
</tbody>
</table>
EXAMPLE F.13  ROUND BAR IN BENDING

Given:
Select an ASTM A36 round bar with a span of 2.50 ft. The bar is braced at end points only. Assume $C_b = 1.0$. Limit the diameter to 2 in. The loads are a concentrated dead load of 0.10 kip and a concentrated live load of 0.25 kip at the center. The weight of the bar is negligible.

Solution:
From AISC Manual Table 2-5, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From Chapter 2 of ASCE/SEI 7 and AISC Manual Table 3-23 diagram 7, the required flexural strength is:

$$Pu_{LRFD} = 1.2(0.10 \text{ kip}) + 1.6(0.25 \text{ kip}) = 0.520 \text{ kip}$$
$$Pu_{ASD} = 0.10 \text{ kip} + 0.25 \text{ kip} = 0.350 \text{ kip}$$

$$Mu_{LRFD} = \frac{(0.520 \text{ kip})(2.50 \text{ ft})}{4} = 0.325 \text{ kip-ft}$$
$$Mu_{ASD} = \frac{(0.350 \text{ kip})(2.50 \text{ ft})}{4} = 0.219 \text{ kip-ft}$$

Try a BAR 1 in. diameter.

From AISC Manual Table 17-27, the geometric properties are as follows:

Round bar

$$S_x = \frac{\pi d^3}{32}$$
$$= \frac{\pi (1.00 \text{ in.})^3}{32}$$
$$= 0.0982 \text{ in.}^3$$
\[ Z_s = \frac{d^3}{6} \]
\[ = \frac{(1.00 \text{ in.})^3}{6} \]
\[ = 0.167 \text{ in.}^3 \]

Nominal Flexural Strength, \( M_n \)

Flexural Yielding

From AISC Specification Section F11.1, the nominal flexural strength based on the limit state of flexural yielding is,

\[ M_n = M_p \]
\[ = F_y Z \leq 1.6M_f \]

\[ 1.6M_f = 1.6F_y S_x \]
\[ = 1.6(36 \text{ ksi})(0.0982 \text{ in.}^3) \]
\[ = 5.66 \text{ kip-in.} \]

\[ F_y Z = 36 \text{ ksi}(0.167 \text{ in.}^3) \]
\[ = 6.01 \text{ kip-in.} > 5.66 \text{ kip-in.} \]

Therefore, \( M_n = 5.66 \text{ kip-in.} \)

The limit state lateral-torsional buckling (AISC Specification Section F11.2) need not be considered for rounds.

The flexural yielding limit state controls.

\( M_n = 5.66 \text{ kip-in. or 0.472 kip-ft} \)

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_n = 0.90(0.472 \text{ kip-ft}) )</td>
<td>( M_n = \frac{0.472 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>= 0.425 kip-ft &gt; 0.325 kip-ft</td>
<td>( \Omega_b = 0.283 \text{ kip-ft} &gt; 0.219 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>
EXAMPLE F.14  POINT-SYMMETRICAL Z-SHAPE IN STRONG-AXIS BENDING

Given:

Determine the available flexural strength of the ASTM A36 Z-shape shown for a simple span of 18 ft. The Z-shape is braced at 6 ft on center. Assume $C_b = 1.0$. The loads are a uniform dead load of 0.025 kip/ft and a uniform live load of 0.10 kip/ft. Assume the beam is loaded through the shear center. The profile of the purlin is shown below.
**Solution:**

From AISC *Manual* Table 2-5, the material properties are as follows:

- ASTM A36
  - $F_s = 36$ ksi
  - $F_u = 58$ ksi

The geometric properties are as follows:

\[
I_y = I_f
\]
\[
= \frac{1}{4} \text{ in.}
\]

\[
A = (2.50 \text{ in.}) (\frac{1}{4} \text{ in.}) (2) + (\frac{1}{4} \text{ in.}) (\frac{1}{4} \text{ in.}) (2) + (11.5 \text{ in.}) (\frac{1}{4} \text{ in.})
\]
\[
= 4.25 \text{ in.}^2
\]

\[
I_x = \left[ \left( \frac{\frac{1}{4} \text{ in.}}{12} \right)^3 + (0.25 \text{ in.})^2 \left( 5.63 \text{ in.} \right)^2 \right] (2)
\]
\[
+ \left[ \left( \frac{2.50 \text{ in.}}{12} \right)^3 + (2.50 \text{ in.}) (\frac{1}{4} \text{ in.}) (5.88 \text{ in.}) \right] (2)
\]
\[
+ \left( \frac{\frac{1}{4} \text{ in.}}{12} \right)^3 (11.5 \text{ in.})^3
\]
\[
= 78.9 \text{ in.}^4
\]

\[
\bar{y} = 6.00 \text{ in.}
\]

\[
S_y = \frac{I_x}{\bar{y}}
\]
\[
= \frac{78.9 \text{ in.}^4}{6.00 \text{ in.}}
\]
\[
= 13.2 \text{ in.}^3
\]

\[
I_y = \left[ \left( \frac{\frac{1}{4} \text{ in.}}{12} \right)^3 (\frac{1}{4} \text{ in.})^2 (2.25 \text{ in.})^2 \right] (2)
\]
\[
+ \left[ \left( \frac{\frac{1}{4} \text{ in.}}{12} \right)^3 + (2.50 \text{ in.}) (\frac{1}{4} \text{ in.}) (1.13 \text{ in.}) \right] (2)
\]
\[
+ \left( \frac{11.5 \text{ in.}}{12} \right)^3 (\frac{1}{4} \text{ in.})^3
\]
\[
= 2.90 \text{ in.}^4
\]

\[
r_y = \sqrt{\frac{I_y}{A}}
\]
\[
= \sqrt{\frac{2.90 \text{ in.}^4}{4.25 \text{ in.}^2}}
\]
\[
= 0.826 \text{ in.}
\]
From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
w_u = 1.2(0.025 \text{ kip/ft}) + 1.6(0.10 \text{ kip/ft}) & w_u = 0.025 \text{ kip/ft} + 0.10 \text{ kip/ft} \\
= 0.190 \text{ kip/ft} & = 0.125 \text{ kip/ft} \\
M_u = \frac{(0.190 \text{ kip/ft})(18.0 \text{ ft})^2}{8} & M_u = \frac{(0.125 \text{ kip/ft})(18.0 \text{ ft})^2}{8} \\
= 7.70 \text{ kip-ft} & = 5.06 \text{ kip-ft} \\
\hline
\end{array}
\]

Nominal Flexural Strength, \( M_n \)

Flexural Yielding

From AISC Specification Section F12.1, the nominal flexural strength based on the limit state of flexural yielding is,

\[
F_n = F_y \quad \text{(Spec. Eq. F12-2)}
\]

\[
= 36 \text{ ksi}
\]

\[
M_n = F_y S_{min} \quad \text{(Spec. Eq. F12-1)}
\]

\[
= 36 \text{ ksi}(13.2 \text{ in.}^3)
\]

\[
= 475 \text{ kip-in.}
\]

Local Buckling

There are no specific local buckling provisions for Z-shapes in the AISC Specification. Use provisions for rolled channels from AISC Specification Table B4.1b, Cases 10 and 15.

Flange Slenderness

Conservatively neglecting the end return,

\[
\lambda = \frac{b}{t_f}
\]

\[
= \frac{2.50 \text{ in.}}{\frac{1}{4} \text{ in.}}
\]

\[
= 10.0
\]

\[
\lambda_p = 0.38 \sqrt{\frac{E}{F_y}} \quad \text{from AISC Specification Table B4.1b case 10}
\]
\[
\lambda = 0.38 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 10.8
\]

\(\lambda < \lambda_p\); therefore, the flange is compact

Web Slenderness

\[
\lambda = \frac{h}{t_c} = \frac{11.5 \text{ in.}}{\frac{3}{8} \text{ in.}} = 46.0
\]

\[
\lambda_p = 3.76 \sqrt{\frac{E}{F_y}} \text{ from AISC Specification Table B4.1b case 15}
\]

\[
= 3.76 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}}
\]

\[
= 107
\]

\(\lambda < \lambda_p\); therefore, the web is compact

Therefore, the local buckling limit state does not apply.

Lateral-Torsional Buckling

Per the User Note in AISC Specification Section F12, take the critical lateral-torsional buckling stress as half that of the equivalent channel. This is a conservative approximation of the lateral-torsional buckling strength which accounts for the rotation between the geometric and principal axes of a Z-shaped cross-section, and is adopted from the North American Specification for the Design of Cold-Formed Steel Structural Members (AISC, 2007).

Calculate limiting unbraced lengths.

For bracing at 6 ft on center,

\[
L_o = 6.00 \text{ ft (12 in./ft)} = 72.0 \text{ in.}
\]

\[
L_p = 1.76r_e \sqrt{\frac{E}{F_y}}
\]

(Spec. Eq. F2-5)

\[
= 1.76(0.826 \text{ in.}) \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}}
\]

\[
= 41.3 \text{ in.} < 72.0 \text{ in.}
\]

\[
L_c = 1.95r_e \left( \frac{E}{0.7F_y} \right) \left( \frac{Jc}{S_yh_0} + \sqrt{\frac{Jc}{S_yh_0}} \right)^2 + 6.76 \left( \frac{0.7F_y}{E} \right)^2
\]

(Spec. Eq. F2-6)
Per the User Note in AISC Specification Section F2, the square root term in AISC Specification Equation F2-4 can conservatively be taken equal to one. Therefore, Equation F2-6 can also be simplified. Substituting 0.7\(F_y\) for \(F_{cr}\) in Equation F2-4 and solving for \(L_b = L_r\), AISC Specification Equation F2-6 becomes:

\[
L_b = \pi r_b \frac{E}{0.7F_y} = \pi (0.543 \text{ in.}) \frac{29,000 \text{ ksi}}{0.7(36 \text{ ksi})} = 57.9 \text{ in.} < 72.0 \text{ in.}
\]

Calculate one half of the critical lateral-torsional buckling stress of the equivalent channel.

\[
L_b > L_r, \text{ therefore,}
\]

\[
F_{cr} = 0.5 \frac{C_b \pi^2 E}{L_r^2} \left( 1 + 0.078 \left( \frac{J_c}{S_n h_0} \right)^2 \left( \frac{L_r}{r_n} \right)^2 \right)
\]

(Spec. Eq. F2-4)

Conservatively taking the square root term as 1.0,

\[
F_{cr} = 0.5 \frac{C_b \pi^2 E}{L_b^2} = 0.5 \frac{1.0}{(57.9 \text{ in.})^2} \left( \frac{29,000 \text{ ksi}}{72.0 \text{ in.}} \right)^2 = 8.14 \text{ ksi}
\]

\(F_n = F_{cr} \leq F_y = 8.14 \text{ ksi} \leq 36 \text{ ksi} \quad \text{o.k.} \quad \text{(Spec. Eq. F12-3)}\)

\[M_a = F_c S_{min}\]

\[= 8.14 \text{ ksi} (13.2 \text{ in.}^3) = 107 \text{ kip-in.} \quad \text{(Spec. Eq. F12-1)}\]

The lateral-torsional buckling limit state controls.

\[M_a = 107 \text{ kip-in. or 8.95 kip-ft}\]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_b = 0.90)</td>
<td>(\Omega_b = 1.67)</td>
</tr>
<tr>
<td>(\phi_b M_a = 0.90(8.95 \text{ kip-ft}))</td>
<td>(M_a = 8.95 \text{ kip-ft})</td>
</tr>
<tr>
<td>= 8.06 \text{ kip-ft} &gt; 7.70 \text{ kip-ft} \quad \text{o.k.}</td>
<td>(\Omega_b = \frac{1.67}{5.36 \text{ kip-ft} &gt; 5.06 \text{ kip-ft}} \quad \text{o.k.}</td>
</tr>
</tbody>
</table>

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
Because the beam is loaded through the shear center, consideration of a torsional moment is unnecessary. If the loading produced torsion, the torsional effects should be evaluated using AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).
CHAPTER F DESIGN EXAMPLE REFERENCES


**Chapter G**

**Design of Members for Shear**

**INTRODUCTION**

This chapter covers webs of singly or doubly symmetric members subject to shear in the plane of the web, single angles, HSS sections, and shear in the weak direction of singly or doubly symmetric shapes.

Most of the equations from this chapter are illustrated by example. Tables for all standard ASTM A992 W-shapes and ASTM A36 channels are included in the AISC *Manual*. In the tables, where applicable, LRFD and ASD shear information is presented side-by-side for quick selection, design and verification.

LRFD and ASD will produce identical designs for the case where the live load effect is approximately three times the dead load effect.

**G1. GENERAL PROVISIONS**

The design shear strength, $\phi V_n$, and the allowable shear strength, $V_{a}/\Omega_v$, are determined as follows:

\[
V_n = \text{nominal shear strength based on shear yielding or shear buckling}
\]

\[
V_v = 0.6 F_v A C
\]

(Spec. Eq. G2-1)

$\phi_v = 0.90$ (LRFD) \hspace{1cm} $\Omega_v = 1.67$ (ASD)

Exception: For all current ASTM A6, W, S and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26 and W12×14 for $F_v = 50$ ksi:

$\phi_v = 1.00$ (LRFD) \hspace{1cm} $\Omega_v = 1.50$ (ASD)

AISC *Specification* Section G2 does not utilize tension field action. AISC *Specification* Section G3 specifically addresses the use of tension field action.

Strong axis shear values are tabulated for W-shapes in AISC *Manual* Tables 3-2 and 3-6, for S-shapes in AISC *Manual* Table 3-7, for C-shapes in AISC *Manual* Table 3-8, and for MC-shapes in AISC *Manual* Table 3-9. Weak axis shear values for W-shapes, S-shapes, C-shapes and MC-shapes, and shear values for angles, rectangular HSS and box members, and round HSS are not tabulated.

**G2. MEMBERS WITH UNSTIFFENED OR STIFFENED WEBS**

As indicated in the User Note of this section, virtually all W, S and HP shapes are not subject to shear buckling and are also eligible for the more liberal safety and resistance factors, $\phi_v = 1.00$ (LRFD) and $\Omega_v = 1.50$ (ASD). This is presented in Example G.1 for a W-shape. A channel shear strength design is presented in Example G.2.

**G3. TENSION FIELD ACTION**

A built-up girder with a thin web and transverse stiffeners is presented in Example G.8.

**G4. SINGLE ANGLES**

Rolled angles are typically made from ASTM A36 steel. A single angle example is illustrated in Example G.3.

**G5. RECTANGULAR HSS AND BOX-SHAPE MEMBERS**
The shear height, \( h \), is taken as the clear distance between the flanges less the inside corner radius on each side. If the corner radii are unknown, \( h \) shall be taken as the corresponding outside dimension minus 3 times the thickness. A rectangular HSS example is provided in Example G.4.

G6. ROUND HSS

For all round HSS and pipes of ordinary length listed in the AISC Manual, \( F_{cr} \) can be taken as 0.6\( F_y \) in AISC Specification Equation G6-1. A round HSS example is illustrated in Example G.5.

G7. WEAK AXIS SHEAR IN DOUBLY SYMMETRIC AND SINGLY SYMMETRIC SHAPES

For examples of weak axis shear, see Example G.6 and Example G.7.

G8. BEAMS AND GIRDERs WITH WEB OPENINGS

For a beam and girder with web openings example, see AISC Design Guide 2, Steel and Composite Beams with Web Openings (Darwin, 1990).
EXAMPLE G.1A  W-SHAPE IN STRONG AXIS SHEAR

Given:

Determine the available shear strength and adequacy of a W24×62 ASTM A992 beam using the AISC Manual with end shears of 48 kips from dead load and 145 kips from live load.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

\[
\begin{align*}
Vu & = 1.2(48.0 \text{ kips}) + 1.6(145 \text{ kips}) \\
& = 290 \text{ kips}
\end{align*}
\]

\[
\begin{align*}
Va & = 48.0 \text{ kips} + 145 \text{ kips} \\
& = 193 \text{ kips}
\end{align*}
\]

From AISC Manual Table 3-2, the available shear strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi V_a = 306 \text{ kips} ]</td>
<td>[ \frac{V_a}{\Omega_e} = 204 \text{ kips} ]</td>
</tr>
<tr>
<td>306 kips &gt; 290 kips</td>
<td>204 kips &gt; 193 kips</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE G.1B  W-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength, which can be easily determined by the tabulated values of the AISC Manual, can be verified by directly applying the provisions of the AISC Specification. Determine the available shear strength for the W-shape in Example G.1A by applying the provisions of the AISC Specification.

Solution:

From AISC Manual Table 1-1, the geometric properties are as follows:

\[ W24 \times 62 \]
\[ d = 23.7 \text{ in.} \]
\[ t_w = 0.430 \text{ in.} \]

Except for very few sections, which are listed in the User Note, AISC Specification Section G2.1(a) is applicable to the I-shaped beams published in the AISC Manual for \( F_y = 50 \text{ ksi} \).

\[ C_v = 1.0 \]  
(spec. Eq. G2-2)

Calculate \( A_w \).

\[ A_w = dt_w \] from AISC Specification Section G2.1b
\[ = 23.7 \text{ in.}(0.430 \text{ in.}) \]
\[ = 10.2 \text{ in.}^2 \]

Calculate \( V_n \).

\[ V_n = 0.6F_yA_wC_v \]  
(spec. Eq. G2-1)
\[ = 0.6(50 \text{ ksi})(10.2 \text{ in.}^2)(1.0) \]
\[ = 306 \text{ kips} \]

From AISC Specification Section G2.1a, the available shear strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_v = 1.00 )</td>
<td>( \Omega_v = 1.50 )</td>
</tr>
<tr>
<td>( \phi_v V_n = 1.00(306 \text{ kips}) )</td>
<td>( V_n = 306 \text{ kips} )</td>
</tr>
</tbody>
</table>
| \( \phi_v V_n = 306 \text{ kips} \) | \( \Omega_v = \frac{1.50}{1.50} \)
|             | = 204 kips   |
EXAMPLE G.2A  C-SHAPE IN STRONG AXIS SHEAR

Given:

Verify the available shear strength and adequacy of a C15×33.9 ASTM A36 channel with end shears of 17.5 kips from dead load and 52.5 kips from live load.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

\[
\begin{align*}
LRFD & \\
V_u &= 1.2(17.5 \text{ kips}) + 1.6(52.5 \text{ kips}) \\
&= 105 \text{ kips}
\end{align*}
\]

\[
\begin{align*}
ASD & \\
V_a &= 17.5 \text{ kips} + 52.5 \text{ kips} \\
&= 70.0 \text{ kips}
\end{align*}
\]

From AISC Manual Table 3-8, the available shear strength is:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi_VV_n = 117 \text{ kips} & \frac{V_n}{\Omega_n} = 77.6 \text{ kips} \\
117 \text{ kips} > 105 \text{ kips} & 77.6 \text{ kips} > 70.0 \text{ kips} \quad \text{o.k.} \\
\hline
\end{array}
\]
EXAMPLE G.2B  C-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength, which can be easily determined by the tabulated values of the AISC Manual, can be verified by directly applying the provisions of the AISC Specification. Determine the available shear strength for the channel in Example G.2A.

Solution:

From AISC Manual Table 1-5, the geometric properties are as follows:

\[ C15 \times 33.9 \]
\[ d = 15.0 \text{ in.} \]
\[ t_w = 0.400 \text{ in.} \]

AISC Specification Equation G2-1 is applicable. All ASTM A36 channels listed in the AISC Manual have \( h/t_w \leq 1.10 \sqrt{k_y E / F_y} \); therefore,

\[ C_v = 1.0 \quad \text{(Spec. Eq. G2-3)} \]

Calculate \( A_w \).

\[ A_w = d t_w \text{ from AISC Specification Section G2.1b} \]
\[ = 15.0 \text{ in.}(0.400 \text{ in.}) \]
\[ = 6.00 \text{ in.}^2 \]

Calculate \( V_n \).

\[ V_n = 0.6 F_y A_w C_v \quad \text{(Spec. Eq. G2-1)} \]
\[ = 0.6(36 \text{ ksi})(6.00 \text{ in.}^2)(1.0) \]
\[ = 130 \text{ kips} \]

Available Shear Strength

The values of \( \phi_v = 1.00 \) (LRFD) and \( \Omega_v = 1.50 \) (ASD) do not apply to channels. The general values \( \phi_v = 0.90 \) (LRFD) and \( \Omega_v = 1.67 \) (ASD) must be used.

<table>
<thead>
<tr>
<th>( \phi_v V_n )</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_v V_n = 0.90(130 \text{ kips}) )</td>
<td>( V_n = 130 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( = 117 \text{ kips} )</td>
<td>( \Omega_v = \frac{1.67}{1.67} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 77.8 \text{ kips} )</td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE G.3 ANGLE IN SHEAR

Given:

Determine the available shear strength and adequacy of a L5×3×¼ (LLV) ASTM A36 with end shears of 3.50 kips from dead load and 10.5 kips from live load.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36

\( F_y = 36 \text{ ksi} \)

\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-7, the geometric properties are as follows:

L5×3×¼

\( b = 5.00 \text{ in.} \)

\( t = \frac{1}{4} \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

\[
Vu = 1.2(3.50 \text{ kips}) + 1.6(10.5 \text{ kips})
\]

\[
= 21.0 \text{ kips}
\]

\[
Va = 3.50 \text{ kips} + 10.5 \text{ kips}
\]

\[
= 14.0 \text{ kips}
\]

Note: There are no tables for angles in shear, but the available shear strength can be calculated according to AISC Specification Section G4, as follows.

AISC Specification Section G4 stipulates \( k_v = 1.2 \).

Calculate \( A_w \).

\[
A_w = bt
\]

\[
= 5.00 \text{ in.}(\frac{1}{4} \text{ in.})
\]

\[
= 1.25 \text{ in.}^2
\]

Determine \( C_v \) from AISC Specification Section G2.1(b).

\[
h/t_w = b/t
\]

\[
= 5.0 \text{ in.}/\frac{1}{4} \text{ in.}
\]

\[
= 20
\]

\[
1.10\sqrt{k_vE/F_y} = 1.10\sqrt{1.2(29,000 \text{ ksi}/36 \text{ ksi})}
\]

\[
= 34.2
\]

\( 20 < 34.2; \) therefore, \( C_v = 1.0 \)

(Spec. Eq. G2-3)

Calculate \( V_n \).

\[
V_n = 0.6F_yA_wC_v
\]

\[
= 0.6(36 \text{ ksi})(1.25 \text{ in.}^2)(1.0)
\]

\[
= 27.0 \text{ kips}
\]

From AISC Specification Section G1, the available shear strength is:
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$\phi_v \cdot V'_n = 0.90(27.0 \text{ kips})$</td>
<td>$V'_n = \frac{27.0 \text{ kips}}{1.67}$</td>
</tr>
<tr>
<td>= 24.3 kips</td>
<td>= 16.2 kips</td>
</tr>
<tr>
<td>24.3 kips &gt; 21.0 kips</td>
<td>16.2 kips &gt; 14.0 kips</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE G.4  RECTANGULAR HSS IN SHEAR

Given:

Determine the available shear strength and adequacy of an HSS6×4×3/4 in. ASTM A500 Grade B member with end shears of 11.0 kips from dead load and 33.0 kips from live load. The beam is oriented with the shear parallel to the 6 in. dimension.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A500 Grade B
  - $F_y = 46$ ksi
  - $F_u = 58$ ksi

From AISC Manual Table 1-11, the geometric properties are as follows:

- HSS6×4×3/4
  - $H = 6.00$ in.
  - $B = 4.00$ in.
  - $t = 0.349$ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

$$V_u = 1.2(11.0 \text{ kips}) + 1.6(33.0 \text{ kips}) = 66.0 \text{ kips}$$

$$V_a = 11.0 \text{ kips} + 33.0 \text{ kips} = 44.0 \text{ kips}$$

Note: There are no AISC Manual Tables for shear in HSS shapes, but the available shear strength can be determined from AISC Specification Section G5, as follows.

Nominal Shear Strength

For rectangular HSS in shear, use AISC Specification Section G2.1 with $A_w = 2ht$ (per AISC Specification Section G5) and $k_v = 5$.

From AISC Specification Section G5, if the exact radius is unknown, $h$ shall be taken as the corresponding outside dimension minus three times the design thickness.

$$h = H - 3t$$

$$= 6.00 \text{ in.} - 3(0.349 \text{ in.})$$

$$= 4.95 \text{ in.}$$

$$h = 4.95 \text{ in.}$$

$$t_w = 0.349 \text{ in.}$$

$$= 14.2$$

$$1.10[k_v E/F_y] = 1.10(5)(29,000 \text{ ksi}/46 \text{ ksi})$$

$$= 61.8$$

$$14.2 \leq 61.8, \text{ therefore, } C_v = 1.0$$

(Spec. Eq. G2-3)

Note: Most standard HSS sections listed in the AISC Manual have $C_v = 1.0$ at $F_y \leq 46$ ksi.
Calculate $A_w$.

$$A_w = 2ht$$

$$= 2(4.95 \text{ in.})(0.349 \text{ in.})$$

$$= 3.46 \text{ in.}^2$$

Calculate $V_n$.

$$V_n = 0.6F_y A_w C_v$$

$$(\text{Spec. Eq. G2-1})$$

$$= 0.6(46 \text{ ksi})(3.46 \text{ in.}^2)(1.0)$$

$$= 95.5 \text{ kips}$$

From AISC Specification Section G1, the available shear strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$V_n = 86.0 \text{ kips}$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$\phi V_n = 0.90(95.5 \text{ kips})$</td>
<td>$V_n = 95.5 \text{ kips}$</td>
<td>$V_n = 1.67$</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>86.0 kips &gt; 66.0 kips</td>
<td>o.k.</td>
<td>$57.2 \text{ kips &gt; 44.0 kips}$</td>
</tr>
</tbody>
</table>
EXAMPLE G.5  ROUND HSS IN SHEAR

Given:

Verify the available shear strength and adequacy of a round HSS16.000×0.375 ASTM A500 Grade B member spanning 32 ft with end shears of 30.0 kips from dead load and 90.0 kips from live load.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B

\[ F_y = 42 \text{ ksi} \]

\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-13, the geometric properties are as follows:

HSS16.000×0.375

\[ D = 16.0 \text{ in.} \]
\[ t = 0.349 \text{ in.} \]
\[ A_g = 17.2 \text{ in}^2 \]

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

\[ V_u = 1.2(30.0 \text{ kips}) + 1.6(90.0 \text{ kips}) = 180 \text{ kips} \]

\[ V_a = 30.0 \text{ kips} + 90.0 \text{ kips} = 120 \text{ kips} \]

There are no AISC Manual tables for round HSS in shear, but the available strength can be determined from AISC Specification Section G6, as follows:

Using AISC Specification Section G6, calculate \( F_{cr} \) as the larger of:

\[ F_{cr} = \frac{1.60E}{\sqrt{\frac{L_v}{D} \left( \frac{D}{t} \right)^2}} \]

where \( L_v = \) half the span = 192 in.

\[ = \frac{1.60(29,000 \text{ ksi})}{\sqrt{\frac{192 \text{ in.}}{16.0 \text{ in.}} \left( \frac{16.0 \text{ in.}}{0.349 \text{ in.}} \right)^2}} \]

\[ = 112 \text{ ksi} \]

or

\[ F_{cr} = \frac{0.78E}{(D/t)^{\frac{3}{2}}} \]

\[ = \frac{0.78(29,000 \text{ ksi})}{\left( \frac{16.0 \text{ in.}}{0.349 \text{ in.}} \right)^{\frac{3}{2}}} \]

\[ = 72.9 \text{ ksi} \]
The maximum value of $F_{cr}$ permitted is,

$$F_{cr} = 0.6F_y$$

$$= 0.6(42 \text{ ksi})$$

$$= 25.2 \text{ ksi} \quad \text{controls}$$

Note: AISC Specification Equations G6-2a and G6-2b will not normally control for the sections published in the AISC Manual except when high strength steel is used or the span is unusually long.

Calculate $V_n$ using AISC Specification Section G6.

$$V_n = \frac{F_{cr} A_g}{2} \quad (\text{Spec. Eq. G6-1})$$

$$= \frac{(25.2 \text{ ksi})(17.2 \text{ in.}^2)}{2}$$

$$= 217 \text{ kips}$$

From AISC Specification Section G1, the available shear strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$\phi_v V_n = 0.90(217 \text{ kips})$</td>
<td>$V_n = 217 \text{ kips}$</td>
</tr>
<tr>
<td>$195 \text{ kips} &gt; 180 \text{ kips}$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$195 \text{ kips} &gt; 180 \text{ kips}$</td>
<td>$130 \text{ kips} &gt; 120 \text{ kips}$</td>
</tr>
</tbody>
</table>

o.k.
EXAMPLE G.6  DOUBLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of a W21×48 ASTM A992 beam with end shears of 20.0 kips from dead load and 60.0 kips from live load in the weak direction.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

Fy = 50 ksi

Fu = 65 ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

W21×48

bf = 8.14 in.

tf = 0.430 in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vu = 1.2(20.0 kips) + 1.6(60.0 kips)</td>
<td>Va = 20.0 kips + 60.0 kips</td>
</tr>
<tr>
<td>= 120 kips</td>
<td>= 80.0 kips</td>
</tr>
</tbody>
</table>

From AISC Specification Section G7, for weak axis shear, use AISC Specification Equation G2-1 and AISC Specification Section G2.1(b) with Aw = bftf for each flange, h/tw = b/tf, b = b/2 and kv = 1.2.

Calculate Aw. (Multiply by 2 for both shear resisting elements.)

Aw = 2bf tf

= 2(8.14 in.)(0.430 in.)

= 7.00 in.²

Calculate Cv.

h/tw = b/tf

= (8.14 in.) / 2

= 0.430 in.

= 9.47

1.10kE/Fy = 1.10(6.2(29,000 ksi/50 ksi)

= 29.0 ≥ 9.47, therefore, Cv = 1.0

(Spec. Eq. G2-3)

Note: For all ASTM A6 W-, S-, M- and HP-shapes when Fy ≤ 50 ksi, Cv = 1.0, except some M-shapes noted in the User Note at the end of AISC Specification Section G2.1.

Calculate Vn.

Vn = 0.6FyAwCv

= 0.6(50 ksi)(7.00 in.²)(1.0)

= 210 kips

(Spec. Eq. G2-1)
From AISC *Specification* Section G1, the available shear strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_v = 0.90 )</td>
<td>( \phi_v V_n = 0.90(210 \text{ kips}) )</td>
<td>( \Omega_v = 1.67 )</td>
</tr>
<tr>
<td></td>
<td>( = 189 \text{ kips} )</td>
<td>( V_n = 210 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( = 189 \text{ kips} )</td>
<td>( \Omega_v = 1.67 )</td>
</tr>
<tr>
<td>189 kips &gt; 120 kips</td>
<td>o.k.</td>
<td>126 kips &gt; 80.0 kips</td>
</tr>
</tbody>
</table>
EXAMPLE G.7 SINGLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of a C9×20 ASTM A36 channel with end shears of 5.00 kips from dead load and 15.0 kips from live load in the weak direction.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-5, the geometric properties are as follows:

C9×20
\( b_f = 2.65 \text{ in.} \)
\( t_f = 0.413 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

\[
\begin{array}{c|c|c}
\text{LRFD} & \text{ASD} \\
\hline
V_u = 1.2(5.00 \text{ kips}) + 1.6(15.0 \text{ kips}) & V_u = 5.00 \text{ kips} + 15.0 \text{ kips} \\
= 30.0 \text{ kips} & = 20.0 \text{ kips} \\
\end{array}
\]

Note: There are no AISC Manual tables for weak-axis shear in channel sections, but the available strength can be determined from AISC Specification Section G7.

From AISC Specification Section G7, for weak axis shear, use AISC Specification Equation G2-1 and AISC Specification Section G2.1(b) with \( A_w = b_f t_f \) for each flange, \( h/t_w = b/t_f \), \( b = b_f \) and \( k_v = 1.2 \).

Calculate \( A_w \). (Multiply by 2 for both shear resisting elements.)

\[
A_w = 2b_f t_f \\
= 2(2.65 \text{ in.})(0.413 \text{ in.}) \\
= 2.19 \text{ in.}^2
\]

Calculate \( C_v \).

\[
\frac{b_f}{t_f} = \frac{2.65 \text{ in.}}{0.413 \text{ in.}} \\
= 6.42
\]

\[
1.10 \sqrt{\frac{E}{F_y}} = 1.10 \sqrt{2(29,000 \text{ ksi})(36 \text{ ksi})} \\
= 34.2 \geq 6.42, \text{ therefore, } C_v = 1.0 \quad (\text{Spec. Eq. G2-3})
\]

Calculate \( V_n \).

\[
V_n = 0.6F_y A_w C_v \\
= 0.6(36 \text{ ksi})(2.19 \text{ in.}^2)(1.0) \\
= 47.3 \text{ kips} \quad (\text{Spec. Eq. G2-1})
\]
From AISC Specification Section G1, the available shear strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$\phi_v V_n = 0.90(47.3$ kips)</td>
<td>$V_n = 47.3$ kips</td>
</tr>
<tr>
<td></td>
<td>$\frac{V_n}{\Omega_v} = 1.67$</td>
</tr>
<tr>
<td>42.6 kips &gt; 30.0 kips</td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE G.8A  BUILT-UP GIRDER WITH TRANSVERSE STIFFENERS

Given:

A built-up ASTM A36 I-shaped girder spanning 56 ft has a uniformly distributed dead load of 0.920 klf and a live load of 2.74 klf in the strong direction. The girder is 36 in. deep with 12-in. × 1½-in. flanges and a ½-in. web. Determine if the member has sufficient available shear strength to support the end shear, without and with tension field action. Use transverse stiffeners, as required.

Note: This built-up girder was purposely selected with a thin web in order to illustrate the design of transverse stiffeners. A more conventionally proportioned plate girder would have at least a ½-in. web and slightly smaller flanges.

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

\[
F_y = 36 \text{ ksi} \\
F_u = 58 \text{ ksi}
\]

The geometric properties are as follows:

Built-up girder

\[
t_w = ½ \text{ in.} \\
d = 36.0 \text{ in.} \\
b_f = b_c = 12.0 \text{ in.} \\
t_f = 1½ \text{ in.} \\
h = 33.0 \text{ in.}
\]

From Chapter 2 of ASCE/SEI 7, the required shear strength at the support is:

\[
R_u = \frac{wL}{2} = \frac{(0.920 \text{ klf} + 2.74 \text{ klf})(56.0 \text{ ft})}{2} = 154 \text{ kips}
\]

\[
R_a = \frac{wL}{2} = \frac{(0.920 \text{ klf} + 2.74 \text{ klf})(56.0 \text{ ft})}{2} = 102 \text{ kips}
\]

Stiffener Requirement Check

\[
A_w = dt_w \text{ from AISC Specification Section G2.1(b)} \\
= 36.0 \text{ in.}(½ \text{ in.}) \\
= 11.3 \text{ in.}^2
\]
\[
\frac{h}{t_w} = \frac{33.0 \text{ in.}}{3/8 \text{ in.}} = 106
\]

106 < 260; therefore \( k_v = 5 \) for webs without transverse stiffeners from AISC *Specification* Section G2.1(b)

\[
1.37 \sqrt{\frac{k_v E}{F_y}} = 1.37 \sqrt{5(29,000 \text{ ksi} / 36 \text{ ksi})} = 86.9
\]

106 > 86.9; therefore, use AISC *Specification* Equation G2-5 to calculate \( C_v \)

\[
C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y}
\]

\[
= \frac{1.51(5)(29,000 \text{ ksi})}{(106)^2(36 \text{ ksi})}
\]

\[= 0.541\]

Calculate \( V_n \).

\[
V_n = 0.6 F_y A_w C_v
\]

\[
= 0.6(36 \text{ ksi})(11.3 \text{ in.}^2)(0.541)
\]

\[= 132 \text{ kips}\]

From AISC *Specification* Section G1, the available shear strength without stiffeners is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_v = 0.90 )</td>
<td>( \Omega_v = 1.67 )</td>
</tr>
<tr>
<td>( \phi_v V_n = 0.90(132 \text{ kips}) )</td>
<td>( V_n = 132 \text{ kips} )</td>
</tr>
<tr>
<td>= 119 kips</td>
<td>( \Omega_v = 1.67 )</td>
</tr>
</tbody>
</table>

119 kips < 154 kips

**Therefore, stiffeners are required.**

**Limits on the Use of Tension Field**

AISC *Manual* Tables 3-16a and 3-16b can be used to select stiffener spacings needed to develop the required stress in the web.

From AISC *Specification* Section G3.1, consideration of tension field action is not permitted for any of the following conditions:

(a) end panels in all members with transverse stiffeners
(b) members when \( a/h > 3.0 \) or \( [260/(h/t_w)]^2 \)
(c) \( 2A_w/(A_{gc} + A_g) > 2.5; 2(11.3)/[2(12 \text{ in.})(1 1/2 \text{ in.})] = 0.628 < 2.5 \)
(d) \( h/b_{fc} \) or \( h/b_{ft} > 6.0; 33 \text{ in.}/12 \text{ in.} = 2.75 < 6.0 \)

Items (c) and (d) are satisfied by the configuration provided. Item (b) is accounted for in AISC *Manual* Tables 3-16a and 3-16b.

**Stiffener Spacing for End Panel**

Tension field action is not permitted for end panels, therefore use AISC *Manual* Table 3-16a.
Use $V_u = \phi V_n$ to determine the required stress in the web by dividing by the web area.

$$\phi \frac{V_n}{A_w} = \frac{V_u}{A_w} = \frac{154 \text{ kips}}{11.3 \text{ in}^2} = 13.6 \text{ ksi}$$

Use $V_u = V_n / \Omega_v$ to determine the required stress in the web by dividing by the web area.

$$\frac{V_n}{\Omega_v A_w} = \frac{V_u}{A_w} = \frac{102 \text{ kips}}{11.3 \text{ in}^2} = 9.03 \text{ ksi}$$

Use Table 3-16a from the AISC Manual to select the required stiffener ratio $a/h$ based on the $h/t_w$ ratio of the girder and the required stress. Interpolate and follow an available stress curve, $\phi \frac{V_n}{A_w} = 13.6 \text{ ksi}$ for LRFD, $V_n / \Omega_v A_w = 9.03 \text{ ksi}$ for ASD, until it intersects the horizontal line for a $h/t_w$ value of 106. Project down from this intersection and take the maximum $a/h$ value of 2.00 from the axis across the bottom. Because $h = 33.0$ in., stiffeners are required at $(2.00)(33.0 \text{ in.}) = 66.0$ in. maximum. Conservatively, use 60.0 in. spacing.

**Stiffener Spacing for the Second Panel**

From AISC Specification Section G3.1, tension field action is allowed because the second panel is not an end panel.

The required shear strength at the start of the second panel, 60 in. from the end is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_u = 154 \text{ kips} - [1.2(0.920 \text{ klf}) + 1.6(2.74 \text{ klf})] \left(\frac{60.0 \text{ in.}}{12 \text{ in./ft}}\right)$</td>
<td>$V_u = 102 \text{ kips} - (0.920 \text{ klf} + 2.74 \text{ klf}) \left(\frac{60.0 \text{ in.}}{12 \text{ in./ft}}\right)$</td>
</tr>
<tr>
<td>$= 127 \text{ kips}$</td>
<td>$= 83.7 \text{ kips}$</td>
</tr>
</tbody>
</table>

From AISC Specification Section G1, the available shear strength without stiffeners is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>From previous calculations, $\phi_v V_u = 119 \text{ kips}$</td>
<td>From previous calculations, $\frac{V_u}{\Omega_v} = 79.0 \text{ kips}$</td>
</tr>
<tr>
<td>119 kips $&lt; 127$ kips</td>
<td>79.0 kips $&lt; 83.7$ kips</td>
</tr>
<tr>
<td><strong>Therefore additional stiffeners are required.</strong></td>
<td><strong>Therefore additional stiffeners are required.</strong></td>
</tr>
</tbody>
</table>

Use $V_u = \phi V_n$ to determine the required stress in the web by dividing by the web area.

$$\phi \frac{V_n}{A_w} = \frac{V_u}{A_w} = \frac{127 \text{ kips}}{11.3 \text{ in}^2} = 11.2 \text{ ksi}$$

Use $V_u = V_n / \Omega_v$ to determine the required stress in the web by dividing by the web area.

$$\frac{V_n}{\Omega_v A_w} = \frac{V_u}{A_w} = \frac{83.7 \text{ kips}}{11.3 \text{ in}^2} = 7.41 \text{ ksi}$$
Use Table 3-16b from the AISC Manual, including tension field action, to select the required stiffener ratio $a/h$ based on the $h/t_w$ ratio of the girder and the required stress. Interpolate and follow an available stress curve, $\phi V_u/A_w = 11.2$ ksi for LRFD, $V_u/\Omega A_w = 7.41$ ksi for ASD, until it intersects the horizontal line for a $h/t_w$ value of 106. Because the available stress does not intersect the $h/t_w$ value of 106, the maximum value of 3.0 for $a/h$ may be used. Because $h = 33.0$ in., an additional stiffener is required at $(3.0)(33.0 \text{ in.}) = 99.0$ in. maximum from the previous one.

**Stiffener Spacing for the Third Panel**

From AISC Specification Section G3.1, tension field action is allowed because the next panel is not an end panel.

The required shear strength at the start of the third panel, 159 in. from the end is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_u$</td>
<td>$154 \text{ kips} - \left[1.2(0.920 \text{ klf}) + 1.6(2.74 \text{ klf}) \right] \times \left(\frac{159 \text{ in.}}{12 \text{ in./ft}}\right)$</td>
<td>$102 \text{ kips} - (0.920 \text{ klf} + 2.74 \text{ klf}) \left(\frac{159 \text{ in.}}{12 \text{ in./ft}}\right)$</td>
</tr>
<tr>
<td></td>
<td>= 81.3 kips</td>
<td>= 53.5 kips</td>
</tr>
</tbody>
</table>

From AISC Specification Section G1, the available shear strength without stiffeners is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi V_u$</td>
<td>0.90</td>
<td>1.67</td>
</tr>
<tr>
<td>From previous calculations, $\phi V_u = 119 \text{ kips}$</td>
<td>From previous calculations, $\frac{V_u}{\Omega_v} = 79.0 \text{ kips}$</td>
<td>$79.0 \text{ kips} &gt; 53.5 \text{ kips}$ o.k.</td>
</tr>
<tr>
<td>119 kips &gt; 81.3 kips o.k.</td>
<td>Therefore additional stiffeners are not required.</td>
<td>Therefore additional stiffeners are not required.</td>
</tr>
</tbody>
</table>

The four Available Shear Stress tables, AISC Manual Tables 3-16a, 3-16b, 3-17a and 3-17b, are useful because they permit a direct solution for the required stiffener spacing. Alternatively, you can select a stiffener spacing and check the resulting strength, although this process is likely to be iterative. In Example G.8B, the stiffener spacings used are taken from this example.
EXAMPLE G.8B  BUILT-UP GIRDER WITH TRANSVERSE STIFFENERS

Given:
Verify the stiffener spacings from Example G.8A, which were easily determined from the tabulated values of the AISC Manual, by directly applying the provisions of the AISC Specification.

Solution:

Shear Strength of End Panel

Determine $k_v$ based on AISC Specification Section G2.1(b) and check $a/h$ limits.

$$a/h = \frac{60.0 \text{ in.}}{33.0 \text{ in.}} = 1.82$$

$$k_v = 5 + \frac{5}{(a/h)^2}$$

$$= 5 + \frac{5}{(1.82)^2}$$

$$= 6.51$$

Based on AISC Specification Section G2.1, $k_v = 5$ when $a/h > 3.0$ or $a/h > \left[ \frac{260}{(h/t_w)} \right]^2$

$$h = 33.0 \text{ in.}$$

$$t_w = \frac{1}{76} \text{ in.}$$

$$= 106$$

$$a/h = 1.82 \leq 3.0$$

$$a/h = 1.82 \leq \left[ \frac{260}{(h/t_w)} \right]^2$$

$$\left[ \frac{260}{(h/t_w)} \right]^2 = \left[ \frac{260}{106} \right]^2$$

$$= 6.02$$

$$1.82 \leq 6.02$$

Therefore, use $k_v = 6.51$.

Tension field action is not allowed because the panel is an end panel.

Because $h/t_w > 1.37 \sqrt{k_vE/F_y}$

$$= 1.37 \sqrt{6.51(29,000 \text{ ksi} / 36 \text{ ksi})}$$

$$= 99.2$$

$$C_v = \frac{1.51k_vE}{(h/t_w)^2F_y}$$

$$= \frac{1.51(6.51)(29,000 \text{ ksi})}{(106)^2(36 \text{ ksi})}$$

$$= 0.705$$


\[ V_n = 0.6F_y A_w C_v \]  
\[ = 0.6(36 \text{ ksi})(11.3 \text{ in.}^2)(0.705) \]  
\[ = 172 \text{ kips} \]  

From AISC Specification Section G1, the available shear strength is:

*LRFD* 
- \( \phi v \) = 0.90
- \( \phi v V_n \) = 155 kips
- 155 kips > 154 kips, o.k.

*ASD* 
- \( \Omega v \) = 1.67
- \( V_n = 172 \text{ kips} \)
- \( \Omega v V_n \) = 155 kips
- 155 kips > 154 kips, o.k.

Shear Strength of the Second Panel (AISC Specification Section G2.1b)

Determine \( k_v \) and check \( a/h \) limits based on AISC Specification Section G2.1(b).

\( a/h \) for the second panel is 3.0

\[ k_v = 5 + \frac{5}{(a/h)^2} \]  
\[ = 5 + \frac{5}{(3.0)^2} \]  
\[ = 5.56 \]  

Check \( a/h \) limits.

\( a/h = 3.00 \leq 3.0 \)

\[ a/h = 3.00 \leq \left[ \frac{260}{(h/t_w)^2} \right] \]  
\[ \leq 6.02 \text{ as previously calculated} \]

Therefore, use \( k_v = 5.56 \).

Because \( h/t_w > 1.37 \sqrt{k_v E / F_y} \)

\[ = 1.37 \sqrt{5.56(29,000 \text{ ksi} / 36 \text{ ksi})} \]  
\[ = 91.7 \]

\[ C_v = \frac{1.51k_v E}{(h/t_w)^2 F_y} \]  
\[ = \frac{1.51(5.56)(29,000 \text{ ksi})}{(106)^2(36 \text{ ksi})} \]  
\[ = 0.602 \]
Check the additional limits from AISC Specification Section G3.1 for the use of tension field action:

Note the limits of $a/h \leq 3.0$ and $a/h \leq [260/(h/t_w)]^2$ have already been calculated.

$$\frac{2A_w}{(A_p + A_r)} = \frac{2(11.3 \text{ in.}^2)}{2(12.0 \text{ in.})(1\frac{1}{2} \text{ in.})} = 0.628 \leq 2.5$$

$$\frac{h}{b_e} = \frac{h}{b_p} = \frac{33.0 \text{ in.}}{12.0 \text{ in.}} = 2.75 \leq 6.0$$

Tension field action is permitted because the panel under consideration is not an end panel and the other limits indicated in AISC Specification Section G3.1 have been met.

From AISC Specification Section G3.2,

$$1.10 \sqrt{k, E/F_v} = 1.10 \sqrt{5.56(29,000 \text{ ksi} / 36 \text{ ksi})} = 73.6$$

because $h/t_w > 73.6$, use AISC Specification Equation G3-2

$$V_n = 0.6F_yA_w \left[ C_v + \frac{1-C_v}{1.15\sqrt{3} + (a/h)^2} \right] \quad \text{(Spec. Eq. G3-2)}$$

$$V_n = 0.6(36 \text{ ksi})(11.3 \text{ in.}^2) \left[ 0.602 + \frac{1-0.602}{1.15\sqrt{3} + (3.00)^2} \right] = 174 \text{ kips}$$

From AISC Specification Section G1, the available shear strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v$</td>
<td>0.90</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$\phi_v V_n$</td>
<td>0.90(174 kips)</td>
<td>$V_n = 174 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>157 kips</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
<td>104 kips</td>
</tr>
<tr>
<td>157 kips $&gt;$ 127 kips</td>
<td>o.k.</td>
<td>104 kips $&gt;$ 83.7 kips</td>
</tr>
</tbody>
</table>
CHAPTER G DESIGN EXAMPLE REFERENCES

Darwin, D. (1990), Steel and Composite Beams with Web Openings, Design Guide 2, AISC, Chicago, IL.
Chapter H
Design of Members for Combined Forces and Torsion

For all interaction equations in AISC Specification Chapter H, the required forces and moments must include second-order effects, as required by Chapter C of the AISC Specification. ASD users of the 1989 AISC Specification are accustomed to using an interaction equation that includes a partial second-order amplification. Second order effects are now calculated in the analysis and are not included in these interaction equations.
EXAMPLE H.1A  W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC Manual Table 6-1, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes $P$-$\delta$ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 400$ kips</td>
<td>$P_a = 267$ kips</td>
</tr>
<tr>
<td>$M_{ux} = 250$ kip-ft</td>
<td>$M_{ax} = 167$ kip-ft</td>
</tr>
<tr>
<td>$M_{uy} = 80.0$ kip-ft</td>
<td>$M_{ay} = 53.3$ kip-ft</td>
</tr>
</tbody>
</table>

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The combined strength parameters from AISC Manual Table 6-1 are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p = 0.887 \times 10^3$ kips at 14.0 ft</td>
<td>$p = 1.33 \times 10^3$ kips at 14.0 ft</td>
</tr>
<tr>
<td>$b_x = 1.38 \times 10^3$ kip-ft at 14.0 ft</td>
<td>$b_x = 2.08 \times 10^3$ kip-ft at 14.0 ft</td>
</tr>
<tr>
<td>$b_y = 2.85 \times 10^3$ kip-ft</td>
<td>$b_y = 4.29 \times 10^3$ kip-ft</td>
</tr>
</tbody>
</table>

Check limit for AISC Specification Equation H1-1a.

From AISC Manual Part 6,

$$ \frac{P_a}{\phi \cdot P_u} = \frac{pP_a}{P_u} = \frac{0.887}{10^3} (400 \text{ kips}) = 0.355 $$

Because $pP_u \geq 0.2$,

$$ pP_a + b_x M_{ux} + b_y M_{uy} \leq 1.0 \quad (\text{Manual Eq. 6-1}) $$

From AISC Manual Part 6,

$$ \frac{P_a}{\phi \cdot P_u} = \frac{pP_a}{P_u} = \frac{1.33}{10^3} (267 \text{ kips}) = 0.355 $$

Because $pP_u \geq 0.2$,

$$ pP_a + b_x M_{ux} + b_y M_{uy} \leq 1.0 \quad (\text{Manual Eq. 6-1}) $$
AISC Manual Table 6-1 simplifies the calculation of AISC Specification Equations H1-1a and H1-1b. A direct application of these equations is shown in Example H.1B.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.355 + \left( \frac{1.38}{10'} \right) (250 \text{ kip-ft}) + \left( \frac{2.85}{10'} \right) (80.0 \text{ kip-ft}) \leq 1.0</td>
<td>0.355 + \left( \frac{2.08}{10'} \right) (167 \text{ kip-ft}) + \left( \frac{4.29}{10'} \right) (53.3 \text{ kip-ft}) \leq 1.0</td>
</tr>
<tr>
<td>0.355 + 0.345 + 0.228</td>
<td>0.355 + 0.347 + 0.229</td>
</tr>
<tr>
<td>= 0.928 \leq 1.0 o.k.</td>
<td>= 0.931 \leq 1.0 o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE H.1B  W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC Manual tables to determine the available compressive and flexural strengths, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The available axial and flexural strengths from AISC Manual Tables 4-1, 3-10 and 3-4 are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 400$ kips</td>
<td>$P_a = 267$ kips</td>
</tr>
<tr>
<td>$M_{ax} = 250$ kip-ft</td>
<td>$M_{ax} = 167$ kip-ft</td>
</tr>
<tr>
<td>$M_{ay} = 80.0$ kip-ft</td>
<td>$M_{ay} = 53.3$ kip-ft</td>
</tr>
</tbody>
</table>

Because $\frac{P_u}{\phi_u P_s} \geq 0.2$, use AISC Specification Equation H1-1a.

$$\frac{P_u}{P_s} + \frac{8}{9}\left(\frac{M_{ax}}{M_{cx}} + \frac{M_{ay}}{M_{cy}}\right) \leq 1.0$$

400 kips + $\frac{8}{9}\left(\frac{250$ kip-ft }{642$ kip-ft } + \frac{80.0$ kip-ft }{311$ kip-ft }\right)$

= 0.354 + $\frac{8}{9}(0.389 + 0.257)$

= 0.928 < 1.0

o.k.
EXAMPLE H.2  W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BY AISC SPECIFICATION SECTION H2)

Given:

Using AISC Specification Section H2, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft. This example is included primarily to illustrate the use of AISC Specification Section H2.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 360$ kips</td>
<td>$P_a = 240$ kips</td>
</tr>
<tr>
<td>$M_{ax} = 250$ kip-ft</td>
<td>$M_{ax} = 167$ kip-ft</td>
</tr>
<tr>
<td>$M_{ay} = 80.0$ kip-ft</td>
<td>$M_{ay} = 53.3$ kip-ft</td>
</tr>
</tbody>
</table>

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

$F_y = 50$ ksi

$F_u = 65$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

W14×99

$A = 29.1$ in.$^2$

$S_x = 157$ in.$^3$

$S_y = 55.2$ in.$^3$

The required flexural and axial stresses are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ra} = \frac{P_u}{A}$</td>
<td>$f_{ra} = \frac{P_a}{A}$</td>
</tr>
<tr>
<td>= $\frac{360 \text{kips}}{29.1 \text{in.}^2}$</td>
<td>= $\frac{240 \text{kips}}{29.1 \text{in.}^2}$</td>
</tr>
<tr>
<td>= 12.4 ksi</td>
<td>= 8.25 ksi</td>
</tr>
<tr>
<td>$f_{she} = \frac{M_{ax}}{S_x}$</td>
<td>$f_{she} = \frac{M_{ax}}{S_x}$</td>
</tr>
<tr>
<td>= $\frac{250 \text{kip-ft} \left( \frac{12 \text{in.}}{\text{ft}} \right)}{157 \text{in.}^3 \left( \frac{12 \text{in.}}{\text{ft}} \right)}$</td>
<td>= $\frac{167 \text{kip-ft} \left( \frac{12 \text{in.}}{\text{ft}} \right)}{157 \text{in.}^3 \left( \frac{12 \text{in.}}{\text{ft}} \right)}$</td>
</tr>
<tr>
<td>= 19.1 ksi</td>
<td>= 12.8 ksi</td>
</tr>
</tbody>
</table>
Calculate the available flexural and axial stresses from the available strengths in Example H.1B.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{cy} = \frac{M_{cy}}{S_y} )</td>
<td>( f_{cy} = \frac{M_{cy}}{S_y} )</td>
</tr>
<tr>
<td>[ \begin{align*} &amp;= \frac{80.0 \text{ kip-ft}}{55.2 \text{ in.}^3} \left( \frac{12 \text{ in.}}{\text{ft}} \right) \ &amp;= 17.4 \text{ ksi} \end{align*} ]</td>
<td>[ \begin{align*} &amp;= \frac{53.3 \text{ kip-ft}}{55.2 \text{ in.}^3} \left( \frac{12 \text{ in.}}{\text{ft}} \right) \ &amp;= 11.6 \text{ ksi} \end{align*} ]</td>
</tr>
</tbody>
</table>

As shown in the LRFD calculation of \( F_{cby} \) in the preceding text, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.

**Combined Stress Ratio**

From AISC Specification Section H2, check the combined stress ratios as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{ca} = \frac{f_c P_n}{A} )</td>
<td>( F_{ca} = \frac{F_c}{\Omega_n} )</td>
</tr>
<tr>
<td>[ \begin{align*} &amp;= \frac{1,130 \text{ kips}}{29.1 \text{ in.}^2} \left( \frac{12 \text{ in.}}{\text{ft}} \right) \ &amp;= 38.8 \text{ ksi} \end{align*} ]</td>
<td>[ \begin{align*} &amp;= \frac{750 \text{ kips}}{29.1 \text{ in.}^2} \ &amp;= 25.8 \text{ ksi} \end{align*} ]</td>
</tr>
</tbody>
</table>

| \( F_{cby} = \frac{f_c M_{cy}}{S_y} \) | \( F_{cby} = \frac{M_{cy}}{\Omega_y S_y} \) |
| \[ \begin{align*} &= \frac{642 \text{ kip-ft}}{157 \text{ in.}^3} \left( \frac{12 \text{ in.}}{\text{ft}} \right) \\ &= 49.1 \text{ ksi} \end{align*} \] | \[ \begin{align*} &= \frac{428 \text{ kip-ft}}{157 \text{ in.}^3} \left( \frac{12 \text{ in.}}{\text{ft}} \right) \\ &= 32.7 \text{ ksi} \end{align*} \] |

| \( F_{cby} = \frac{f_c M_{cy}}{S_y} \) | \( F_{cby} = \frac{M_{cy}}{\Omega_y S_y} \) |
| \[ \begin{align*} &= \frac{311 \text{ kip-ft}}{55.2 \text{ in.}^3} \left( \frac{12 \text{ in.}}{\text{ft}} \right) \\ &= 67.6 \text{ ksi} \end{align*} \] | \[ \begin{align*} &= \frac{207 \text{ kip-ft}}{55.2 \text{ in.}^3} \left( \frac{12 \text{ in.}}{\text{ft}} \right) \\ &= 45.0 \text{ ksi} \end{align*} \] |

As shown in the LRFD calculation of \( F_{cby} \) in the preceding text, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.

\[ \frac{f_{ca}}{F_{ca}} + \frac{f_{cha}}{F_{cby}} + \frac{f_{cy}}{F_{cby}} \leq 1.0 \quad \text{(from Spec. Eq. H2-1)} \]

\[ \frac{f_{ca}}{F_{ca}} + \frac{f_{cha}}{F_{cby}} + \frac{f_{cy}}{F_{cby}} \leq 1.0 \quad \text{(from Spec. Eq. H2-1)} \]

\[ \begin{align*} 
12.4 \text{ ksi} + 19.1 \text{ ksi} + 17.4 \text{ ksi} &= 0.966 \leq 1.0 \quad \text{o.k.} \\
38.8 \text{ ksi} + 49.1 \text{ ksi} + 67.6 \text{ ksi} &= 0.969 \leq 1.0 \quad \text{o.k.}
\end{align*} \]
A comparison of these results with those from Example H.1B shows that AISC Specification Equation H1-1a will produce less conservative results than AISC Specification Equation H2-1 when its use is permitted.

Note: This check is made at a point on the cross-section (extreme fiber, in this example). The designer must therefore determine which point on the cross-section is critical, or check multiple points if the critical point cannot be readily determined.
EXAMPLE H.3 W-SHAPE SUBJECT TO COMBINED AXIAL TENSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 14-in. nominal depth to carry forces of 29.0 kips from dead load and 87.0 kips from live load in axial tension, as well as the following moments due to uniformly distributed loads:

- $M_{x_D} = 32.0$ kip-ft
- $M_{x_L} = 96.0$ kip-ft
- $M_{y_D} = 11.3$ kip-ft
- $M_{y_L} = 33.8$ kip-ft

The unbraced length is 30.0 ft and the ends are pinned. Assume the connections are made with no holes.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_a = 1.2(29.0 \text{ kips}) + 1.6(87.0 \text{ kips})$</td>
<td>$P_a = 29.0 \text{ kips} + 87.0 \text{ kips}$</td>
</tr>
<tr>
<td>= 174 kips</td>
<td>= 116 kips</td>
</tr>
<tr>
<td>$M_{ax} = 1.2(32.0 \text{ kip-ft}) + 1.6(96.0 \text{ kip-ft})$</td>
<td>$M_{ax} = 32.0 \text{ kip-ft} + 96 \text{ kip-ft}$</td>
</tr>
<tr>
<td>= 192 kip-ft</td>
<td>= 128 kip-ft</td>
</tr>
<tr>
<td>$M_{ay} = 1.2(11.3 \text{ kip-ft}) + 1.6(33.8 \text{ kip-ft})$</td>
<td>$M_{ay} = 11.3 \text{ kip-ft} + 33.8 \text{ kip-ft}$</td>
</tr>
<tr>
<td>= 67.6 kip-ft</td>
<td>= 45.1 kip-ft</td>
</tr>
</tbody>
</table>

Try a W14×82.

From AISC Manual Tables 1-1 and 3-2, the geometric properties are as follows:

<table>
<thead>
<tr>
<th>W14×82</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = 24.0 \text{ in.}^2$</td>
</tr>
<tr>
<td>$S_x = 123 \text{ in.}^3$</td>
</tr>
<tr>
<td>$Z_x = 139 \text{ in.}^3$</td>
</tr>
<tr>
<td>$S_y = 29.3 \text{ in.}^3$</td>
</tr>
<tr>
<td>$Z_y = 44.8 \text{ in.}^3$</td>
</tr>
<tr>
<td>$I_x = 148 \text{ in.}^4$</td>
</tr>
<tr>
<td>$L_p = 8.76 \text{ ft}$</td>
</tr>
<tr>
<td>$L_r = 33.2 \text{ ft}$</td>
</tr>
</tbody>
</table>

Nominal Tensile Strength

From AISC Specification Section D2(a), the nominal tensile strength due to tensile yielding on the gross section is:

$$ P_n = F_y A_g $$

(\text{Spec. Eq. D2-1})

$$ P_n = 50 \text{ ksi}(24.0 \text{ in.}^2) $$

$$ P_n = 1,200 \text{ kips} $$
Note that for a member with holes, the rupture strength of the member would also have to be computed using AISC Specification Equation D2-2.

**Nominal Flexural Strength for Bending About the X-X Axis**

**Yielding**

From AISC Specification Section F2.1, the nominal flexural strength due to yielding (plastic moment) is:

\[ M_{nx} = M_p \]
\[ = F_y Z_x \]
\[ = 50 \text{ ksi}(139 \text{ in.}^3) \]
\[ = 6,950 \text{ kip-in.} \]  

**Lateral-Torsional Buckling**

From AISC Specification Section F2.2, the nominal flexural strength due to lateral-torsional buckling is determined as follows:

Because \( L_p < L_{x} \leq L_{r} \), i.e., 8.76 ft < 30.0 ft < 33.2 ft, AISC Specification Equation F2-2 applies.

Lateral-Torsional Buckling Modification Factor, \( C_b \)

From AISC Manual Table 3-1, \( C_b = 1.14 \), without considering the beneficial effects of the tension force. However, per AISC Specification Section H1.2, \( C_b \) may be increased because the column is in axial tension.

\[ P_{pl} = \frac{\pi^2 E I_y}{L_b^2} = \frac{\pi^2 (29,000 \text{ ksi})(148 \text{ in.}^4)}{[30.0 \text{ ft} (12.0 \text{ in./ft})]^2} = 327 \text{ kips} \]

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td>( \frac{1 + \alpha P_{pl}}{P_{pl}} ) = \frac{1 + 1.0 (174 \text{ kips})}{327 \text{ kips}} = 1.24 )</td>
<td>( \frac{1 + \alpha P_{pl}}{P_{pl}} = \frac{1 + 1.6 (116 \text{ kips})}{327 \text{ kips}} = 1.25 )</td>
</tr>
</tbody>
</table>
| ASD      | \( \frac{1 + \alpha P_{pl}}{P_{pl}} \) = \frac{1 + 1.6 (116 \text{ kips})}{327 \text{ kips}} = 1.25 \) | \( \frac{1 + \alpha P_{pl}}{P_{pl}} \) = \frac{1 + 1.6 (116 \text{ kips})}{327 \text{ kips}} = 1.25 \)

\( C_b = 1.24(1.14) \)
\[ = 1.41 \]

\[ M_a = C_b \left[ M_p - (M_p - 0.7 F_y S_y) \left( \frac{L_b - L_p}{L_b - L_p} \right) \right] \leq M_p \]  

\( (\text{Spec. Eq. F2-2}) \)

\[ = 1.41 \left[ 6,950 \text{ kip-in.} - \left( 6,950 \text{ kip-in.} - 0.7(50 \text{ ksi})(123 \text{ in.}^3) \right) \left( \frac{30.0 \text{ ft}}{33.2 \text{ ft} - 8.76 \text{ ft}} \right) \right] \]

\[ = 6,560 \text{ kip-in.} \leq M_p \]

Therefore, use \( M_a = 6,560 \text{ kip-in.} \) or 547 kip-ft **controls**

**Local Buckling**
Per AISC Manual Table 1-1, the cross section is compact at $F_y = 50$ ksi; therefore, the local buckling limit state does not apply.

**Nominal Flexural Strength for Bending About the Y-Y Axis and the Interaction of Flexure and Tension**

Because a W14×82 has compact flanges, only the limit state of yielding applies for bending about the y-y axis.

\[
M_{ny} = M_p = F_y Z_y \leq 1.6 F_y S_y
\]

\[
= 50 \text{ksi}(44.8 \text{in.}^3) \leq 1.6(50 \text{ksi})(29.3 \text{in.}^3)
\]

\[
= 2,240 \text{kip-in.} \leq 2,340 \text{kip-in.}
\]

Therefore, use $M_{ny} = 2,240$ kip-in. or 187 kip-ft

**Available Strength**

From AISC Specification Sections D2 and F1, the available strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = \phi_t = 0.90$</td>
<td></td>
<td>$\Omega_b = \Omega_t = 1.67$</td>
</tr>
<tr>
<td>$P_c = \phi P_n$</td>
<td>$P_c = \frac{P_n}{\Omega_t}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.90(1,200 \text{ kips})$</td>
<td>$= 1,200 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$= 1,080 \text{ kips}$</td>
<td>$= 719 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$M_{cx} = \phi_b M_{nxy}$</td>
<td>$M_{cx} = \frac{M_{nxy}}{\Omega_t}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.90(547 \text{ kip-ft})$</td>
<td>$= 547 \text{ kip-ft}$</td>
<td></td>
</tr>
<tr>
<td>$= 492 \text{ kip-ft}$</td>
<td>$= 328 \text{ kip-ft}$</td>
<td></td>
</tr>
<tr>
<td>$M_{cy} = \phi_b M_{ny}$</td>
<td>$M_{cy} = \frac{M_{nxy}}{\Omega_t}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.90(187 \text{ kip-ft})$</td>
<td>$= 187 \text{ kip-ft}$</td>
<td></td>
</tr>
<tr>
<td>$= 168 \text{ kip-ft}$</td>
<td>$= 112 \text{ kip-ft}$</td>
<td></td>
</tr>
</tbody>
</table>

**Interaction of Tension and Flexure**

Check limit for AISC Specification Equation H1-1a.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{P}{P_n} = \frac{P}{\phi P_n}$</td>
<td>$\frac{P}{P_n} = \frac{P}{\phi P_n}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.174 \text{ kips}$</td>
<td>$= 0.161 &lt; 0.2$</td>
<td></td>
</tr>
<tr>
<td>$= 1,080 \text{ kips}$</td>
<td>$= 719 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.161 &lt; 0.2$</td>
<td>$= 0.161 &lt; 0.2$</td>
<td></td>
</tr>
</tbody>
</table>
Therefore, AISC Specification Equation H1-1b applies.

\[
\frac{P}{2P_c} + \left( \frac{M_{ca}}{M_{ca}} + \frac{M_{a}}{M_{cy}} \right) \leq 1.0
\]

(Spec. Eq. H1-1b)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>174 kips + 192 kip-ft + 67.6 kip-ft \leq 1.0</td>
<td>116 kips + 128 kip-ft + 45.1 kip-ft \leq 1.0</td>
</tr>
<tr>
<td>2(1,080 kips) + 492 kip-ft + 168 kip-ft \leq 1.0</td>
<td>2(719 kips) + 328 kip-ft + 112 kip-ft \leq 1.0</td>
</tr>
<tr>
<td>0.873 \leq 1.0</td>
<td>0.874 \leq 1.0</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
EXAMPLE H.4  W-SHAPE SUBJECT TO COMBINED AXIAL COMPRESSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 10-in. nominal depth to carry axial compression forces of 5.00 kips from dead load and 15.0 kips from live load. The unbraced length is 14.0 ft and the ends are pinned. The member also has the following required moment strengths due to uniformly distributed loads, not including second-order effects:

\[ M_{DL} = 15 \text{ kip-ft} \]
\[ M_{LH} = 45 \text{ kip-ft} \]
\[ M_{DR} = 2 \text{ kip-ft} \]
\[ M_{LR} = 6 \text{ kip-ft} \]

The member is not subject to sidesway (no lateral translation).

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required strength (not considering second-order effects) is:

\[ P_u = 1.2(5.00 \text{ kips}) + 1.6(15.0 \text{ kips}) = 30.0 \text{ kips} \]
\[ M_{ux} = 1.2(15.0 \text{ kip-ft}) + 1.6(45.0 \text{ kip-ft}) = 90.0 \text{ kip-ft} \]
\[ M_{uy} = 1.2(2.00 \text{ kip-ft}) + 1.6(6.00 \text{ kip-ft}) = 12.0 \text{ kip-ft} \]
\[ P_a = 5.00 \text{ kips} + 15.0 \text{ kips} = 20.0 \text{ kips} \]
\[ M_{ax} = 15.0 \text{ kip-ft} + 45.0 \text{ kip-ft} = 60.0 \text{ kip-ft} \]
\[ M_{ay} = 2.00 \text{ kip-ft} + 6.00 \text{ kip-ft} = 8.00 \text{ kip-ft} \]

Try a W10×33.

From AISC Manual Tables 1-1 and 3-2, the geometric properties are as follows:

- W10×33
  - \( A = 9.71 \text{ in.}^2 \)
  - \( S_x = 35.0 \text{ in.}^3 \)
  - \( Z_x = 38.8 \text{ in.}^3 \)
  - \( I_x = 171 \text{ in.}^4 \)
  - \( r_x = 4.19 \text{ in.} \)
  - \( S_y = 9.20 \text{ in.}^3 \)
  - \( Z_y = 14.0 \text{ in.}^3 \)
  - \( I_y = 36.6 \text{ in.}^4 \)
  - \( r_y = 1.94 \text{ in.} \)
  - \( L_p = 6.85 \text{ ft} \)
  - \( L_r = 21.8 \text{ ft} \)
Available Axial Strength

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \).

Because \( KL_x = KL_y = 14.0 \) ft and \( r_x > r_y \), the \( y-y \) axis will govern.

From AISC Manual Table 4-1, the available axial strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_c = \phi_c P_n ) = 253 kips</td>
<td>( P_c = \frac{P_n}{\Omega} = 168 ) kips</td>
</tr>
</tbody>
</table>

Required Flexural Strength (including second-order amplification)

Use the approximate method of second-order analysis procedure from AISC Specification Appendix 8. Because the member is not subject to sidesway, only \( P-\delta \) amplifiers need to be added.

\[
B_i = \frac{C_m}{1 - \alpha P_i / P_{ci}} \geq 1 \tag{Spec. Eq. A-8-3}
\]

\( C_m = 1.0 \)

The \( x-x \) axis flexural magnifier is,

\[
P_{ci} = \frac{\pi^2 EI_i}{(K_i L_i)^2} \tag{from Spec. Eq. A-8-5}
\]

\[
= \frac{\pi^2 (29,000 \text{ ksi})(171 \text{ in.}^4)}{[(1.0)(14.0 \text{ ft})(12 \text{ in./ft})]^2}
\]

\[= 1,730 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 1.0 )</td>
<td>( \alpha = 1.6 )</td>
</tr>
<tr>
<td>( B_i = \frac{1.0}{1 - 1.0(30.0 \text{ kips} / 1,730 \text{ kips})} = 1.02 )</td>
<td>( B_i = \frac{1.0}{1 - 1.6(20.0 \text{ kips} / 1,730 \text{ kips})} = 1.02 )</td>
</tr>
<tr>
<td>( M_{ci} = 1.02(90.0 \text{ kip-ft}) = 91.8 \text{ kip-ft} )</td>
<td>( M_{ci} = 1.02(60.0 \text{ kip-ft}) = 61.2 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

The \( y-y \) axis flexural magnifier is,

\[
P_{ci} = \frac{\pi^2 EI_i}{(K_i L_i)^2} \tag{from Spec. Eq. A-8-5}
\]

\[
= \frac{\pi^2 (29,000 \text{ ksi})(36.6 \text{ in.}^4)}{[(1.0)(14.0 \text{ ft})(12 \text{ in./ft})]^2}
\]

\[= 371 \text{ kips} \]
Nominal Flexural Strength about the X-X Axis

Yielding

\[ M_{xx} = M_p = F_y Z_x \]
\[ = 50 \text{ ksi}(38.8 \text{ in.}^3) \]
\[ = 1,940 \text{ kip-in} \]

Lateral-Torsional Buckling

Because \( L_p < L_b < L_r \), i.e., \( 6.85 \text{ ft} < 14.0 \text{ ft} < 21.8 \text{ ft} \), AISC *Specification* Equation F2-2 applies.

From AISC *Manual* Table 3-1, \( C_b = 1.14 \)

\[ M_{xx} = C_b \left[ M_p - \left( M_p - 0.7F_y S_y \right) \left( L_b - L_p \right) \right] \leq M_p \]  
\[ \text{Spec. Eq. F2-2} \]

\[ = 1.14 \left[ 1,940 \text{ kip-in.} - \left[ 1,940 \text{ kip-in.} - 0.7(50 \text{ ksi})(35.0 \text{ in.}^3) \right] \right] = 1,820 \text{ kip-in.} \leq 1,940 \text{ kip-in.} \]

Therefore, use \( M_{xx} = 1,820 \text{ kip-in.} \) or 152 kip-ft controls

Local Buckling

Per AISC *Manual* Table 1-1, the member is compact for \( F_y = 50 \text{ ksi} \), so the local buckling limit state does not apply.

Nominal Flexural Strength about the Y-Y Axis

Determine the nominal flexural strength for bending about the \( y-y \) axis from AISC *Specification* Section F6. Because a W10\( \times 33 \) has compact flanges, only the yielding limit state applies.

From AISC *Specification* Section F6.2,

\[ M_{yy} = M_p = F_y Z_y \leq 1.6F_y S_y \]  
\[ = 50 \text{ ksi}(14.0 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(9.20 \text{ in.}^3) \]
\[ = 700 \text{ kip-in.} \leq 736 \text{ kip-in.} \]

Therefore, use \( M_{yy} = 700 \text{ kip-in.} \) or 58.3 kip-ft

From AISC *Specification* Section F1, the available flexural strength is:
Check limit for AISC Specification Equations H1-1a and H1-1b.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( M_{cx} = \phi_b M_{ax} )</td>
<td>( M_{cx} = \frac{M_{ax}}{\Omega_b} )</td>
</tr>
<tr>
<td>( = 0.90(152 \text{ kip-ft}) )</td>
<td>( = \frac{152 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>( = 137 \text{ kip-ft} )</td>
<td>( = 91.0 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( M_{cy} = \phi_b M_{ay} )</td>
<td>( M_{cy} = \frac{M_{ay}}{\Omega_b} )</td>
</tr>
<tr>
<td>( = 0.90(58.3 \text{ kip-ft}) )</td>
<td>( = \frac{58.3 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>( = 52.5 \text{ kip-ft} )</td>
<td>( = 34.9 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

\[ \frac{P_r}{2P_c} + \left( \frac{M_{cx}}{M_{cy}} \right) \leq 1.0 \quad \text{(Spec. Eq. H1-1b)} \]

LRFD: \[ \frac{30.0 \text{ kips}}{2(253 \text{ kips})} + \left( \frac{91.8 \text{ kip-ft}}{137 \text{ kip-ft}} \right) = 0.0593 + 0.920 = 0.979 \leq 1.0 \quad \text{o.k.} \]

ASD: \[ \frac{20.0 \text{ kips}}{2(168 \text{ kips})} + \left( \frac{61.2 \text{ kip-ft}}{91.0 \text{ kip-ft}} \right) = 0.0595 + 0.922 = 0.982 \leq 1.0 \quad \text{o.k.} \]
EXAMPLE H.5A  RECTANGULAR HSS TORSIONAL STRENGTH

Given:

Determine the available torsional strength of an ASTM A500 Grade B HSS6×4×¾.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B

\( F_y = 46 \) ksi
\( F_u = 58 \) ksi

From AISC Manual Table 1-11, the geometric properties are as follows:

HSS6×4×¾

\( h/t = 22.8 \)
\( b/t = 14.2 \)
\( t = 0.233 \) in.
\( C = 10.1 \) in.\(^3\)

The available torsional strength for rectangular HSS is stipulated in AISC Specification Section H3.1(b).

\[ h/t > b/t, \text{ therefore, } h/t \text{ governs} \]

\[ h/t \leq 2.45 \sqrt[2]{\frac{E}{F_y}} \]

\[ 22.8 \leq 2.45 \sqrt[2]{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \]

\[ = 61.5, \text{ therefore, use AISC Specification Equation H3-3} \]

\[ F_{cr} = 0.6F_y \text{ (Spec. Eq. H3-3)} \]

\[ = 0.6(46 \text{ ksi}) \]
\[ = 27.6 \text{ ksi} \]

The nominal torsional strength is,

\[ T_n = F_{cr}C \text{ (Spec. Eq. H3-1)} \]

\[ = 27.6 \text{ ksi (10.1 in.}^3) \]
\[ = 279 \text{ kip-in.} \]

From AISC Specification Section H3.1, the available torsional strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( \phi_T = 0.90 \) | \( \phi_TT_n = 0.90(279 \text{ kip-in.}) \) | \( \Omega_T = 1.67 \)
|                | 251 kip-in.   | \( T_n = 279 \text{ kip-in.} \) |
|                |               | \( \Omega_T = 1.67 \)
|                |               | \( = 167 \text{ kip-in.} \)

Note: For more complete guidance on designing for torsion, see AISC Design Guide 9, Torsional Analysis of Structural Steel Members (Seaburg and Carter, 1997).
EXAMPLE H.5B  ROUND HSS TORSIONAL STRENGTH

Given:

Determine the available torsional strength of an ASTM A500 Grade B HSS 5.000×0.250 that is 14 ft long.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B

$F_y = 42$ ksi

$F_u = 58$ ksi

From AISC Manual Table 1-13, the geometric properties are as follows:

HSS 5.000×0.250

$D/t = 21.5$

$t = 0.233$ in.

$D = 5.00$ in.

$C = 7.95$ in.$^3$

The available torsional strength for round HSS is stipulated in AISC Specification Section H3.1(a).

Calculate the critical stress as the larger of:

$$F_{cr} = \frac{1.23E}{\sqrt{\frac{L}{D}}} \left(\frac{D}{t}\right)^{\frac{5}{2}}$$

$$= \frac{1.23(29,000 \text{ ksi})}{\sqrt{\frac{14.0 \text{ ft}(12 \text{ in./ft})}{5.00 \text{ in.}}} \left(\frac{21.5}{5.00}\right)^{\frac{5}{2}}}$$

$$= 133 \text{ ksi}$$

and

$$F_{cr} = \frac{0.60E}{\left(\frac{D}{t}\right)^{\frac{3}{2}}}$$

$$= \frac{0.60(29,000 \text{ ksi})}{(21.5)^{\frac{3}{2}}}$$

$$= 175 \text{ ksi}$$

However, $F_{cr}$ shall not exceed the following:

$$0.6F_y = 0.6(42 \text{ ksi})$$

$$= 25.2 \text{ ksi}$$

Therefore, $F_{cr} = 25.2$ ksi.
The nominal torsional strength is,

\[ T_n = F_{\alpha}C \]

\[ = 25.2 \text{ ksi (7.95 in.}^3) \]

\[ = 200 \text{ kip-in.} \]

From AISC Specification Section H3.1, the available torsional strength is:

\[ \phi_T T_n = 0.90(200 \text{ kip-in.}) \]

\[ = 180 \text{ kip-in.} \]

Note: For more complete guidance on designing for torsion, see AISC Design Guide 9, Torsional Analysis of Structural Steel Members (Seaburg and Carter, 1997).
EXAMPLE H.5C RECTANGULAR HSS COMBINED TORSIONAL AND FLEXURAL STRENGTH

Given:

Verify the strength of an ASTM A500 Grade B HSS6×4×¾ loaded as shown. The beam is simply supported and is torsionally fixed at the ends. Bending is about the strong axis.

\[ w_G = 0.460 \text{ kip/ft applied 6 in. off centerline} \]
\[ w_L = 1.38 \text{ kip/ft applied 6 in. off centerline} \]

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B
\[ F_y = 46 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-11, the geometric properties are as follows:

HSS6×4×¾
\[ \frac{h}{t} = 22.8 \]
\[ \frac{b}{t} = 14.2 \]
\[ t = 0.233 \text{ in.} \]
\[ Z_x = 8.53 \text{ in.}^3 \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ w_u = 1.2(0.460 \text{ kip/ft}) + 1.6(1.38 \text{ kip/ft}) \]  
\[ = 2.76 \text{ kip/ft} \]  
\[ w_a = 0.460 \text{ kip/ft} + 1.38 \text{ kip/ft} \]  
\[ = 1.84 \text{ kip/ft} \] |

Calculate the maximum shear (at the supports) using AISC Manual Table 3-23, Case 1.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ V_r = V_u \]  
\[ = \frac{w_u l}{2} \]  
\[ = \frac{2.76 \text{ kip/ft} (8.00 \text{ ft})}{2} \]  
\[ = 11.0 \text{ kips} \]  
\[ V_r = V_a \]  
\[ = \frac{w_u l}{2} \]  
\[ = \frac{1.84 \text{ kip/ft} (8.00 \text{ ft})}{2} \]  
\[ = 7.36 \text{ kips} \] |

Calculate the maximum torsion (at the supports).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ T_r = T_u \]  
\[ = \frac{w_u l e}{2} \]  
\[ = \frac{w_u l e}{2} \] |
Available Shear Strength

Determine the available shear strength from AISC Specification Section G5.

\[ h = 6.00 \text{ in.} - 3(0.233 \text{ in.}) \]
\[ = 5.30 \text{ in.} \]

\[ A_w = 2ht \text{ from AISC Specification Section G5} \]
\[ = 2(5.30 \text{ in.})(0.233 \text{ in.}) \]
\[ = 2.47 \text{ in.}^2 \]

\[ k_v = 5 \]

The web shear coefficient is determined from AISC Specification Section G2.1(b).

\[ \frac{h}{t_w} = 22.8 \leq 1.10 \sqrt{\frac{k_v E}{F_y}} \]
\[ = 1.10 \sqrt{\frac{5(29,000 \text{ksi})}{46 \text{ ksi}}} \]
\[ = 61.8, \text{ therefore, } C_v = 1.0 \]  
(Spec. Eq. G2-3)

The nominal shear strength from AISC Specification Section G2.1 is,

\[ V_n = 0.6F_y A_w C_v \]
\[ = 0.6(46 \text{ ksi})(2.47 \text{ in.}^2)(1.0) \]
\[ = 68.2 \text{ kips} \]

From AISC Specification Section G1, the available shear strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_v = 0.90 )</td>
<td>( \Omega_v = 1.67 )</td>
</tr>
<tr>
<td>( V_{c} = \phi_v V_n )</td>
<td>( V_c = \frac{V_n}{\Omega_v} )</td>
</tr>
</tbody>
</table>
| \( \frac{68.2 \text{ kips}}{} \) | \( \frac{68.2 \text{ kips}}{1.67} \)
| \( = 40.8 \text{ kips} \) | \( = 40.8 \text{ kips} \)

Available Flexural Strength

The available flexural strength is determined from AISC Specification Section F7 for rectangular HSS. For the limit state of flexural yielding, the nominal flexural strength is,

\[ M_n = M_{p} = F_y Z_a \]
\[ = 46 \text{ ksi}(8.53 \text{ in.}^3) \]
\[ = 392 \text{ kip-in.} \]

(Spec. Eq. F7-1)
Determine if the limit state of flange local buckling applies as follows:

\[
\lambda = \frac{b}{t} = 14.2
\]

Determine the flange compact slenderness limit from AISC Specification Table B4.1b Case 17.

\[
\lambda_p = 1.12 \sqrt{\frac{E}{F_y}} = 1.12 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 28.1
\]

\[\lambda < \lambda_p; \text{ therefore, the flange is compact and the flange local buckling limit state does not apply.}\]

Determine if the limit state of web local buckling applies as follows:

\[
\lambda = \frac{h}{t} = 22.8
\]

Determine the web compact slenderness limit from AISC Specification Table B4.1b Case 19.

\[
\lambda_p = 2.42 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 60.8
\]

\[\lambda < \lambda_p; \text{ therefore, the web is compact and the web local buckling limit state does not apply.}\]

Therefore, \(M_n = 392 \text{ kip-in.}\), controlled by the flexural yielding limit state.

From AISC Specification Section F1, the available flexural strength is:

\[
\begin{array}{c|c|c|c}
\hline
& \text{LRFD} & \text{ASD} \\
\hline
\phi_b & 0.90 & \Omega_b = 1.67 \\
\Omega_b M_n & = 0.90(392 \text{ kip-in.}) & M_c = \frac{M_n}{\Omega_b} = \frac{392 \text{ kip-in.}}{1.67} = 235 \text{ kip-in.} \\
\hline
\end{array}
\]

From Example H.5A, the available torsional strength is:

\[
\begin{array}{c|c|c}
\hline
& \text{LRFD} & \text{ASD} \\
\hline
T_c = \phi_T T_s & = 0.90(279 \text{ kip-in.}) & T_n = \frac{T_s}{\Omega_T} = \frac{279 \text{ kip-in.}}{1.67} \\
& = 251 \text{ kip-in.} & \\
\hline
\end{array}
\]
Using AISC Specification Section H3.2, check combined strength at several locations where $T_r > 0.2T_c$.

Check at the supports, the point of maximum shear and torsion.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_r = 66.2$ kip-in.</td>
<td></td>
</tr>
<tr>
<td>$T_r = 251$ kip-in.</td>
<td></td>
</tr>
<tr>
<td>$= 0.264 &gt; 0.20$</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, use AISC Specification Equation H3-6

\[
\left( \frac{P}{P_c} + \frac{M}{M_c} \right) + \left( \frac{V}{V_c} + \frac{T_r}{T_c} \right)^2 \leq 1.0
\]

\[
(0 + 0) + \left( \frac{11.0}{61.4} \right)^2 + \left( \frac{66.2}{251} \right)^2
\]

\[
= 0.196 \leq 1.0 \quad \text{o.k.}
\]

Therefore, use AISC Specification Equation H3-6

\[
\left( \frac{P}{P_c} + \frac{M}{M_c} \right) + \left( \frac{V}{V_c} + \frac{T_r}{T_c} \right)^2 \leq 1.0
\]

\[
(0 + 0) + \left( \frac{7.36}{40.8} \right)^2 + \left( \frac{44.2}{167} \right)^2
\]

\[
= 0.198 \leq 1.0 \quad \text{o.k.}
\]

Check near the location where $T_r = 0.2T_c$. This is the location with the largest bending moment required to be considered in the interaction.

Calculate the shear and moment at this location, $x$.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \frac{66.2 \text{ kip-in.} - (0.20)(251 \text{ kip-in.})}{2.76 \text{ kip/ft} (6.00 \text{ in.})}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.966 \text{ ft}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{T_r}{T_c} = 0.20$</td>
<td></td>
</tr>
<tr>
<td>$V_r = 11.0 \text{ kips} - 0.966 \text{ ft} (2.76 \text{ kips/ft})$</td>
<td></td>
</tr>
<tr>
<td>$= 8.33 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$M_r = \frac{2.76 \text{ kip/ft} (0.966 \text{ ft})^2}{2} + 8.33 \text{ kips} (0.966 \text{ ft})$</td>
<td></td>
</tr>
<tr>
<td>$= 9.33 \text{ kip-ft} = 112 \text{ kip-in.}$</td>
<td></td>
</tr>
</tbody>
</table>

\[
\left( 0 + \frac{112 \text{ kip-in.}}{353 \text{ kip-in.}} \right) + \left( 8.33 \text{ kips} + 0.20 \right)^2
\]

\[
= 0.430 \leq 1.0 \quad \text{o.k.}
\]

<table>
<thead>
<tr>
<th>ASD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \frac{44.2 \text{ kip-in.} - (0.20)(167 \text{ kip-in.})}{1.84 \text{ kip/ft} (6.00 \text{ in.})}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.978 \text{ ft}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{T_r}{T_c} = 0.20$</td>
<td></td>
</tr>
<tr>
<td>$V_r = 7.36 \text{ kips} - 0.978 \text{ ft} (1.84 \text{ kips/ft})$</td>
<td></td>
</tr>
<tr>
<td>$= 5.56 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$M_r = \frac{1.84 \text{ kip/ft} (0.978 \text{ ft})^2}{2} + 5.56 \text{ kips} (0.978 \text{ ft})$</td>
<td></td>
</tr>
<tr>
<td>$= 6.32 \text{ kip-ft} = 75.8 \text{ kip-in.}$</td>
<td></td>
</tr>
</tbody>
</table>

\[
\left( 0 + \frac{75.8 \text{ kip-in.}}{235 \text{ kip-in.}} \right) + \left( 5.56 \text{ kips} + 0.20 \right)^2
\]

\[
= 0.436 \leq 1.0 \quad \text{o.k.}
\]

Note: The remainder of the beam, where $T_r \leq T_c$, must also be checked to determine if the strength without torsion controls over the interaction with torsion.
EXAMPLE H.6  W-SHAPE TORSIONAL STRENGTH

Given:

This design example is taken from AISC Design Guide 9, *Torsional Analysis of Structural Steel Members*. As shown in the following diagram, an ASTM A992 W10×49 spans 15 ft and supports concentrated loads at midspan that act at a 6-in. eccentricity with respect to the shear center. Determine the stresses on the cross section, the adequacy of the section to support the loads, and the maximum rotation.

The end conditions are assumed to be flexurally pinned and unrestrained for warping torsion. The eccentric load can be resolved into a torsional moment and a load applied through the shear center.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

- W10×49
  - $I_x = 272$ in.$^4$
  - $S_x = 54.6$ in.$^3$
  - $t_f = 0.560$ in.
  - $t_w = 0.340$ in.
  - $J = 1.39$ in.$^4$
  - $C_w = 2.070$ in.$^6$
  - $Z_x = 60.4$ in.$^3$

From the AISC Shapes Database, the additional torsional properties are as follows:

- W10×49
  - $S_{w1} = 33.0$ in.$^4$
  - $W_{wo} = 23.6$ in.$^2$
  - $Q_{f} = 12.8$ in.$^3$
  - $Q_{w} = 29.8$ in.$^3$

From AISC Design Guide 9 (Seaburg and Carter, 1997), the torsional property, $a$, is calculated as follows:
From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(2.50 \text{ kips}) + 1.6(7.50 \text{ kips}) )</td>
<td>( P_u = 2.50 \text{ kips} + 7.50 \text{ kips} )</td>
</tr>
<tr>
<td>= 15.0 kips</td>
<td>= 10.0 kips</td>
</tr>
<tr>
<td>( V_u = \frac{P_u}{2} )</td>
<td>( V_u = \frac{P_u}{2} )</td>
</tr>
<tr>
<td>= ( \frac{15.0 \text{ kips}}{2} )</td>
<td>= ( \frac{10.0 \text{ kips}}{2} )</td>
</tr>
<tr>
<td>= 7.50 kips</td>
<td>= 5.00 kips</td>
</tr>
<tr>
<td>( M_u = \frac{P_u l}{4} )</td>
<td>( M_u = \frac{P_u l}{4} )</td>
</tr>
<tr>
<td>= ( \frac{15.0 \text{ kips}(15.0 \text{ ft})(12 \text{ in./ft})}{4} )</td>
<td>= ( \frac{10.0 \text{ kips}(15.0 \text{ ft})(12 \text{ in./ft})}{4} )</td>
</tr>
<tr>
<td>= 675 kip-in.</td>
<td>= 450 kip-in.</td>
</tr>
<tr>
<td>( T_u = P_u e )</td>
<td>( T_u = P_u e )</td>
</tr>
<tr>
<td>= 15.0 kips(6.00 in.)</td>
<td>= 10.0 kips(6.00 in.)</td>
</tr>
<tr>
<td>= 90.0 kip-in.</td>
<td>= 60.0 kip-in.</td>
</tr>
</tbody>
</table>

**Normal and Shear Stresses from Flexure**

The normal and shear stresses from flexure are determined from AISC Design Guide 9, as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{ab} = \frac{M_u}{S_x} ) (from Design Guide 9 Eq. 4.5)</td>
<td>( \sigma_{ab} = \frac{M_u}{S_x} ) (from Design Guide 9 Eq. 4.5)</td>
</tr>
<tr>
<td>= ( \frac{675 \text{ kip-in.}}{54.6 \text{ in.}^3} )</td>
<td>= ( \frac{450 \text{ kip-in.}}{54.6 \text{ in.}^3} )</td>
</tr>
<tr>
<td>= 12.4 ksi (compression at top, tension at bottom)</td>
<td>= 8.24 ksi (compression at top, tension at bottom)</td>
</tr>
<tr>
<td>( \tau_{ab, web} = \frac{V_u Q_w}{I_{t_w}} ) (from Design Guide 9 Eq. 4.6)</td>
<td>( \tau_{ab, web} = \frac{V_u Q_w}{I_{t_w}} ) (from Design Guide 9 Eq. 4.6)</td>
</tr>
<tr>
<td>= ( \frac{7.50 \text{ kips}(29.8 \text{ in.}^3)}{272 \text{ in.}^4(0.340 \text{ in.})} )</td>
<td>= ( \frac{5.00 \text{ kips}(29.8 \text{ in.}^3)}{272 \text{ in.}^4(0.340 \text{ in.})} )</td>
</tr>
<tr>
<td>= 2.42 ksi</td>
<td>= 1.61 ksi</td>
</tr>
<tr>
<td>( \tau_{ab, flange} = \frac{V_u Q_f}{I_{t_f}} ) (from Design Guide 9 Eq. 4.6)</td>
<td>( \tau_{ab, flange} = \frac{V_u Q_f}{I_{t_f}} ) (from Design Guide 9 Eq. 4.6)</td>
</tr>
</tbody>
</table>


### Shear Stresses Due to Pure Torsion

The shear stresses due to pure torsion are determined from AISC Design Guide 9 as follows:

\[
\frac{T_s}{GJ} = \frac{-90.0 \text{ kip-in.}}{11,200 \text{ ksi} \left(1.39 \text{ in.}^4\right)} = -5.78 \times 10^{-3} \text{ rad/in.}
\]

\[
\frac{T_s}{GJ} = \frac{-60.0 \text{ kip-in.}}{11,200 \text{ ksi} \left(1.39 \text{ in.}^4\right)} = -3.85 \times 10^{-3} \text{ rad/in.}
\]
\[ \tau_r = G\theta' \]  
(Design Guide 9 Eq. 4.1)

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At midspan:</strong></td>
<td>( \theta' = 0; \ \tau_{aw} = 0 )</td>
<td>( \theta' = 0; \ \tau_{aw} = 0 )</td>
</tr>
<tr>
<td></td>
<td>[ \tau_{aw} = 11,200 \text{ ksi}(0.340 \text{ in.})(0.28) \left( \frac{-5.78 \text{ rad}}{10^3 \text{ in.}} \right) ]</td>
<td>[ \tau_{aw} = 11,200 \text{ ksi}(0.340 \text{ in.})(0.28) \left( \frac{-3.85 \text{ rad}}{10^3 \text{ in.}} \right) ]</td>
</tr>
<tr>
<td></td>
<td>= (-6.16 \text{ ksi})</td>
<td>= (-4.11 \text{ ksi})</td>
</tr>
<tr>
<td><strong>At the support:</strong></td>
<td>( \theta' = 0; \ \tau_{aw} = 0 )</td>
<td>( \theta' = 0; \ \tau_{aw} = 0 )</td>
</tr>
<tr>
<td></td>
<td>[ \tau_{aw} = 11,200 \text{ ksi}(0.560 \text{ in.})(0.28) \left( \frac{-5.78 \text{ rad}}{10^3 \text{ in.}} \right) ]</td>
<td>[ \tau_{aw} = 11,200 \text{ ksi}(0.560 \text{ in.})(0.28) \left( \frac{-3.85 \text{ rad}}{10^3 \text{ in.}} \right) ]</td>
</tr>
<tr>
<td></td>
<td>= (-10.2 \text{ ksi})</td>
<td>= (-6.76 \text{ ksi})</td>
</tr>
</tbody>
</table>

**Shear Stresses Due to Warping**

The shear stresses due to warping are determined from AISC Design Guide 9 as follows:

\[ \tau_{aw} = \frac{-ES_n \theta''}{t_f} \]  
(from Design Guide 9 Eq. 4.2a)

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At midspan:</strong></td>
<td>[ \tau_{aw} = -29,000 \text{ ksi}(33.0 \text{ in.}^3) \left[ \frac{-0.50(-5.78 \text{ rad})}{62.1 \text{ in.}^2(10^3 \text{ in.})} \right] ]</td>
<td>[ \tau_{aw} = -29,000 \text{ ksi}(33.0 \text{ in.}^3) \left[ \frac{-0.50(-3.85 \text{ rad})}{62.1 \text{ in.}^2(10^3 \text{ in.})} \right] ]</td>
</tr>
<tr>
<td></td>
<td>= (-1.28 \text{ ksi})</td>
<td>= (-0.853 \text{ ksi})</td>
</tr>
<tr>
<td><strong>At the support:</strong></td>
<td>[ \tau_{aw} = -29,000 \text{ ksi}(33.0 \text{ in.}^3) \left[ \frac{-0.22(-5.78 \text{ rad})}{62.1 \text{ in.}^2(10^3 \text{ in.})} \right] ]</td>
<td>[ \tau_{aw} = -29,000 \text{ ksi}(33.0 \text{ in.}^3) \left[ \frac{-0.22(-3.85 \text{ rad})}{62.1 \text{ in.}^2(10^3 \text{ in.})} \right] ]</td>
</tr>
<tr>
<td></td>
<td>= (-0.563 \text{ ksi})</td>
<td>= (-0.375 \text{ ksi})</td>
</tr>
</tbody>
</table>

**Normal Stresses Due to Warping**

The normal stresses due to warping are determined from AISC Design Guide 9 as follows:

\[ \sigma_n = EW_n \theta'' \]  
(from Design Guide 9 Eq. 4.3a)

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At midspan:</strong></td>
<td>[ \sigma_{nw} = 29,000 \text{ ksi}(23.6 \text{ in.}^2) \left[ \frac{-0.44(-5.78 \text{ rad})}{62.1 \text{ in.}(10^3 \text{ in.})} \right] ]</td>
<td>[ \sigma_{nw} = 29,000 \text{ ksi}(23.6 \text{ in.}^2) \left[ \frac{-0.44(-3.85 \text{ rad})}{62.1 \text{ in.}(10^3 \text{ in.})} \right] ]</td>
</tr>
<tr>
<td></td>
<td>= 28.0 ksi</td>
<td>= 18.7 ksi</td>
</tr>
<tr>
<td><strong>At the support:</strong></td>
<td>Because ( \theta'' = 0 ), ( \sigma_{nw} = 0 )</td>
<td>Because ( \theta'' = 0 ), ( \sigma_{nw} = 0 )</td>
</tr>
</tbody>
</table>
Combined Stresses

The stresses are summarized in the following table and shown in Figure H.6-1.

<table>
<thead>
<tr>
<th>Location</th>
<th>Normal Stresses</th>
<th>Shear Stresses</th>
<th>Normal Stresses</th>
<th>Shear Stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{uw}$</td>
<td>$\tau_{uw}$</td>
<td>$\sigma_{uv}$</td>
<td>$\tau_{uv}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{ub}$</td>
<td>$\tau_{ub}$</td>
<td>$\sigma_{ab}$</td>
<td>$\tau_{ab}$</td>
</tr>
<tr>
<td></td>
<td>$f_{uw}$</td>
<td>$f_{uv}$</td>
<td>$f_{ab}$</td>
<td>$f_{av}$</td>
</tr>
</tbody>
</table>

**LRFD**

- $-28.0$
- $-12.4$
- $-40.4$ (max comp.)
- $+28.0$
- $+12.4$
- $+40.4$ (max ten.)
- $+15.6$ (ten.)

**ASD**

- $-18.7$
- $-8.24$
- $-26.9$ (max comp.)
- $+18.7$
- $+8.24$
- $+26.9$ (max ten.)
- $+10.5$ (ten.)

**LRFD**

- $T_o = 90$ kip-in.

**ASD**

- $T_o = 60$ kip-in.

**Fig. H.6-1. Stresses due to flexure and torsion.**
The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 40.4 ksi.

The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 11.4 ksi.

The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 26.9 ksi.

The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 7.56 ksi.

### Available Torsional Strength

The available torsional strength is the lowest value determined for the limit states of yielding under normal stress, shear yielding under shear stress, or buckling in accordance with AISC Specification Section H3.3. The nominal torsional strength due to the limit states of yielding under normal stress and shear yielding under shear stress are compared to the applicable buckling limit states.

### Buckling

For the buckling limit state, lateral-torsional buckling and local buckling must be evaluated. The nominal torsional strength due to the limit state of lateral-torsional buckling is determined as follows:

\[
C_b = 1.32 \text{ from AISC Manual Table 3-1.}
\]

Compute \( F_n \) for a \( W10 \times 49 \) using values from AISC Manual Table 3-10 with \( L_b = 15.0 \) ft and \( C_b = 1.0 \).

\[
\phi_b M_n = 204 \text{ kip-ft}
\]

\[
F_n = F_{cr} \quad (\text{Spec. Eq. H3-9})
\]

\[
= C_b \frac{\phi_b M_n}{\phi_b S_i}
\]

\[
= 1.32 \frac{204 \text{ kip-ft}}{0.90 \left( 54.6 \text{ in.}^2 \right)} \left( \frac{12 \text{ in.}}{\text{ft}} \right)
\]

\[
= 65.8 \text{ ksi}
\]

The limit state of local buckling does not apply because a \( W10 \times 49 \) is compact in flexure per the user note in AISC Specification Section F2.

### Yielding Under Normal Stress

The nominal torsional strength due to the limit state of yielding under normal stress is determined as follows:

\[
F_n = F_y
\]

\[
= 50 \text{ ksi} \quad (\text{Spec. Eq. H3-7})
\]

Therefore, the limit state of yielding under normal stress controls over buckling. The available torsional strength for yielding under normal stress is determined as follows, from AISC Specification Section H3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 40.4 ksi.</td>
<td>The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 26.9 ksi.</td>
</tr>
<tr>
<td>The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 11.4 ksi.</td>
<td>The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 7.56 ksi.</td>
</tr>
</tbody>
</table>

Design Examples V14.1
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Shear Yielding Under Shear Stress

The nominal torsional strength due to the limit state of shear yielding under shear stress is:

$$ F_n = 0.6F_y $$

$$ = 0.6(50 \text{ ksi}) $$

$$ = 30 \text{ ksi} $$

$$(Spec. \text{ Eq. H3-8)}$$

The limit state of shear yielding under shear stress controls over buckling. The available torsional strength for shear yielding under shear stress determined as follows, from AISC Specification Section H3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_T = 0.90$</td>
<td>$\Omega_T = 1.67$</td>
</tr>
<tr>
<td>$\phi_T F_n = 0.90(0.6)(50 \text{ ksi})$</td>
<td>$F_a = 0.6(50 \text{ ksi})$</td>
</tr>
<tr>
<td>$= 27.0 \text{ ksi} &gt; 11.4 \text{ ksi}$</td>
<td>$\Omega_T = 1.67$</td>
</tr>
<tr>
<td>$\text{ o.k.}$</td>
<td>$= 18.0 \text{ ksi} &gt; 7.56 \text{ ksi}$</td>
</tr>
<tr>
<td>$\text{ o.k.}$</td>
<td>$\text{ o.k.}$</td>
</tr>
</tbody>
</table>

Maximum Rotation at Service Load

The maximum rotation occurs at midspan. The service load torque is:

$$ T = Pe $$

$$ = -(2.50 \text{ kips} + 7.50 \text{ kips})(6.00 \text{ in.}) $$

$$ = -60.0 \text{ kip-in.} $$

From AISC Design Guide 9, Appendix B, Case 3 with $\alpha = 0.5$, the maximum rotation is:

$$ \theta = +0.09 \frac{Tl}{GJ} $$

$$ = 0.09\left(-60.0 \text{ kip-in.}\right)(180 \text{ in.}) $$

$$ = \frac{11,200 \text{ ksi}(1.39 \text{ in.}^4)}{11,200 \text{ ksi}(1.39 \text{ in.}^4)} $$

$$ = -0.0624 \text{ rads or } -3.58^\circ $$

See AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* for additional guidance.
CHAPTER H DESIGN EXAMPLE REFERENCES

Chapter I
Design of Composite Members

I1. GENERAL PROVISIONS

Design, detailing, and material properties related to the concrete and steel reinforcing portions of composite members are governed by ACI 318 as modified with composite-specific provisions by the AISC Specification.

The available strength of composite sections may be calculated by one of two methods; the plastic stress distribution method, or the strain-compatibility method. The composite design tables in the Steel Construction Manual and the Examples are based on the plastic stress distribution method.

Filled composite sections are classified for local buckling according to the slenderness of the compression steel elements as illustrated in AISC Specification Table I1.1 and Examples I.4, I.6 and I.7. Local buckling effects do not need to be considered for encased composite members.

Terminology used within the Examples for filled composite section geometry is illustrated in Figure I-2.

I2. AXIAL FORCE

The available compressive strength of a composite member is based on a summation of the strengths of all of the components of the column with reductions applied for member slenderness and local buckling effects where applicable.

For tension members, the concrete tensile strength is ignored and only the strength of the steel member and properly connected reinforcing is permitted to be used in the calculation of available tensile strength.

The available compressive strengths given in AISC Manual Tables 4-13 through 4-20 reflect the requirements given in AISC Specification Sections I1.4 and I2.2. The design of filled composite compression and tension members is presented in Examples I.4 and I.5.

The design of encased composite compression and tension members is presented in Examples I.9 and I.10. There are no tables in the Manual for the design of these members.

Note that the AISC Specification stipulates that the available compressive strength need not be less than that specified for the bare steel member.

I3. FLEXURE

The design of typical composite beams with steel anchors is illustrated in Examples I.1 and I.2. AISC Manual Table 3-19 provides available flexural strengths for composite beams, Table 3-20 provides lower-bound moments of inertia for plastic composite sections, and Table 3-21 provides shear strengths of steel stud anchors utilized for composite action in composite beams.

The design of filled composite members for flexure is illustrated within Examples I.6 and I.7, and the design of encased composite members for flexure is illustrated within Example I.11.

I4. SHEAR

For composite beams with formed steel deck, the available shear strength is based upon the properties of the steel section alone in accordance with AISC Specification Chapter G as illustrated in Examples I.1 and I.2.

For filled and encased composite members, either the shear strength of the steel section alone, the steel section plus the reinforcing steel, or the reinforced concrete alone are permitted to be used in the calculation of
available shear strength. The calculation of shear strength for filled composite members is illustrated within Examples I.6 and I.7 and for encased composite members within Example I.11.

I5. COMBINED FLEXURE AND AXIAL FORCE

Design for combined axial force and flexure may be accomplished using either the strain compatibility method or the plastic-distribution method. Several different procedures for employing the plastic-distribution method are outlined in the Commentary, and each of these procedures is demonstrated for concrete filled members in Example I.6 and for concrete encased members in Example I.11. Interaction calculations for non-compact and slender concrete filled members are illustrated in Example I.7.

To assist in developing the interaction curves illustrated within the design examples, a series of equations is provided in Figure I-1 (Geschwindner, 2010). These equations define selected points on the interaction curve, without consideration of slenderness effects. Figures I-1a through I-1d outline specific cases, and the applicability of the equations to a cross-section that differs should be carefully considered. As an example, the equations in Figure I-1a are appropriate for the case of side bars located at the centerline, but not for other side bar locations. In contrast, these equations are appropriate for any amount of reinforcing at the extreme reinforcing bar location. In Figure I-1b, the equations are appropriate only for the case of 4 reinforcing bars at the corners of the encased section. When design cases deviate from those presented the appropriate interaction equations can be derived from first principles.

I6. LOAD TRANSFER

The AISC Specification provides several requirements to ensure that the concrete and steel portions of the section act together. These requirements address both force allocation - how much of the applied loads are resisted by the steel versus the reinforced concrete, and force transfer mechanisms - how the force is transferred between the two materials. These requirements are illustrated in Example I.3 for concrete filled members and Example I.8 for encased composite members.

I7. COMPOSITE DIAPHRAGMS AND COLLECTOR BEAMS

The Commentary provides guidance on design methodologies for both composite diaphragms and composite collector beams.

I8. STEEL ANCHORS

AISC Specification Section I8 addresses the strength of steel anchors in composite beams and in composite components. Examples I.1 and I.2 illustrates the design of composite beams with steel headed stud anchors.

The application of steel anchors in composite component provisions have strict limitations as summarized in the User Note provided at the beginning of AISC Specification Section I8.3. These provisions do not apply to typical composite beam designs nor do they apply to hybrid construction where the steel and concrete do not resist loads together via composite action such as in embed plates. The most common application for these provisions is for the transfer of longitudinal shear within the load introduction length of composite columns as demonstrated in Example I.8. The application of these provisions to an isolated anchor within an applicable composite system is illustrated in Example I.12.
Plastic Capacities for Rectangular, Encased W-Shapes Bent About the X-X Axis

<table>
<thead>
<tr>
<th>Section</th>
<th>Stress Distribution</th>
<th>Pt.</th>
<th>Defining Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td>$P_n = A_y F_y + A_{fy} F_{fy} + 0.85f_y A_{ci}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_{nA} = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A_s = $ area of steel shape</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A_{cf} = $ area of all continuous reinforcing bars</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A_{cr} = h_t h_2 - A_s - A_{cf}$</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>$P_D = 0.85f_c A_{ci}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_{cd} = M_{cd}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{ci} = (A_s - A_{cf}) \left( \frac{b_2 - c}{2} \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{ci} = \frac{h_t h_2}{4} - Z_s - Z_{ef}$</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td>$P_D = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_{cd} = M_{cd} - Z_{ef} F_y - \frac{Z_{ef}}{2} (0.85f_c)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{ef} = h_t h_2 - Z_{ef}$</td>
</tr>
</tbody>
</table>

For $h_{cy}$ below the flange ($h_{cy} < \frac{d}{2} - t_i$)

$Z_{ef} = \frac{0.85f_c (A_s + A_{cf}) - 2F_y A_{cr}}{2 \left[ 0.85f_c (h_t - t_i) + 2F_y t_i \right]}$

For $h_{cy}$ within the flange ($\frac{d}{2} - t_i < h_{cy} < \frac{d}{2}$)

$Z_{ef} = Z_s - t_i \left( \frac{d}{2} - h_{cy} \right) \left( \frac{d}{2} + h_{cy} \right)$

For $h_{cy}$ above the flange ($h_{cy} > \frac{d}{2}$)

$Z_{ef} = \frac{0.85f_c (A_s + A_{cf}) - 2F_y A_s - 2F_y A_{cr}}{2 \left[ 0.85f_c h_t \right]}$

Fig. I-1a. W-shapes, strong-axis anchor points.
Plastic Capacities for Rectangular, Encased W-Shapes Bent About the Y-Y Axis

<table>
<thead>
<tr>
<th>Section</th>
<th>Stress Distribution</th>
<th>Pt.</th>
<th>Defining Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$0.85f_y F_y F_{yY}$</td>
<td>$P_A = A_t F_y + A_{tr} F_{yY} + 0.85 f_y A_t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_J = 0$</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>$A_t$ = area of steel shape</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>$A_{tr}$ = area of continuous reinforcing bars</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>$A_t = h_2 b_2 - A_t - A_{tr}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>$P_E = A_t F_y + (0.85 f_y \left( A_t - \frac{h_2}{2} (b_2 - b_3) + \frac{A_{tr}}{2} \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>$M_E = M_J - Z_{PL} F_y - \frac{Z_{PE}}{2} \cdot (0.85 f_y)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>$Z_{PE} = Z_{pl} - \text{full y-axis plastic section modulus of steel shape}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>$Z_{PE} = -\frac{h_2 b_2^2}{4} - Z_{pl}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>$P_C = 0.85 f_t A_t$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>$M_C = M_J$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>$P_D = \frac{0.85 f_t A_t}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>$M_D = Z_{pl} F_y + Z_{PE} (0.85 f_y)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>$Z_{PF} = A_{tr} \left( \frac{h_2}{2} - c \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>$Z_{pl} = \frac{h_2 b_2^2}{4} - Z_{PF}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$P_B = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$M_B = M_J - Z_{pl} F_y - \frac{Z_{con}}{2} \cdot (0.85 f_y)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$Z_{con} = -h_2 b_2^2 - Z_{con}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>For $h_2$, below the flange $\left( \frac{h_2}{2} &lt; h_2 &lt; \frac{b_2}{2} \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$h_2 = \frac{0.85 f_t \left( A_t + A_{tr} - 2t_h b_2 \right) - 2F_y (A_t - 2t_h b_2)}{2t_h f_y + \left( h_2 - 2t_h \right) \cdot 0.85 f_y}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$Z_{con} = Z_{pl} - 2t_h \left( \frac{b_2}{2} + h_2 \right) \left( \frac{b_2}{2} - h_2 \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>For $h_2$, above the flange $\left( h_2 &gt; \frac{b_2}{2} \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$h_2 = \frac{0.85 f_t \left( A_t + A_{tr} \right) - 2F_y A_t}{2 \cdot 0.85 f_t h_2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$Z_{con} = Z_{pl}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. I-1b. W-shapes, weak-axis anchor points.
Plastic Capacities for Composite
Filled HSS Bent About Either Principal Axis

<table>
<thead>
<tr>
<th>Section</th>
<th>Stress Distribution</th>
<th>Pt.</th>
<th>Defining Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td>( P_A = F_y A_y + 0.85f_y A_{eh} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( M_A = 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( A_{eh} = b_t h - 0.85b_t^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( b_t = B - 2t )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( h_t = H - 2t )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( t_t = t )</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td>( P_E = \frac{0.85f_y A_{eh}}{2} + 0.85f_y b_t h_t + 4F_y t h_t )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( M_E = M_D - F_y Z_{eh} - \frac{0.85f_y Z_{eh}}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( Z_{eh} = b_t h_t^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( Z_{eh} = 2t h_t^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( h_t = \frac{h_{eh} + H}{2} )</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>( P_C = 0.85f_y A_{eh} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( M_C = M_D )</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td>( P_D = \frac{0.85f_y A_{eh}}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( M_D = F_y Z_{eh} + \frac{0.85f_y Z_{eh}}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( Z_{eh} = \text{full } x\text{-axis plastic section modulus of HSS} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( Z_{eh} = b_t h_t^2 - 0.192b_t^3 )</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>( P_B = 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( M_B = M_D - F_y Z_{eh} - \frac{0.85f_y Z_{eh}}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( Z_{eh} = 2t h_t^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( Z_{eh} = b_t h_t^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( h_t = \frac{0.85f_y A_{eh}}{2} )</td>
</tr>
</tbody>
</table>

Note: Equations in this table are equally applicable to bending about the shape’s X-X axis (when H ≤ B) and to bending about the shape’s Y-Y axis (when B > H).

Fig. I-1c. Filled rectangular or square HSS, strong-axis anchor points.
Plastic Capacities for Composite
Filled Round HSS Bent About Any Axis

<table>
<thead>
<tr>
<th>Section</th>
<th>Stress Distribution</th>
<th>Pt.</th>
<th>Defining Equations</th>
</tr>
</thead>
</table>
| A       | $0.95F_y$           | A   | $P_A = F_y A_d + 0.95F_y A_z$
|         |                     |     | $M_A = 0$         |
|         |                     | A   | $A_d = \pi (d - t)^2$
|         |                     | A   | $A_z = \frac{nh^2}{4}$
|         |                     | E   | $P_E = P_A - \frac{F_y}{2} \left( d^2 - h^2 \right) + 0.95F_y h^2 \left( \theta_2 - \sin \theta_2 \right)$
|         |                     | E   | $M_E = F_y \bar{Z_{IE}} + 0.95F_y Z_{IE}$
|         |                     | E   | $Z_{IE} = \frac{h^3}{6} \sin \left( \frac{\theta_2}{2} \right)$
|         |                     | E   | $\theta_2 = \frac{h^2}{h} - \frac{h}{4}$
|         |                     | E   | $\theta_2 = -2 \arcsin \left( \frac{2 \bar{Z}_{IE}}{h} \right)$
| C       | $0.95F_y A_c$       | C   | $P_C = 0$         |
|         |                     | C   | $M_C = M_0$       |
| D       | $0.95F_y A_c$       | D   | $P_D = 0$         |
|         |                     | D   | $M_D = F_y Z_d + 0.95F_y Z_{d0}$
|         |                     | D   | $Z_d = \frac{d^2 - h^2}{6} - Z_{c}$
|         |                     | D   | $Z_{d0} = \frac{h^3}{6}$
| B       |                     | B   | $P_B = 0$         |
|         |                     | B   | $M_B = F_y Z_{d0} + 0.95F_y Z_{d0}$
|         |                     | B   | $Z_{d0} = \frac{6}{2} \sin \left( \frac{\theta}{2} \right)$
|         |                     | B   | $\theta = \frac{0.0260K_o - 2K_s}{0.0849K_o} + \sqrt{\left( \frac{0.0260K_o + 2K_s}{0.0849K_o} \right)^2 + 0.857K_s K_o}$ (rad)
|         |                     | B   | $K_o = 4t h^2$
|         |                     | B   | $K_s = F_y \left( \frac{d - t}{2} \right)$ *(thin" HSS wall assumed)*
|         |                     | B   | $h_f = \frac{h}{2} \sin \left( \frac{n - \theta}{2} \right) \leq \frac{h}{2}$

Fig. I-1d. Filled round HSS anchor points.
Fig. I-2. Terminology used for filled members.

REFERENCES

EXAMPLE I.1 COMPOSITE BEAM DESIGN

Given:

A typical bay of a composite floor system is illustrated in Figure I.1-1. Select an appropriate ASTM A992 W-shaped beam and determine the required number of 3/4-in.-diameter steel headed stud anchors. The beam will not be shored during construction.

![](image)

**Fig. I.1-1. Composite bay and beam section.**

To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4 1/2 in. of normal weight (145 lb/ft³) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, $f'_c = 4$ ksi.

Applied loads are as follows:

**Dead Loads:**
- Pre-composite:
  - Slab = 75 lb/ft² (in accordance with metal deck manufacturer’s data)
  - Self weight = 5 lb/ft² (assumed uniform load to account for beam weight)

- Composite (applied after composite action has been achieved):
  - Miscellaneous = 10 lb/ft² (HVAC, ceiling, floor covering, etc.)

**Live Loads:**
- Pre-composite:
  - Construction = 25 lb/ft² (temporary loads during concrete placement)

- Composite (applied after composite action has been achieved):
  - Non-reducible = 100 lb/ft² (assembly occupancy)
Solution:

From AISC Manual Table 2-4, the material properties are as follows:

\[
\begin{align*}
F_y &= 50 \text{ ksi} \\
F_u &= 65 \text{ ksi}
\end{align*}
\]

Applied Loads

For slabs that are to be placed at a constant elevation, AISC Design Guide 3 (West and Fisher, 2003) recommends an additional 10% of the nominal slab weight be applied to account for concrete ponding due to deflections resulting from the wet weight of the concrete during placement. For the slab under consideration, this would result in an additional load of 8 lb/ft²; however, for this design the slab will be placed at a constant thickness thus no additional weight for concrete ponding is required.

For pre-composite construction live loading, 25 lb/ft² will be applied in accordance with recommendations from ASCE/SEI 37-02 Design Loads on Structures During Construction (ASCE, 2002) for a light duty operational class which includes concrete transport and placement by hose.

Composite Deck and Anchor Requirements

Check composite deck and anchor requirements stipulated in AISC Specification Sections I1.3, I3.2c and I8.

1. Concrete Strength: \( 3 \text{ ksi} \leq f'_c \leq 10 \text{ ksi} \)  
   \( f'_c = 4 \text{ ksi} \) o.k.

2. Rib height: \( h \leq 3 \text{ in.} \)  
   \( h = 3 \text{ in.} \) o.k.

3. Average rib width: \( w_r \geq 2 \text{ in.} \)  
   \( w_r = 6 \text{ in.} \) (from deck manufacturer’s literature) o.k.

4. Use steel headed stud anchors \( \frac{3}{8} \text{ in.} \) or less in diameter.
   Use \( \frac{3}{8} \text{-in.-diameter} \) steel anchors per problem statement o.k.

5. Steel headed stud anchor diameter: \( d_{wa} \leq 2.5(t_r) \)
   
   In accordance with AISC Specification Section I8.1, this limit only applies if steel headed stud anchors are not welded to the flange directly over the web. The \( \frac{3}{8} \text{-in.-diameter} \) anchors will be placed in pairs transverse to the web in some locations, thus this limit must be satisfied. Select a beam size with a minimum flange thickness of 0.30 in., as determined below:

   \[
   t_r \geq \frac{d_{wa}}{2.5}
   \]

   \[
   \begin{align*}
   d_{wa} &\geq \frac{2.5}{2.5} \\
   &= 0.30 \text{ in.}
   \end{align*}
   \]

6. Steel headed stud anchors, after installation, shall extend not less than \( 1\frac{1}{2} \text{ in.} \) above the top of the steel deck.
A minimum anchor length of 4½ in. is required to meet this requirement for 3 in. deep deck. From steel headed stud anchor manufacturer’s data, a standard stock length of 4⅛ in. is selected. Using a ¾-in. length reduction to account for burn off during anchor installation through the deck yields a final installed length of 4½ in.

4⅛ in. = 4½ in.  o.k.

(7) Minimum length of stud anchors = 4d,ad

4½ in. > 4(¾ in.) = 3.00 in.  o.k.

(8) There shall be at least ½ in. of specified concrete cover above the top of the headed stud anchors.

As discussed in AISC Specification Commentary to Section I3.2c, it is advisable to provide greater than ½ in. minimum cover to assure anchors are not exposed in the final condition, particularly for intentionally cambered beams.

7½ in. − 4½ in. = 3.00 in. > ½ in.  o.k.

(9) Slab thickness above steel deck ≥ 2 in.

4½ in. > 2 in.  o.k.

Design for Pre-Composite Condition

Construction (Pre-Composite) Loads

The beam is uniformly loaded by its tributary width as follows:

\[ w_D = \left[ (10 \text{ ft}) \left( 75 \text{ lb/ft}^2 + 5 \text{ lb/ft}^2 \right) \right] (0.001 \text{ kip/lb}) \]

\[ = 0.800 \text{ kip/ft} \]

\[ w_L = \left[ (10 \text{ ft}) \left( 25 \text{ lb/ft}^2 \right) \right] (0.001 \text{ kip/lb}) \]

\[ = 0.250 \text{ kip/ft} \]

Construction (Pre-Composite) Flexural Strength

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u ) = 1.2(0.800 kip/ft) + 1.6(0.250 kip/ft)</td>
<td>( w_u = 0.800 \text{ kip/ft} + 0.250 \text{ kip/ft} )</td>
<td></td>
</tr>
<tr>
<td>( = 1.36 \text{ kip/ft} )</td>
<td>( = 1.05 \text{ kip/ft} )</td>
<td></td>
</tr>
<tr>
<td>( M_u = \frac{w_u L^2}{8} )</td>
<td>( M_u = \frac{w_u L^2}{8} )</td>
<td></td>
</tr>
<tr>
<td>( = \frac{(1.36 \text{ kip/ft})(45 \text{ ft})^2}{8} )</td>
<td>( = \frac{(1.05 \text{ kip/ft})(45 \text{ ft})^2}{8} )</td>
<td></td>
</tr>
<tr>
<td>( = 344 \text{ kip-ft} )</td>
<td>( = 266 \text{ kip-ft} )</td>
<td></td>
</tr>
</tbody>
</table>
Beam Selection

Assume that attachment of the deck perpendicular to the beam provides adequate bracing to the compression flange during construction, thus the beam can develop its full plastic moment capacity. The required plastic section modulus, $Z_x$, is determined as follows, from AISC Specification Equation F2-1:

$$Z_{x,\text{min}} = \frac{M_u}{\phi_b F_y} = \frac{(344 \text{ kip-ft})(12 \text{ in./ft})}{0.90(50 \text{ ksi})} = 91.7 \text{ in.}^3$$

From AISC Manual Table 3-2 select a W21×50 with a $Z_x$ value of 110 in.$^3$

Note that for the member size chosen, the self weight on a pounds per square foot basis is 50 plf/10 ft = 5.00 psf; thus the initial self weight assumption is adequate.

From AISC Manual Table 1-1, the geometric properties are as follows:

$$W21 \times 50$$
$$A = 14.7 \text{ in.}^2$$
$$I_x = 984 \text{ in.}^4$$

Pre-Composite Deflections

AISC Design Guide 3 recommends deflections due to concrete plus self weight not exceed the minimum of $L/360$ or 1.0 in.

From AISC Manual Table 3-23, Case 1:

$$\Delta_{wc} = \frac{5wL^4}{384EI}$$

Substituting for the moment of inertia of the non-composite section, $I = 984 \text{ in.}^4$, yields a dead load deflection of:

$$\Delta_{wc} = \frac{5 \left[ \frac{(0.800 \text{ kip/ft})}{12 \text{ in./ft}} \right] \left[ (45.0 \text{ ft})(12 \text{ in./ft}) \right]^4}{384(29,000 \text{ ksi})(984 \text{ in.}^4)} = 2.59 \text{ in.}$$

Pre-composite deflections exceed the recommended limit. One possible solution is to increase the member size. A second solution is to induce camber into the member. For this example, the second solution is selected, and the beam will be cambered to reduce the net pre-composite deflections.
Reducing the estimated simple span deflections to 80% of the calculated value to reflect the partial restraint of the end connections as recommended in AISC Design Guide 3 yields a camber of:

\[
\text{Camber} = 0.8(2.59 \text{ in.}) \\
= 2.07 \text{ in.}
\]

Rounding down to the nearest \(\frac{1}{4}\)-in. increment yields a specified camber of 2 in.

Select a \(W21 \times 50\) with 2 in. of camber.

**Design for Composite Condition**

**Required Flexural Strength**

Using tributary area calculations, the total uniform loads (including pre-composite dead loads in addition to dead and live loads applied after composite action has been achieved) are determined as:

\[
\begin{align*}
w_d &= \left[(10.0 \text{ ft})(75 \text{ lb/ft}^2 + 5 \text{ lb/ft}^2 + 10 \text{ lb/ft}^2)\right](0.001 \text{ kip/lb}) \\
&= 0.900 \text{ kip/ft} \\

w_L &= \left[(10.0 \text{ ft})(100 \text{ lb/ft}^2)\right](0.001 \text{ kip/lb}) \\
&= 1.00 \text{ kip/ft}
\end{align*}
\]

From ASCE/SEI 7 Chapter 2, the required flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_u)</td>
<td>= 1.2(0.900 kip/ft) + 1.6(1.00 kip/ft)</td>
<td>(w_u) = 0.900 kip/ft + 1.00 kip/ft</td>
</tr>
<tr>
<td></td>
<td>= 2.68 kip/ft</td>
<td>= 1.90 kip/ft</td>
</tr>
<tr>
<td>(M_u)</td>
<td>(= \frac{w_u L^2}{8})</td>
<td>(M_u = \frac{w_u L^2}{8})</td>
</tr>
<tr>
<td></td>
<td>(= \frac{(2.68 \text{ kip/ft})(45.0 \text{ ft})^2}{8})</td>
<td>(= \frac{(1.90 \text{ kip/ft})(45.0 \text{ ft})^2}{8})</td>
</tr>
<tr>
<td></td>
<td>= 678 kip-ft</td>
<td>= 481 kip-ft</td>
</tr>
</tbody>
</table>

**Determine \(b\)**

The effective width of the concrete slab is the sum of the effective widths to each side of the beam centerline as determined by the minimum value of the three widths set forth in AISC Specification Section 13.1a:

1. one-eighth of the beam span center-to-center of supports
   \[
   \frac{45.0 \text{ ft}}{8} \text{ (2 sides)} = 11.3 \text{ ft}
   \]

2. one-half the distance to the centerline of the adjacent beam
   \[
   \frac{10.0 \text{ ft}}{2} \text{ (2 sides)} = 10.0 \text{ ft} \quad \text{controls}
   \]

3. distance to the edge of the slab
   not applicable for an interior member
Available Flexural Strength

According to AISC Specification Section I3.2a, the nominal flexural strength shall be determined from the plastic stress distribution on the composite section when \( h/t_w \leq 3.76\sqrt{E/F_y} \).

From AISC Manual Table 1-1, \( h/t_w \) for a W21\( \times \)50 = 49.4.

\[
49.4 \leq 3.76\sqrt{29,000 \text{ ksi} / 50 \text{ ksi}} \\
\leq 90.6
\]

Therefore, use the plastic stress distribution to determine the nominal flexural strength.

According to the User Note in AISC Specification Section I3.2a, this check is generally unnecessary as all current W-shapes satisfy this limit for \( F_y \leq 50 \text{ ksi} \).

Flexural strength can be determined using AISC Manual Table 3-19 or calculated directly using the provisions of AISC Specification Chapter I. This design example illustrates the use of the Manual table only. For an illustration of the direct calculation procedure, refer to Design Example I.2.

To utilize AISC Manual Table 3-19, the distance from the compressive concrete flange force to beam top flange, \( Y_2 \), must first be determined as illustrated by Manual Figure 3-3. Fifty percent composite action \([\Sigma Q_a \approx 0.50(A_f F_y)]\) is used to calculate a trial value of the compression block depth, \( a_{\text{trial}} \), for determining \( Y_2 \) as follows:

\[
a_{\text{trial}} = \frac{\Sigma Q_a}{0.85 f_y b}
\]

\[
= \frac{0.50(A_f F_y)}{0.85 f_y b}
\]

\[
= \frac{0.50(14.7 \text{ in.}^2)(50 \text{ ksi})}{0.85(4 \text{ ksi})(10.0 \text{ ft})(12 \text{ in./ft})}
\]

\[
= 0.90 \text{ in.} \rightarrow \text{say } 1.0 \text{ in.}
\]

Note that a trial value of \( a = 1.0 \text{ in.} \) is a common starting point in many design problems.

\[
Y_2 = Y_{\text{con}} - \frac{a_{\text{trial}}}{2}
\]

where

\[
Y_{\text{con}} = \text{distance from top of steel beam to top of slab, in.}
\]

\[
= 7.50 \text{ in.}
\]

\[
Y_2 = 7.50 \text{ in.} - \frac{1.0 \text{ in.}}{2}
\]

\[
= 7.00 \text{ in.}
\]

Enter AISC Manual Table 3-19 with the required strength and \( Y_2 = 7.00 \text{ in.} \) to select a plastic neutral axis location for the W21\( \times \)50 that provides sufficient available strength.

Selecting PNA location 5 (BFL) with \( \Sigma Q_a = 386 \text{ kips} \) provides a flexural strength of:
Based on the available flexural strength provided in Table 3-19, the required PNA location for ASD and LRFD design methodologies differ. This discrepancy is due to the live to dead load ratio in this example, which is not equal to the ratio of 3 at which ASD and LRFD design methodologies produce equivalent results as discussed in AISC Specification Commentary Section B3.4. The selected PNA location 5 is acceptable for ASD design, and more conservative for LRFD design.

The actual value for the compression block depth, \( a \), is determined as follows:

\[
a = \frac{\sum Q_a}{0.85 f_y b} = \frac{386 \text{ kips}}{0.85(4 \text{ ksi})(10 \text{ ft})(12 \text{ in./ft})} = 0.946 \text{ in.}
\]

\( a = 0.946 \text{ in.} < a_{\text{total}} = 1.0 \text{ in.} \) \text{o.k.}

**Live Load Deflection**

Deflections due to live load applied after composite action has been achieved will be limited to \( L / 360 \) under the design live load as required by Table 1604.3 of the 2009 International Building Code (IBC) (ICC, 2009), or 1 in. using a 50% reduction in design live load as recommended by AISC Design Guide 3.

Deflections for composite members may be determined using the lower bound moment of inertia provided by Specification Commentary Equation C-I3-1 and tabulated in AISC Manual Table 3-20. The Specification Commentary also provides an alternate method for determining deflections of a composite member through the calculation of an effective moment of inertia. This design example illustrates the use of the Manual table. For an illustration of the direct calculation procedure for each method, refer to Design Example I.2.

Entering Table 3-20, for a \( W21 \times 50 \) with PNA location 5 and \( Y2 = 7.00 \text{ in.} \), provides a lower bound moment of inertia of \( I_{LB} = 2,520 \text{ in.}^4 \).

Inserting \( I_{LB} \) into AISC Manual Table 3-23, Case 1, to determine the live load deflection under the full design live load for comparison to the IBC limit yields:

\[
\Delta_c = \frac{5W_L L^4}{384EI_{LB}} = \frac{5 \left( 1.00 \text{ kip/ft} \right) \left( 45.0 \text{ ft} \right)\left( 12 \text{ in./ft} \right)^4}{384 \left( 29,000 \text{ ksi} \right) \left( 2,520 \text{ in.}^4 \right)}
\]

\[
= 1.26 \text{ in.}
\]

\( = L / 429 < L / 360 \) \text{o.k.}

Performing the same check with 50% of the design live load for comparison to the AISC Design Guide 3 limit yields:

---

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_u \geq M_u )</td>
<td>( M_u / \Omega_b \geq M_u )</td>
</tr>
<tr>
<td>( \phi_b M_u = 769 \text{ kip-ft} \geq 678 \text{ kip-ft} \text{ o.k.} )</td>
<td>( M_u / \Omega_b = 512 \text{ kip-ft} \geq 481 \text{ kip-ft} \text{ o.k.} )</td>
</tr>
</tbody>
</table>
Steel Anchor Strength

Steel headed stud anchor strengths are tabulated in AISC Manual Table 3-21 for typical conditions. Conservatively assuming that all anchors are placed in the weak position, the strength for \( \frac{3}{4}\)-in.-diameter anchors in normal weight concrete with \( f'_c = 4 \) ksi and deck oriented perpendicular to the beam is:

1 anchor per rib: \( Q_a = 17.2 \) kips/anchor
2 anchors per rib: \( Q_a = 14.6 \) kips/anchor

Number and Spacing of Anchors

Deck flutes are spaced at 12 in. on center according to the deck manufacturer’s literature. The minimum number of deck flutes along each half of the 45-ft-long beam, assuming the first flute begins a maximum of 12 in. from the support line at each end, is:

\[
\left( \frac{45.0 \text{ ft}}{2} - 2(12 \text{ in.})(1 \text{ ft/12 in.}) \right) + 1
\]

\[
= \frac{45.0 \text{ ft} - 2(12 \text{ in.})(1 \text{ ft/12 in.})}{2(1 \text{ ft per space})} + 1
\]

\[
= 22.5 \rightarrow \text{say 22 flutes}
\]

According to AISC Specification Section I8.2c, the number of steel headed stud anchors required between the section of maximum bending moment and the nearest point of zero moment is determined by dividing the required horizontal shear, \( \Sigma Q_a \), by the nominal shear strength per anchor, \( Q_a \). Assuming one anchor per flute:

\[
n_{\text{anchors}} = \frac{\Sigma Q_a}{Q_a}
\]

\[
= \frac{386 \text{ kips}}{17.2 \text{ kips/anchor}}
\]

\[
= 22.4 \rightarrow \text{place 23 anchors on each side of the beam centerline}
\]

As the number of anchors exceeds the number of available flutes by one, place two anchors in the first flute. The revised horizontal shear capacity of the anchors taking into account the reduced strength for two anchors in one flute is:

\[
\Sigma Q_a = 2(14.6 \text{ kips}) + 21(17.2 \text{ kips})
\]

\[
= 390 \text{ kips} \geq 386 \text{ kips} \quad \text{o.k.}
\]

The final anchor pattern chosen is illustrated in Figure I.1-2.
Review steel headed stud anchor spacing requirements of AISC Specification Sections I8.2d and I3.2c.

(1) Maximum anchor spacing along beam: \[ 8t_{\text{slab}} = 8(7.5 \text{ in.}) = 60.0 \text{ in.} \text{ or } 36 \text{ in.} \]
   12 in. < 36 in. \( \text{o.k.} \)

(2) Minimum anchor spacing along beam: \[ 6d_{\omega} = 6(\frac{3}{4} \text{ in.}) = 4.50 \text{ in.} \]
   12 in. > 4.50 in. \( \text{o.k.} \)

(3) Minimum transverse spacing between anchor pairs: \[ 4d_{\omega} = 4(\frac{3}{4} \text{ in.}) = 3.00 \text{ in.} \]
   3.00 in. = 3.00 in. \( \text{o.k.} \)

(4) Minimum distance to free edge in the direction of the horizontal shear force:

   AISC Specification Section I8.2d requires that the distance from the center of an anchor to a free edge in the direction of the shear force be a minimum of 8 in. for normal weight concrete slabs.

(5) Maximum spacing of deck attachment:

   AISC Specification Section I3.2c(4) requires that steel deck be anchored to all supporting members at a maximum spacing of 18 in. The stud anchors are welded through the metal deck at a maximum spacing of 12 inches in this example, thus this limit is met without the need for additional puddle welds or mechanical fasteners.

**Available Shear Strength**

According to AISC Specification Section I4.2, the beam should be assessed for available shear strength as a bare steel beam using the provisions of Chapter G.

Applying the loads previously determined for the governing ASCE/SEI 7-10 load combinations and using available shear strengths from AISC Manual Table 3-2 for a W21×50 yields the following:
**Serviceability**

Depending on the intended use of this bay, vibrations might need to be considered. See AISC Design Guide 11 (Murray et al., 1997) for additional information.

**Summary**

From Figure I.1-2, the total number of stud anchors used is equal to \((2)(2 + 21) = 46\). A plan layout illustrating the final beam design is provided in Figure I.1-3:

A W21×50 with 2 in. of camber and 46, 3/4-in.-diameter by 4%-in.-long steel headed stud anchors is adequate to resist the imposed loads.
EXAMPLE I.2 COMPOSITE GIRDER DESIGN

Given:

Two typical bays of a composite floor system are illustrated in Figure I.2-1. Select an appropriate ASTM A992 W-shaped girder and determine the required number of steel headed stud anchors. The girder will not be shored during construction.

![Composite bay and girder section](image)

Fig. I.2-1. Composite bay and girder section.

To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4\(\frac{1}{2}\) in. of normal weight (145 lb/ft\(^3\)) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, \(f'_c = 4\) ksi.

Applied loads are as follows:

Dead Loads:
- Pre-composite:
  - Slab = 75 lb/ft\(^2\) (in accordance with metal deck manufacturer’s data)
  - Self weight = 80 lb/ft (trial girder weight)
  - = 50 lb/ft (beam weight from Design Example I.1)

- Composite (applied after composite action has been achieved):
  - Miscellaneous = 10 lb/ft\(^2\) (HVAC, ceiling, floor covering, etc.)

Live Loads:
- Pre-composite:
  - Construction = 25 lb/ft\(^2\) (temporary loads during concrete placement)

- Composite (applied after composite action has been achieved):
  - Non-reducible = 100 lb/ft\(^2\) (assembly occupancy)
Solution:

From AISC Manual Table 2-4, the material properties are as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Strength ($F_y$)</th>
<th>Tensile Strength ($F_u$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
</tbody>
</table>

**Applied Loads**

For slabs that are to be placed at a constant elevation, AISC Design Guide 3 (West and Fisher, 2003) recommends an additional 10% of the nominal slab weight be applied to account for concrete ponding due to deflections resulting from the wet weight of the concrete during placement. For the slab under consideration, this would result in an additional load of 8 lb/ft$^2$; however, for this design the slab will be placed at a constant thickness thus no additional weight for concrete ponding is required.

For pre-composite construction live loading, 25 lb/ft$^2$ will be applied in accordance with recommendations from ASCE/SEI 37-02 Design Loads on Structures During Construction (ASCE, 2002) for a light duty operational class which includes concrete transport and placement by hose.

**Composite Deck and Anchor Requirements**

Check composite deck and anchor requirements stipulated in AISC Specification Sections I1.3, I3.2c and I8.

1. **Concrete Strength:** $3 \text{ ksi} \leq f'_{c} \leq 10 \text{ ksi}$
   
   $f'_{c} = 4 \text{ ksi} \quad \text{o.k.}$

2. **Rib height:** $h_r \leq 3$ in.
   
   $h_r = 3$ in. \quad \text{o.k.}$

3. **Average rib width:** $w_r \geq 2$ in.
   
   $w_r = 6$ in. (See Figure I.2-1) \quad \text{o.k.}$

4. **Use steel headed stud anchors $\frac{3}{8}$ in. or less in diameter.**
   
   Select $\frac{3}{8}$-in.-diameter steel anchors \quad \text{o.k.}$

5. **Steel headed stud anchor diameter:** $d_{wa} \leq 2.5 (t_f)$
   
   In accordance with AISC Specification Section I8.1, this limit only applies if steel headed stud anchors are not welded to the flange directly over the web. The $\frac{3}{8}$-in.-diameter anchors will be attached in a staggered pattern, thus this limit must be satisfied. Select a girder size with a minimum flange thickness of 0.30 in., as determined below:

   $$t_f \geq \frac{d_{wa}}{2.5}$$

   $$d_{wa} = \frac{3}{8} \text{ in.}$$

   $$\frac{3}{8} \div 2.5 = 0.30 \text{ in.}$$

6. **Steel headed stud anchors, after installation, shall extend not less than 1$\frac{1}{2}$ in. above the top of the steel deck.**
A minimum anchor length of 4½ in. is required to meet this requirement for 3 in. deep deck. From steel headed stud anchor manufacturer’s data, a standard stock length of 4⅛ in. is selected. Using a ⅛-in. length reduction to account for burn off during anchor installation directly to the girder flange yields a final installed length of 4⅛ in.

4⅛ in. > 4½ in.  o.k.

(7) Minimum length of stud anchors = 4dsa

4⅛ in. > 4(¼ in.) = 3.00 in.  o.k.

(8) There shall be at least ½ in. of specified concrete cover above the top of the headed stud anchors.

As discussed in the Specification Commentary to Section I3.2c, it is advisable to provide greater than ½ in. minimum cover to assure anchors are not exposed in the final condition.

7½ in. − 4⅛ in. = 2⅜ in. > ½ in.  o.k.

(9) Slab thickness above steel deck ≥ 2 in.

4½ in. > 2 in.  o.k.

Design for Pre-Composite Condition

Construction (Pre-Composite) Loads

The girder will be loaded at third points by the supported beams. Determine point loads using tributary areas.

\[
P_2 = \left( 45.0 \text{ ft} \right) \left( 10.0 \text{ ft} \right) \left( 75 \text{ lb/ft}^2 \right) + \left( 45.0 \text{ ft} \right) \left( 50 \text{ lb/ft} \right) \left( 0.001 \text{ kip/lb} \right) = 36.0 \text{ kips} \\
P_1 = \left( 45.0 \text{ ft} \right) \left( 10.0 \text{ ft} \right) \left( 25 \text{ lb/ft}^2 \right) \left( 0.001 \text{ kip/lb} \right) = 11.3 \text{ kips}
\]

Construction (Pre-Composite) Flexural Strength

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_u = 1.2(36.0 \text{ kips}) + 1.6(11.3 \text{ kips}) ] = 61.3 kips</td>
<td>[ P_a = 36.0 \text{ kips} + 11.3 \text{ kips} ] = 47.3 kips</td>
</tr>
<tr>
<td>[ w_u = 1.2(80 \text{ lb/ft})(0.001 \text{ kip/lb}) ] = 0.0960 kip/ft</td>
<td>[ w_a = (80 \text{ lb/ft})(0.001 \text{ kip/lb}) ] = 0.0800 kip/ft</td>
</tr>
<tr>
<td>[ M_u = P_u a + \frac{w_u L^2}{8} ] = (61.3 kips)(10.0 ft) + \left( \frac{0.0960 \text{ kip/ft}}{8} \right)(30.0 \text{ ft})^2 = 624 \text{ kip-ft}</td>
<td>[ M_a = P_a a + \frac{w_a L^2}{8} ] = (47.3 kips)(10.0 ft) + \left( \frac{0.0800 \text{ kip/ft}}{8} \right)(30.0 \text{ ft})^2 = 482 \text{ kip-ft}</td>
</tr>
</tbody>
</table>

Girder Selection
Based on the required flexural strength under construction loading, a trial member can be selected utilizing AISC Manual Table 3-2. For the purposes of this example, the unbraced length of the girder prior to hardening of the concrete is taken as the distance between supported beams (one third of the girder length).

Try a W24×76

\[ L_o = 10.0 \text{ ft} \]
\[ L_p = 6.78 \text{ ft} \]
\[ L_r = 19.5 \text{ ft} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_p BF = 22.6 \text{ kips} )</td>
<td>( BF / \Omega_b = 15.1 \text{ kips} )</td>
</tr>
<tr>
<td>( \phi_p M_{ps} = 750 \text{ kip-ft} )</td>
<td>( M_{ps} / \Omega_b = 499 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( \phi_p M_{cr} = 462 \text{ kip-ft} )</td>
<td>( M_{cr} / \Omega_b = 307 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Because \( L_p < L_o < L_r \), use AISC Manual Equations 3-4a and 3-4b with \( C_o = 1.0 \) within the center girder segment in accordance with Manual Table 3-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ \phi_p M_o = C_o \left[ \phi_p M_{ps} - \phi_p BF \delta (L_o - L_p) \right] \leq \phi_p M_{ps} \]  
\[ = 1.0[750 \text{ kip-ft} - 22.6 \text{ kip} (10.0 \text{ ft} - 6.78 \text{ ft})] \]  
\[ = 677 \text{ kip-ft} \leq 750 \text{ kip-ft} \] | \[ M_o = C_o \left[ \frac{M_{ps}}{\Omega_b} - \frac{BF}{\Omega_b} (L_o - L_p) \right] \leq \frac{M_{ps}}{\Omega_b} \]  
\[ = 1.0[499 \text{ kip-ft} - 15.1 \text{ kips} (10.0 \text{ ft} - 6.78 \text{ ft})] \]  
\[ = 450 \text{ kip-ft} \leq 499 \text{ kip-ft} \] |
| \( \phi_p M_o \geq M_o \) | \( M_o \geq M_o \) |
| 677 kip-ft > 624 kip-ft \text{ o.k.} | 450 kip-ft < 482 kip-ft \text{ n.g.} |

For this example, the relatively low live load to dead load ratio results in a lighter member when LRFD methodology is employed. When ASD methodology is employed, a heavier member is required, and it can be shown that a W24×84 is adequate for pre-composite flexural strength. This example uses a W24×76 member to illustrate the determination of flexural strength of the composite section using both LRFD and ASD methodologies; however, this is done for comparison purposes only, and calculations for a W24×84 would be required to provide a satisfactory ASD design. Calculations for the heavier section are not shown as they would essentially be a duplication of the calculations provided for the W24×76 member.

Note that for the member size chosen, 76 lb/ft ≤ 80 lb/ft, thus the initial weight assumption is adequate.

From AISC Manual Table 1-1, the geometric properties are as follows:

\[ \text{W24×76} \]
\[ A = 22.4 \text{ in.}^2 \]
\[ I_i = 2.100 \text{ in.}^4 \]
\[ b_f = 8.99 \text{ in.} \]
\[ t_f = 0.680 \text{ in.} \]
\[ d = 23.9 \text{ in.} \]

Pre-Composite Deflections
AISC Design Guide 3 recommends deflections due to concrete plus self weight not exceed the minimum of \( L/360 \) or 1.0 in.

From the superposition of AISC Manual Table 3-23, Cases 1 and 9:

\[
\Delta_{nc} = \frac{P_d L^3}{28EI} + \frac{5w_p L^4}{384EI}
\]

Substituting for the moment of inertia of the non-composite section, \( I = 2,100 \text{ in.}^4 \), yields a dead load deflection of:

\[
\Delta_{nc} = \frac{36.0 \text{ kips} \left[ (30.0 \text{ ft})(12 \text{ in.}/\text{ft}) \right]^3}{28(29,000 \text{ ksi})(2,100 \text{ in.}^4)} + \frac{5 \left[ \frac{0.0760 \text{ kip/ft}}{12 \text{ in./ft}} \right]^2 \left[ (30.0 \text{ ft})(12 \text{ in.}/\text{ft}) \right]^3}{384(29,000 \text{ ksi})(2,100 \text{ in.}^4)} = 1.01 \text{ in.} \]

Pre-composite deflections slightly exceed the recommended value. One possible solution is to increase the member size. A second solution is to induce camber into the member. For this example, the second solution is selected, and the girder will be cambered to reduce pre-composite deflections.

Reducing the estimated simple span deflections to 80% of the calculated value to reflect the partial restraint of the end connections as recommended in AISC Design Guide 3 yields a camber of:

\[
\text{Camber} = 0.8(1.01 \text{ in.}) = 0.808 \text{ in.}
\]

Rounding down to the nearest \( \frac{1}{6} \)-in. increment yields a specified camber of \( \frac{3}{4} \) in.

Select a W24×76 with \( \frac{3}{4} \) in. of camber.

**Design for Composite Flexural Strength**

**Required Flexural Strength**

Using tributary area calculations, the total applied point loads (including pre-composite dead loads in addition to dead and live loads applied after composite action has been achieved) are determined as:

\[
P_d = \left[ (45.0 \text{ ft})(10.0 \text{ ft})(75 \text{ lb/ft}^2 + 10 \text{ lb/ft}^2) + (45.0 \text{ ft})(50 \text{ lb/ft}) \right](0.001 \text{ kip/lb}) = 40.5 \text{ kips}
\]

\[
P_L = \left[ (45.0 \text{ ft})(10.0 \text{ ft})(100 \text{ lb/ft}^2) \right](0.001 \text{ kip/lb}) = 45.0 \text{ kips}
\]

The required flexural strength diagram is illustrated by Figure I.2-2:
From ASCE/SEI 7-10 Chapter 2, the required flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| $P_r = P_a$
  $= 1.2(40.5 \text{ kips}) + 1.6(45.0 \text{ kips})$
  $= 121 \text{ kips}$
| $P_r = P_a$
  $= 40.5 \text{ kips} + 45.0 \text{ kips}$
  $= 85.5 \text{ kips}$
| $w_u = 1.2(0.0760 \text{ kip/ft})$ 
  $= 0.0912 \text{ kip/ft}$ from self weight of $W_{24\times76}$
| $w_u = 0.0760 \text{ kip/ft}$ from self weight of $W_{24\times76}$

From AISC Manual Table 3-23, Case 1 and 9. $M_{a1} = M_{a3}$

$$M_{a1} = M_{a3} = P_a a + \frac{w_u a (L - a)}{2}$$

$$= (121 \text{ kips})(10.0 \text{ ft})$$
  $$+ \frac{(0.0912 \text{ kip/ft})(10.0 \text{ ft})}{2}(30.0 \text{ ft} - 10.0 \text{ ft})$$
  $$= 1,220 \text{ kip-ft}$$

$$M_{a2} = P_a a + \frac{w_u L^2}{8}$$

$$= (121 \text{ kips})(10.0 \text{ ft})$$
  $$+ \frac{(0.0912 \text{ kip/ft})(30.0 \text{ ft})^2}{8}$$
  $$= 1,220 \text{ kip-ft}$$

$$M_{a3} = P_a a + \frac{w_u L^2}{8}$$

$$= (85.5 \text{ kips})(10.0 \text{ ft})$$
  $$+ \frac{(0.0760 \text{ kip/ft})(10.0 \text{ ft})}{2}(30.0 \text{ ft} - 10.0 \text{ ft})$$
  $$= 863 \text{ kip-ft}$$

$$M_{a2} = P_a a + \frac{w_u L^2}{8}$$

$$= (85.5 \text{ kips})(10.0 \text{ ft})$$
  $$+ \frac{(0.0760 \text{ kip/ft})(30.0 \text{ ft})^2}{8}$$
  $$= 864 \text{ kip-ft}$$

**Determine b**

The effective width of the concrete slab is the sum of the effective widths to each side of the beam centerline as determined by the minimum value of the three conditions set forth in AISC Specification Section I3.1a:

1. one-eighth of the girder span center-to-center of supports

$$\frac{30.0 \text{ ft}}{8}(2 \text{ sides}) = 7.50 \text{ ft} \quad \text{controls}$$
(2) one-half the distance to the centerline of the adjacent girder
\[
\frac{45 \text{ ft}}{2} \times 2 \text{ sides} = 45.0 \text{ ft}
\]

(3) distance to the edge of the slab
not applicable for an interior member

Available Flexural Strength

According to AISC Specification Section I3.2a, the nominal flexural strength shall be determined from the plastic stress distribution on the composite section when \( \frac{h}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}} \).

From AISC Manual Table 1-1, \( \frac{h}{t_w} \) for a W24×76 = 49.0.

\[
49.0 \leq 3.76 \sqrt{29,000 \text{ ksi}} / 50 \text{ ksi} \\
\leq 90.6
\]

Therefore, use the plastic stress distribution to determine the nominal flexural strength.

According to the User Note in AISC Specification Section I3.2a, this check is generally unnecessary as all current W-shapes satisfy this limit for \( F_y \leq 50 \text{ ksi} \).

AISC Manual Table 3-19 can be used to facilitate the calculation of flexural strength for composite beams. Alternately, the available flexural strength can be determined directly using the provisions of AISC Specification Chapter I. Both methods will be illustrated for comparison in the following calculations.

Method 1: AISC Manual

To utilize AISC Manual Table 3-19, the distance from the compressive concrete flange force to beam top flange, \( Y_2 \), must first be determined as illustrated by Manual Figure 3-3. Fifty percent composite action \( [\Sigma Q_n \approx 0.50(A,F_y)] \) is used to calculate a trial value of the compression block depth, \( a_{trial} \), for determining \( Y_2 \) as follows:

\[
a_{trial} = \frac{\sum Q_n}{0.85 f_y b} \\
= 0.50(A,F_y) / 0.85 f_y b \\
= \frac{0.50(22.4 \text{ in.}^2)(50 \text{ ksi})}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})} \\
= 1.83 \text{ in.}
\]

\[
Y_2 = Y_{con} - \frac{a_{trial}}{2}
\]

where
\( Y_{con} = \) distance from top of steel beam to top of slab
\( = 7.50 \text{ in.} \)

(2) (from Manual. Eq. 3-7)

(3) (from Manual. Eq. 3-6)
Enter AISC Manual Table 3-19 with the required strength and \( Y2 = 6.59 \) in. to select a plastic neutral axis location for the W24×76 that provides sufficient available strength. Based on the available flexural strength provided in Table 3-19, the required PNA location for ASD and LRFD design methodologies differ. This discrepancy is due to the live to dead load ratio in this example, which is not equal to the ratio of 3 at which ASD and LRFD design methodologies produce equivalent results as discussed in AISC Specification Commentary Section B3.4.

Selecting PNA location 5 (BFL) with \( \sum Q_n = 509 \) kips provides a flexural strength of:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_a \geq M_a )</td>
<td>( M_a / \Omega_b \geq M_a )</td>
</tr>
<tr>
<td>( \phi_b M_a = 1,240 \text{ kip-ft} &gt; 1,220 \text{ kip-ft} ) o.k.</td>
<td>( M_a / \Omega_b = 823 \text{ kip-ft} &lt; 864 \text{ kip-ft} ) n.g.</td>
</tr>
</tbody>
</table>

The selected PNA location 5 is acceptable for LRFD design, but inadequate for ASD design. For ASD design, it can be shown that a W24×76 is adequate if a higher composite percentage of approximately 60% is employed. However, as discussed previously, this beam size is not adequate for construction loading and a larger section is necessary when designing utilizing ASD.

The actual value for the compression block depth, \( a \), for the chosen PNA location is determined as follows:

\[
a = \frac{\sum Q_n}{0.85 f_c b} \quad \text{(Manual Eq. 3-7)}
\]

\[
= \frac{509 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})}
\]

\[
= 1.66 \text{ in.}
\]

\[
a = 1.66 \text{ in.} < a_{trial} = 1.83 \text{ in.} \quad \text{o.k. for LRFD design}
\]

Method 2: Direct Calculation

According to AISC Specification Commentary Section I3.2a, the number and strength of steel headed stud anchors will govern the compressive force, \( C \), for a partially composite beam. The composite percentage is based on the minimum of the limit states of concrete crushing and steel yielding as follows:

(1) Concrete crushing

\[
A_c = \text{Area of concrete slab within effective width. Assume that the deck profile is 50% void and 50% concrete fill.}
\]

\[
b_{eff} (4\frac{1}{2} \text{ in.}) + \left( \frac{b_{eff}}{2} \right) (3.00 \text{ in.})
\]

\[
= (7.50 \text{ ft})(12 \text{ in./ft})(4\frac{1}{2} \text{ in.}) + \left( \frac{(7.50 \text{ ft})(12 \text{ in./ft})}{2} \right) (3.00 \text{ in.})
\]

\[
= 540 \text{ in.}^2
\]

\[
C = 0.85 f'_c A_c \quad \text{(Comm. Eq. C-I3-7)}
\]

\[
= 0.85(4 \text{ ksi})(540 \text{ in.}^2)
\]

\[
= 1,840 \text{ kips}
\]

(2) Steel yielding
\[ C = A_Fy \]
\[ = (22.4 \text{ in.}^2)(50 \text{ ksi}) \]
\[ = 1,120 \text{ kips} \]

(3) Shear transfer

Fifty percent is used as a trial percentage of composite action as follows:

\[ C = \sum Q_i \]
\[ = 50\% \left( \min \left\{ \frac{1,840 \text{ kips}}{1,120 \text{ kips}} \right\} \right) \]
\[ = 560 \text{ kips to achieve 50\% composite action} \]

Location of the Plastic Neutral Axis

The plastic neutral axis (PNA) is located by determining the axis above and below which the sum of horizontal forces is equal. This concept is illustrated in Figure I.2-3, assuming the trial PNA location is within the top flange of the girder.

\[ \sum F_{\text{above PNA}} = \sum F_{\text{below PNA}} \]
\[ C + xb_Fy = (A_e - bx)x_Fy \]

Solving for \( x \):

\[ x = \frac{A_Fy - C}{2bx_Fy} \]
\[ = \frac{(22.4 \text{ in.}^2)(50 \text{ ksi}) - 560 \text{ kips}}{2(8.99 \text{ in.})(50 \text{ ksi})} \]
\[ = 0.623 \text{ in.} \]
\[ x = 0.623 \text{ in.} \leq t_f = 0.680 \text{ in.} \quad \text{PNA in flange} \]

\[ \frac{C}{d_1 = 6.59^\circ} \]
\[ xb_Fy \]
\[ \frac{d_2 = 0.312^\circ}{(A_e - bx)x_Fy} \]

\[ \text{Fig. I.2-3. Plastic neutral axis location.} \]
Determine the nominal moment resistance of the composite section following the procedure in *Specification Commentary Section I3.2a* as illustrated in Figure C-I3.3.

\[
M_a = C(d_1 + d_2) + P_x(d_3 - d_z) \quad \text{(Comm. Eq. C-I3-10)}
\]

\[
a = \frac{C}{0.85 f_y/b} \quad \text{(Comm. Eq. C-I3-9)}
\]

\[
\begin{align*}
d_1 &= t_{slab} - a/2 \\
&= 7.50 \text{ in.} - 1.83 \text{ in.}/2 \\
&= 6.59 \text{ in.}
\end{align*}
\]

\[
d_2 &= x/2 \\
&= 0.623 \text{ in.}/2 \\
&= 0.312 \text{ in.}
\]

\[
d_3 = d/2 \\
&= 23.9 \text{ in.}/2 \\
&= 12.0 \text{ in.}
\]

\[
P_x = A_p f_y \\
&= 22.4 \text{ in.}^2 (50 \text{ ksi}) \\
&= 1,120 \text{ kips}
\]

\[
M_a = \left[ (560 \text{ kips})(6.59 \text{ in.} + 0.312 \text{ in.}) + (1,120 \text{ kips})(12.0 \text{ in.} - 0.312 \text{ in.}) \right]/12 \text{ in./ft}
\]

\[
= 17,000 \text{ kip-in.} \\
12 \text{ in./ft} \\
= 1,420 \text{ kip-ft}
\]

Note that Equation C-I3-10 is based on summation of moments about the centroid of the compression force in the steel; however, the same answer may be obtained by summing moments about any arbitrary point.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b ) = 0.90</td>
<td>( \Omega_b ) = 1.67</td>
</tr>
<tr>
<td>( \phi_b M_a \geq M_u )</td>
<td>( M_u / \Omega_b \geq M_u )</td>
</tr>
<tr>
<td>( \phi_b M_a = 0.90(1,420 \text{ kip-ft}) )</td>
<td>( M_u / \Omega_b = \frac{1,420 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>= 1,280 kip-ft &gt; 1,220 kip-ft o.k.</td>
<td>= 850 kip-ft &lt; 864 kip-ft n.g.</td>
</tr>
</tbody>
</table>

As was determined previously using the Manual Tables, a W24×76 with 50% composite action is acceptable when LRFD methodology is employed, while for ASD design the beam is inadequate at this level of composite action.

Continue with the design using a W24×76 with 50% composite action.

**Steel Anchor Strength**

Steel headed stud anchor strengths are tabulated in AISC *Manual* Table 3-21 for typical conditions and may be calculated according to AISC *Specification* Section I8.2a as follows:
\[ Q_a = 0.5 A_u \sqrt{f_c' E_c} \leq R_g R_p A_u F_a \quad \text{(Spec. Eq. I8-1)} \]

\[ A_u = \pi d_{u3}^2 / 4 \]
\[ = \pi (\frac{3}{8} \text{ in.})^2 / 4 \]
\[ = 0.442 \text{ in.}^2 \]
\[ f_c' = 4 \text{ ksi} \]
\[ E_c = w_c^{1.5} \sqrt{f_c'} \]
\[ = \left(145 \text{ lb/ft}^3\right)^{1.5} \sqrt{4 \text{ ksi}} \]
\[ = 3,490 \text{ ksi} \]
\[ R_g = 1.0 \quad \text{Stud anchors welded directly to the steel shape within the slab haunch} \]
\[ R_p = 0.75 \quad \text{Stud anchors welded directly to the steel shape} \]
\[ F_c = 65 \text{ ksi} \quad \text{From AISC Manual Table 2-6 for ASTM A108 steel anchors} \]
\[ Q_a = (0.5)\left(0.442 \text{ in.}^2\right)\sqrt{4 \text{ ksi}}(3,490 \text{ ksi}) \leq (1.0)(0.75)(0.442 \text{ in.}^2)(65 \text{ ksi}) \]
\[ = 26.1 \text{ kips} > 21.5 \text{ kips} \]
\[ \text{use } Q_a = 21.5 \text{ kips} \]

**Number and Spacing of Anchors**

According to AISC Specification Section I8.2c, the number of steel headed stud anchors required between any concentrated load and the nearest point of zero moment shall be sufficient to develop the maximum moment required at the concentrated load point.

From Figure I.2-2 the moment at the concentrated load points, \( M_{r1} \) and \( M_{r3} \), is approximately equal to the maximum beam moment, \( M_{r2} \). The number of anchors between the beam ends and the point loads should therefore be adequate to develop the required compressive force associated with the maximum moment, \( C \), previously determined to be 560 kips.

\[ N_{\text{anchors}} = \frac{\sum Q_a}{Q_a} \]
\[ = \frac{C}{Q_a} \]
\[ = \frac{560 \text{ kips}}{21.5 \text{ kips/anchor}} \]
\[ = 26 \text{ anchors from each end to concentrated load points} \]

In accordance with AISC Specification Section I8.2d, anchors between point loads should be spaced at a maximum of:

\[ 8t_{slab} = 60.0 \text{ in.} \]
\[ \text{or } 36 \text{ in. controls} \]

For beams with deck running parallel to the span such as the one under consideration, spacing of the stud anchors is independent of the flute spacing of the deck. Single anchors can therefore be spaced as needed along the beam length provided a minimum longitudinal spacing of six anchor diameters in accordance with AISC Specification Section I8.2d is maintained. Anchors can also be placed in aligned or staggered pairs provided a minimum transverse spacing of four stud diameters = 3 in. is maintained. For this design, it was chosen to use pairs of anchors along each end of the girder to meet strength requirements and single anchors along the center section of the girder to meet maximum spacing requirements as illustrated in Figure I.2-4.
AISC Specification Section I8.2d requires that the distance from the center of an anchor to a free edge in the direction of the shear force be a minimum of 8 in. for normal weight concrete slabs. For simply-supported composite beams this provision could apply to the distance between the slab edge and the first anchor at each end of the beam. Assuming the slab edge is coincident to the centerline of support, Figure I.2-4 illustrates an acceptable edge distance of 9 in., though in this case the column flange would prevent breakout and negate the need for this check. The slab edge is often uniformly supported by a column flange or pour stop in typical composite construction thus preventing the possibility of a concrete breakout failure and nullifying the edge distance requirement as discussed in AISC Specification Commentary Section I8.3.

For this example, the minimum number of headed stud anchors required to meet the maximum spacing limit previously calculated is used within the middle third of the girder span. Note also that AISC Specification Section I3.2c(1)(4) requires that steel deck be anchored to all supporting members at a maximum spacing of 18 in. Additionally, ANSI/SDI C1.0-2006, Standard for Composite Steel Floor Deck (SDI, 2006), requires deck attachment at an average of 12 in. but no more than 18 in.

From the previous discussion and Figure I.2-4, the total number of stud anchors used is equal to $(13)(2) + 3 + (13)(2) = 55$. A plan layout illustrating the final girder design is provided in Figure I.2-5.
Live Load Deflection Criteria

Deflections due to live load applied after composite action has been achieved will be limited to \( L / 360 \) under the design live load as required by Table 1604.3 of the 2009 International Building Code (IBC) (ICC, 2009), or 1 in. using a 50% reduction in design live load as recommended by AISC Design Guide 3.

Deflections for composite members may be determined using the lower bound moment of inertia provided in AISC Specification Commentary Equation C-I3-1 and tabulated in AISC Manual Table 3-20. The Specification Commentary also provides an alternate method for determining deflections through the calculation of an effective moment of inertia. Both methods are acceptable and are illustrated in the following calculations for comparison purposes:

Method 1: Calculation of the lower bound moment of inertia, \( I_{LB} \)

\[
I_{LB} = I_s + A_s \left( Y_{ENA} - d_3 \right)^2 + \left( \frac{\Sigma Q_a}{F_y} \right) \left( 2d_3 + d_1 - Y_{ENA} \right)^2
\]

(Comm. Eq. C-I3-1)

Variables \( d_1, d_2 \) and \( d_3 \) in AISC Specification Commentary Equation C-I3-1 are determined using the same procedure previously illustrated for calculating nominal moment resistance. However, for the determination of \( I_{LB} \) the nominal strength of steel anchors is calculated between the point of maximum positive moment and the point of zero moment as opposed to between the concentrated load and point of zero moment used previously. The maximum moment is located at the center of the span and it can be seen from Figure I.2-4 that 27 anchors are located between the midpoint of the beam and each end.

\[
\Sigma Q_a = (27 \text{ anchors})(21.5 \text{ kips/anchor})
\]

= 581 kips

\[
a = \frac{C}{0.85 f'c b}
\]

\[
= \frac{\Sigma Q_a}{0.85 f'c b}
\]

\[= \frac{581 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})} = 1.90 \text{ in.}
\]

\[d_1 = t_{slab} - a / 2 = 7.50 \text{ in.} - 1.90 \text{ in.} / 2 = 6.55 \text{ in.}
\]

\[x = \frac{A_s F_y - \Sigma Q_a}{2b_y F_y}
\]

\[= \frac{(22.4 \text{ in.}^2)(50 \text{ ksi}) - 581 \text{ kips}}{2(8.99 \text{ in.})(50 \text{ ksi})} = 0.600 \text{ in.} < t_f = 0.680 \text{ in.} \quad \text{(PNA within flange)}
\]

\[d_2 = x / 2 = 0.600 \text{ in.} / 2 = 0.300 \text{ in.}
\]

\[d_3 = d / 2 = 23.9 \text{ in.} / 2 = 12.0 \text{ in.}
\]
The distance from the top of the steel section to the elastic neutral axis, \( Y_{ENA} \), for use in Equation C-I3-1 is calculated using the procedure provided in AISC Specification Commentary Section I3.2 as follows:

\[
Y_{ENA} = \frac{A_d d_3 + \left( \frac{\sum Q_y}{F_y} \right)(2d_3 + d_t)}{A_d + \left( \frac{\sum Q_y}{F_y} \right)} \quad (Comm. Eq. C-I3-2)
\]

\[
= \left( 22.4 \text{ in.}^2 \right)(12.0 \text{ in.}) + \left( \frac{581 \text{ kips}}{50 \text{ ksi}} \right) \left[ 2(12.0 \text{ in.}) + 6.55 \text{ in.} \right] \over \left( 22.4 \text{ in.}^2 \right) + \left( \frac{581 \text{ kips}}{50 \text{ ksi}} \right) = 18.3 \text{ in.}
\]

Substituting these values into AISC Specification Commentary Equation C-I3-1 yields the following lower bound moment of inertia:

\[
I_{LB} = 2,100 \text{ in.}^4 + (22.4 \text{ in.})(18.3 \text{ in.} - 12.0 \text{ in.})^2 + \left( \frac{581 \text{ kips}}{50 \text{ ksi}} \right) \left[ 2(12.0 \text{ in.}) + 6.55 \text{ in.} - 18.3 \text{ in.} \right]^2
\]

\[
= 4,730 \text{ in.}^4
\]

Alternately, this value can be determined directly from AISC Manual Table 3-20 as illustrated in Design Example I.1.

Method 2: Calculation of the effective moment of inertia, \( I_{eff} \)

An alternate procedure for determining a moment of inertia for deflections of the composite section is presented in AISC Specification Commentary Section I3.2 as follows:

\textit{Transformed Moment of Inertia, } \( I_t \)

The effective width of the concrete below the top of the deck may be approximated with the deck profile resulting in a 50% effective width as depicted in Figure I.2-6. The effective width, \( b_{eff} = 7.50 \text{ ft}(12 \text{ in./ft}) = 90.0 \text{ in.} \)

Transformed slab widths are calculated as follows:

\[
n = \frac{E_s}{E_c} = 29,000 \text{ ksi} / 3,490 \text{ ksi} = 8.31
\]

\[
b_{tr} = b_{eff} / n = 90.0 \text{ in.} / 8.31 = 10.8 \text{ in.}
\]

\[
b_{tr} = 0.5b_{eff} / n = 0.5(90.0 \text{ in.}) / 8.31 = 5.42 \text{ in.}
\]

The transformed model is illustrated in Figure I.2-7.
Determine the elastic neutral axis of the transformed section (assuming fully composite action) and calculate the transformed moment of inertia using the information provided in Table I.2-1 and Figure I.2-7. For this problem, a trial location for the elastic neutral axis (ENA) is assumed to be within the depth of the composite deck.

<table>
<thead>
<tr>
<th>Part</th>
<th>$A$ (in.$^2$)</th>
<th>$y$ (in.)</th>
<th>$I$ (in.$^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>48.6</td>
<td>$2.25 + x$</td>
<td>82.0</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$5.42x$</td>
<td>$x/2$</td>
<td>$0.452x^3$</td>
</tr>
<tr>
<td>W24×76</td>
<td>22.4</td>
<td>$x - 15.0$</td>
<td>2,100</td>
</tr>
</tbody>
</table>

![Fig. I.2-6. Effective concrete width.](image1)

![Fig. I.2-7. Transformed area model.](image2)
\[ \sum Ay \text{ about Elastic Neutral Axis} = 0 \]
\[ (48.6 \text{ in}^2)(2.25 \text{ in.} + x) + (5.42 \text{ in.})\left(\frac{x^2}{2}\right) + (22.4 \text{ in}^2)(x - 15 \text{ in.}) = 0 \]
solve for \( x \) \( \rightarrow x = 2.88 \text{ in.} \)

Verify trial location:

2.88 in. < \( h_r = 3 \text{ in.} \)  \textbf{Elastic Neutral Axis within composite deck}

Utilizing the parallel axis theorem and substituting for \( x \) yields:

\[ I_w = \Sigma I + \Sigma Ay^2 \]
\[ = 82.0 \text{ in}^4 + (0.452 \text{ in.})(2.88 \text{ in.})^3 + 2,100 \text{ in}^4 + \left(48.6 \text{ in}^2\right)(2.25 \text{ in.} + 2.88 \text{ in.})^2 \]
\[ + \left(\frac{5.42 \text{ in.}}{4}\right)(2.88 \text{ in.})^3 + (22.4 \text{ in}^2)(2.88 \text{ in.} - 15.0 \text{ in.})^2 \]
\[ = 6,800 \text{ in}^4 \]

**Determine the equivalent moment of inertia, \( I_{\text{equiv}} \)**

\[ I_{\text{equiv}} = I_x + \left(\Sigma Q_n / C_f\right)(I_w - I_x) \]  \( (\text{Comm. Eq. C-13-4}) \)
\[ \Sigma Q_n = 581 \text{ kips (previously determined in Method 1)} \]
\[ C_f \] = Compression force for fully composite beam previously determined to be controlled by \( A F_y = 1,120 \text{ kips} \)
\[ I_{\text{equiv}} = 2,100 \text{ in}^4 + \sqrt{(581 \text{ kips} / 1,120 \text{ kips}) (6,800 \text{ in}^4 - 2,100 \text{ in}^4)} \]
\[ = 5,490 \text{ in}^4 \]

According to Specification Commentary Section 13.2:

\[ I_{\text{eff}} = 0.75 I_{\text{equiv}} \]
\[ = 0.75(5,490 \text{ in}^4) \]
\[ = 4,120 \text{ in}^4 \]

**Comparison of Methods and Final Deflection Calculation**

\( I_{LB} \) was determined to be 4,730 in.\(^4 \) and \( I_{\text{eff}} \) was determined to be 4,120 in.\(^4 \) \( I_{LB} \) will be used for the remainder of this example.

From AISC Manual Table 3-23, Case 9:

\[ \Delta_{LL} = \frac{P L^3}{28 E I_{LB}} \]
\[ = \left(45.0 \text{ kips}\right)\left[(30.0 \text{ ft})(12 \text{ in./ft})\right]^3 \]
\[ = \frac{28(29,000 \text{ ksi})(4,730 \text{ in}^4)}{28(29,000 \text{ ksi})(4,730 \text{ in}^4)} \]
Available Shear Strength

According to AISC Specification Section I4.2, the girder should be assessed for available shear strength as a bare steel beam using the provisions of Chapter G.

Applying the loads previously determined for the governing load combination of ASCE/SEI 7-10 and obtaining available shear strengths from AISC Manual Table 3-2 for a W24×76 yields the following:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_u = 121 \text{ kips} + (0.0912 \text{ kip/ft})(30.0 \text{ ft/2}) )</td>
<td>( V_u = 85.5 \text{ kips} + (0.0760 \text{ kip/ft})(30.0 \text{ ft/2}) )</td>
</tr>
<tr>
<td>( = 122 \text{ kips} )</td>
<td>( = 86.6 \text{ kips} )</td>
</tr>
<tr>
<td>( \phi V_u \geq V' )</td>
<td>( V_u / \Omega \geq V' )</td>
</tr>
<tr>
<td>( \phi V_u = 315 \text{ kips} &gt; 122 \text{ kips} \text{ o.k.} )</td>
<td>( V_u / \Omega = 210 \text{ kips} &gt; 86.6 \text{ kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

Serviceability

Depending on the intended use of this bay, vibrations might need to be considered. See AISC Design Guide 11 (Murray et al., 1997) for additional information.

It has been observed that cracking of composite slabs can occur over girder lines. The addition of top reinforcing steel transverse to the girder span will aid in mitigating this effect.

Summary

Using LRFD design methodology, it has been determined that a W24×76 with \( \frac{3}{4} \text{ in.} \) of camber and 55, \( \frac{3}{4} \)-in.-diameter by 4%-in.-long steel headed stud anchors as depicted in Figure I.2-4, is adequate for the imposed loads and deflection criteria. Using ASD design methodology, a W24×84 with a steel headed stud anchor layout determined using a procedure analogous to the one demonstrated in this example would be required.
EXAMPLE I.3  FILLED COMPOSITE MEMBER FORCE ALLOCATION AND LOAD TRANSFER

Given:
Refer to Figure I.3-1.

**Part I:** For each loading condition (a) through (c) determine the required longitudinal shear force, \( V' \), to be transferred between the steel section and concrete fill.

**Part II:** For loading condition (a), investigate the force transfer mechanisms of direct bearing, shear connection, and direct bond interaction.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft\(^3\)) concrete fill having a specified concrete compressive strength, \( f'_c = 5 \) ksi. Use ASTM A36 material for the bearing plate.

Applied loading, \( P_r \), for each condition illustrated in Figure I.3-1 is composed of the following nominal loads:

\[
\begin{align*}
P_D &= 32.0 \text{ kips} \\
P_L &= 84.0 \text{ kips}
\end{align*}
\]

Fig. I.3-1. Concrete filled member in compression.
Solution:

Part I—Force Allocation

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A500 Grade B

\( F_y = 46 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-11 and Figure I.3-1, the geometric properties are as follows:

HSS10×6×\( \frac{3}{16} \)
\( A_s = 10.4 \text{ in.}^2 \)
\( H = 10.0 \text{ in.} \)
\( B = 6.00 \text{ in.} \)
\( t_{\text{nom}} = \frac{3}{16} \text{ in.} \) (nominal wall thickness)
\( t = 0.349 \text{ in.} \) (design wall thickness in accordance with AISC Specification Section B4.2)
\( h/t = 25.7 \)
\( b/t = 14.2 \)

Calculate the concrete area using geometry compatible with that used in the calculation of the steel area in AISC Manual Table 1-11 (taking into account the design wall thickness and a corner radii of two times the design wall thickness in accordance with AISC Manual Part 1), as follows:

\[
\begin{align*}
    h_i &= H - 2t \\
    &= 10.0 \text{ in.} - 2(0.349 \text{ in.}) \\
    &= 9.30 \text{ in.} \\
    b_i &= B - 2t \\
    &= 6.00 \text{ in.} - 2(0.349 \text{ in.}) \\
    &= 5.30 \text{ in.} \\
    A_c &= b_i h_i - t^2 (4 - \pi) \\
    &= (5.30 \text{ in.})(9.30 \text{ in.}) - (0.349 \text{ in.})^2 (4 - \pi) \\
    &= 49.2 \text{ in.}^2
\end{align*}
\]

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_i = P_e )</td>
<td>1.2(32.0 kips) + 1.6(84.0 kips)</td>
<td>( P_i = P_e ) 32.0 kips + 84.0 kips</td>
</tr>
<tr>
<td></td>
<td>173 kips</td>
<td>116 kips</td>
</tr>
</tbody>
</table>

Composite Section Strength for Force Allocation

In order to determine the composite section strength, the member is first classified as compact, noncompact or slender in accordance with AISC Specification Table I1.1a. However, the results of this check do not affect force allocation calculations as Specification Section I6.2 requires the use of Equation I2-9a regardless of the local buckling classification, thus this calculation is omitted for this example. The nominal axial compressive strength without consideration of length effects, \( P_{n0} \), used for force allocation calculations is therefore determined as:
\[ P_{\text{nom}} = P \]
\[ = F'_{c} A_{t} + C_{z} f'_{c} \left( A_{s} + A_{w} \frac{E_{s}}{E_{c}} \right) \]

where
\[ C_{z} = 0.85 \text{ for rectangular sections} \]
\[ A_{w} = 0 \text{ when no reinforcing steel is present within the HSS} \]

\[ P_{\text{nom}} = (46 \text{ ksi}) (10.4 \text{ in.}^{2}) + 0.85(5 \text{ ksi})(49.2 \text{ in.}^{2} + 0.0 \text{ in.}^{2}) \]
\[ = 688 \text{ kips} \]

**Transfer Force for Condition (a)**

Refer to Figure I.3-1(a). For this condition, the entire external force is applied to the steel section only, and the provisions of AISC Specification Section I6.2a apply.

\[ V'_{r} = P \left( 1 - \frac{F'_{c} A_{t}}{P_{\text{nom}}} \right) \]
\[ = P \left[ 1 - \frac{(46 \text{ ksi})(10.4 \text{ in.}^{2})}{688 \text{ kips}} \right] \]
\[ = 0.305P \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V'_{r} = 0.305(173 \text{ kips}) )</td>
<td>( V'_{r} = 0.305(116 \text{ kips}) )</td>
</tr>
<tr>
<td>= 52.8 kips</td>
<td>= 35.4 kips</td>
</tr>
</tbody>
</table>

**Transfer Force for Condition (b)**

Refer to Figure I.3-1(b). For this condition, the entire external force is applied to the concrete fill only, and the provisions of AISC Specification Section I6.2b apply.

\[ V'_{r} = P \left( \frac{F'_{c} A_{t}}{P_{\text{nom}}} \right) \]
\[ = P \left[ \frac{(46 \text{ ksi})(10.4 \text{ in.}^{2})}{688 \text{ kips}} \right] \]
\[ = 0.695P \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V'_{r} = 0.695(173 \text{ kips}) )</td>
<td>( V'_{r} = 0.695(116 \text{ kips}) )</td>
</tr>
<tr>
<td>= 120 kips</td>
<td>= 80.6 kips</td>
</tr>
</tbody>
</table>

**Transfer Force for Condition (c)**

Refer to Figure I.3-1(c). For this condition, external force is applied to the steel section and concrete fill concurrently, and the provisions of AISC Specification Section I6.2c apply.

AISC Specification Commentary Section I6.2 states that when loads are applied to both the steel section and concrete fill concurrently, \( V'_{r} \) can be taken as the difference in magnitudes between the portion of the external force
applied directly to the steel section and that required by Equation I6-2. Using the plastic distribution approach employed in Specification Equations I6-1 and I6-2, this concept can be written in equation form as follows:

$$V'_r = P_{rs} - P_r \left( \frac{A_r F_r}{P_{mo}} \right)$$

(Eq. 1)

where

$$P_{rs} = \text{portion of external force applied directly to the steel section, kips}$$

Currently the Specification provides no specific requirements for determining the distribution of the applied force for the determination of $$P_{rs}$$, so it is left to engineering judgment. For a bearing plate condition such as the one represented in Figure I.3-1(c), one possible method for determining the distribution of applied forces is to use an elastic distribution based on the material axial stiffness ratios as follows:

$$E_c = \frac{E_c}{E_c} \sqrt{J_c}$$

$$= \left( 145 \text{ lb/ft} \right)^{1.5} \sqrt{5 \text{ ksi}}$$

$$= 3,900 \text{ ksi}$$

$$P_{rs} = \left( \frac{E_c A_c}{E_c A_c + E_s A_s} \right) \left( P_r \right)$$

$$= \left[ \frac{\left( 29,000 \text{ ksi} \right) \left( 10.4 \text{ in.}^2 \right)}{\left( 29,000 \text{ ksi} \right) \left( 10.4 \text{ in.}^2 \right) + \left( 3,900 \text{ ksi} \right) \left( 49.2 \text{ in.}^2 \right)} \right] \left( P_r \right)$$

$$= 0.611 P_r$$

Substituting the results into Equation 1 yields:

$$V'_r = 0.611 P_r - P_r \left( \frac{A_r F_r}{P_{mo}} \right)$$

$$= 0.611 P_r - P_r \left( \frac{10.4 \text{ in.}^2 \left( 46 \text{ ksi} \right)}{688 \text{ kips}} \right)$$

$$= 0.0843 P_r$$

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$V'_r = 0.0843 \times 173 \text{ kips}$$</td>
<td>= 14.6 kips</td>
<td>$$V'_r = 0.0843 \times 116 \text{ kips}$$</td>
</tr>
</tbody>
</table>

An alternate approach would be the use of a plastic distribution method whereby the load is partitioned to each material in accordance with their contribution to the composite section strength given in Equation I2-9b. This method eliminates the need for longitudinal shear transfer provided the local bearing strength of the concrete and steel are adequate to resist the forces resulting from this distribution.

**Additional Discussion**

- The design and detailing of the connections required to deliver external forces to the composite member should be performed according to the applicable sections of AISC Specification Chapters J and K. Note that for checking bearing strength on concrete confined by a steel HSS or box member, the $$\sqrt{A_c / A_s}$$ term in Equation J8-2 may be taken as 2.0 according to the User Note in Specification Section I6.2.
The connection cases illustrated by Figure I.3-1 are idealized conditions representative of the mechanics of actual connections. For instance, a standard shear connection welded to the face of an HSS column is an example of a condition where all external force is applied directly to the steel section only. Note that the connection configuration can also impact the strength of the force transfer mechanism as illustrated in Part II of this example.

Solution:

Part II—Load Transfer

The required longitudinal force to be transferred, \( V' \), determined in Part I condition (a) will be used to investigate the three applicable force transfer mechanisms of AISC Specification Section I6.3: direct bearing, shear connection, and direct bond interaction. As indicated in the Specification, these force transfer mechanisms may not be superimposed; however, the mechanism providing the greatest nominal strength may be used.

Direct Bearing

Trial Layout of Bearing Plate

For investigating the direct bearing load transfer mechanism, the external force is delivered directly to the HSS section by standard shear connections on each side of the member as illustrated in Figure I.3-2. One method for utilizing direct bearing in this instance is through the use of an internal bearing plate. Given the small clearance within the HSS section under consideration, internal access for welding is limited to the open ends of the HSS; therefore, the HSS section will be spliced at the bearing plate location. Additionally, it is a practical consideration that no more than 50% of the internal width of the HSS section be obstructed by the bearing plate in order to facilitate concrete placement. It is essential that concrete mix proportions and installation of concrete fill produce full bearing above and below the projecting plate. Based on these considerations, the trial bearing plate layout depicted in Figure I.3-2 was selected using an internal plate protrusion, \( L_p \), of 1.0 in.

Location of Bearing Plate

The bearing plate is placed within the load introduction length discussed in AISC Specification Section I6.4b. The load introduction length is defined as two times the minimum transverse dimension of the HSS both above and below the load transfer region. The load transfer region is defined in Specification Commentary Section I6.4 as the depth of the connection. For the configuration under consideration, the bearing plate should be located within \( 2(B = 6 \text{ in.}) = 12 \text{ in.} \) of the bottom of the shear connection. From Figure I.3-2, the location of the bearing plate is 6 in. from the bottom of the shear connection and is therefore adequate.
Available Strength for the Limit State of Direct Bearing

The contact area between the bearing plate and concrete, \( A_1 \), may be determined as follows:

\[
A_1 = A_0 - (h_0 - 2L_p)(h_0 - 2L_p)
\]

where

\[
L_p = \text{typical protrusion of bearing plate inside HSS} = 1.0 \text{ in.}
\]

Substituting for the appropriate geometric properties previously determined in Part I into Equation 2 yields:

\[
A_1 = 49.2 \text{ in.}^2 - [5.30\text{ in.} - 2(1.0\text{ in.})][9.30\text{ in.} - 2(1.0\text{ in.})] = 25.1 \text{ in.}^2
\]

The available strength for the direct bearing force transfer mechanism is:

\[
R_n = 1.7 f'_{y} A_1
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>0.65</td>
</tr>
<tr>
<td>( \phi R_n \geq V'_c )</td>
<td>( \Omega_g = 2.31 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.65(1.7)(5 \text{ ksi})(25.1 \text{ in.}^2) )</td>
<td>( R_n / \Omega_b \geq V'_c )</td>
</tr>
<tr>
<td>= 139 kips &gt; 52.8 kips o.k.</td>
<td>( R_n / \Omega_b = \frac{1.7(5 \text{ ksi})(25.1 \text{ in.}^2)}{2.31} )</td>
</tr>
<tr>
<td></td>
<td>= 92.4 kips &gt; 35.4 kips o.k.</td>
</tr>
</tbody>
</table>

Required Thickness of Internal Bearing Plate

There are several methods available for determining the bearing plate thickness. For round HSS sections with circular bearing plate openings, a closed-form elastic solution such as those found in Roark’s Formulas for Stress and Strain (Young and Budynas, 2002) may be used. Alternately, the use of computational methods such as finite element analysis may be employed.

For this example, yield line theory can be employed to determine a plastic collapse mechanism of the plate. In this case, the walls of the HSS lack sufficient stiffness and strength to develop plastic hinges at the perimeter of the bearing plate. Utilizing only the plate material located within the HSS walls, and ignoring the HSS corner radii, the yield line pattern is as depicted in Figure I.3-3.
Utilizing the results of the yield line analysis with $F_y = 36$ ksi plate material, the plate thickness may be determined as follows:

$$t_p = \sqrt{\frac{w_u}{2\phi F_y} \left[ L_p \left( h_i + h_u \right) - \frac{8L_p^2}{3} \right]}$$

where

- $w_u$ = bearing pressure on plate determined using LRFD load combinations
- $t_p$ = plate thickness

$$t_p = \sqrt{\frac{52.8 \text{ kips}}{25.1 \text{ in.}^2} \left[ 1.0 \text{ in.} \left( 5.30 \text{ in.} + 9.30 \text{ in.} \right) - \frac{8 \left( 1.0 \text{ in.} \right)^3}{3} \right]}$$

$$t_p = 0.622 \text{ in.}$$

$$t_p = \sqrt{\frac{35.4 \text{ kips}}{25.1 \text{ in.}^2} \left[ 1.0 \text{ in.} \left( 5.30 \text{ in.} + 9.30 \text{ in.} \right) - \frac{8 \left( 1.0 \text{ in.} \right)^3}{3} \right]}$$

$$t_p = 0.625 \text{ in.}$$

Thus, select a $\frac{3}{16}$-in.-thick bearing plate.

**Splice Weld**

The HSS is in compression due to the imposed loads, therefore the splice weld indicated in Figure I.3-2 is sized according to the minimum weld size requirements of Chapter J. Should uplift or flexure be applied in other loading conditions, the splice should be designed to resist these forces using the applicable provisions of AISC Specification Chapters J and K.

**Shear Connection**

Shear connection involves the use of steel headed stud or channel anchors placed within the HSS section to transfer the required longitudinal shear force. The use of the shear connection mechanism for force transfer in filled HSS is usually limited to large HSS sections and built-up box shapes, and is not practical for the composite member in question. Consultation with the fabricator regarding their specific capabilities is recommended to determine the feasibility of shear connection for HSS and box members. Should shear connection be a feasible load transfer mechanism, AISC Specification Section I6.3b in conjunction with the steel anchors in composite component provisions of Section I8.3 apply.

**Direct Bond Interaction**

The use of direct bond interaction for load transfer is limited to filled HSS and depends upon the location of the load transfer point within the length of the member being considered (end or interior) as well as the number of faces to which load is being transferred.

From AISC Specification Section I6.3c, the nominal bond strength for a rectangular section is:

$$R_u = B^2 C_{iu} F_{in}$$

*(Spec. Eq. 16-5)*
where
\[ B = \text{overall width of rectangular steel section along face transferring load, in.} \]
\[ C_n = \begin{cases} 2 & \text{if the filled composite member extends to one side of the point of force transfer} \\ 4 & \text{if the filled composite member extends to both sides of the point of force transfer} \end{cases} \]
\[ F_n = 0.06 \text{ ksi} \]

For the design of this load transfer mechanism, two possible cases will be considered:

Case 1: End Condition – Load Transferred to Member from Four Sides Simultaneously

For this case the member is loaded at an end condition (the composite member only extends to one side of the point of force transfer). Force is applied to all four sides of the section simultaneously thus allowing the full perimeter of the section to be mobilized for bond strength.

From AISC Specification Equation I6-5:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td>ASD</td>
<td></td>
</tr>
<tr>
<td>\phi \leq 0.45</td>
<td>\phi R_n \geq V'_n</td>
<td>\Omega \leq 3.33</td>
</tr>
<tr>
<td>\phi R_n = 0.45 \left[ 2 \left( \frac{6.00 \text{ in.}}{2} \right)^2 + 2 \left( \frac{10.0 \text{ in.}}{2} \right)^2 \right] (2)(0.06 \text{ ksi})</td>
<td>\Omega = \beta \frac{R_n}{\Omega} \geq V'_n</td>
<td></td>
</tr>
<tr>
<td>\phi R_n = 14.7 \text{ kips} &lt; 52.8 \text{ kips}</td>
<td>\phi R_n / \Omega = \frac{3.33}{9.80 \text{ kips}} \geq 9.78 \text{ kips}</td>
<td>n.g.</td>
</tr>
</tbody>
</table>

Bond strength is inadequate and another force transfer mechanism such as direct bearing must be used to meet the load transfer provisions of AISC Specification Section I6.

Alternately, the detail could be revised so that the external force is applied to both the steel section and concrete fill concurrently as schematically illustrated in Figure I.3-1(c). Comparing bond strength to the load transfer requirements for concurrent loading determined in Part I of this example yields:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td>ASD</td>
<td></td>
</tr>
<tr>
<td>LRFD</td>
<td>ASD</td>
<td></td>
</tr>
<tr>
<td>\phi \leq 0.45</td>
<td>\phi R_n \geq V'_n</td>
<td>\Omega \leq 3.33</td>
</tr>
<tr>
<td>\phi R_n = 0.45 \left[ 2 \left( \frac{6.00 \text{ in.}}{2} \right)^2 + 2 \left( \frac{10.0 \text{ in.}}{2} \right)^2 \right] (2)(0.06 \text{ ksi})</td>
<td>\Omega = \beta \frac{R_n}{\Omega} \geq V'_n</td>
<td></td>
</tr>
<tr>
<td>\phi R_n = 14.7 \text{ kips} &gt; 14.6 \text{ kips}</td>
<td>\phi R_n / \Omega = \frac{9.80 \text{ kips}}{9.80 \text{ kips}} &gt; 9.78 \text{ kips}</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Case 2: Interior Condition – Load Transferred to Three Faces

For this case the composite member is loaded from three sides away from the end of the member (the composite member extends to both sides of the point of load transfer) as indicated in Figure I.3-4.
Longitudinal shear forces to be transferred at each face of the HSS are calculated using the relationship to external forces determined in Part I of the example for condition (a) shown in Figure I.3-1, and the applicable ASCE/SEI 7-10 load combinations as follows:

\[
\begin{align*}
\text{Face 1:} & \\
\ P_{11} &= P_u \\
\ &= 1.2(2.00 \text{ kips}) + 1.6(6.00 \text{ kips}) \\
\ &= 12.0 \text{ kips} \\
\ V'_{11} &= 0.305P_{11} \\
\ &= 0.305(12.0 \text{ kips}) \\
\ &= 3.66 \text{ kips} \\
\ \text{Faces 2 and 3:} & \\
\ P_{2-3} &= P_u \\
\ &= 1.2(15.0 \text{ kips}) + 1.6(39.0 \text{ kips}) \\
\ &= 80.4 \text{ kips} \\
\ V'_{2-3} &= 0.305P_{2-3} \\
\ &= 0.305(80.4 \text{ kips}) \\
\ &= 24.5 \text{ kips}
\end{align*}
\]

Load transfer at each face of the section is checked separately for the longitudinal shear at that face using Equation I6-5 as follows:

\[
\begin{align*}
\text{LRFD} & \\
\bar{\phi} &= 0.45 \\
\text{Face 1:} & \\
\phi R_{st} & \geq V'_{11} \\
\ &= 0.45(6.00 \text{ in.})^2(4)(0.06 \text{ ksi}) \\
\ &= 3.89 \text{ kips} \geq 3.66 \text{ kips} \quad \text{o.k.} \\
\text{SRFD} & \\
\Omega &= 3.33 \\
\text{Face 1:} & \\
\frac{R_{st}}{\Omega} & \geq V'_{11} \\
\ & = (6.00 \text{ in.)}^2(4)(0.06 \text{ ksi}) \\
\ & = 3.33 \\
\ & = 2.59 \text{ kips} \geq 2.44 \text{ kips} \quad \text{o.k.}
\end{align*}
\]
The calculations indicate that the bond strength is inadequate for two of the three loaded faces, thus an alternate means of load transfer such as the use of internal bearing plates as demonstrated previously in this example is necessary.

As demonstrated by this example, direct bond interaction provides limited available strength for transfer of longitudinal shears and is generally only acceptable for lightly loaded columns or columns with low shear transfer requirements such as those with loads applied to both concrete fill and steel encasement simultaneously.
EXAMPLE I.4 FILLED COMPOSITE MEMBER IN AXIAL COMPRESSION

Given:
Determine if the 14 ft long, filled composite member illustrated in Figure I.4-1 is adequate for the indicated dead and live loads.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft³) concrete fill having a specified concrete compressive strength, $f'_c = 5$ ksi.

![Composite Member Section and Applied Loading](image)

**Solution:**
From AISC Manual Table 2-4, the material properties are:

- ASTM A500 Grade B
  - $F_y = 46$ ksi
  - $F_u = 58$ ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

$$P_{ru} = 1.2(32.0 \text{ kips}) + 1.6(84.0 \text{ kips}) = 173 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_t = P_u$</td>
<td>$P_t = P_u$</td>
</tr>
<tr>
<td>$= 173 \text{ kips}$</td>
<td>$= 32.0 \text{ kips} + 84.0 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 116 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Method 1: AISC Manual Tables**

The most direct method of calculating the available compressive strength is through the use of AISC Manual Table 4-14. A $K$ factor of 1.0 is used for a pin-ended member. Because the unbraced length is the same in both the $x-x$ and $y-y$ directions, and $I_x$ exceeds $I_y$, $y-y$ axis buckling will govern.
Entering Table 4-14 with $KL_y = 14$ ft yields:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{u} / \phi P_{u}$</td>
<td>354 kips</td>
<td>$P_{u} / \phi P_{u}$</td>
</tr>
<tr>
<td>$P_{u} / \Omega_{x}$</td>
<td>$\geq 143$ kips</td>
<td>$P_{u} / \Omega_{x}$</td>
</tr>
<tr>
<td>$\phi P_{u}$</td>
<td>354 kips &gt; 173 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Method 2: AISC Specification Calculations**

As an alternate to the AISC Manual tables, the available compressive strength can be calculated directly using the provisions of AISC Specification Chapter I.

From AISC Manual Table 1-11 and Figure I.4-1, the geometric properties of an HSS 10×6×a are as follows:

- $A_s = 10.4 \text{ in.}^2$
- $H = 10.0 \text{ in.}$
- $B = 6.00 \text{ in.}$
- $t_{nom} = \frac{a}{6} \text{ in.} \quad \text{(nominal wall thickness)}$
- $t = 0.349 \text{ in.} \quad \text{(design wall thickness in accordance with AISC Specification Section B4.2)}$
- $h/t = 25.7$
- $b/t = 14.2$
- $I_{sx} = 137 \text{ in.}^4$
- $I_{sy} = 61.8 \text{ in.}^4$

Internal clear distances are determined as:

- $h_i = H - 2t$
  
  $= 10.0 \text{ in.} - 2(0.349 \text{ in.})$
  
  $= 9.30 \text{ in.}$

- $b_i = B - 2t$
  
  $= 6.0 \text{ in.} - 2(0.349 \text{ in.})$
  
  $= 5.30 \text{ in.}$

From Design Example I.3, the area of concrete, $A_c$, equals 49.2 in.$^2$. The steel and concrete areas can be used to calculate the gross cross-sectional area as follows:

- $A_g = A_s + A_c$
- $= 10.4 \text{ in.}^2 + 49.2 \text{ in.}^2$
- $= 59.6 \text{ in.}^2$

Calculate the concrete moment of inertia using geometry compatible with that used in the calculation of the steel area in AISC Manual Table 1-11 (taking into account the design wall thickness and corner radii of two times the design wall thickness in accordance with AISC Manual Part 1), the following equations may be used, based on the terminology given in Figure I-2 of the introduction to these examples:

For bending about the x-x axis:

$$I_{cx} = \frac{(B - 4t) h^3}{12} + \frac{t (H - 4t)^3}{6} + \left(\frac{9 \pi^2 - 64}{36 \pi}\right) t^4 + \pi t \left(\frac{H - 4t}{2} + \frac{4t}{3 \pi}\right)^2$$
\[
\begin{align*}
&= \frac{[6.00 \text{ in.} - 4(0.349 \text{ in.})]}{12} (9.30 \text{ in.})^3 + \frac{(0.349 \text{ in.})[10.0 \text{ in.} - 4(0.349 \text{ in.})]}{6} + \frac{(9\pi^2 - 64)(0.349 \text{ in.})^4}{36\pi} \\
&\quad + \pi (0.349 \text{ in.})^2 \left( \frac{10.0 \text{ in.} - 4(0.349 \text{ in.})}{2} + \frac{4(0.349 \text{ in.})}{3\pi} \right)^2 \\
&= 353 \text{ in.}^4
\end{align*}
\]

For bending about the y-y axis:
\[
I_{cy} = \frac{(H - 4t)h^3}{12} + \frac{t(B - 4t)}{6} + \frac{(9\pi^2 - 64)t^4}{36\pi} + \pi t^2 \left( \frac{B - 4t}{2} + \frac{4t}{3\pi} \right)^2 \\
= \frac{[10.0 \text{ in.} - 4(0.349 \text{ in.})]}{12} (5.30 \text{ in.})^3 + \frac{(0.349 \text{ in.})[6.00 \text{ in.} - 4(0.349 \text{ in.})]}{6} + \frac{(9\pi^2 - 64)(0.349 \text{ in.})^4}{36\pi} \\
\quad + \pi (0.349 \text{ in.})^2 \left( \frac{6.00 \text{ in.} - 4(0.349 \text{ in.})}{2} + \frac{4(0.349 \text{ in.})}{3\pi} \right)^2 \\
= 115 \text{ in.}^4
\]

Limitations of AISC Specification Sections I1.3 and I2.2a

1. Concrete Strength: \( f_c' \leq 10 \text{ ksi} \), \( f_c' = 5 \text{ ksi} \) \text{ o.k.}

2. Specified minimum yield stress of structural steel: \( F_y \leq 75 \text{ ksi} \), \( F_y = 46 \text{ ksi} \) \text{ o.k.}

3. Cross-sectional area of steel section: \( A_s \geq 0.01A_g \)
\[
10.4 \text{ in.}^2 \geq (0.01)(59.6 \text{ in.}^2) \\
> 0.596 \text{ in.}^2 \text{ o.k.}
\]

There are no minimum longitudinal reinforcement requirements in the AISC Specification within filled composite members; therefore, the area of reinforcing bars, \( A_{sr} \), for this example is zero.

Classify Section for Local Buckling

In order to determine the strength of the composite section subject to axial compression, the member is first classified as compact, noncompact or slender in accordance with AISC Specification Table I1.1A.

\[
\lambda_p = 2.26 \sqrt{\frac{E}{F_y}} \\
= 2.26 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \\
= 56.7
\]
\[
\lambda_{controlling} = \max \left( \frac{h}{t} = 25.7 \right) \\
= 25.7
\]
\[
\lambda_{controlling} \leq \lambda_p \text{ section is compact}
\]
Available Compressive Strength

The nominal axial compressive strength for compact sections without consideration of length effects, \( P_{no} \), is determined from AISC Specification Section I2.2b as:

\[
P_{no} = P_p \\
= F_a A_t + C_2 f_y \left( A_t + A_w \frac{E_s}{E_t} \right)
\]

where \( C_2 = 0.85 \) for rectangular sections

\[
P_{no} = (46 \text{ ksi})(10.4 \text{ in.}^2) + 0.85(5 \text{ ksi})(49.2 \text{ in.}^2 + 0.0 \text{ in.}^2)
\]

\[
= 688 \text{ kips}
\]

Because the unbraced length is the same in both the \( x-x \) and \( y-y \) directions, the column will buckle about the weaker \( y-y \) axis (the axis having the lower moment of inertia). \( I_y \) and \( I_x \) will therefore be used for calculation of length effects in accordance with AISC Specification Sections I2.2b and I2.1b as follows:

\[
C_3 = 0.6 + 2 \left( \frac{A_t}{A_t + A_w} \right) \leq 0.9
\]

\[
= 0.6 + 2 \left( \frac{10.4 \text{ in.}^2}{49.2 \text{ in.}^2 + 10.4 \text{ in.}^2} \right) \leq 0.9
\]

\[
= 0.949 > 0.9 \quad 0.9 \text{ controls}
\]

\[
E_c = w^{0.5} \sqrt{f_y^2}
\]

\[
= (145 \text{ lb/ft}^3)^{0.5} \sqrt{5 \text{ ksi}}
\]

\[
= 3,900 \text{ ksi}
\]

\[
EI_{eff} = E_s I_y + E_s I_y + C_3 E_s I_y
\]

\[
= (29,000 \text{ ksi})(61.8 \text{ in.}^4) + 0 + 0.9(3,900 \text{ ksi})(115 \text{ in.}^4)
\]

\[
= 2,200,000 \text{ kip-in.}^2
\]

\[
P_e = \pi^2 (EI_{eff}) / (KL)^2
\]

where \( K=1.0 \) for a pin-ended member

\[
P_e = \pi^2 \left( 2,200,000 \text{ kip-in.}^2 \right) \left[ \left[ (1.0)(14.0 \text{ ft})(12 \text{ in./ft}) \right]^2 \right]
\]

\[
= 769 \text{ kips}
\]

\[
P_{no} = \frac{688 \text{ kips}}{769 \text{ kips}} = 0.895 < 2.25
\]

Therefore, use AISC Specification Equation I2-2.

\[
P_n = P_{no} \left[ 0.658 \frac{P_n}{P_{no}} \right]^{0.895}
\]

\[
= (688 \text{ kips})(0.658)^{0.895}
\]

\[
= 473 \text{ kips}
\]
Check adequacy of the composite column for the required axial compressive strength:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c = 0.75$</td>
<td>$\Omega_c = 2.00$</td>
</tr>
<tr>
<td>$\phi_c P_n \geq P_n$</td>
<td>$P_n / \Omega_c \geq P_n$</td>
</tr>
<tr>
<td>$\phi_c P_s = 0.75(473 \text{ kips})$</td>
<td>$P_s / \Omega_c = \frac{473 \text{ kips}}{2.00}$</td>
</tr>
<tr>
<td>$= 355 \text{ kips} &gt; 173 \text{ kips}$</td>
<td>$= 237 \text{ kips} &gt; 116 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

The slight differences between these values and those tabulated in the AISC Manual are due to the number of significant digits carried through the calculations.

**Available Compressive Strength of Bare Steel Section**

Due to the differences in resistance and safety factors between composite and noncomposite column provisions, it is possible to calculate a lower available compressive strength for a composite column than one would calculate for the corresponding bare steel section. However, in accordance with AISC Specification Section I2.1b, the available compressive strength need not be less than that calculated for the bare steel member in accordance with Chapter E.

From AISC Manual Table 4-3, for an HSS10×6×3/8, $KL_y = 14.0$ ft:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c P_n = 313 \text{ kips}$</td>
<td>$P_n / \Omega_c = 208 \text{ kips}$</td>
</tr>
<tr>
<td>313 kips $&lt; 355$ kips</td>
<td>208 kips $&lt; 237$ kips</td>
</tr>
</tbody>
</table>

Thus, the composite section strength controls and is adequate for the required axial compressive strength as previously demonstrated.

**Force Allocation and Load Transfer**

Load transfer calculations for external axial forces should be performed in accordance with AISC Specification Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.
EXAMPLE I.5  FILLED COMPOSITE MEMBER IN AXIAL TENSION

Given:
Determine if the 14 ft long, filled composite member illustrated in Figure I.5-1 is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the steel section.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft³) concrete fill having a specified concrete compressive strength, \( f'_{c} = 5 \text{ ksi} \).

Solution:

From AISC Manual Table 2-4, the material properties are:
- ASTM A500 Grade B
  \( F_y = 46 \text{ ksi} \)
  \( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-11, the geometric properties are as follows:
- HSS10×6×\( b \frac{3}{8} \)
  \( A_s = 10.4 \text{ in.}^2 \)

There are no minimum requirements for longitudinal reinforcement in the AISC Specification; therefore it is common industry practice to use filled shapes without longitudinal reinforcement, thus \( A_{sr} = 0 \).

From Chapter 2 of ASCE/SEI 7, the required compressive strength is (taking compression as negative and tension as positive):
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<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Governing Uplift Load Combination = 0.9D + 1.0W</td>
<td>Governing Uplift Load Combination = 0.6D + 0.6W</td>
</tr>
<tr>
<td>( P_t = P_u )</td>
<td>( P_t = P_u )</td>
</tr>
<tr>
<td>( = 0.9(-32.0 \text{ kips}) + 1.0(100 \text{ kips}) )</td>
<td>( = 0.6(-32.0 \text{ kips}) + 0.6(100 \text{ kips}) )</td>
</tr>
<tr>
<td>( = 71.2 \text{ kips} )</td>
<td>( = 40.8 \text{ kips} )</td>
</tr>
</tbody>
</table>

Available Tensile Strength

Available tensile strength for a filled composite member is determined in accordance with AISC Specification Section I2.2c.

\[
P_n = A_s F_y + A_w F_{yw} \]

\[
= (10.4 \text{ in.}^2)(46 \text{ ksi}) + (0.0 \text{ in.}^2)(60 \text{ksi})
\]

\( = 478 \text{ kips} \)  \((Spec. \text{ Eq. I2-14)}\)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi P_u \geq P_u )</td>
<td>( P_t / \Omega \geq P_u )</td>
</tr>
</tbody>
</table>
| \( \phi P_n = 0.90(478 \text{ kips}) \) | \( P_t / \Omega = \frac{478 \text{ kips}}{1.67} \)
| \( = 430 \text{ kips} > 71.2 \text{ kips} \text{ o.k.} \) | \( = 286 \text{ kips} > 40.8 \text{ kips} \text{ o.k.} \)

For concrete filled HSS members with no internal longitudinal reinforcing, the values for available tensile strength may also be taken directly from AISC Manual Table 5-4.

Force Allocation and Load Transfer

Load transfer calculations are not required for concrete filled members in axial tension that do not contain longitudinal reinforcement, such as the one under investigation, as only the steel section resists tension.
EXAMPLE I.6 FILLED COMPOSITE MEMBER IN COMBINED AXIAL COMPRESSION, FLEXURE AND SHEAR

Given:

Determine if the 14 ft long, filled composite member illustrated in Figure I.6-1 is adequate for the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC Specification Chapter C for the controlling ASCE/SEI 7-10 load combinations.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft³) concrete fill having a specified concrete compressive strength, $f'_c = 5$ ksi.

Solution:

From AISC Manual Table 2-4, the material properties are:

- ASTM A500 Grade B
- $F_y = 46$ ksi
- $F_u = 58$ ksi

From AISC Manual Table 1-11 and Figure I.6-1, the geometric properties are as follows:

- HSS10×6×3/8
- $H = 10.0$ in.
- $B = 6.00$ in.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_r$ (kips)</td>
<td>129</td>
<td>98.2</td>
</tr>
<tr>
<td>$M_r$ (kip-ft)</td>
<td>120</td>
<td>54.0</td>
</tr>
<tr>
<td>$V_r$ (kips)</td>
<td>17.1</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Fig. I.6-1. Concrete filled member section and member forces.
\[ t_{\text{nom}} = \frac{3}{8} \text{ in. (nominal wall thickness)} \]
\[ t = 0.349 \text{ in. (design wall thickness)} \]
\[ h/t = 25.7 \]
\[ b/t = 14.2 \]
\[ A_s = 10.4 \text{ in.}^2 \]
\[ I_{sx} = 137 \text{ in.}^4 \]
\[ I_{sy} = 61.8 \text{ in.}^4 \]
\[ Z_{sx} = 33.8 \text{ in.}^3 \]

Additional geometric properties used for composite design are determined in Design Examples I.3 and I.4 as follows:

\[ h_i = 9.30 \text{ in.} \quad \text{clear distance between HSS walls (longer side)} \]
\[ b_i = 5.30 \text{ in.} \quad \text{clear distance between HSS walls (shorter side)} \]
\[ A_c = 49.2 \text{ in.}^2 \quad \text{cross-sectional area of concrete fill} \]
\[ A_g = 59.6 \text{ in.}^2 \quad \text{gross cross-sectional area of composite member} \]
\[ A_{sr} = 0 \text{ in.}^2 \quad \text{area of longitudinal reinforcement} \]
\[ E_c = 3,900 \text{ ksi} \quad \text{modulus of elasticity of concrete} \]
\[ I_{cx} = 353 \text{ in.}^4 \quad \text{moment of inertia of concrete fill about the} \text{-}\text{-} x-x \text{ axis} \]
\[ I_{cy} = 115 \text{ in.}^4 \quad \text{moment of inertia of concrete fill about the} \text{-}\text{-} y-y \text{ axis} \]

**Limitations of AISC Specification Sections I1.3 and I2.2a**

1. Concrete Strength: \[ 3 \text{ ksi} \leq f'_c \leq 10 \text{ ksi} \]
   \[ f'_c = 5 \text{ ksi} \quad \text{o.k.} \]

2. Specified minimum yield stress of structural steel: \[ F_y \leq 75 \text{ ksi} \]
   \[ F_y = 46 \text{ ksi} \quad \text{o.k.} \]

3. Cross-sectional area of steel section: \[ A_i \geq 0.01 A_g \]
   \[ 10.4 \text{ in.}^2 \geq (0.01)(59.6 \text{ in.}^2) \]
   \[ > 0.596 \text{ in.}^2 \quad \text{o.k.} \]

**Classify Section for Local Buckling**

The composite member in question was shown to be compact for pure compression in Design Example I.4 in accordance with AISC Specification Table I1.1a. The section must also be classified for local buckling due to flexure in accordance with Specification Table I1.1b; however, since the limits for members subject to flexure are equal to or less stringent than those for members subject to compression, the member is compact for flexure.

**Interaction of Axial Force and Flexure**

The interaction between axial forces and flexure in composite members is governed by AISC Specification Section I5 which, for compact members, permits the use of a strain compatibility method or plastic stress distribution method, with the option to use the interaction equations of Section H1.1.

The strain compatibility method is a generalized approach that allows for the construction of an interaction diagram based upon the same concepts used for reinforced concrete design. Application of the strain compatibility method is required for irregular/nonsymmetrical sections, and its general application may be found in reinforced concrete design texts and will not be discussed further here.

Plastic stress distribution methods are discussed in AISC Specification Commentary Section I5 which provides three acceptable procedures for filled members. The first procedure, Method 1, invokes the interaction equations of
Section H1. This is the only method applicable to sections with noncompact or slender elements. The second procedure, Method 2, involves the construction of a piecewise-linear interaction curve using the plastic strength equations provided in Figure I.1c located within the front matter of the Chapter I Design Examples. The third procedure, Method 2 – Simplified, is a reduction of the piecewise-linear interaction curve that allows for the use of less conservative interaction equations than those presented in Chapter H.

For this design example, each of the three applicable plastic stress distribution procedures are reviewed and compared.

**Method 1: Interaction Equations of Section H1**

The most direct and conservative method of assessing interaction effects is through the use of the interaction equations of AISC specification Section H1. For HSS shapes, both the available compressive and flexural strengths can be determined from Manual Table 4-14. In accordance with the direct analysis method, a $K$ factor of 1 is used. Because the unbraced length is the same in both the $x$- and $y$-directions, and $I_x$ exceeds $I_y$, $y$-axis buckling will govern for the compressive strength. Flexural strength is determined for the $x$-axis to resist the applied moment indicated in Figure I.6-1.

Entering Table 4-14 with $K L_y = 14$ ft yields:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_a = 354$ kips</td>
<td>$P_a / \Omega_e = 236$ kips</td>
</tr>
<tr>
<td>$\phi M_{ax} = 130$ kip-ft</td>
<td>$M_{ax} / \Omega_e = 86.6$ kip-ft</td>
</tr>
<tr>
<td>$\frac{P_f}{P_c} = \frac{P_u}{\phi P_a}$</td>
<td>$\frac{P_f}{P_c} = \frac{P_a}{\phi M_{ax}}$</td>
</tr>
<tr>
<td>= 129 kips</td>
<td>= 98.2 kips</td>
</tr>
<tr>
<td>354 kips</td>
<td>236 kips</td>
</tr>
<tr>
<td>0.364 ≥ 0.2</td>
<td>0.416 ≥ 0.2</td>
</tr>
</tbody>
</table>

Therefore, use AISC Specification Equation H1-1a.

Using LRFD methodology, Method 1 indicates that the section is inadequate for the applied loads. The designer can elect to choose a new section that passes the interaction check or re-analyze the current section using a less conservative design method such as Method 2. The use of Method 2 is illustrated in the following section.

**Method 2: Interaction Curves from the Plastic Stress Distribution Model**

The procedure for creating an interaction curve using the plastic stress distribution model is illustrated graphically in Figure I.6-2.
Referencing Figure I.6-2, the nominal strength interaction surface A, B, C, D, E is first determined using the equations of Figure I-1c found in the introduction of the Chapter I Design Examples. This curve is representative of the short column member strength without consideration of length effects. A slenderness reduction factor, \( \lambda \), is then calculated and applied to each point to create surface A', B', C', D', E'. The appropriate resistance or safety factors are then applied to create the design surface A'', B'', C'', D'', E''. Finally, the required axial and flexural strengths from the applicable load combinations of ASCE/SEI 7-10 are plotted on the design surface, and the member is acceptable for the applied loading if all points fall within the design surface. These steps are illustrated in detail by the following calculations.

**Step 1:** Construct nominal strength interaction surface A, B, C, D, E without length effects

Using the equations provided in Figure I-1c for bending about the x-x axis yields:

**Point A (pure axial compression):**

\[
P_a = f'cA_c + 0.85f'cA_e
\]

\[
= (46 \text{ ksi})(10.4 \text{ in.}^2) + 0.85(5 \text{ ksi})(49.2 \text{ in.}^2)
\]

\[
= 688 \text{ kips}
\]

\[
M_a = 0 \text{ kip-ft}
\]

**Point D (maximum nominal moment strength):**

\[
P_D = \frac{0.85f'cA_e}{2}
\]

\[
= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2}
\]

\[
= 105 \text{ kips}
\]

\[
Z_{ax} = 33.8 \text{ in.}^3
\]
\[ Z_c = \frac{bh_c^2}{4} - 0.192r_c^3 \quad \text{where} \quad r_c = t \]
\[ = \frac{(5.30 \text{ in.})(9.30 \text{ in.})^2}{4} - 0.192(0.349 \text{ in.})^3 \]
\[ = 115 \text{ in.}^3 \]
\[ M_D = F_y Z_{cs} + \frac{0.85 f'_c Z_c}{2} \]
\[ = (46 \text{ ksi})(33.8 \text{ in.}^3) + \frac{0.85(5 \text{ ksi})(115 \text{ in.}^3)}{2} \]
\[ = 1,800 \text{ kip-in.} \]
\[ = 150 \text{ kip-ft} \]

Point B (pure flexure):

\[ P_B = 0 \text{ kips} \]
\[ h_n = \frac{0.85 f'_c A_c}{2(0.85 f'_c h_n + 4tF_y)} \leq \frac{h_n}{2} \]
\[ = \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2[0.85(5 \text{ ksi})(5.30 \text{ in.}) + 4(0.349 \text{ in.})(46 \text{ ksi})]} \leq \frac{9.30 \text{ in.}}{2} \]
\[ = 1.21 \text{ in.} \leq 4.65 \text{ in.} \]
\[ = 1.21 \text{ in.} \]
\[ Z_{sn} = 2th_n^2 \]
\[ = 2(0.349 \text{ in.})(1.21 \text{ in.})^2 \]
\[ = 1.02 \text{ in.}^3 \]
\[ Z_{cn} = bh_n^2 \]
\[ = (5.30 \text{ in.})(1.21 \text{ in.})^2 \]
\[ = 7.76 \text{ in.}^3 \]
\[ M_B = M_D - F_y Z_{sn} - \frac{0.85 f'_c Z_{cn}}{2} \]
\[ = 1,800 \text{ kip-in.} - (46 \text{ ksi})(1.02 \text{ in.}^3) - \frac{0.85(5 \text{ ksi})(7.76 \text{ in.}^3)}{2} \]
\[ = 1,740 \text{ kip-in.} \]
\[ = 145 \text{ kip-ft} \]

Point C (intermediate point):

\[ P_C = 0.85 f'_c A_c \]
\[ = 0.85(5 \text{ ksi})(49.2 \text{ in.}^2) \]
\[ = 209 \text{ kips} \]
\[ M_C = M_B \]
\[ = 145 \text{ kip-ft} \]
Point E (optional):

Point E is an optional point that helps better define the interaction curve.

\[
h_E = \frac{h_e + H}{2}
\]

where \( h_e = 1.21 \) in. from Point B

\[
= \frac{1.21 \text{ in.} + 10.0 \text{ in.}}{2}
\]

\[
= 3.11 \text{ in.}
\]

\[
P_E = \frac{0.85 f'_{y} A_e}{2} + 0.85 f'_{y} h_E + 4 F_{y} t h_E
\]

\[
= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2} + 0.85(5 \text{ ksi})(5.30 \text{ in.})(3.11 \text{ in.}) + 4(46 \text{ ksi})(0.349 \text{ in.})(3.11 \text{ in.})
\]

\[
= 374 \text{ kips}
\]

\[
Z_{cE} = h_E
\]

\[
= 51.3 \text{ in.}^3
\]

\[
Z_{sE} = 2th_E^2
\]

\[
= 2(0.349 \text{ in.})(3.11 \text{ in.})^2
\]

\[
= 6.75 \text{ in.}^3
\]

\[
M_E = M_D - F_{y} Z_{sE} - \frac{0.85 f'_{y} Z_{cE}}{2}
\]

\[
= 1,800 \text{ kip-in.} - (46 \text{ ksi})(6.75 \text{ in.}^3) - \frac{0.85(5 \text{ ksi})(51.3 \text{ in.}^3)}{2}
\]

\[
= 1,380 \text{ kip-in.}
\]

\[
= \frac{12 \text{ in./ft}}{12 \text{ in./ft}}
\]

\[
= 115 \text{ kip-ft}
\]

The calculated points are plotted to construct the nominal strength interaction surface without length effects as depicted in Figure I.6-3.
Fig. I.6-3. Nominal strength interaction surface without length effects.

Step 2: Construct nominal strength interaction surface A', B', C', D', E' with length effects

The slenderness reduction factor, \( \lambda \), is calculated for Point A using AISC Specification Section I2.2 in accordance with Specification Commentary Section I5.

\[
P_{\text{no}} = P_t = 688 \text{ kips}
\]
\[
C_3 = 0.6 + 2 \left( \frac{A_s}{A_s + A_t} \right) \leq 0.9
\]
\[
= 0.6 + 2 \left( \frac{10.4 \text{ in}^2}{49.2 \text{ in}^2 + 10.4 \text{ in}^2} \right) \leq 0.9
\]
\[
= 0.949 > 0.9 \hspace{1cm} \text{0.9 controls}
\]
\[
EI_{\text{eff}} = E_s I_{s} + E_t I_{t} + C_3 E_s I_{cy}
\]
\[
= (29,000 \text{ ksi})(61.8 \text{ in}^4) + 0 + 0.9(3,900 \text{ ksi})(115 \text{ in}^4)
\]
\[
= 2,200,000 \text{ ksi}
\]
\[
P_e = \pi^2 \left( \frac{EI_{\text{eff}}}{KL} \right)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method}
\]
\[
= \pi^2 \left( \frac{2,200,000 \text{ ksi}}{(14.0 \text{ ft})(12 \text{ in./ft})} \right)^2
\]
\[
= 769 \text{ kips}
\]
\[
\frac{P_n}{P_e} = \frac{688 \text{ kips}}{769 \text{ kips}} = 0.895 < 2.25
\]

Use AISC Specification Equation I2-2.
\[ P_\lambda = P_{\infty} \left( 0.658 \frac{A}{I} \right)^{0.895} \]  

\[ = 688 \text{ kips} \times (0.658)^{0.895} \]
\[ = 473 \text{ kips} \]

\[ \lambda = \frac{P_\lambda}{P_{\infty}} \]
\[ = \frac{473 \text{ kips}}{688 \text{ kips}} \]
\[ = 0.688 \]

In accordance with AISC Specification Commentary Section I5, the same slenderness reduction is applied to each of the remaining points on the interaction surface as follows:

\[ P_x = \lambda P_a \]
\[ = 0.688 \times 688 \text{ kips} \]
\[ = 473 \text{ kips} \]

\[ P_y = \lambda P_b \]
\[ = 0.688 \times 0 \text{ kips} \]
\[ = 0 \text{ kips} \]

\[ P_c = \lambda P_c \]
\[ = 0.688 \times 209 \text{ kips} \]
\[ = 144 \text{ kips} \]

\[ P_d = \lambda P_d \]
\[ = 0.688 \times 105 \text{ kips} \]
\[ = 72.2 \text{ kips} \]

\[ P_e = \lambda P_e \]
\[ = 0.688 \times 374 \text{ kips} \]
\[ = 257 \text{ kips} \]

The modified axial strength values are plotted with the flexural strength values previously calculated to construct the nominal strength interaction surface including length effects. These values are superimposed on the nominal strength surface not including length effects for comparison purposes in Figure I.6-4.
Fig. I.6-4. Nominal strength interaction surfaces (with and without length effects).

Step 3: Construct design interaction surface $A^*$, $B^*$, $C^*$, $D^*$, $E^*$ and verify member adequacy

The final step in the Method 2 procedure is to reduce the interaction surface for design using the appropriate resistance or safety factors.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design compressive strength:</strong></td>
<td><strong>Allowable compressive strength:</strong></td>
</tr>
<tr>
<td>$\phi_c = 0.75$</td>
<td>$\Omega_x = 2.00$</td>
</tr>
<tr>
<td>$P_{c^*} = \phi_c P_c$</td>
<td>$P_{c^*} = P_c / \Omega_x$</td>
</tr>
<tr>
<td>where $X = A, B, C, D$ or $E$</td>
<td>where $X = A, B, C, D$ or $E$</td>
</tr>
<tr>
<td>$P_{c^*} = 0.75(473$ kips)</td>
<td>$P_{c^*} = 473$ kips / 2.00</td>
</tr>
<tr>
<td>= 355$ kips</td>
<td>= 237$ kips</td>
</tr>
<tr>
<td>$P_{y^*} = 0.75(0$ kips)</td>
<td>$P_{y^*} = 0$ kips / 2.00</td>
</tr>
<tr>
<td>= 0$ kips</td>
<td>= 0$ kips</td>
</tr>
<tr>
<td>$P_{z^*} = 0.75(144$ kips)</td>
<td>$P_{z^*} = 144$ kips / 2.00</td>
</tr>
<tr>
<td>= 108$ kips</td>
<td>= 72$ kips</td>
</tr>
<tr>
<td>$P_{y^*} = 0.75(72.2$ kips)</td>
<td>$P_{y^*} = 72.2$ kips / 2.00</td>
</tr>
<tr>
<td>= 54.2$ kips</td>
<td>= 36.1$ kips</td>
</tr>
<tr>
<td>$P_{z^*} = 0.75(257$ kips)</td>
<td>$P_{z^*} = 257$ kips / 2.00</td>
</tr>
<tr>
<td>= 193$ kips</td>
<td>= 129$ kips</td>
</tr>
<tr>
<td><strong>Design flexural strength:</strong></td>
<td><strong>Allowable flexural strength:</strong></td>
</tr>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$M_{x^*} = \phi_b M_x$</td>
<td>$M_{x^*} = M_x / \Omega_b$</td>
</tr>
<tr>
<td>where $X = A, B, C, D$ or $E$</td>
<td>where $X = A, B, C, D$ or $E$</td>
</tr>
</tbody>
</table>
The available strength values for each design method can now be plotted. These values are superimposed on the nominal strength surfaces (with and without length effects) previously calculated for comparison purposes in Figure I.6-5.

By plotting the required axial and flexural strength values determined for the governing load combinations on the available strength surfaces indicated in Figure I.6-5, it can be seen that both ASD \( (M_a, P_a) \) and LRFD \( (M_e, P_e) \) points lie within their respective design surfaces. The member in question is therefore adequate for the applied loads.

Designers should carefully review the proximity of the available strength values in relation to point \( D'' \) on Figure I.6-5 as it is possible for point \( D'' \) to fall outside of the nominal strength curve, thus resulting in an unsafe design. This possibility is discussed further in AISC Commentary Section I5 and is avoided through the use of Method 2 – Simplified as illustrated in the following section.

**Method 2: Simplified**

The simplified version of Method 2 involves the removal of points \( D'' \) and \( E' \) from the Method 2 interaction surface leaving only points \( A'', B'' \) and \( C'' \) as illustrated in the comparison of the two methods in Figure I.6-6.
Reducing the number of interaction points allows for a bilinear interaction check defined by AISC Specification Commentary Equations C-I5-1a and C-I5-1b to be performed. Using the available strength values previously calculated in conjunction with the Commentary equations, interaction ratios are determined as follows:

\[
\begin{align*}
LRFD & \\
\frac{P_r - P_a}{P_a - P_c} + \frac{M_e}{M_c} & \leq 1.0 \\
\frac{98.2 - 72}{72 - 54} + \frac{54}{54} & \leq 1.0 \\
& \leq 1.0 \\
& \text{Use AISC Specification Commentary Equation C-I5-1b.}\n\end{align*}
\]

\[
\begin{align*}
ASD & \\
\frac{P_r - P_a}{P_a - P_c} + \frac{M_e}{M_c} & \leq 1.0 \\
\frac{72.0 - 54}{54 - 46} + \frac{46}{46} & \leq 1.0 \\
& \leq 1.0 \\
& \text{Use AISC Specification Commentary Equation C-I5-1b.}\n\end{align*}
\]

Thus, the member is adequate for the applied loads.

**Comparison of Methods**

The composite member was found to be inadequate using Method 1—Chapter H interaction equations, but was found to be adequate using both Method 2 and Method 2—Simplified procedures. A comparison between the methods is most easily made by overlaying the design curves from each method as illustrated in Figure I.6-7 for LRFD design.
From Figure I.6-7, the conservative nature of the Chapter H interaction equations can be seen. Method 2 provides the highest available strength; however, the Method 2—Simplified procedure also provides a good representation of the complete design curve. By using Part 4 of the AISC Manual to determine the available strength of the composite member in compression and flexure (Points A* and B* respectively), the modest additional effort required to calculate the available compressive strength at Point C* can result in appreciable gains in member strength when using Method 2—Simplified as opposed to Method 1.

Available Shear Strength

AISC Specification Section I4.1 provides three methods for determining the available shear strength of a filled member: available shear strength of the steel section alone in accordance with Chapter G, available shear strength of the reinforced concrete portion alone per ACI 318, or available shear strength of the steel section plus the reinforcing steel ignoring the contribution of the concrete.

Available Shear Strength of Steel Section

From AISC Specification Section G5, the nominal shear strength, \( V_n \), of HSS members is determined using the provisions of Section G2.1(b) with \( k_v = 5 \). The provisions define the width of web resisting the shear force, \( h \), as the outside dimension minus three times the design wall thickness.

\[
h = H - 3t = 10.0 \text{ in.} - 3(0.349 \text{ in.}) = 8.95 \text{ in.}
\]

\[
A_w = 2ht = 2(8.95 \text{ in.})(0.349 \text{ in.}) = 6.25 \text{ in.}^2
\]

The slenderness value, \( h/t_w \), used to determine the web shear coefficient, \( C_w \), is provided in AISC Manual Table 1-11 as 25.7.

\[ C_v = 1.0 \]  

The nominal shear strength is calculated as:

\[ V_n = 0.6 F_v A_n C_v \]
\[ = 0.6 \left( 46 \text{ ksi} \right) \left( 6.25 \text{ in.}^2 \right) (1.0) \]
\[ = 173 \text{ kips} \]

The available shear strength of the steel section is:

\[ \frac{V_n}{\Omega_n} \geq V_u \]

\[ \phi V_n = 0.90 \left( 173 \text{ kips} \right) \]
\[ = 156 \text{ kips} > 17.1 \text{ kips} \textbf{ o.k.} \]

Available Shear Strength of the Reinforced Concrete

The available shear strength of the steel section alone has been shown to be sufficient, but the available shear strength of the concrete will be calculated for demonstration purposes. Considering that the member does not have longitudinal reinforcing, the method of shear strength calculation involving reinforced concrete is not valid; however, the design shear strength of the plain concrete using Chapter 22 of ACI 318 can be determined as follows:

\[ \phi = 0.60 \text{ for plain concrete design from ACI 318 Section 9.3.5} \]
\[ \lambda = 1.0 \text{ for normal weight concrete from ACI 318 Section 8.6.1} \]

\[ V_n = \left( \frac{4}{3} \right) \lambda \sqrt{f_{c} b_n h} \]
\[ b_n = b \]
\[ h = h \]

\[ V_n = \left( \frac{4}{3} \right) (1.0) \sqrt{5,000 \text{ psi} \times 5.30 \text{ in.} \times 9.30 \text{ in.}} \left( \frac{1 \text{ kip}}{1,000 \text{ lb}} \right) \]
\[ = 4.65 \text{ kips} \]
\[ \phi V_n = 0.60 \times 4.65 \text{ kips} \]
\[ = 2.79 \text{ kips} \]
\[ \phi V_n \geq V_u \]
\[ 2.79 \text{ kips} < 17.1 \text{ kips} \textbf{ n.g.} \]
As can be seen from this calculation, the shear resistance provided by plain concrete is small and the strength of the steel section alone is generally sufficient.

**Force Allocation and Load Transfer**

Load transfer calculations for applied axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.
EXAMPLE I.7  FILLED BOX COLUMN WITH NONCOMPACT/SLENDER ELEMENTS

Given:

Determine the required ASTM A36 plate thickness of the 30 ft long, composite box column illustrated in Figure I.7-1 to resist the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC Specification Chapter C for the controlling ASCE/SEI 7-10 load combinations. The core is composed of normal weight (145 lb/ft³) concrete fill having a specified concrete compressive strength, $f'_{c} = 7$ ksi.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_r$ (kips)</td>
<td>1,310</td>
</tr>
<tr>
<td>$M_r$ (kip-ft)</td>
<td>552</td>
</tr>
<tr>
<td>$V_r$ (kips)</td>
<td>36.8</td>
</tr>
</tbody>
</table>

Fig. I.7-1. Composite box column section and member forces.

Solution:

From AISC Manual Table 2-4, the material properties are:

ASTM A36

$F_y = 36$ ksi

$F_u = 58$ ksi

Trial Size 1 (Noncompact)

For ease of calculation the contribution of the plate extensions to the member strength will be ignored as illustrated by the analytical model in Figure I.7-1.
Trial Plate Thickness and Geometric Section Properties of the Composite Member

Select a trial plate thickness, \( t \), of \( \frac{3}{8} \) in. Note that the design wall thickness reduction of AISC Specification Section B4.2 applies only to electric-resistance-welded HSS members and does not apply to built-up sections such as the one under consideration.

The calculated geometric properties of the 30 in. by 30 in. steel box column are:

\[
\begin{align*}
B & = 30.0 \text{ in.} & A_g & = 900 \text{ in.}^2 & E_c & = w_c^{1.5} \sqrt{f_c'} \\
H & = 30.0 \text{ in.} & A_c & = 856 \text{ in.}^2 & = \left(145 \text{ lb/ft}^3\right)^{1.5} \sqrt{7 \text{ ksi}} \\
& & A_s & = 44.4 \text{ in.}^2 & = 4620 \text{ ksi} \\
b_i & = B - 2t = 29.25 \text{ in.} & A_i & = 29.25 \text{ in.} \\
h_i & = H - 2t = 29.25 \text{ in.} & b & = 29.25 \text{ in.} \\
I_{gs} & = BH^3 / 12 & I_{cs} & = b_i h_i^3 / 12 & I_{cs} & = I_{gs} - I_{cs} \\
& = 67,500 \text{ in.}^4 & & = 61,000 \text{ in.}^4 & & = 6,500 \text{ in.}^4
\end{align*}
\]

Limitations of AISC Specification Sections I1.3 and I2.2a

1. Concrete Strength: \( 3 \text{ ksi} \leq f_c' \leq 10 \text{ ksi} \)
   \( f_c' = 7 \text{ ksi} \) \textbf{o.k.}

2. Specified minimum yield stress of structural steel: \( F_y \leq 75 \text{ ksi} \)
   \( F_y = 36 \text{ ksi} \) \textbf{o.k.}

3. Cross-sectional area of steel section: \( A_s \geq 0.01A_g \)
   \( 44.4 \text{ in.}^2 \geq (0.01)(900 \text{ in.}^2) \)
   \( > 9.00 \text{ in.}^2 \) \textbf{o.k.}

Classify Section for Local Buckling

Classification of the section for local buckling is performed in accordance with AISC Specification Table I1.1A for compression and Table I1.1B for flexure. As noted in Specification Section I1.4, the definitions of width, depth and thickness used in the evaluation of slenderness are provided in Tables B4.1a and B4.1b.

For box columns, the widths of the stiffened compression elements used for slenderness checks, \( b_i \) and \( h_i \), are equal to the clear distances between the column walls, \( b \) and \( h \). The slenderness ratios are determined as follows:

\[
\lambda = \frac{b_i}{t} = \frac{h_i}{t} = \frac{29.25 \text{ in.}}{\frac{3}{8} \text{ in.}} = 78.0
\]

Classify section for local buckling in steel elements subject to axial compression from AISC Specification Table I1.1A:
\[ \lambda_p = 2.26 \sqrt{\frac{E}{F_y}} \]
\[ = 2.26 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} \]
\[ = 64.1 \]
\[ \lambda_r = 3.00 \sqrt{\frac{E}{F_y}} \]
\[ = 3.00 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} \]
\[ = 85.1 \]

\[ \lambda_p \leq \lambda \leq \lambda_r \]
\[ 64.1 \leq 78.0 \leq 85.1; \text{ therefore, the section is noncompact for compression.} \]

According to AISC Specification Section I1.4, if any side of the section in question is noncompact or slender, then the entire section is treated as noncompact or slender. For the square section under investigation; however, this distinction is unnecessary as all sides are equal in length.

Classification of the section for local buckling in elements subject to flexure is performed in accordance with AISC Specification Table I1.1B. Note that flanges and webs are treated separately; however, for the case of a square section only the most stringent limitations, those of the flange, need be applied. Noting that the flange limitations for bending are the same as those for compression,

\[ \lambda_p \leq \lambda \leq \lambda_r \]
\[ 64.1 \leq 78.0 \leq 85.1; \text{ therefore, the section is noncompact for flexure} \]

**Available Compressive Strength**

Compressive strength for noncompact filled members is determined in accordance with AISC Specification Section I2.2b(b).

\[ P_p = F_y A_v + C_2 f_v \left( A_i + A_w \frac{E_i}{E_v} \right) \text{ where } C_2 = 0.85 \text{ for rectangular sections} \]
\[ = (36 \text{ ksi})(44.4 \text{ in.}^2) + 0.85(7 \text{ ksi})(856 \text{ in.}^2 + 0) \]
\[ = 6,690 \text{ kips} \]

\[ P_y = F_y A_v + 0.7 f_v \left( A_i + A_w \frac{E_i}{E_v} \right) \]
\[ = (36 \text{ ksi})(44.4 \text{ in.}^2) + 0.7(7 \text{ ksi})(856 \text{ in.}^2 + 0) \]
\[ = 5,790 \text{ kips} \]

\[ P_{sw} = P_p - \frac{P_p - P_y}{(\lambda_r - \lambda_p)^2} (\lambda - \lambda_p)^2 \]
\[ = 6,690 \text{ kips} - \frac{6,690 \text{ kips} - 5,790 \text{ kips}}{(85.1 - 64.1)^2} (78.0 - 64.1)^2 \]
\[ = 6,300 \text{ kips} \]
\[ C_3 = 0.6 + 2 \left( \frac{A_i}{A_i + A_r} \right) \leq 0.9 \]  
\[ = 0.6 + 2 \left( \frac{44.4 \text{ in.}^2}{856 \text{ in.}^2 + 44.4 \text{ in.}^2} \right) \leq 0.9 \]  
\[ = 0.699 \leq 0.9 \]  
\[ E_{I_{\text{eff}}} = E_i I_i + E_r I_r + C_3 E_i I_c \]  
\[ = \left( 29,000 \text{ ksi} \right) \left( 6,500 \text{ in.}^4 \right) + 0.0 + 0.699 \left( 4,620 \text{ ksi} \right) \left( 61,000 \text{ in.}^4 \right) \]  
\[ = 385,000,000 \text{ ksi} \]  
\[ P_r = \pi^2 \left( E_{I_{\text{eff}}} \right) / (KL)^2 \]  
\[ \text{where } K = 1.0 \text{ in accordance with the direct analysis method} \]  
\[ = \pi^2 \left( 385,000,000 \text{ ksi} \right) / \left[ (30.0 \text{ ft}) (12 \text{ in./ft}) \right]^2 \]  
\[ = 29,300 \text{ kips} \]  
\[ P_{\text{max}} = 6,300 \text{ kips} \]  
\[ P_r / P_{\text{max}} = 0.215 < 2.25 \]  

Therefore, use AISC Specification Equation I2-2.  
\[ P_a = P_{\text{max}} \left( 0.658 \frac{K}{\pi} \right) \]  
\[ = 6,300 \text{ kips} \left( 0.658 \right)^{0.215} \]  
\[ = 5,760 \text{ kips} \]

According to AISC Specification Section I2.2b, the compression strength need not be less than that specified for the bare steel member as determined by Specification Chapter E. It can be shown that the compression strength of the bare steel for this section is equal to 955 kips, thus the strength of the composite section controls.

The available compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c )</td>
<td>0.75</td>
<td>( \Omega_c ) = 2.00</td>
</tr>
<tr>
<td>( \phi_r P_a )</td>
<td>0.75(5,760 kips)</td>
<td>( P_r / \Omega_c = 5,760 \text{ kips} / 2.00 )</td>
</tr>
<tr>
<td></td>
<td>4,320 kips</td>
<td>( = 2,880 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Available Flexural Strength**

Flexural strength of noncompact filled composite members is determined in accordance with AISC Specification Section I3.4b(b):

\[ M_a = M_p - \left( M_p - M_y \right) \left( \lambda - \lambda_p \right) / \left( \lambda_{cr} - \lambda_p \right) \]  
\[ \text{(Spec. Eq. I3-3b)} \]

In order to utilize Equation I3-3b, both the plastic moment strength of the section, \( M_p \), and the yield moment strength of the section, \( M_y \), must be calculated.
Plastic Moment Strength

The first step in determining the available flexural strength of a noncompact section is to calculate the moment corresponding to the plastic stress distribution over the composite cross section. This concept is illustrated graphically in AISC Specification Commentary Figure C-I3.7(a) and follows the force distribution depicted in Figure I.7-2 and detailed in Table I.7-1.

**Figure I.7-2. Plastic moment stress blocks and force distribution.**

<table>
<thead>
<tr>
<th>Table I.7-1. Plastic Moment Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Component</strong></td>
</tr>
<tr>
<td>Compression in steel flange</td>
</tr>
<tr>
<td>Compression in concrete</td>
</tr>
<tr>
<td>Compression in steel web</td>
</tr>
<tr>
<td>Tension in steel web</td>
</tr>
<tr>
<td>Tension in steel flange</td>
</tr>
</tbody>
</table>

where:

$$a_p = \frac{2 F_y H t_w + 0.85 f'_c b t_f}{4 t_w F_y + 0.85 f'_c b_t}$$

$$M_p = \sum (\text{force })(\text{moment arm})$$

Using the equations provided in Table I.7-1 for the section in question results in the following:

$$a_p = \frac{2(36 \text{ ksi})(30.0 \text{ in.})(\% \text{ in.}) + 0.85(7 \text{ ksi})(29.25 \text{ in.})(\% \text{ in.})}{4(\% \text{ in.})(36 \text{ ksi}) + 0.85(7 \text{ ksi})(29.25 \text{ in.})}$$

$$= 3.84 \text{ in.}$$
Yield Moment Strength

The next step in determining the available flexural strength of a noncompact filled member is to determine the yield moment strength. The yield moment is defined in AISC Specification Section I3.4b(b) as the moment corresponding to first yield of the compression flange calculated using a linear elastic stress distribution with a maximum concrete compressive stress of $0.7f_c'$. This concept is illustrated diagrammatically in Specification Commentary Figure C-I3.7(b) and follows the force distribution depicted in Figure I.7-3 and detailed in Table I.7-2.

![Figure I.7-3. Yield moment stress blocks and force distribution.](image-url)
Table I.7-2. Yield Moment Equations

<table>
<thead>
<tr>
<th>Component</th>
<th>Force</th>
<th>Moment Arm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression in steel flange</td>
<td>$C_1 = b_t f_y$</td>
<td>$y_{c1} = a_y - \frac{t_y}{2}$</td>
</tr>
<tr>
<td>Compression in concrete</td>
<td>$C_2 = 0.35 f'_y (a_y - t_y) b_t$</td>
<td>$y_{c2} = \frac{2(a_y - t_y)}{3}$</td>
</tr>
<tr>
<td>Compression in steel web</td>
<td>$C_3 = a_y 2t_w 0.5F_y$</td>
<td>$y_{c3} = \frac{2a_y}{3}$</td>
</tr>
<tr>
<td>Tension in steel web</td>
<td>$T_1 = a_y 2t_w 0.5F_y$</td>
<td>$y_{r1} = \frac{2a_y}{3}$</td>
</tr>
<tr>
<td></td>
<td>$T_2 = (H - 2a_y) 2t_w F_y$</td>
<td>$y_{r2} = \frac{H}{2}$</td>
</tr>
<tr>
<td>Tension in steel flange</td>
<td>$T_3 = b_t f_y$</td>
<td>$y_{r3} = H - a_y - \frac{t_y}{2}$</td>
</tr>
</tbody>
</table>

where:

$\frac{a_y}{a_y} = \frac{2F_y H t_w + 0.35 f'_y b_t}{4t_w F_y + 0.35 f'_y b_t}$

$M_y = \sum \text{(force)(moment arm)}$

Using the equations provided in Table I.7-2 for the section in question results in the following:

$a_y = \frac{2(36 \text{ ksi})(30.0 \text{ in.})(\frac{1}{8} \text{ in.}) + 0.35(7 \text{ ksi})(29.25 \text{ in.})(\frac{1}{8} \text{ in.})}{4(\frac{1}{8} \text{ in.})(36 \text{ ksi}) + 0.35(7 \text{ ksi})(29.25 \text{ in.})}$

$= 6.66 \text{ in.}$

<table>
<thead>
<tr>
<th>Force</th>
<th>Moment Arm</th>
<th>Force × Moment Arm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1 = (29.25 \text{ in.})(\frac{1}{8} \text{ in.})(36 \text{ ksi})$</td>
<td>$y_{c1} = 6.66 \text{ in.} - \frac{\frac{1}{8} \text{ in.}}{2}$</td>
<td>$C_1 y_{c1} = 2,560 \text{ kip-in.}$</td>
</tr>
<tr>
<td>= 395 kips</td>
<td>$y_{c1} = 6.47 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$C_2 = 0.35(7 \text{ ksi})(6.66\text{ in.} - \frac{1}{8} \text{ in.})(29.25 \text{ in.})$</td>
<td>$y_{c2} = \frac{2(6.66 \text{ in.} - \frac{1}{8} \text{ in.})}{3}$</td>
<td>$C_2 y_{c2} = 1,890 \text{ kip-in.}$</td>
</tr>
<tr>
<td>= 450 kips</td>
<td>$y_{c2} = 4.19 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$C_3 = (6.66 \text{ in.})(2)(\frac{1}{8} \text{ in.})(0.5)(36 \text{ ksi})$</td>
<td>$y_{c3} = \frac{2(6.66 \text{ in.})}{3}$</td>
<td>$C_3 y_{c3} = 399 \text{ kip-in.}$</td>
</tr>
<tr>
<td>= 89.9 kips</td>
<td>$y_{c3} = 4.44 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$T_1 = (6.66 \text{ in.})(2)(\frac{1}{8} \text{ in.})(0.5)(36 \text{ ksi})$</td>
<td>$y_{r1} = \frac{2(6.66 \text{ in.})}{3}$</td>
<td>$T_1 y_{r1} = 399 \text{ kip-in.}$</td>
</tr>
<tr>
<td>= 89.9 kips</td>
<td>$y_{r1} = 4.44 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$T_2 = <a href="2">30.0 - 2(6.66 \text{ in.})</a>(\frac{1}{8} \text{ in.})(36 \text{ ksi})$</td>
<td>$y_{r2} = \frac{30.0 \text{ in.}}{2}$</td>
<td>$T_2 y_{r2} = 6,750 \text{ kip-in.}$</td>
</tr>
<tr>
<td>= 450 kips</td>
<td>$y_{r2} = 15.0 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$T_3 = (29.25 \text{ in.})(\frac{1}{8} \text{ in.})(36 \text{ ksi})$</td>
<td>$y_{r3} = 30.0 \text{ in.} - 6.66 \text{ in.} - \frac{\frac{1}{8} \text{ in.}}{2}$</td>
<td>$T_3 y_{r3} = 9,160 \text{ kip-in.}$</td>
</tr>
<tr>
<td>= 395 kips</td>
<td>$y_{r3} = 23.2 \text{ in.}$</td>
<td></td>
</tr>
</tbody>
</table>
\[ M_y = \sum \text{(force)} \times \text{(moment arm)} \]
\[ = \frac{2,560 \text{ kip-in.} + 1,890 \text{ kip-in.} + 399 \text{ kip-in.} + 399 \text{ kip-in.} + 6,750 \text{ kip-in.} + 9,160 \text{ kip-in.}}{12 \text{ in./ft}} \]
\[ = 1,760 \text{ kip-ft} \]

Now that both \( M_p \) and \( M_y \) have been determined, Equation I3-3b may be used in conjunction with the flexural slenderness values previously calculated to determine the nominal flexural strength of the composite section as follows:

\[ M_s = M_p - \left( M_p - M_y \right) \left( \frac{\lambda_c - \lambda_p}{\lambda_c - \lambda_p} \right) \]

\[ = 1,850 \text{ kip-ft} - \left( 1,850 \text{ kip-ft} - 1,760 \text{ kip-ft} \right) \left( \frac{78.0 - 64.1}{85.1 - 64.1} \right) \]
\[ = 1,790 \text{ kip-ft} \]

The available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_s = 0.90 (1,790 \text{ kip-ft}) )</td>
<td>( M_u / \Omega_b = 1,790 \text{ kip-ft} / 1.67 )</td>
</tr>
<tr>
<td>= 1,610 kip-ft</td>
<td>= 1,070 kip-ft</td>
</tr>
</tbody>
</table>

**Interaction of Flexure and Compression**

Design of members for combined forces is performed in accordance with AISC *Specification* Section I5. For filled composite members with noncompact or slender sections, interaction is determined in accordance with Section H1.1 as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_c = 1,310 \text{ kips} )</td>
<td>( P_c = 1,370 \text{ kips} )</td>
</tr>
<tr>
<td>( M_u = 552 \text{ kip-ft} )</td>
<td>( M_u = 248 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( \frac{P_c}{P_c} = \frac{P_e}{\phi_b P_c} )</td>
<td>( \frac{P_c}{P_c} = \frac{P_e}{\phi_b P_c} )</td>
</tr>
<tr>
<td>( = 1,310 \text{ kips} )</td>
<td>( = 1,370 \text{ kips} )</td>
</tr>
<tr>
<td>( = 4,320 \text{ kips} )</td>
<td>( = 2,880 \text{ kips} )</td>
</tr>
<tr>
<td>( = 0.303 \geq 0.2 )</td>
<td>( = 0.476 \geq 0.2 )</td>
</tr>
</tbody>
</table>

Use *Specification* Equation H1-1a.

\[ \frac{P_e}{\phi_b P_c} + \frac{8}{9} \left( \frac{M_u}{\phi_b M_u} \right) \leq 1.0 \]
\[ = \frac{1,310 \text{ kips}}{4,320 \text{ kips}} + \frac{8}{9} \left( \frac{552 \text{ kip-ft}}{1,610 \text{ kip-ft}} \right) \leq 1.0 \]
\[ 0.608 < 1.0 \text{ o.k.} \]
The composite section is adequate; however, as there is available strength remaining for the trial plate thickness chosen, re-analyze the section to determine the adequacy of a reduced plate thickness.

**Trial Size 2 (Slender)**

The calculated geometric section properties using a reduced plate thickness of \( t = \frac{1}{4} \) in. are:

\[
B = 30.0 \text{ in.} \quad A_g = 900 \text{ in}^2 \quad E_c = \nu c^{1.5} \left( \frac{f_c'}{1.5} \right)^{1.5} \sqrt{1 \text{ ksi}}
\]

\[
H = 30.0 \text{ in.} \quad A_e = 870 \text{ in}^2
\]

\[
b_t = B - 2t = 29.50 \text{ in.} \quad A_e = 29.8 \text{ in}^2
\]

\[
h_t = H - 2t = 29.50 \text{ in.}
\]

\[
I_{gg} = BH^3 / 12 \quad I_{es} = b_t h_t^3 / 12 \quad I_{ex} = I_{gg} - I_{es}
\]

\[
= 67,500 \text{ in}^4 \quad = 63,100 \text{ in}^4 \quad = 4,400 \text{ in}^4
\]

**Limitations of AISC Specification Sections I1.3 and I2.2a**

1. Concrete Strength: \( 3 \text{ ksi} \leq f_c' \leq 10 \text{ ksi} \)
   \( f_c' = 7 \text{ ksi} \) \textbf{o.k.}

2. Specified minimum yield stress of structural steel: \( F_y \leq 75 \text{ ksi} \)
   \( F_y = 36 \text{ ksi} \) \textbf{o.k.}

3. Cross sectional area of steel section: \( A_e \geq 0.01A_g \)
   \( 29.8 \text{ in}^2 \geq 0.01 \left( 900 \text{ in}^2 \right) \)
   \( > 9.00 \text{ in}^2 \) \textbf{o.k.}

**Classify Section for Local Buckling**

As noted previously, the definitions of width, depth and thickness used in the evaluation of slenderness are provided in AISC Specification Tables B4.1a and B4.1b.

For a box column, the slenderness ratio is determined as the ratio of clear distance to wall thickness:

\[
\lambda = \frac{b_t}{t} = \frac{h_t}{t}
\]

\[
= \frac{29.5}{\frac{1}{4}} \text{ in.} = 118
\]

Classify section for local buckling in steel elements subject to axial compression from AISC Specification Table I1.1A. As determined previously, \( \lambda_c = 85.1 \).
\[
\lambda_{max} = 5.00 \sqrt{\frac{E}{F_s'}} \\
= 5.00 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} \\
= 142 \\
\lambda_s \leq \lambda \leq \lambda_{max} \\
85.1 \leq 118 \leq 142; \text{ therefore, the section is slender for compression}
\]

Classification of the section for local buckling in elements subject to flexure occurs separately per AISC Specification Table I1.1B. Because the flange limitations for bending are the same as those for compression,

\[
\lambda_s \leq \lambda \leq \lambda_{max} \\
85.1 \leq 118 \leq 142; \text{ therefore, the section is slender for flexure}
\]

**Available Compressive Strength**

Compressive strength for a slender filled member is determined in accordance with AISC Specification Section I2.2b(c).

\[
F_{cr} = \frac{9E_s}{\left(\frac{b}{t}\right)^2} \\
= \frac{9(29,000 \text{ ksi})}{(118)^2} \\
= 18.7 \text{ ksi}
\]

\[
P_{cr} = F_{cr}A_s + 0.7f'c\left(A_s + A_w\frac{E_s}{E_c}\right) \\
= (18.7 \text{ ksi})(29.8 \text{ in.}^2) + 0.7(7 \text{ ksi})(870 \text{ in.}^2 + 0) \\
= 4,820 \text{ kips}
\]

\[
C_3 = 0.6 + 2\left(\frac{A_i}{A_s + A_j}\right) \leq 0.9 \\
= 0.6 + 2\left(\frac{29.8 \text{ in.}^2}{870 \text{ in.}^2 + 29.8 \text{ in.}^2}\right) \leq 0.9 \\
= 0.666 < 0.9
\]

\[
EI_{eff} = E_sI_s + E_wI_w + C_3E_cI_c \\
= (29,000 \text{ ksi})(4,400 \text{ in.}^4) + 0.666(4,620 \text{ ksi})(63,100 \text{ in.}^4) \\
= 322,000,000 \text{ ksi}
\]

\[
P_s = \pi^2 \left(\frac{EI_{eff}}{KL}\right)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method} \\
= \pi^2 \left(\frac{322,000,000 \text{ ksi}}{(30.0 \text{ ft})(12 \text{ in./ft})}\right)^2 \\
= 24,500 \text{ kips}
\]
\[ \frac{P_{\text{we}}}{P_c} = \frac{4,820 \text{ kips}}{24,500 \text{ kips}} = 0.197 < 2.25 \]

Therefore, use AISC Specification Equation I2-2.

\[ P_u = P_{\text{we}} \left( \frac{P_{\text{we}}}{P_c} \right)^{0.658} \]

\[ = 4,820 \text{ kips} \times (0.658)^{0.197} \]

\[ = 4,440 \text{ kips} \]

According to AISC Specification Section I2.2b the compression strength need not be less than that determined for the bare steel member using Specification Chapter E. It can be shown that the compression strength of the bare steel for this section is equal to 450 kips, thus the strength of the composite section controls.

The available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_n ) = 0.75</td>
<td>( \Omega_n ) = 2.00</td>
</tr>
<tr>
<td>( \phi_n \cdot P_n = 0.75 \times 4,440 \text{ kips} )</td>
<td>( P_n / \Omega_n = 4,440 \text{ kips} / 2.00 )</td>
</tr>
<tr>
<td>= 3,330 kips</td>
<td>= 2,220 kips</td>
</tr>
</tbody>
</table>

**Available Flexural Strength**

Flexural strength of slender filled composite members is determined in accordance with AISC Specification Section I3.4b(c). The nominal flexural strength is determined as the first yield moment, \( M_{\text{cr}} \), corresponding to a flange compression stress of \( F_{\text{cr}} \) using a linear elastic stress distribution with a maximum concrete compressive stress of \( 0.7f'_c \). This concept is illustrated diagrammatically in Specification Commentary Figure C-I3.7(c) and follows the force distribution depicted in Figure I.7-4 and detailed in Table I.7-3.
### Table I.7-3. First Yield Moment Equations

<table>
<thead>
<tr>
<th>Component</th>
<th>Force</th>
<th>Moment arm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression in steel flange</td>
<td>$C_1 = b_t f_{cr}$</td>
<td>$y_{c1} = a_{cr} - \frac{t_f}{2}$</td>
</tr>
<tr>
<td>Compression in concrete</td>
<td>$C_2 = 0.35 f'<em>c (a</em>{cr} - t_f) b_t$</td>
<td>$y_{c2} = \frac{2(a_{cr} - t_f)}{3}$</td>
</tr>
<tr>
<td>Compression in steel web</td>
<td>$C_3 = a_{cr} 2t_o 0.5F_y$</td>
<td>$y_{c3} = \frac{2a_{cr}}{3}$</td>
</tr>
<tr>
<td>Tension in steel web</td>
<td>$T_t = (H - a_{cr}) 2t_o 0.5F_y$</td>
<td>$y_{t1} = \frac{2(H - a_{cr})}{3}$</td>
</tr>
<tr>
<td>Tension in steel flange</td>
<td>$T_2 = b_t F_y$</td>
<td>$y_{t2} = H - a_{cr} - \frac{t_f}{2}$</td>
</tr>
</tbody>
</table>

where:

$$a_{cr} = \frac{F_c H_{cr} + (0.35 f'_c + F_y - F_{cr}) b_t t_f}{t_o (F_{cr} + F_y) + 0.35 f'_c b_t}$$

$$M_{cr} = \sum (\text{force} \times (\text{moment arm})$$

Using the equations provided in Table I.7-3 for the section in question results in the following:

$$a_{cr} = \frac{(36 \text{ ksi})(30.0 \text{ in.})(\frac{1}{4} \text{ in.}) + [(0.35 (7 ksi) + 36 ksi - 18.7 ksi) \times (29.5 \text{ in.})(\frac{1}{4} \text{ in.})]}{(\frac{1}{4} \text{ in.})(18.7 ksi + 36 ksi) + 0.35 (7 ksi)(29.5 \text{ in.})}$$

$$= 4.84 \text{ in.}$$

<table>
<thead>
<tr>
<th>Force</th>
<th>Moment Arm</th>
<th>Force x Moment Arm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1 = (29.5 \text{ in.})(\frac{1}{4} \text{ in.})(18.7 \text{ ksi})$</td>
<td>$y_{c1} = 4.84 \text{ in.} - \frac{\frac{1}{4} \text{ in.}}{2}$</td>
<td>$C_1 y_{c1} = 651 \text{ kip-in.}$</td>
</tr>
<tr>
<td>$= 138 \text{ kips}$</td>
<td>$= 4.72 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$C_2 = 0.35(7 ksi)(4.84 \text{ in.} - \frac{1}{4} \text{ in.})(29.5 \text{ in.})$</td>
<td>$y_{c2} = \frac{2(4.84 \text{ in.} - \frac{1}{4} \text{ in.})}{3}$</td>
<td>$C_2 y_{c2} = 1,020 \text{ kip-in.}$</td>
</tr>
<tr>
<td>$= 332 \text{ kips}$</td>
<td>$= 3.06 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$C_3 = (4.84 \text{ in.})(2)(\frac{1}{4} \text{ in.})(0.5)(18.7 \text{ ksi})$</td>
<td>$y_{c3} = \frac{2(4.84 \text{ in.})}{3}$</td>
<td>$C_3 y_{c3} = 73.0 \text{ kip-in.}$</td>
</tr>
<tr>
<td>$= 22.6 \text{ kips}$</td>
<td>$= 3.23 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$T_t = (30.0 \text{ in.} - 4.84 \text{ in.})(2)(\frac{1}{4} \text{ in.})(0.5)(36 \text{ ksi})$</td>
<td>$y_{t1} = \frac{2(30.0 \text{ in.} - 4.84 \text{ in.})}{3}$</td>
<td>$T_t y_{t1} = 3,800 \text{ kip-in.}$</td>
</tr>
<tr>
<td>$= 226 \text{ kips}$</td>
<td>$= 16.8 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>$T_2 = (29.5 \text{ in.})(\frac{1}{4} \text{ in.})(36 \text{ ksi})$</td>
<td>$y_{t2} = 30.0 \text{ in.} - 4.84 \text{ in.} - \frac{\frac{1}{4} \text{ in.}}{2}$</td>
<td>$T_2 y_{t2} = 6,650 \text{ kip-in.}$</td>
</tr>
<tr>
<td>$= 266 \text{ kips}$</td>
<td>$= 25.0 \text{ in.}$</td>
<td></td>
</tr>
</tbody>
</table>

$$M_{cr} = \sum (\text{force component}) (\text{moment arm})$$

$$= \frac{651 \text{ kip-in.} + 1,020 \text{ kip-in.} + 73.0 \text{ kip-in.} + 3,800 \text{ kip-in.} + 6,650 \text{ kip-in.}}{12 \text{ in./ft}}$$

$$= 1,020 \text{ kip-ft}$$
The available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b$</td>
<td>$0.90$</td>
<td>$\Omega_b$</td>
</tr>
<tr>
<td>$M_n$</td>
<td>$0.90(1,020 \text{ kip-ft})$</td>
<td>$M_n/\Omega_b = 1,020 \text{ kip-ft}/1.67$</td>
</tr>
<tr>
<td></td>
<td>$= 918 \text{ kip-ft}$</td>
<td>$= 611 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Interaction of Flexure and Compression

The interaction of flexure and compression is determined in accordance with AISC Specification Section H1.1 as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>$1,310 \text{ kips}$</td>
<td>$P_u = 1,370 \text{ kips}$</td>
</tr>
<tr>
<td>$M_u$</td>
<td>$552 \text{ kip-ft}$</td>
<td>$M_u = 248 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>$P_u$</td>
<td>$P_t = P_u/\Omega$</td>
</tr>
<tr>
<td>$\phi_P$</td>
<td>$\phi_P = P_u$</td>
<td></td>
</tr>
<tr>
<td>$P_t/\phi_P$</td>
<td>$= 1,310 \text{ kips}$</td>
<td>$= 1,370 \text{ kips}$</td>
</tr>
<tr>
<td>$P_t/\phi_P$</td>
<td>$= 3,330 \text{ kips}$</td>
<td>$= 2,220 \text{ kips}$</td>
</tr>
<tr>
<td>$= 0.393 \geq 0.2$</td>
<td>$= 0.617 \geq 0.2$</td>
<td></td>
</tr>
</tbody>
</table>

Use AISC Specification Equation H1-1a.

Thus, a plate thickness of $\frac{1}{4}$ in. is adequate.

Note that in addition to the design checks performed for the composite condition, design checks for other load stages should be performed as required by AISC Specification Section I1. These checks should take into account the effect of hydrostatic loads from concrete placement as well as the strength of the steel section alone prior to composite action.

Available Shear Strength

According to AISC Specification Section I4.1 there are three acceptable methods for determining the available shear strength of the member: available shear strength of the steel section alone in accordance with Chapter G, available shear strength of the reinforced concrete portion alone per ACI 318, or available shear strength of the steel section in addition to the reinforcing steel ignoring the contribution of the concrete. Considering that the member in question does not have longitudinal reinforcing, it is determined by inspection that the shear strength will be controlled by the steel section alone using the provisions of Chapter G.

From AISC Specification Section G5 the nominal shear strength, $V_n$, of box members is determined using the provisions of Section G2.1 with $k_v = 5$. As opposed to HSS sections which require the use of a reduced web area to take into account the corner radii, the full web area of a box section may be used as follows:
\[ A_v = 2dt_w \] where \( d = \) full depth of section parallel to the required shear force
\[ = 2(30.0 \text{ in.})(\frac{1}{4} \text{ in.}) \\
= 15.0 \text{ in.}^2 \]

The slenderness value, \( h/t_w \), for the web used in Specification Section G2.1(b) is the same as that calculated previously for use in local buckling classification, \( \lambda = 118 \).

\[ \frac{h}{t_w} > 1.37 \sqrt{k_c E / F_y} \]
\[ = 1.37 \sqrt{5 \left( \frac{29,000 \text{ ksi}}{36 \text{ ksi}} \right)} \\
118 > 86.9 \]

Therefore, use AISC Specification Equation G2-5.

The web shear coefficient and nominal shear strength are calculated as:

\[ C_v = \frac{1.51 k_c E}{(h/t_w)^2 F_y} \] (Spec. Eq. G2-5)
\[ = \frac{1.51(5)(29,000 \text{ ksi})}{(118)^2(36 \text{ ksi})} \\
= 0.437 \]
\[ V_n = 0.6 F_y A_w C_v \] (Spec. Eq. G2-1)
\[ = 0.6(36 \text{ ksi})(15.0 \text{ in.}^2)(0.437) \\
= 142 \text{ kips} \]

The available shear strength is checked as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_a = 36.8 \text{ kips} )</td>
<td>( V_a = 22.1 \text{ kips} )</td>
</tr>
<tr>
<td>( \phi, V_n \geq V_a )</td>
<td>( \Omega, = 1.67 )</td>
</tr>
<tr>
<td>( \phi, V_n = 0.9 \times 142 \text{ kips} )</td>
<td>( V_n / \Omega \geq V_a )</td>
</tr>
<tr>
<td>= 128 kips ( &gt; 36.8 \text{ kips} ) o.k.</td>
<td>( V_n / \Omega = \frac{142 \text{ kips}}{1.67} )</td>
</tr>
<tr>
<td></td>
<td>= 85.0 kips ( &gt; 22.1 \text{ kips} ) o.k.</td>
</tr>
</tbody>
</table>

**Force allocation and load transfer**

Load transfer calculations for applied axial forces should be performed in accordance with AISC Specification Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

**Summary**

It has been determined that a 30 in. \( \times \) 30 in. composite box column composed of \( \frac{1}{4} \)-in.-thick plate is adequate for the imposed loads.
EXAMPLE I.8  ENCASED COMPOSITE MEMBER FORCE ALLOCATION AND LOAD TRANSFER

Given:
Refer to Figure I.8-1.

Part I:  For each loading condition (a) through (c) determine the required longitudinal shear force, \( V' \), to be transferred between the embedded steel section and concrete encasement.

Part II:  For loading condition (b), investigate the force transfer mechanisms of direct bearing and shear connection.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft\(^3\)) reinforced concrete having a specified concrete compressive strength, \( f'_c = 5 \) ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, \( F_{yR} \), of 60 ksi.

Applied loading, \( P_r \), for each condition illustrated in Figure I.8-1 is composed of the following loads:

\[
P_D = 260 \text{ kips} \\
P_L = 780 \text{ kips}
\]

Solution:

Part I—Force Allocation

Fig. I.8-1. Encased composite member in compression.
From AISC Manual Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC Manual Table 1-1 and Figure I.8-1, the geometric properties of the encased W10×45 are as follows:

$A_e = 13.3$ in.$^2$
$t_e = 0.350$ in.
$h_e = 24.0$ in.
$b_f = 8.02$ in.
$d = 10.1$ in.
$h_f = 24.0$ in.
$t_f = 0.620$ in.

Additional geometric properties of the composite section used for force allocation and load transfer are calculated as follows:

\[
A_s = h_fh_t = (24.0\text{ in.})(24.0\text{ in.}) = 576\text{ in.}^2
\]

\[
A_{ori} = 0.79\text{ in.}^2\text{ for a No. 8 bar}
\]

where

- $A_e =$ cross-sectional area of concrete encasement, in.$^2$
- $A_g =$ gross cross-sectional area of composite section, in.$^2$
- $A_{ori} =$ cross-sectional area of reinforcing bar $i$, in.$^2$
- $A_{sr} =$ cross-sectional area of continuous reinforcing bars, in.$^2$
- $n =$ number of continuous reinforcing bars in composite section

From Chapter 2 of ASCE/SEI 7, the required strength is:

\[
P_r = P_u + (1.2(260\text{ kips}) + 1.6(780\text{ kips})) = 1,560\text{ kips}
\]

\[
P_r = P_u + (260\text{ kips} + 780\text{ kips}) = 1,040\text{ kips}
\]

**Composite Section Strength for Force Allocation**

In accordance with AISC Specification Section I6, force allocation calculations are based on the nominal axial compressive strength of the encased composite member without length effects, $P_{no}$. This section strength is defined in Section I2.1b as:

\[
P_{no} = F_y A_e + F_y A_{ori} + 0.85 f_y A_e
\]

\[
= (50\text{ ksi})(13.3\text{ in.}^2) + (60\text{ ksi})(6.32\text{ in.}^2) + 0.85(5\text{ ksi})(556\text{ in.}^2)
\]

\[
= 3,410\text{ kips}
\]

**Transfer Force for Condition (a)**

Refer to Figure I.8-1(a). For this condition, the entire external force is applied to the steel section only, and the provisions of AISC Specification Section I6.2a apply.
Transfer Force for Condition (b)

Refer to Figure I.8-1(b). For this condition, the entire external force is applied to the concrete encasement only, and the provisions of AISC Specification Section I6.2b apply.

\[
V_r' = P_r \left( 1 - \frac{F_r A_r}{P_{mo}} \right) = P_r \left[ 1 - \frac{50\text{ ksi}(13.3\text{ in.}^2)}{3,410\text{ kips}} \right] = 0.805 P_r \]

\[
V_r' = 0.805(1,560\text{ kips}) = 1,260\text{ kips} \\
V_r' = 0.805(1,040\text{ kips}) = 837\text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_r' = 0.805(1,560\text{ kips}))</td>
<td>(V_r' = 0.805(1,040\text{ kips}))</td>
</tr>
</tbody>
</table>

Transfer Force for Condition (c)

Refer to Figure I.8-1(c). For this condition, external force is applied to the steel section and concrete encasement concurrently, and the provisions of AISC Specification Section I6.2c apply.

\[
V_r' = P_r \left( \frac{F_r A_r}{P_{mo}} \right) = P_r \left[ \frac{50\text{ ksi}(13.3\text{ in.}^2)}{3,410\text{ kips}} \right] = 0.195 P_r
\]

\[
V_r' = 0.195(1,560\text{ kips}) = 304\text{ kips} \\
V_r' = 0.195(1,040\text{ kips}) = 203\text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_r' = 0.195(1,560\text{ kips}))</td>
<td>(V_r' = 0.195(1,040\text{ kips}))</td>
</tr>
</tbody>
</table>

AISC Specification Commentary Section I6.2 states that when loads are applied to both the steel section and concrete encasement concurrently, \(V_r'\) can be taken as the difference in magnitudes between the portion of the external force applied directly to the steel section and that required by Equation I6-2. This concept can be written in equation form as follows:

\[
V_r' = \left| P_{rs} - P_r \left( \frac{F_r A_r}{P_{mo}} \right) \right|
\]

where

\(P_{rs} = \) portion of external force applied directly to the steel section (kips)

Currently the Specification provides no specific requirements for determining the distribution of the applied force for the determination of \(P_{rs}\), so it is left to engineering judgment. For a bearing plate condition such as the one represented in Figure I.8-1(c), one possible method for determining the distribution of applied forces is to use an elastic distribution based on the material axial stiffness ratios as follows:
\[ E_r = w_r^{0.5} \sqrt{f'_{u}} \]
\[ = (145 \text{ lb/ft}^{1.5})^{0.5} \sqrt{5 \text{ ksi}} \]
\[ = 3,900 \text{ ksi} \]
\[ P_{cr} = \frac{E_r A_r}{E_r A_r + E_s A_s + E_c A_c} P_r \]
\[ = \left[ \frac{(29,000 \text{ ksi})(13.3 \text{ in.}^2)}{(29,000 \text{ ksi})(13.3 \text{ in.}^2) + (3,900 \text{ ksi})(556 \text{ in.}^2) + (29,000 \text{ ksi})(6.32 \text{ in.}^2)} \right] P_r \]
\[ = 0.141P_r \]

Substituting the results into Equation 1 yields:

\[ V_{cr}' = 0.141P_r - P_r \left( \frac{F_r A_r}{P_{cr}} \right) \]
\[ = 0.141P_r - P_r \left( \frac{50 \text{ ksi}(13.3 \text{ in.}^2)}{3,410 \text{ kips}} \right) \]
\[ = 0.0540P_r \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ V_{cr}' = 0.0540(1,560 \text{ kips}) ]</td>
<td>[ V_{cr}' = 0.0540(1,040 \text{ kips}) ]</td>
</tr>
<tr>
<td>= 84.2 kips</td>
<td>= 56.2 kips</td>
</tr>
</tbody>
</table>

An alternate approach would be use of a plastic distribution method whereby the load is partitioned to each material in accordance with their contribution to the composite section strength given in Equation I2-4. This method eliminates the need for longitudinal shear transfer provided the local bearing strength of the concrete and steel are adequate to resist the forces resulting from this distribution.

**Additional Discussion**

- The design and detailing of the connections required to deliver external forces to the composite member should be performed according to the applicable sections of AISC Specification Chapters J and K.

- The connection cases illustrated by Figure I.8-1 are idealized conditions representative of the mechanics of actual connections. For instance, an extended single plate connection welded to the flange of the W10 and extending out beyond the face of concrete to attach to a steel beam is an example of a condition where it may be assumed that all external force is applied directly to the steel section only.

**Solution:**

**Part II—Load Transfer**

The required longitudinal force to be transferred, \( V_{cr}' \), determined in Part I condition (b) is used to investigate the applicable force transfer mechanisms of AISC Specification Section I6.3: direct bearing and shear connection. As indicated in the Specification, these force transfer mechanisms may not be superimposed; however, the mechanism providing the greatest nominal strength may be used. Note that direct bond interaction is not applicable to encased composite members as the variability of column sections and connection configurations makes confinement and bond strength more difficult to quantify than in filled HSS.
**Direct Bearing**

Determine Layout of Bearing Plates

One method of utilizing direct bearing as a load transfer mechanism is through the use of internal bearing plates welded between the flanges of the encased W-shape as indicated in Figure I.8-2.

![Fig. I.8-2. Composite member with internal bearing plates.](image)

When using bearing plates in this manner, it is essential that concrete mix proportions and installation techniques produce full bearing at the plates. Where multiple sets of bearing plates are used as illustrated in Figure I.8-2, it is recommended that the minimum spacing between plates be equal to the depth of the encased steel member to enhance constructability and concrete consolidation. For the configuration under consideration, this guideline is met with a plate spacing of 24 in. ≥ \( d = 10.1 \) in.

Bearing plates should be located within the load introduction length given in AISC Specification Section I6.4a. The load introduction length is defined as two times the minimum transverse dimension of the composite member both above and below the load transfer region. The load transfer region is defined in Specification Commentary Section I6.4 as the depth of the connection. For the connection configuration under consideration, where the majority of the required force is being applied from the concrete column above, the depth of connection is conservatively taken as zero. Because the composite member only extends to one side of the point of force transfer, the bearing plates should be located within \( 2h_2 = 48 \) in. of the top of the composite member as indicated in Figure I.8-2.

Available Strength for the Limit State of Direct Bearing

Assuming two sets of bearing plates are to be used as indicated in Figure I.8-2, the total contact area between the bearing plates and the concrete, \( A_1 \), may be determined as follows:
\[
a = \frac{b_I - t_w}{2} = \frac{8.02 \text{ in.} - 0.350 \text{ in.}}{2} = 3.84 \text{ in.}
\]

\[
b = d - 2t_f = 10.1 \text{ in.} - 2(0.620 \text{ in.}) = 8.86 \text{ in.}
\]

\[
c = \text{width of clipped corners} = \frac{3}{8} \text{ in.}
\]

\[
A_t = \left(2ab - 2c^2\right) \text{(number of bearing plate sets)} = \left[2(3.84 \text{ in.})(8.86 \text{ in.}) - 2\left(\frac{3}{8} \text{ in.}\right)^2\right] (2) = 134 \text{ in.}^2
\]

The available strength for the direct bearing force transfer mechanism is:

\[
R_a = 1.7 f'_c A_t = 1.7(5 \text{ ksi})(134 \text{ in.}^2) = 1,140 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.65$</td>
<td>$\Omega_b = 2.31$</td>
</tr>
<tr>
<td>$\phi_b R_a \geq V'_c$</td>
<td>$R_a / \Omega_b \geq V'_c$</td>
</tr>
<tr>
<td>$\phi_a R_a = 0.65(1,140 \text{ kips}) = 741 \text{ kips} &gt; 304 \text{ kips} \ o.k.$</td>
<td>$R_a / \Omega_b = \frac{1,140 \text{ kips}}{2.31} = 494 \text{ kips} &gt; 203 \text{ kips} \ o.k.$</td>
</tr>
</tbody>
</table>

Thus two sets of bearing plates are adequate. From these calculations it can be seen that one set of bearing plates are adequate for force transfer purposes; however, the use of two sets of bearing plates serves to reduce the bearing plate thickness calculated in the following section.

**Required Bearing Plate Thickness**

There are several methods available for determining the bearing plate thickness. For rectangular plates supported on three sides, elastic solutions for plate stresses such as those found in *Roark’s Formulas for Stress and Strain* (Young and Budynas, 2002) may be used in conjunction with AISC Specification Section F12 for thickness calculations. Alternately, yield line theory or computational methods such as finite element analysis may be employed.

For this example, yield line theory is employed. Results of the yield line analysis depend on an assumption of column flange strength versus bearing plate strength in order to estimate the fixity of the bearing plate to column flange connection. In general, if the thickness of the bearing plate is less than the column flange thickness, fixity and plastic hinging can occur at this interface; otherwise, the use of a pinned condition is conservative. Ignoring the fillets of the W-shape and clipped corners of the bearing plate, the yield line pattern chosen for the fixed condition is depicted in Figure I.8-3. Note that the simplifying assumption of 45° yield lines illustrated in Figure I.8-3 has been shown to provide reasonably accurate results (Park and Gamble, 2000), and that this yield line pattern is only valid where $b \geq 2a$. 

---

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The plate thickness using $F_y = 36$ ksi material may be determined as:

$$ t_p = \frac{2a^2w_u (3b - 2a)}{3\phi F_y (4a + b)} $$

where

$w_u =$ bearing pressure on plate determined using LRFD load combinations

$$ w_u = \frac{V'}{A_t} $$

= 304 kips

= 134 in.$^2$

= 2.27 ksi

Assuming $t_p \geq t_f$

$$ t_r = \sqrt{\frac{2(3.84 \text{ in.})^2 (2.27 \text{ ksi}) [3(8.86 \text{ in.}) - 2(3.84 \text{ in.})]}{3(0.90)(36 \text{ ksi})[4(3.84 \text{ in.}) + 8.86 \text{ in.}]}} $$

= 0.733 in.

Select $\frac{3}{4}$-in. plate. $t_p = \frac{3}{4} \text{ in.} > t_f = 0.620 \text{ in.}$ **assumption o.k.**

Thus, select $\frac{3}{4}$-in.-thick bearing plates.

**Bearing Plate to Encased Steel Member Weld**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>If $t_p \geq t_f$ :</td>
<td>If $t_p \geq t_f$ :</td>
</tr>
<tr>
<td>$t_p = \frac{2a^2w_u (3b - 2a)}{3\phi F_y (4a + b)}$</td>
<td>$t_p = \sqrt{\frac{2\Omega}{3F_y}[\frac{a^2w_u (3b - 2a)}{(4a + b)}]}$</td>
</tr>
<tr>
<td>If $t_p &lt; t_f$ :</td>
<td>If $t_p &lt; t_f$ :</td>
</tr>
<tr>
<td>$t_p = \frac{2a^2w_u (3b - 2a)}{3\phi F_y (6a + b)}$</td>
<td>$t_p = \sqrt{\frac{2\Omega}{3F_y}[\frac{a^2w_u (3b - 2a)}{(6a + b)}]}$</td>
</tr>
</tbody>
</table>

where

$w_u =$ bearing pressure on plate determined using ASD load combinations

$$ w_u = \frac{V'}{A_t} $$

= 203 kips

= 134 in.$^2$

= 1.51 ksi

Assuming $t_p \geq t_f$

$$ t_r = \sqrt{\frac{2(1.67)(3.84 \text{ in.})^2 (1.51 \text{ ksi}) [3(8.86 \text{ in.}) - 2(3.84 \text{ in.})]}{3(36 \text{ ksi})[4(3.84 \text{ in.}) + 8.86 \text{ in.}]}} $$

= 0.733 in.

Select $\frac{3}{4}$-in. plate $t_p = \frac{3}{4} \text{ in.} > t_f = 0.620 \text{ in.}$ **assumption o.k.**
The bearing plates should be connected to the encased steel member using welds designed in accordance with AISC Specification Chapter J to develop the full strength of the plate. For fillet welds, a weld size of $\frac{3}{4}t_p$ will serve to develop the strength of either a 36- or 50-ksi plate as discussed in AISC Manual Part 10.

Shear Connection

Shear connection involves the use of steel headed stud or channel anchors placed on at least two faces of the steel shape in a generally symmetric configuration to transfer the required longitudinal shear force. For this example, $\frac{3}{8}$-in.-diameter $\times$ 4\(\frac{3}{16}\)-in.-long steel headed stud anchors composed of ASTM A108 material are selected. From AISC Manual Table 2-6, the specified minimum tensile strength, $F_{u}$, of ASTM A108 material is 65 ksi.

Available Shear Strength of Steel Headed Stud Anchors

The available shear strength of an individual steel headed stud anchor is determined in accordance with the composite component provisions of AISC Specification Section I8.3 as directed by Section I6.3b.

$$Q_{av} = F_{u}A_{sa}$$  
$$A_{sa} = \frac{\pi(\frac{3}{8} \text{ in.})^2}{4} = 0.442 \text{ in.}^2$$  

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.65$</td>
<td>$\Omega_v = 2.31$</td>
</tr>
<tr>
<td>$\phi_v Q_{av} = 0.65(65 \text{ ksi})(0.442 \text{ in.}^2)$</td>
<td>$Q_{av}/\Omega_v = \frac{(65 \text{ ksi})(0.442 \text{ in.}^2)}{2.31}$</td>
</tr>
<tr>
<td>=18.7 kips per steel headed stud anchor</td>
<td>=12.4 kips per steel headed stud anchor</td>
</tr>
</tbody>
</table>

Required Number of Steel Headed Stud Anchors

The number of steel headed stud anchors required to transfer the longitudinal shear is calculated as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{anchors} = \frac{V'}{\phi_v Q_{av}}$</td>
<td>$n_{anchors} = \frac{V'}{Q_{av}/\Omega_v}$</td>
</tr>
<tr>
<td>=304 kips</td>
<td>=203 kips</td>
</tr>
<tr>
<td>=18.7 kips</td>
<td>=12.4 kips</td>
</tr>
<tr>
<td>=16.3 steel headed stud anchors</td>
<td>=16.4 steel headed stud anchors</td>
</tr>
</tbody>
</table>

With anchors placed in pairs on each flange, select 20 anchors to satisfy the symmetry provisions of AISC Specification Section I6.4a.

Placement of Steel Headed Stud Anchors

Steel headed stud anchors are placed within the load introduction length in accordance with AISC Specification Section I6.4a. Since the composite member only extends to one side of the point of force transfer, the steel anchors are located within $2h_2 = 48$ in. of the top of the composite member.

Placing two anchors on each flange provides four anchors per group, and maximum stud spacing within the load introduction length is determined as:
Use 10.0 in. spacing beginning 6 in. from top of encased member.

In addition to anchors placed within the load introduction length, anchors must also be placed along the remainder of the composite member at a maximum spacing of 32 times the anchor shank diameter = 24 in. in accordance with AISC Specification Sections I6.4a and I8.3e.

The chosen anchor layout and spacing is illustrated in Figure I.8-4.
Steel Headed Stud Anchor Detailing Limitations of AISC Specification Sections I6.4a, I8.1 and I8.3

Steel headed stud anchor detailing limitations are reviewed in this section with reference to the anchor configuration provided in Figure I.8-4 for anchors having a shank diameter, \( d_{sa} \), of \( \frac{3}{4} \) in. Note that these provisions are specific to the detailing of the anchors themselves and that additional limitations for the structural steel, concrete and reinforcing components of composite members should be reviewed as demonstrated in Design Example I.9.

1. Anchors must be placed on at least two faces of the steel shape in a generally symmetric configuration:
   Anchors are located in pairs on both faces. o.k.

2. Maximum anchor diameter:  
   \[ d_{sa} \leq 2.5(t_f) \]
   \( \frac{3}{4} \) in. \( < 2.5(0.620 \text{ in.}) = 1.55 \text{ in.} \) o.k.

3. Minimum steel headed stud anchor height-to-diameter ratio: \( h / d_{sa} \geq 5 \)
   The minimum ratio of installed anchor height (base to top of head), \( h \), to shank diameter, \( d_{sa} \), must meet the provisions of AISC Specification Section I8.3 as summarized in the User Note table at the end of the section. For shear in normal weight concrete the limiting ratio is five. As previously discussed, a \( 4\frac{1}{4}\)-in.-long anchor was selected from anchor manufacturer’s data. As the \( h/d_{sa} \) ratio is based on the installed length, a length reduction for burn off during installation of \( \frac{3}{4} \) in. is taken to yield the final installed length of 4 in.
   \[ \frac{h}{d_{sa}} = \frac{4 \text{ in.}}{\frac{3}{4} \text{ in.}} = 5.33 > 5 \text{ o.k.} \]

4. Minimum lateral clear concrete cover = 1 in.
   From AWS D1.1 Figure 7.1, the head diameter of a \( \frac{3}{4}\)-in.-diameter stud anchor is equal to 1.25 in.
   
   \[
   \text{lateral clear cover} = \left( \frac{h}{2} \right) - \left( \frac{\text{lateral spacing between anchor centerlines}}{2} \right) - \left( \frac{\text{anchor head diameter}}{2} \right)
   \]
   
   \[
   = \left( \frac{24 \text{ in.}}{2} \right) - \left( \frac{4 \text{ in.}}{2} \right) - \left( \frac{1.25 \text{ in.}}{2} \right)
   \]
   
   \[
   = 9.38 \text{ in.} > 1.0 \text{ in.} \text{ o.k.}
   \]

5. Minimum anchor spacing:
   \[ s_{min} = 4d_{sa} \]
   \[ = 4(\frac{3}{4} \text{ in.}) \]
   \[ = 3.00 \text{ in.} \]
   In accordance with AISC Specification Section I8.3e, this spacing limit applies in any direction.
   \[ s_{transverse} = 4 \text{ in.} \geq s_{min} \text{ o.k.} \]
   \[ s_{longitudinal} = 10 \text{ in.} \geq s_{min} \text{ o.k.} \]

6. Maximum anchor spacing:
   \[ s_{max} = 32d_{sa} \]
   \[ = 32(\frac{3}{4} \text{ in.}) \]
   \[ = 24.0 \text{ in.} \]
In accordance with AISC Specification Section I6.4a, the spacing limits of Section I8.1 apply to steel anchor spacing both within and outside of the load introduction region.

\[ s = 24.0 \text{ in.} \leq s_{\text{max}} \quad \text{o.k.} \]

(7) Clear cover above the top of the steel headed stud anchors:

Minimum clear cover over the top of the steel headed stud anchors is not explicitly specified for steel anchors in composite components; however, in keeping with the intent of AISC Specification Section II.1, it is recommended that the clear cover over the top of the anchor head follow the cover requirements of ACI 318 Section 7.7. For concrete columns, ACI 318 specifies a clear cover of 1½ in.

\[
\text{clear cover above anchor} = \frac{h_2 - d}{2} - \text{installed anchor length} \\
= \frac{24 \text{ in.}}{2} - \frac{10.1 \text{ in.}}{2} - 4 \text{ in.} \\
= 2.95 \text{ in.} > 1\frac{1}{2} \text{ in.} \quad \text{o.k.}
\]

**Concrete Breakout**

AISC Specification Section I8.3a states that in order to use Equation I8-3 for shear strength calculations as previously demonstrated, concrete breakout strength in shear must not be an applicable limit state. If concrete breakout is deemed to be an applicable limit state, the Specification provides two alternatives: either the concrete breakout strength can be determined explicitly using ACI 318 Appendix D in accordance with Specification Section I8.3a(2), or anchor reinforcement can be provided to resist the breakout force as discussed in Specification Section I8.3a(1).

Determining whether concrete breakout is a viable failure mode is left to the engineer. According to AISC Specification Commentary Section I8.3, “it is important that it be deemed by the engineer that a concrete breakout failure mode in shear is directly avoided through having the edges perpendicular to the line of force supported, and the edges parallel to the line of force sufficiently distant that concrete breakout through a side edge is not deemed viable.”

For the composite member being designed, no free edge exists in the direction of shear transfer along the length of the column, and concrete breakout in this direction is not an applicable limit state. However, it is still incumbent upon the engineer to review the possibility of concrete breakout through a side edge parallel to the line of force.

One method for explicitly performing this check is through the use of the provisions of ACI 318 Appendix D as follows:

ACI 318 Section D.6.2.1(c) specifies that concrete breakout shall be checked for shear force parallel to the edge of a group of anchors using twice the value for the nominal breakout strength provided by ACI 318 Equation D-22 when the shear force in question acts perpendicular to the edge.

For the composite member being designed, symmetrical concrete breakout planes form to each side of the encased shape, one of which is illustrated in Figure I.8-5.
Fig. I.8-5. Concrete breakout check for shear force parallel to an edge.

\[ \phi = 0.75 \] for anchors governed by concrete breakout with supplemental reinforcement (provided by tie reinforcement) in accordance with ACI 318 Section D.4.4(c).

\[ V_{c,bg,ec} = \frac{V_{co}}{A_{c}} \] for shear force parallel to an edge

\[ A_{c,cs} = 4.5(c_{a})^2 \]
\[ = 4.5(10 \text{ in.})^2 \]
\[ = 450 \text{ in.}^2 \]

\[ A_{c,c} = (15 \text{ in.} + 40 \text{ in.}+ 15 \text{ in.})(24 \text{ in.}) \text{ from Figure I.8-5} \]
\[ = 1,680 \text{ in.}^2 \]

\[ \Psi_{w,c} = 1.0 \] no eccentricity

\[ \Psi_{w,cd,c} = 1.0 \] in accordance with ACI 318 Section D.6.2.1(c)

\[ \Psi_{e,c} = 1.4 \] compression-only member assumed uncracked

\[ \Psi_{a,c} = 1.0 \]

\[ V_{b} = 8 \left( \frac{f_{c}}{d_{a}} \right)^{0.2} \sqrt{d_{a}} \lambda \sqrt{f_{c}} (c_{a})^{1.5} \]
where

\[ l_a = 4 \text{ in.} - \frac{3}{4}\text{-in. anchor head thickness from AWS D1.1, Figure 7.1} \]
\[ d_{wa} = \frac{3}{4}\text{-in. anchor diameter} \]
\[ \lambda = 1.0 \text{ from ACI 318 Section 8.6.1 for normal weight concrete} \]

\[ V_a = \left[ 8 \left( \frac{3.63 \text{ in.}}{\frac{3}{4} \text{ in.}} \right)^{0.2} \sqrt{\frac{3}{4} \text{ in.}} \right] (1.0) \sqrt{\frac{5,000 \text{ psi}}{1,000 \text{ lb/kip}}} \left( 10 \text{ in.} \right)^{1.5} \]
\[ = 21.2 \text{ kips} \]

\[ V_{cbg} = 2 \left[ \frac{1,680 \text{ in.}^2}{450 \text{ in.}^2} \right] (1.0)(1.4)(1.0)(21.2 \text{ kips}) \]
\[ = 222 \text{ kips} \]

\[ \phi V_{cbg} = 0.75 \left( 222 \text{ kips} \right) \]
\[ = 167 \text{ kips per breakout plane} \]

\[ \phi V_{cbg} = \left( 2 \text{ breakout planes} \right) \left( 167 \text{ kips/plane} \right) \]
\[ = 334 \text{ kips} \]

\[ \phi V_{cbg} \geq V' = 304 \text{ kips} \quad \text{O.K.} \]

Thus, concrete breakout along an edge parallel to the direction of the longitudinal shear transfer is not a controlling limit state, and Equation I8-3 is appropriate for determining available anchor strength.

Encased beam-column members with reinforcing detailed in accordance with the AISC Specification have demonstrated adequate confinement in tests to prevent concrete breakout along a parallel edge from occurring; however, it is still incumbent upon the engineer to review the project-specific detailing used for susceptibility to this limit state.

If concrete breakout was determined to be a controlling limit state, transverse reinforcing ties could be analyzed as anchor reinforcement in accordance with AISC Specification Section I8.3a(1), and tie spacing through the load introduction length adjusted as required to prevent breakout. Alternately, the steel headed stud anchors could be relocated to the web of the encased member where breakout is prevented by confinement between the column flanges.
EXAMPLE I.9 ENCASED COMPOSITE MEMBER IN AXIAL COMPRESSION

Given:

Determine if the 14 ft long, encased composite member illustrated in Figure I.9-1 is adequate for the indicated dead and live loads.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft³) reinforced concrete having a specified concrete compressive strength, $f'_c = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, $F_{ys}$, of 60 ksi.

Solution:

From AISC Manual Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC Manual Table 1-1, Figure I.9-1, and Design Example I.8, geometric and material properties of the composite section are:

\[
\begin{align*}
A_s &= 13.3 \text{ in.}^2 \\
h_f &= 8.02 \text{ in.} \\
t_f &= 0.620 \text{ in.} \\
t_e &= 0.350 \text{ in.} \\
E_c &= 3,900 \text{ ksi}
\end{align*}
\]

\[
\begin{align*}
I_{x} &= 248 \text{ in.}^4 \\
I_{y} &= 53.4 \text{ in.}^4 \\
h_1 &= 24.0 \text{ in.} \\
h_2 &= 24.0 \text{ in.} \\
A_w &= 0.79 \text{ in.}^2 \\
A_v &= 6.32 \text{ in.}^2 \\
A_r &= 556 \text{ in.}^2
\end{align*}
\]
The moment of inertia of the reinforcing bars about the elastic neutral axis of the composite section, $I_{sr}$, is required for composite member design and is calculated as follows:

$$
I_{sr} = \sum_{i=1}^{n} A_{sr_i} \cdot e_i^2
$$

where
- $A_{sr_i}$ = cross-sectional area of reinforcing bar $i$, in.$^2$
- $I_{sr_i}$ = moment of inertia of reinforcing bar $i$ about its elastic neutral axis, in.$^4$
- $I_s$ = moment of inertia of the reinforcing bars about the elastic neutral axis of the composite section, in.$^4$
- $d_b$ = nominal diameter of reinforcing bar, in.
- $e_i$ = eccentricity of reinforcing bar $i$ with respect to the elastic neutral axis of the composite section, in.
- $n$ = number of reinforcing bars in composite section

Note that the elastic neutral axis for each direction of the section in question is located at the $x$-x and $y$-y axes illustrated in Figure I.9-1, and that the moment of inertia calculated for the longitudinal reinforcement is valid about either axis due to symmetry.

The moment of inertia values for the concrete about each axis are determined as:

$$
I_{cx} = I_{cy} - I_{xs} - I_{ys}
$$

$$
= \frac{(24.0 \text{ in.})^4}{12} - 248 \text{ in.}^4 - 428 \text{ in.}^4
$$

$$
= 27,000 \text{ in.}^4
$$

$$
I_{cy} = I_{cx} - I_{xy}
$$

$$
= \frac{(24.0 \text{ in.})^4}{12} - 53.4 \text{ in.}^4 - 428 \text{ in.}^4
$$

$$
= 27,200 \text{ in.}^4
$$

**Classify Section for Local Buckling**

In accordance with AISC Specification Section I1.2, local buckling effects need not be considered for encased composite members, thus all encased sections are treated as compact sections for strength calculations.

**Material and Detailing Limitations**

According to the User Note at the end of AISC Specification Section I1.1, the intent of the Specification is to implement the noncomposite detailing provisions of ACI 318 in conjunction with the composite-specific provisions of Specification Chapter I. Detailing provisions may be grouped into material related limits, transverse reinforcement provisions, and longitudinal and structural steel reinforcement provisions as illustrated in the following discussion.
Material limits are provided in AISC Specification Sections I1.1(2) and I1.3 as follows:

1. Concrete strength: \( 3 \text{ ksi} \leq f' \leq 10 \text{ ksi} \)
   \( f' = 5 \text{ ksi} \quad \text{o.k.} \)

2. Specified minimum yield stress of structural steel: \( F_y \leq 75 \text{ ksi} \)
   \( F_y = 50 \text{ ksi} \quad \text{o.k.} \)

3. Specified minimum yield stress of reinforcing bars: \( F_{y'} \leq 75 \text{ ksi} \)
   \( F_{y'} = 60 \text{ ksi} \quad \text{o.k.} \)

Transverse reinforcement limitations are provided in AISC Specification Section I1.1(3), I2.1a(2) and ACI 318 as follows:

1. Tie size and spacing limitations:

   The AISC Specification requires that either lateral ties or spirals be used for transverse reinforcement. Where lateral ties are used, a minimum of either No. 3 bars spaced at a maximum of 12 in. on center or No. 4 bars or larger spaced at a maximum of 16 in. on center are required.

   No. 3 lateral ties at 12 in. o.c. are provided. \text{o.k.}

   Note that AISC Specification Section I1.1(1) specifically excludes the composite column provisions of ACI 318 Section 10.13, so it is unnecessary to meet the tie reinforcement provisions of ACI 318 Section 10.13.8 when designing composite columns using the provisions of AISC Specification Chapter I.

   If spirals are used, the requirements of ACI 318 Sections 7.10 and 10.9.3 should be met according to the User Note at the end of AISC Specification Section I2.1a.

2. Additional tie size limitation:

   No. 4 ties or larger are required where No. 11 or larger bars are used as longitudinal reinforcement in accordance with ACI 318 Section 7.10.5.1.

   No. 3 lateral ties are provided for No. 8 longitudinal bars. \text{o.k.}

3. Maximum tie spacing should not exceed 0.5 times the least column dimension:

   \[ s_{\text{max}} = 0.5 \min \left( \frac{h_1}{2}, \frac{h_2}{2} \right) \]
   \[ = 12.0 \text{ in.} \]
   \[ s = 12.0 \text{ in.} \leq s_{\text{max}} \quad \text{o.k.} \]

4. Concrete cover:

   ACI 318 Section 7.7 contains concrete cover requirements. For concrete not exposed to weather or in contact with ground, the required cover for column ties is 1½ in.

   \[ \text{cover} = 2.5 \text{ in.} - \frac{d_b}{2} \text{ diameter of No. 3 tie} \]
   \[ = 2.5 \text{ in.} - \frac{1}{2} \text{ in.} - \frac{1}{8} \text{ in.} \]
   \[ = 1.63 \text{ in.} > 1\frac{1}{2} \text{ in.} \quad \text{o.k.} \]
(5) Provide ties as required for lateral support of longitudinal bars:

AISC Specification Commentary Section I2.1a references Chapter 7 of ACI 318 for additional transverse tie requirements. In accordance with ACI 318 Section 7.10.5.3 and Figure R7.10.5, ties are required to support longitudinal bars located farther than 6 in. clear on each side from a laterally supported bar. For corner bars, support is typically provided by the main perimeter ties. For intermediate bars, Figure I.9-1 illustrates one method for providing support through the use of a diamond-shaped tie.

Longitudinal and structural steel reinforcement limits are provided in AISC Specification Sections I1.1(4), I2.1 and ACI 318 as follows:

(1) Structural steel minimum reinforcement ratio: \( \frac{A_f}{A_g} \geq 0.01 \)

\[
\frac{13.3 \text{ in.}^2}{576 \text{ in.}^2} = 0.0231 \quad \text{o.k.}
\]

An explicit maximum reinforcement ratio for the encased steel shape is not provided in the AISC Specification; however, a range of 8 to 12% has been noted in the literature to result in economic composite members for the resistance of gravity loads (Leon and Hajjar, 2008).

(2) Minimum longitudinal reinforcement ratio: \( \frac{A_f}{A_g} \geq 0.004 \)

\[
\frac{6.32 \text{ in.}^2}{576 \text{ in.}^2} = 0.0110 \quad \text{o.k.}
\]

As discussed in AISC Specification Commentary Section I2.1a(3), only continuously developed longitudinal reinforcement is included in the minimum reinforcement ratio, so longitudinal restraining bars and other discontinuous longitudinal reinforcement is excluded. Note that this limitation is used in lieu of the minimum ratio provided in ACI 318 as discussed in Specification Commentary Section I1.1(4).

(3) Maximum longitudinal reinforcement ratio: \( \frac{A_f}{A_g} \leq 0.08 \)

\[
\frac{6.32 \text{ in.}^2}{576 \text{ in.}^2} = 0.0110 \quad \text{o.k.}
\]

This longitudinal reinforcement limitation is provided in ACI 318 Section 10.9.1. It is recommended that all longitudinal reinforcement, including discontinuous reinforcement not used in strength calculations, be included in this ratio as it is considered a practical limitation to mitigate congestion of reinforcement. If longitudinal reinforcement is lap spliced as opposed to mechanically coupled, this limit is effectively reduced to 4% in areas away from the splice location.

(4) Minimum number of longitudinal bars:

ACI 318 Section 10.9.2 requires a minimum of four longitudinal bars within rectangular or circular members with ties and six bars for columns utilizing spiral ties. The intent for rectangular sections is to provide a minimum of one bar in each corner, so irregular geometries with multiple corners require additional longitudinal bars.

8 bars provided. \( \text{o.k.} \)

(5) Clear spacing between longitudinal bars:

ACI 318 Section 7.6.3 requires a clear distance between bars of \( 1.5d_b \) or \( 1\frac{1}{2} \) in.
Clear spacing between longitudinal bars and the steel core:

AISC Specification Section I2.1e requires a minimum clear spacing between the steel core and longitudinal reinforcement of 1.5 reinforcing bar diameters, but not less than 1 1/2 in.

\[ s_{\text{min}} = \max \left\{ 1.5d_b = 1\frac{1}{2} \text{ in.} \right\} \]

\[ = 1\frac{1}{2} \text{ in. clear} \]

Closest reinforcing bars to the encased section are the center bars adjacent to each flange:

\[ s = \frac{h_2}{2} - \frac{d}{2} - 2.5 \text{ in.} - \frac{d_b}{2} \]

\[ = \frac{24 \text{ in.}}{2} - 10.1 \text{ in.} - \frac{2.5 \text{ in.}}{2} - \frac{1 \text{ in.}}{2} \]

\[ = 3.95 \text{ in.} \]

\[ s = 3.95 \text{ in.} \geq s_{\text{min}} = 1\frac{1}{2} \text{ in.} \quad \text{ o.k.} \]

Concrete cover for longitudinal reinforcement:

ACI 318 Section 7.7 provides concrete cover requirements for reinforcement. The cover requirements for column ties and primary reinforcement are the same, and the tie cover was previously determined to be acceptable, thus the longitudinal reinforcement cover is acceptable by inspection.

From Chapter 2 of ASCE/SEI, the required compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r = P_c )</td>
<td>1.2(260 kips) + 1.6(780 kips)</td>
<td>( P_r = P_c ) = 260 kips + 780 kips</td>
</tr>
<tr>
<td>( = 1,560 \text{ kips} )</td>
<td>( = 1,040 \text{ kips} )</td>
<td></td>
</tr>
</tbody>
</table>

Available Compressive Strength

The nominal axial compressive strength without consideration of length effects, \( P_{nom} \), is determined from AISC Specification Section I2.1b as:

\[
P_{nom} = F_y A_s + F_{ys} A_{sr} + 0.85 f'c A_c \quad \text{(Spec. Eq. I2-4)}
\]

\[
= (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2) + 0.85(5 \text{ ksi})(556 \text{ in.}^2)
\]

\[ = 3,410 \text{ kips} \]

Because the unbraced length is the same in both the \( x-x \) and \( y-y \) directions, the column will buckle about the axis having the smaller effective composite section stiffness, \( EI_{eff} \). Noting the moment of inertia values determined previously for the concrete and reinforcing steel are similar about each axis, the column will buckle about the weak...
axis of the steel shape by inspection. \( I_{cy}, I_{sy}, \) and \( I_{sy} \) are therefore used for calculation of length effects in accordance with AISC Specification Section I2.1b as follows:

\[
C_1 = 0.1 + 2 \left( \frac{A_y}{A_y + A_t} \right) \leq 0.3
\]

\[
= 0.1 + 2 \left( \frac{13.3 \text{ in.}^2}{556 \text{ in.}^2 + 13.3 \text{ in.}^2} \right) \leq 0.3
\]

\[
= 0.147 < 0.3
\]

0.147 controls  

\[\begin{align*}
EI_{eff} &= E_s I_{sy} + 0.5 E_s I_{sy} + C_1 E_s I_{sy} \\
&= (29,000 \text{ ksi})(53.4 \text{ in.}^4) + 0.5(29,000 \text{ ksi})(428 \text{ in.}^4) \\
&\quad + 0.147(3,900 \text{ ksi})(27,200 \text{ in.}^4)
\end{align*}\]

\[= 23,300,000 \text{ ksi}\]

\[P_c = \pi^2 \left( \frac{EI_{eff}}{KL} \right)^2 \] where \( K = 1.0 \) for a pin-ended member  

\[
= \frac{\pi^2 (23,300,000 \text{ ksi})}{[(1.0)(14 \text{ ft})(12 \text{ in./ft})]^2}
\]

\[= 8,150 \text{ kips}\]

\[
\frac{P_u}{P_e} = \frac{3,410 \text{ kips}}{8,150 \text{ kips}}
\]

\[= 0.418 < 2.25\]

Therefore, use AISC Specification Equation I2-2.

\[
P_u = P_e \left[ 0.658 \frac{P_u}{P_e} \right]
\]

\[
= (3,410 \text{ kips})(0.658)^{0.418}
\]

\[= 2,860 \text{ kips}\]

Check adequacy of the composite column for the required axial compressive strength:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_e) = 0.75</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>(\phi_e P_e \geq P_u)</td>
<td>(P_u / \Omega \geq P_u)</td>
</tr>
<tr>
<td>(\phi_e P_e = 0.75(2,860 \text{ kips}))</td>
<td>(P_u / \Omega = \frac{2,860 \text{ kips}}{2.00})</td>
</tr>
<tr>
<td>= 2,150 kips &gt; 1,560 kips o.k.</td>
<td>= 1,430 kips &gt; 1,040 kips o.k.</td>
</tr>
</tbody>
</table>

Available Compressive Strength of Composite Section Versus Bare Steel Section

Due to the differences in resistance and safety factors between composite and noncomposite column provisions, it is possible in rare instances to calculate a lower available compressive strength for an encased composite column than one would calculate for the corresponding bare steel section. However, in accordance with AISC Specification Section I2.1b, the available compressive strength need not be less than that calculated for the bare steel member in accordance with Chapter E.
From AISC Manual Table 4-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_u = 359$ kips</td>
<td>$P_u / \Omega_c = 239$ kips</td>
</tr>
<tr>
<td>$359$ kips $&lt; 2,150$ kips</td>
<td>$239$ kips $&lt; 1,430$ kips</td>
</tr>
</tbody>
</table>

Thus, the composite section strength controls and is adequate for the required axial compressive strength as previously demonstrated.

**Force Allocation and Load Transfer**

Load transfer calculations for external axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions for encased composite members is provided in Design Example I.8.

**Typical Detailing Convention**

Designers are directed to AISC Design Guide 6 (Griffis, 1992) for additional discussion and typical details of encased composite columns not explicitly covered in this example.
EXAMPLE I.10  ENCASED COMPOSITE MEMBER IN AXIAL TENSION

Given:
Determine if the 14 ft long, encased composite member illustrated in Figure I.10-1 is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the encased steel section.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft³) reinforced concrete having a specified concrete compressive strength, \( f'_c = 5 \text{ ksi} \).

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, \( F_{y'r} \), of 60 ksi.

Solution:

From AISC Manual Table 2-4, the steel material properties are:

- ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

From AISC Manual Table 1-1 and Figure I.10-1, the relevant properties of the composite section are:

\[
A_r = 13.3 \text{ in.}^2
\]
\[
A_{sr} = 6.32 \text{ in.}^2 \quad \text{(area of eight No. 8 bars)}
\]

Material and Detailing Limitations

Refer to Design Example I.9 for a check of material and detailing limitations specified in AISC Specification Chapter I for encased composite members.

Taking compression as negative and tension as positive, from Chapter 2 of ASCE/SEI 7, the required strength is:
Available Tensile Strength

Available tensile strength for an encased composite member is determined in accordance with AISC Specification Section I2.1c.

\[ P_n = F_y A_s + F_{yr} A_o \]
\[ = (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2) \]
\[ = 1,040 \text{ kips} \]

\[ \phi, P_n \geq P_u \]
\[ \phi, P_n = 0.90(1,040 \text{ kips}) \]
\[ = 936 \text{ kips} > 746 \text{ kips} \quad \text{o.k.} \]

Force Allocation and Load Transfer

In cases where all of the tension is applied to either the reinforcing steel or the encased steel shape, and the available strength of the reinforcing steel or encased steel shape by itself is adequate, no additional load transfer calculations are required.

In cases such as the one under consideration, where the available strength of both the reinforcing steel and the encased steel shape are needed to provide adequate tension resistance, AISC Specification Section I6 can be modified for tensile load transfer requirements by replacing the \( P_{no} \) term in Equations I6-1 and I6-2 with the nominal tensile strength, \( P_n \), determined from Equation I2-8.

For external tensile force applied to the encased steel section:

\[ V'_r = P_e \left(1 - \frac{F_r A_r}{P_n}\right) \]
\[ \text{(Eq. 1)} \]

For external tensile force applied to the longitudinal reinforcement of the concrete encasement:

\[ V'_r = P_e \left(\frac{F_r A_r}{P_n}\right) \]
\[ \text{(Eq. 2)} \]

where
\[ P_e = \text{required external tensile force applied to the composite member, kips} \]
\[ P_n = \text{nominal tensile strength of encased composite member from Equation I2-8, kips} \]
Per the problem statement, the entire external force is applied to the encased steel section, thus Equation 1 is used as follows:

\[
V' = P \left[ 1 - \frac{(50 \text{ ksi})(13.3 \text{ in.}^2)}{1,040 \text{ kips}} \right]
\]

\[
= 0.361P
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V' = 0.361(746 \text{ kips}))</td>
<td>(V' = 0.361(432 \text{ kips}))</td>
</tr>
<tr>
<td>= 269 kips</td>
<td>= 156 kips</td>
</tr>
</tbody>
</table>

The longitudinal shear force must be transferred between the encased steel shape and longitudinal reinforcing using the force transfer mechanisms of direct bearing or shear connection in accordance with AISC Specification Section I6.3 as illustrated in Design Example I.8.
EXAMPLE I.11 ENCASED COMPOSITE MEMBER IN COMBINED AXIAL COMPRESSION, FLEXURE AND SHEAR

Given:

Determine if the 14 ft long, encased composite member illustrated in Figure I.11-1 is adequate for the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC Specification Chapter C for the controlling ASCE/SEI 7-10 load combinations.

The composite member consists of an ASTM A992 W-shape encased by normal weight (3145 lb/ft\(^2\)) reinforced concrete having a specified concrete compressive strength, \(f'_c = 5\) ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, \(F_{yr}\), of 60 ksi.

Solution:

From AISC Manual Table 2-4, the steel material properties are:

\[
\begin{align*}
F_y &= 50\text{ ksi} \\
F_u &= 65\text{ ksi}
\end{align*}
\]

From AISC Manual Table 1-1, Figure I.11-1, and Design Examples I.8 and I.9, the geometric and material properties of the composite section are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_r) (kips)</td>
<td>1,170</td>
<td>879</td>
</tr>
<tr>
<td>(M_r) (kip-ft)</td>
<td>670</td>
<td>302</td>
</tr>
<tr>
<td>(V_r) (kips)</td>
<td>95.7</td>
<td>57.4</td>
</tr>
</tbody>
</table>

Fig. I.11-1. Encased composite member section and member forces.
The area of continuous reinforcing located at the centerline of the composite section, $A_{srs}$, is determined from Figure I.11-1 as follows:

$$A_{srs} = 2 \left( A_{sri} \right)$$

$$= 2 \left( 0.79 \text{ in.}^2 \right)$$

$$= 1.58 \text{ in.}^2$$

where

$A_{sri} = $ area of reinforcing bar $i$ at centerline of composite section

$= 0.79 \text{ in.}^2$ for a No. 8 bar

For the section under consideration, $A_{srs}$ is equal about both the $x$- and $y$-axis.

Classify Section for Local Buckling

In accordance with AISC Specification Section I1.2, local buckling effects need not be considered for encased composite members, thus all encased sections are treated as compact sections for strength calculations.

Material and Detailing Limitations

Refer to Design Example I.9 for a check of material and detailing limitations.

Interaction of Axial Force and Flexure

Interaction between axial forces and flexure in composite members is governed by AISC Specification Section I5 which permits the use of a strain compatibility method or plastic stress distribution method.

The strain compatibility method is a generalized approach that allows for the construction of an interaction diagram based upon the same concepts used for reinforced concrete design. Application of the strain compatibility method is required for irregular/nonsymmetrical sections, and its general implementation may be found in reinforced concrete design texts and will not be discussed further here.

Plastic stress distribution methods are discussed in AISC Specification Commentary Section I5 which provides four procedures. The first procedure, Method 1, invokes the interaction equations of Section H1. The second procedure, Method 2, involves the construction of a piecewise-linear interaction curve using the plastic strength equations provided in Figure I-1 located within the front matter of the Chapter I Design Examples. The third procedure, Method 2—Simplified, is a reduction of the piecewise-linear interaction curve that allows for the use of less conservative interaction equations than those presented in Chapter H. The fourth and final procedure, Method 3, utilizes AISC Design Guide 6 (Griffis, 1992).

For this design example, three of the available plastic stress distribution procedures are reviewed and compared. Method 3 is not demonstrated as it is not applicable to the section under consideration due to the area of the encased steel section being smaller than the minimum limit of 4% of the gross area of the composite section provided in the earlier Specification upon which Design Guide 6 is based.
Method 1—Interaction Equations of Section H1

The most direct and conservative method of assessing interaction effects is through the use of the interaction equations of AISC Specification Section H1. Unlike concrete filled HSS shapes, the available compressive and flexural strengths of encased members are not tabulated in the AISC Manual due to the large variety of possible combinations. Calculations must therefore be performed explicitly using the provisions of Chapter I.

Available Compressive Strength

The available compressive strength is calculated as illustrated in Design Example I.9.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_a$</td>
<td>2,150 kips</td>
<td>$P_e / \Omega_e = 1,430$ kips</td>
</tr>
</tbody>
</table>

Nominal Flexural Strength

The applied moment illustrated in Figure I.11-1 is resisted by the flexural strength of the composite section about its strong ($x-x$) axis. The strength of the section in pure flexure is calculated using the equations of Figure I-1a found in the front matter of the Chapter I Design Examples for Point B. Note that the calculation of the flexural strength at Point B first requires calculation of the flexural strength at Point D as follows:

$Z_c = (A_c - A_m) \left( \frac{h_f}{2} - c \right)$

$= (6.32 \text{ in.}^2 - 1.58 \text{ in.}^2) \left( \frac{24.0 \text{ in.}}{2} - 2.5 \text{ in.} \right)$

$= 45.0 \text{ in.}^3$

$Z_c = \frac{h_fh_s^2}{4} - Z_s - Z_r$

$= \left( \frac{(24.0 \text{ in.})(24.0 \text{ in.})^2}{4} - 54.9 \text{ in.}^3 - 45.0 \text{ in.}^3 \right)$

$= 3,360 \text{ in.}^3$

$M_D = Z_c F_r + Z_c F_{rs} + \frac{Z_c}{2} (0.85 f')$

$= (54.9 \text{ in.}^3)(50 \text{ ksi}) + (45.0 \text{ in.}^3)(60 \text{ ksi}) + \frac{3,360 \text{ in.}^3}{2} (0.85)(5 \text{ ksi})$

$= 12,600 \text{ kip-in.}$

Assuming $h_a$ is within the flange $\left( \frac{d}{2} - t_f < h_a \leq \frac{d}{2} \right)$:

$h_a = \frac{0.85 f'_{r'}(A_c + A_m - db_f + A_{rs}) - 2F_{rs}(A_c - db_f) - 2F_{rs}A_{rs}}{2[0.85 f'(h_f - b_f) + 2F_{rs}b_f]}$

$= \left[ 0.85 (5 \text{ ksi}) \left[ 556 \text{ in.}^2 + 13.3 \text{ in.}^2 - (10.1 \text{ in.})(8.02 \text{ in.}) + 1.58 \text{ in.}^2 \right] - 2(50 \text{ ksi}) \left[ 13.3 \text{ in.}^2 - (10.1 \text{ in.})(8.02 \text{ in.}) \right] 
- 2(60 \text{ ksi})(1.58 \text{ in.}^2) \right] / 2\left[ 0.85(5 \text{ ksi})(24.0 \text{ in.} - 8.02 \text{ in.}) + 2(50 \text{ ksi})(8.02 \text{ in.}) \right]$

$= 4.98 \text{ in.}$
Check assumption:
\[
\left(\frac{10.1 \text{ in.}}{2} - 0.620 \text{ in.}\right) \leq h_r \leq \frac{10.1 \text{ in.}}{2}
\]
4.43 in. < \( h_r \) = 4.98 in. < 5.05 in.  assumption o.k.

\[
Z_{cr} = Z_t - b_f \left(\frac{d}{2} - h_r\right) \left(\frac{d}{2} + h_r\right)
\]

\[
= 54.9 \text{ in.}^3 - \left(8.02 \text{ in.}\right) \left(\frac{10.1 \text{ in.}}{2} - 4.98 \text{ in.}\right) \left(\frac{10.1 \text{ in.}}{2} + 4.98 \text{ in.}\right)
\]

\[
= 49.3 \text{ in.}^3
\]

\[
Z_{cr} = h_r h_a^3 - Z_{cr}
\]

\[
= \left(24.0 \text{ in.}\right)(4.98 \text{ in.})^3 - 49.3 \text{ in.}^3
\]

\[
= 546 \text{ in.}^3
\]

\[
M_a = M_D - Z_{cr} F_v - \frac{Z_{cr} (0.85 f_y')}{2}
\]

\[
= 12,600 \text{ kip-in.} - \left(49.3 \text{ in.}^3\right)(50 \text{ ksi}) - \frac{\left(546 \text{ in.}^3\right)(0.85)(5 \text{ ksi})}{2}
\]

\[
= 8,970 \text{ kip-in.}
\]

\[
= 12 \text{ in./ft}
\]

\[
= 748 \text{ kip-ft}
\]

Available Flexural Strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b )</td>
<td>0.90</td>
<td>( 0.90 )</td>
</tr>
<tr>
<td>( \phi_b M_n )</td>
<td>0.90(748 kip-ft)</td>
<td>( 0.90 )</td>
</tr>
<tr>
<td></td>
<td>673 kip-ft</td>
<td>( 673 )</td>
</tr>
<tr>
<td>( \Omega_b )</td>
<td>1.67</td>
<td>( 1.67 )</td>
</tr>
<tr>
<td>( M_a )</td>
<td>748 kip-ft</td>
<td>( 748 )</td>
</tr>
<tr>
<td></td>
<td>( 448 ) kip-ft</td>
<td>( 448 )</td>
</tr>
</tbody>
</table>

Interaction of Axial Compression and Flexure

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi, P_a )</td>
<td>2,150 kips</td>
<td>( 2,150 ) kips</td>
</tr>
<tr>
<td>( \phi, M_n )</td>
<td>673 kip-ft</td>
<td>( 673 ) kip-ft</td>
</tr>
<tr>
<td>( P_a )</td>
<td>( P_a ) kips</td>
<td>( P_a ) kips</td>
</tr>
<tr>
<td>( P_a / P_t )</td>
<td>( P_a / P_t )</td>
<td>( P_a / P_t )</td>
</tr>
<tr>
<td>( P_a / P_t )</td>
<td>( 1,170 ) kips</td>
<td>( 1,170 ) kips</td>
</tr>
<tr>
<td>( P_a / P_t )</td>
<td>( 2,150 ) kips</td>
<td>( 2,150 ) kips</td>
</tr>
<tr>
<td>( P_a / P_t )</td>
<td>( 0.544 &gt; 0.2 )</td>
<td>( 0.544 &gt; 0.2 )</td>
</tr>
</tbody>
</table>

Use AISC Specification Equation H1-1a.

\[
\frac{P_a}{\phi, P_a} + 8 \left(\frac{M_a}{\phi, M_n}\right) \leq 1.0
\]

Use AISC Specification Equation H1-1a.

\[
\frac{P_a}{\phi, P_a} + 8 \left(\frac{M_a}{\phi, M_n}\right) \leq 1.0
\]
Method 1 indicates that the section is inadequate for the applied loads. The designer can elect to choose a new section that passes the interaction check or re-analyze the current section using a less conservative design method such as Method 2. The use of Method 2 is illustrated in the following section.

Method 2—Interaction Curves from the Plastic Stress Distribution Model

The procedure for creating an interaction curve using the plastic stress distribution model is illustrated graphically in AISC Specification Commentary Figure C-I5.2, and repeated here.

![Fig. C-I5.2. Interaction diagram for composite beam-columns – Method 2.](image)

Referencing Figure C.I5.2, the nominal strength interaction surface A, B, C, D is first determined using the equations of Figure I-1a found in the front matter of the Chapter I Design Examples. This curve is representative of the short column member strength without consideration of length effects. A slenderness reduction factor, $\lambda$, is then calculated and applied to each point to create surface $A'$, $B'$, $C'$, $D'$. The appropriate resistance or safety factors are then applied to create the design surface $A''$, $B''$, $C''$, $D''$. Finally, the required axial and flexural strengths from the applicable load combinations of ASCE/SEI 7-10 are plotted on the design surface. The member is then deemed acceptable for the applied loading if all points fall within the design surface. These steps are illustrated in detail by the following calculations.

**Step 1: Construct nominal strength interaction surface A, B, C, D without length effects**

Using the equations provided in Figure I-1a for bending about the x-x axis yields:

Point A (pure axial compression):

$$P_A = A_f f_y + A_c f_y + 0.85 f_y A_c$$

$$= (13.3 \text{ in.}^2)(50 \text{ ksi}) + (6.32 \text{ in.}^2)(60 \text{ ksi}) + 0.85(5 \text{ ksi})(556 \text{ in.}^2)$$

$$= 3,410 \text{ kips}$$

$$M_A = 0 \text{ kip-ft}$$
Point D (maximum nominal moment strength):

\[ P_D = \frac{0.85 f'c A_c}{2} = \frac{0.85(5 \text{ ksi})(556 \text{ in.}^2)}{2} = 1,180 \text{ kips} \]

Calculation of \( M_D \) was demonstrated previously in Method 1.

\[ M_D = 1,050 \text{ kip-ft} \]

Point B (pure flexure):

\[ P_B = 0 \text{ kips} \]

Calculation of \( M_B \) was demonstrated previously in Method 1.

\[ M_B = 748 \text{ kip-ft} \]

Point C (intermediate point):

\[ P_C = 0.85 f'c A_c = 0.85(5 \text{ ksi})(556 \text{ in.}^2) = 2,360 \text{ kips} \]

\[ M_C = M_B = 748 \text{ kip-ft} \]

The calculated points are plotted to construct the nominal strength interaction surface without length effects as depicted in Figure I.11-2.

![Fig. I.11-2. Nominal strength interaction surface without length effects.](image-url)
Step 2: Construct nominal strength interaction surface $A'$, $B'$, $C'$, $D'$ with length effects

The slenderness reduction factor, $\lambda$, is calculated for Point A using AISC *Specification* Section I2.1 in accordance with *Specification* Commentary Section I5.

Because the unbraced length is the same in both the $x$- and $y$-directions, the column will buckle about the axis having the smaller effective composite section stiffness, $EI_{eff}$. Noting the moment of inertia values for the concrete and reinforcing steel are similar about each axis, the column will buckle about the weak axis of the steel shape by inspection. $I_{x}$, $I_{y}$ and $I_{xy}$ are therefore used for calculation of length effects in accordance with AISC *Specification* Section I2.1b.

\[
P_{\text{no}} = P_{A}
\]
\[
= 3,410 \text{ kips}
\]

\[
C_1 = 0.1 + 2 \left( \frac{A_s}{A_c + A_s} \right) \leq 0.3
\]
\[
= 0.1 + 2 \left( \frac{13.3 \text{ in}^2}{556 \text{ in}^2 + 13.3 \text{ in}^2} \right) \leq 0.3
\]
\[
= 0.147 < 0.3; \text{ therefore } C_1 = 0.147.
\]

\[
EI_{\text{eff}} = E_s I_{xy} + 0.5 E_s I_{yy} + C_s E_s I_{xy}
\]
\[
= (29,000 \text{ ksi})(53.4 \text{ in}^2) + 0.5(29,000 \text{ ksi})(428 \text{ in}^4)
\]
\[
+ 0.147(3,900 \text{ ksi})(27,200 \text{ in}^4)
\]
\[
= 23,300,000 \text{ ksi}
\]

\[
P = \pi^2 \left( \frac{EI_{\text{eff}}}{KL} \right)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method}
\]
\[
= \pi^2 \left( \frac{23,300,000 \text{ ksi}}{[(1.0)(14 \text{ ft})(12 \text{ in./ft})]^2} \right)
\]
\[
= 8,150 \text{ kips}
\]

\[
P'_{\text{no}} = 3,410 \text{ kips}
\]
\[
P'_{P} = 8,150 \text{ kips}
\]
\[
= 0.418 < 2.25
\]

Therefore, use AISC *Specification* Equation I2-2.

\[
P_n = P_{no} \left[ \frac{P_{P}}{P_{no}} \right]^{\frac{0.658}{0.418}}
\]
\[
= 3,410 \text{ kips}(0.658)^{0.418}
\]
\[
= 2,860 \text{ kips}
\]

\[
\lambda = \frac{P_n}{P_{no}}
\]
\[
= \frac{2,860 \text{ kips}}{3,410 \text{ kips}}
\]
\[
= 0.839
\]

In accordance with AISC *Specification* Commentary Section I5, the same slenderness reduction is applied to each of the remaining points on the interaction surface as follows:
The modified axial strength values are plotted with the flexural strength values previously calculated to construct the nominal strength interaction surface including length effects. These values are superimposed on the nominal strength surface not including length effects for comparison purposes in Figure I.11-3.

The consideration of length effects results in a vertical reduction of the nominal strength curve as illustrated by Figure I.11-3. This vertical movement creates an unsafe zone within the shaded area of the figure where flexural capacities of the nominal strength (with length effects) curve exceed the section capacity. Application of resistance or safety factors reduces this unsafe zone as illustrated in the following step; however, designers should be cognizant of the potential for unsafe designs with loads approaching the predicted flexural capacity of the section. Alternately, the use of Method 2—Simplified eliminates this possibility altogether.

Step 3: Construct design interaction surface A′′, B′′, C′′, D′′ and verify member adequacy

The final step in the Method 2 procedure is to reduce the interaction surface for design using the appropriate resistance or safety factors.

Fig. I.11-3. Nominal strength interaction surfaces (with and without length effects).
The available compressive and flexural strengths are determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>Allowable compressive strength:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \phi = 0.75 )</td>
</tr>
<tr>
<td></td>
<td>( P_{c} = \phi P_{c'} )</td>
</tr>
<tr>
<td>where ( X = A, B, C ) or ( D )</td>
<td>where ( X = A, B, C ) or ( D )</td>
</tr>
<tr>
<td></td>
<td>( P_{c'} = 0.75 \times 2,860 ) kips</td>
</tr>
<tr>
<td></td>
<td>= 2,150 kips</td>
</tr>
<tr>
<td></td>
<td>( P_{c'} = 0.75 \times 0 ) kips</td>
</tr>
<tr>
<td></td>
<td>= 0 kips</td>
</tr>
<tr>
<td></td>
<td>( P_{c'} = 0.75 \times 1,980 ) kips</td>
</tr>
<tr>
<td></td>
<td>= 1,490 kips</td>
</tr>
<tr>
<td></td>
<td>( P_{c'} = 0.75 \times 990 ) kips</td>
</tr>
<tr>
<td></td>
<td>= 743 kips</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design flexural strength:</th>
<th>Allowable flexural strength:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega ) = 1.67</td>
</tr>
<tr>
<td>( M_{x'} = \phi M_{x} )</td>
<td>( M_{x'} = M_{x} / \Omega )</td>
</tr>
<tr>
<td>where ( X = A, B, C ) or ( D )</td>
<td>where ( X = A, B, C ) or ( D )</td>
</tr>
<tr>
<td>( M_{x} = 0.90 \times 0 ) kip-ft</td>
<td>( M_{x'} = 0 ) kip-ft / 1.67</td>
</tr>
<tr>
<td>= 0 kip-ft</td>
<td>= 0 kip-ft</td>
</tr>
<tr>
<td>( M_{x} = 0.90 \times 748 ) kip-ft</td>
<td>( M_{x'} = 748 ) kip-ft / 1.67</td>
</tr>
<tr>
<td>= 673 kip-ft</td>
<td>= 448 kip-ft</td>
</tr>
<tr>
<td>( M_{x} = 0.90 \times 748 ) kip-ft</td>
<td>( M_{x'} = 748 ) kip-ft / 1.67</td>
</tr>
<tr>
<td>= 673 kip-ft</td>
<td>= 448 kip-ft</td>
</tr>
<tr>
<td>( M_{x} = 0.90 \times 1,050 ) kip-ft</td>
<td>( M_{x'} = 1,050 ) kip-ft / 1.67</td>
</tr>
<tr>
<td>= 945 kip-ft</td>
<td>= 629 kip-ft</td>
</tr>
</tbody>
</table>

The available strength values for each design method can now be plotted. These values are superimposed on the nominal strength surfaces (with and without length effects) previously calculated for comparison purposes in Figure I.11-4.
By plotting the required axial and flexural strength values on the available strength surfaces indicated in Figure I.11-4, it can be seen that both ASD ($M_a, P_a$) and LRFD ($M_u, P_u$) points lie within their respective design surfaces. The member in question is therefore adequate for the applied loads.

As discussed previously in Step 2 as well as in AISC Specification Commentary Section I5, when reducing the flexural strength of Point D for length effects and resistance or safety factors, an unsafe situation could result whereby additional flexural strength is permitted at a lower axial compressive strength than predicted by the cross section strength of the member. This effect is highlighted by the magnified portion of Figure I.11-4, where LRFD design point D'' falls slightly below the nominal strength curve. Designs falling within this zone are unsafe and not permitted.

**Method 2—Simplified**

The unsafe zone discussed in the previous section for Method 2 is avoided in the Method 2—Simplified procedure by the removal of Point D'' from the Method 2 interaction surface leaving only points A'', B'' and C'' as illustrated in Figure I.11-5. Reducing the number of interaction points also allows for a bilinear interaction check defined by AISC Specification Commentary Equations C-I5-1a and C-I5-1b to be performed.
Using the available strength values previously calculated in conjunction with the Commentary equations, interaction ratios are determined as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_t = P_{uc}$</td>
<td>$1,170 \text{ kips}$</td>
<td>$879 \text{ kips}$</td>
</tr>
<tr>
<td>$P_t &lt; P_{uc}$</td>
<td>$&lt; 1,490 \text{ kips}$</td>
<td>$&lt; 990 \text{ kips}$</td>
</tr>
<tr>
<td>Therefore, use Commentary Equation C-I5-1a.</td>
<td>$M_f/M_C \leq 1.0$</td>
<td>$M_f/M_C \leq 1.0$</td>
</tr>
<tr>
<td>$670 \text{ kip-ft}$</td>
<td>$\leq 1.0$</td>
<td>$302 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$673 \text{ kip-ft}$</td>
<td>$= 1.0$</td>
<td>$448 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$1.0 = 1.0$</td>
<td>o.k.</td>
<td>$0.67 &lt; 1.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Thus, the member is adequate for the applied loads.

**Comparison of Methods**

The composite member was found to be inadequate using Method 1—Chapter H interaction equations, but was found to be adequate using both Method 2 and Method 2—Simplified procedures. A comparison between the methods is most easily made by overlaying the design curves from each method as illustrated in Figure I.11-6 for LRFD design.
From Figure I.11-6, the conservative nature of the Chapter H interaction equations can be seen. Method 2 provides the highest available strength; however, the Method 2—Simplified procedure also provides a good representation of the design curve. The procedure in Figure I-1 for calculating the flexural strength of Point C'' first requires the calculation of the flexural strength for Point D''. The design effort required for the Method 2—Simplified procedure, which utilizes Point C'', is therefore not greatly reduced from Method 2.

**Available Shear Strength**

According to AISC Specification Section I4.1, there are three acceptable options for determining the available shear strength of an encased composite member:

- **Option 1**—Available shear strength of the steel section alone in accordance with AISC Specification Chapter G.
- **Option 2**—Available shear strength of the reinforced concrete portion alone per ACI 318.
- **Option 3**—Available shear strength of the steel section in addition to the reinforcing steel ignoring the contribution of the concrete.

Option 1—Available Shear Strength of Steel Section

A W10×45 member meets the criteria of AISC Specification Section G2.1(a) according to the User Note at the end of the section. As demonstrated in Design Example I.9, No. 3 ties at 12 in. on center as illustrated in Figure I.11-1 satisfy the minimum detailing requirements of the Specification. The nominal shear strength may therefore be determined as:
The available shear strength of the steel section is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_u = 95.7$ kips</td>
<td>$V_a = 57.4$ kips</td>
</tr>
<tr>
<td>$\phi_v = 1.0$</td>
<td>$\Omega_v = 1.50$</td>
</tr>
<tr>
<td>$\phi_v V_u \geq V_u$</td>
<td>$V_u / \Omega_v \geq V_a$</td>
</tr>
<tr>
<td>$\phi_v V_u = 1.0(106$ kips$)$</td>
<td>$V_u / \Omega_v = \frac{106$ kips$}{1.50}$</td>
</tr>
<tr>
<td>106 kips &gt; 95.7 kips o.k.</td>
<td>70.7 kips &gt; 57.4 kips o.k.</td>
</tr>
</tbody>
</table>

Option 2—Available Shear Strength of the Reinforced Concrete (Concrete and Transverse Steel Reinforcement)

The available shear strength of the steel section alone has been shown to be sufficient; however, the amount of transverse reinforcement required for shear resistance in accordance with AISC Specification Section 14.1(b) will be determined for demonstration purposes.

**Tie Requirements for Shear Resistance**

The nominal concrete shear strength is:

$$V_c = 2\lambda \sqrt{f'_c b_v d}$$

where

- $\lambda = 1.0$ for normal weight concrete from ACI 318 Section 8.6.1
- $b_v = h_i$
- $d =$ distance from extreme compression fiber to centroid of longitudinal tension reinforcement
  - $= 24$ in. − $2\frac{1}{2}$ in.
  - $= 21.5$ in.

$$V_c = 2(1.0) \sqrt{5,000 \text{ psi}} (24 \text{ in.})(21.5 \text{ in.}) \left( \frac{1 \text{ kip}}{1,000 \text{ lb}} \right)$$

$$= 73.0 \text{ kips}$$

The tie requirements for shear resistance are determined from ACI 318 Chapter 11 and AISC Specification Section I4.1(b), as follows:
Minimum Reinforcing Limits

Check that the minimum shear reinforcement is provided as required by ACI 318, Section 11.4.6.3.

\[
A_{v,\min} = 0.75 \sqrt{f'_c \left( \frac{b_v s}{f_y} \right)} \geq \frac{50 b_v s}{f_y} \\
A_{v,\min} = \frac{0.75 \sqrt{5,000 \text{ psi} (24 \text{ in.})}}{60,000 \text{ psi}} \geq \frac{50 (24 \text{ in.})}{60,000 \text{ psi}}
\]

\[
= 0.0212 \geq 0.0200
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_v = 95.7 \text{ kips} )</td>
<td>( V_a = 57.4 \text{ kips} )</td>
</tr>
<tr>
<td>( \phi_v = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( A_v = \frac{V_a - \phi_v V_c}{\phi f_y d} )</td>
<td>( A_v = \frac{V_a - (V_c/\Omega)}{f_y d/\Omega} )</td>
</tr>
<tr>
<td>= 95.7 kips ( -0.75(73.0 \text{ kips}) )</td>
<td>= ( \frac{57.4 \text{ kips} - (73.0 \text{ kips})}{2.00} )</td>
</tr>
<tr>
<td>( 0.75(60 \text{ ksi})(21.5 \text{ in.}) )</td>
<td>= ( \frac{60 \text{ ksi}(21.5 \text{ in.})}{2.00} )</td>
</tr>
<tr>
<td>= 0.0423 in.</td>
<td>= 0.0324 in.</td>
</tr>
</tbody>
</table>

Using two legs of No. 3 ties with \( A_v = 0.11 \text{ in.}^2 \) from ACI 318 Appendix E:

\[
2(0.11 \text{ in.}^2) = 0.0423 \text{ in.}
\]

\( s = 5.20 \text{ in.} \)

Using two legs of the No. 4 ties with \( A_v = 0.20 \text{ in.}^2 \):

\[
2(0.20 \text{ in.}^2) = 0.0423 \text{ in.}
\]

\( s = 9.46 \text{ in.} \)

From ACI 318 Section 11.4.5.1, the maximum spacing is:

\[
s_{\text{max}} = \frac{d}{2} = \frac{21.5 \text{ in.}}{2} = 10.8 \text{ in.}
\]

Use No. 3 ties at 5 in. o.c. or No. 4 ties at 9 in. o.c.

From ACI 318 Section 11.4.5.1, the maximum spacing is:

\[
s_{\text{max}} = \frac{d}{2} = \frac{21.5 \text{ in.}}{2} = 10.8 \text{ in.}
\]

Use No. 3 ties at 6 in. o.c. or No. 4 ties at 10 in. o.c.
Maximum Reinforcing Limits

From ACI 318 Section 11.4.5.3, maximum stirrup spacing is reduced to \( d/4 \) if \( V_s \geq 4\sqrt{f_y b_n d} \). If No. 4 ties at 9 in. on center are selected:

\[
V_s = \frac{A_v f_y d}{s}
\]

\[
= \frac{2(0.20 \text{ in.}^2)(60 \text{ ksi})(21.5 \text{ in.)}}{9 \text{ in.}}
\]

\[
= 57.3 \text{ kips}
\]

\[
V_{s,\text{max}} = 4\sqrt{f_y b_n d}
\]

\[
= 4\sqrt{5,000 \text{ psi}(24 \text{ in.})(21.5 \text{ in.})\left(\frac{1 \text{ kip}}{1,000 \text{ lb}}\right)}
\]

\[
= 146 \text{ kips} > 57.3 \text{ kips}
\]

Therefore, the stirrup spacing is acceptable.

Option 3—Determine Available Shear Strength of the Steel Section Plus Reinforcing Steel

The third procedure combines the shear strength of the reinforcing steel with that of the encased steel section, ignoring the contribution of the concrete. AISC Specification Section 14.1(c) provides a combined resistance and safety factor for this procedure. Note that the combined resistance and safety factor takes precedence over the factors in Chapter G used for the encased steel section alone in Option 1. The amount of transverse reinforcement required for shear resistance is determined as follows:

Tie Requirements for Shear Resistance

The nominal shear strength of the encased steel section was previously determined to be:

\[
V_{n,\text{steel}} = 106 \text{ kips}
\]

The tie requirements for shear resistance are determined from ACI 318 Chapter 11 and AISC Specification Section 14.1(c), as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_a = 95.7 \text{ kips as} )</td>
<td>( V_a = 57.4 \text{ kips} )</td>
</tr>
<tr>
<td>( \phi_e = 0.75 )</td>
<td>( \Omega_s = 2.00 )</td>
</tr>
<tr>
<td>( A_v = V_a - \phi_e V_{n,\text{steel}} )</td>
<td>( A_v = V_a - \left(V_{n,\text{steel}} / \Omega_s\right) )</td>
</tr>
<tr>
<td>( \frac{V_a}{s} = \frac{V_a - \phi_e V_{n,\text{steel}}}{s} )</td>
<td>( \frac{A_v}{s} = \frac{V_a - \left(V_{n,\text{steel}} / \Omega_s\right)}{f_y d / \Omega_s} )</td>
</tr>
<tr>
<td>95.7 kips - 0.75(106 kips) ( = \frac{0.75(60 \text{ ksi})(21.5 \text{ in.)}}{60 \text{ ksi})(21.5 \text{ in.)}})</td>
<td>57.4 kips - ( \left(106 \text{ kips}/2.00\right) )</td>
</tr>
<tr>
<td>0.0167 in.</td>
<td>0.00682 in.</td>
</tr>
</tbody>
</table>
As determined in Option 2, the minimum value of \( A_v / s = 0.0212 \), and the maximum tie spacing for shear resistance is 10.8 in. Using two legs of No. 3 ties for \( A_v \):

\[
\frac{2(0.11 \text{ in.}^2)}{s} = 0.0212 \text{ in.}
\]

\[
s = 10.4 \text{ in.} < s_{\text{ut}} = 10.8 \text{ in.}
\]

Use No. 3 ties at 10 in. o.c.

*Summary and Comparison of Available Shear Strength Calculations*

The use of the steel section alone is the most expedient method for calculating available shear strength and allows the use of a tie spacing which may be greater than that required for shear resistance by ACI 318. Where the strength of the steel section alone is not adequate, Option 3 will generally result in reduced tie reinforcement requirements as compared to Option 2.

*Force Allocation and Load Transfer*

Load transfer calculations should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions for encased composite members is provided in Design Example I.8 and AISC Design Guide 6.
EXAMPLE I.12 STEEL ANCHORS IN COMPOSITE COMPONENTS

Given:
Select an appropriate ¾-in.-diameter, Type B steel headed stud anchor to resist the dead and live loads indicated in Figure I.12-1. The anchor is part of a composite system that may be designed using the steel anchor in composite components provisions of AISC Specification Section I8.3.

![Fig. I.12-1. Steel headed stud anchor and applied loading.](image)

The steel headed stud anchor is encased by normal weight (3145 lb/ft³) reinforced concrete having a specified concrete compressive strength, \( f'_c = 5 \text{ ksi} \). In accordance with AWS D1.1, steel headed stud anchors shall be made from material conforming to the requirements of ASTM A108. From AISC Manual Table 2-6, the specified minimum tensile stress, \( F_u \), for ASTM A108 material is 65 ksi.

The anchor is located away from edges such that concrete breakout in shear is not a viable limit state, and the nearest anchor is located 24 in. away. The concrete is considered to be uncracked.

Solution:

Minimum Anchor Length

AISC Specification Section I8.3 provides minimum length to shank diameter ratios for anchors subjected to shear, tension, and interaction of shear and tension in both normal weight and lightweight concrete. These ratios are also summarized in the User Note provided within Section I8.3. For normal weight concrete subject to shear and tension, \( h/d \geq 8 \), thus:

\[
\begin{align*}
  h & \geq 8d \\
  & \geq 8(\frac{3}{4} \text{ in.}) \\
  & \geq 6.00 \text{ in.}
\end{align*}
\]

This length is measured from the base of the steel headed stud anchor to the top of the head after installation. From anchor manufacturer’s data, a standard stock length of 6\( \frac{1}{2} \) in. is selected. Using a \( \frac{1}{60} \)-in. length reduction to account for burn off during installation yields a final installed length of 6.00 in.

6.00 in. = 6.00 in. o.k.

Select a ¾-in.-diameter × 6\( \frac{1}{2} \)-in.-long headed stud anchor.
**Required Shear and Tensile Strength**

From Chapter 2 of ASCE/SEI 7, the required shear and tensile strengths are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Governing Load Combination for interaction:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_{sv}$ = 1.2(2.00 kips) + 1.6(5.00 kips)</td>
<td>10.4 kips (shear)</td>
<td>7.00 kips (shear)</td>
</tr>
<tr>
<td>$Q_{tu}$ = 1.2(3.00 kips) + 1.6(7.50 kips)</td>
<td>15.6 kips (tension)</td>
<td>10.5 kips (tension)</td>
</tr>
</tbody>
</table>

**Available Shear Strength**

Per the problem statement, concrete breakout is not considered to be an applicable limit state. AISC Equation I8-3 may therefore be used to determine the available shear strength of the steel headed stud anchor as follows:

$Q_{sv} = F_v A_w \tag{Spec. Eq. I8-3}$

where

- $A_w$ = cross-sectional area of steel headed stud anchor

$\pi \left( \frac{3}{4} \text{ in.} \right)^2$

$= \frac{\pi}{4}$

$= 0.442 \text{ in.}^2$

$Q_{sv} = (65 \text{ ksi})(0.442 \text{ in.}^2)$

$= 28.7 \text{ kips}$

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_v$</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>$\phi_v Q_{sv} = 0.65(28.7 \text{ kips})$</td>
<td>18.7 kips</td>
<td></td>
</tr>
<tr>
<td>$\Omega_v$</td>
<td>2.31</td>
<td></td>
</tr>
<tr>
<td>$Q_{sv} / \Omega_v$</td>
<td>$\frac{28.7 \text{ kips}}{2.31}$</td>
<td>$12.4 \text{ kips}$</td>
</tr>
</tbody>
</table>

Alternately, available shear strengths can be selected directly from Table I.12-1 located at the end of this example.

**Available Tensile Strength**

The nominal tensile strength of a steel headed stud anchor is determined using AISC Specification Equation I8-4 provided the edge and spacing limitations of AISC Specification Section I8.3b are met as follows:

1. Minimum distance from centerline of anchor to free edge: $1.5h = 1.5(6.00 \text{ in.}) = 9.00 \text{ in.}$

   There are no free edges, therefore this limitation does not apply.

2. Minimum distance between centerlines of adjacent anchors: $3h = 3(6.00 \text{ in.}) = 18.0 \text{ in.}$

   18.0 in. < 24 in.  o.k.
Equation I8-4 may therefore be used as follows:

\[ Q_{aw} = F_a A_{aw} \]
\[ Q_{aw} = (65 \text{ ksi})(0.442 \text{ in.}^2) \]
\[ = 28.7 \text{ kips} \]

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi, = 0.75 & \Omega, = 2.00 \\
\phi, Q_{aw} = 0.75(28.7 \text{ kips}) & Q_{aw} = 28.7 \text{ kips} \\
= 21.5 \text{ kips} & \Omega, = 2.00 \\
\hline
\end{array}
\]

Alternately, available tension strengths can be selected directly from Table I.12-1 located at the end of this example.

**Interaction of Shear and Tension**

The detailing limits on edge distances and spacing imposed by AISC *Specification* Section I8.3c for shear and tension interaction are the same as those previously reviewed separately for tension and shear alone. Tension and shear interaction is checked using *Specification* Equation I8-5 which can be written in terms of LRFD and ASD design as follows:

\[
\begin{align*}
\left[ \frac{Q_{aw}}{\phi, Q_{aw}} \right]^{5/3} + \left[ \frac{Q_{aw}}{\phi, Q_{aw}} \right]^{5/3} & \leq 1.0 \\
\left[ \frac{15.6 \text{ kips}}{21.5 \text{ kips}} \right]^{5/3} + \left[ \frac{10.4 \text{ kips}}{18.7 \text{ kips}} \right]^{5/3} & = 0.96
\end{align*}
\]

Thus, 0.96 < 1.0 o.k.

\[
\begin{align*}
\left[ \frac{Q_{aw}}{Q_{aw} / \Omega,} \right]^{5/3} + \left[ \frac{Q_{aw}}{Q_{aw} / \Omega,} \right]^{5/3} & \leq 1.0 \\
\left[ \frac{10.5 \text{ kips}}{14.4 \text{ kips}} \right]^{5/3} + \left[ \frac{7.00 \text{ kips}}{12.4 \text{ kips}} \right]^{5/3} & = 0.98
\end{align*}
\]

Thus, 0.98 < 1.0 o.k.

Thus, a \( \frac{3}{8} \)-in.-diameter \( \times 6\frac{1}{8} \)-in.-long headed stud anchor is adequate for the applied loads.

**Limits of Application**

The application of the steel anchors in composite component provisions have strict limitations as summarized in the User Note provided at the beginning of AISC *Specification* Section I8.3. These provisions do not apply to typical composite beam designs nor do they apply to hybrid construction where the steel and concrete do not resist loads together via composite action such as in embed plates. This design example is intended solely to illustrate the calculations associated with an isolated anchor that is part of an applicable composite system.

**Available Strength Table**

Table I.12-1 provides available shear and tension strengths for standard Type B steel headed stud anchors conforming to the requirements of AWS D1.1 for use in composite components.
### Table I.12-1. Steel Headed Stud Anchor Available Strengths

<table>
<thead>
<tr>
<th>Anchor Shank Diameter</th>
<th>$A_{sv}$</th>
<th>$Q_{sv} / \Omega_v$</th>
<th>$\phi_v Q_{sv}$</th>
<th>$Q_{atl} / \Omega_t$</th>
<th>$\phi_t Q_{atl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>in.</td>
<td>in.$^2$</td>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
</tr>
<tr>
<td>¼</td>
<td>0.199</td>
<td>5.52</td>
<td>8.30</td>
<td>6.83</td>
<td>9.57</td>
</tr>
<tr>
<td>½</td>
<td>0.307</td>
<td>8.83</td>
<td>13.0</td>
<td>9.97</td>
<td>15.0</td>
</tr>
<tr>
<td>¾</td>
<td>0.442</td>
<td>12.4</td>
<td>18.7</td>
<td>14.4</td>
<td>21.5</td>
</tr>
<tr>
<td>¾</td>
<td>0.601</td>
<td>16.9</td>
<td>25.4</td>
<td>N/A*</td>
<td>N/A*</td>
</tr>
<tr>
<td>1</td>
<td>0.785</td>
<td>22.1</td>
<td>33.2</td>
<td>25.5</td>
<td>38.3</td>
</tr>
</tbody>
</table>

| ASD | LRFD | *¼-in.-diameter anchors conforming to AWS D1.1

- $\Omega_v = 2.31$
- $\phi_v = 0.65$
- $\Omega_t = 2.00$
- $\phi_t = 0.75$

Figure 7.1 do not meet the minimum head-to-shank diameter ratio of 1.6 as required for tensile resistance per AISC Specification Section 18.3.
CHAPTER I DESIGN EXAMPLE REFERENCES

ASCE (2002), Design Loads on Structures During Construction, SEI/ASCE 37-02, American Society of Civil Engineers, Reston, VA.


ICC (2009), International Building Code, International Code Council, Falls Church, VA.


SDI (2006), Standard for Composite Steel Floor Deck, ANSI/SDI C1.0-2006, Fox River Grove, IL.


Chapter J
Design of Connections

Chapter J of the AISC Specification addresses the design and review of connections. The chapter’s primary focus is the design of welded and bolted connections. Design requirements for fillers, splices, column bases, concentrated forces, anchors rods and other threaded parts are also covered. Special requirements for connections subject to fatigue are not covered in this chapter.
EXAMPLE J.1  FILLET WELD IN LONGITUDINAL SHEAR

Given:

A ¼-in. × 18-in. wide plate is fillet welded to a ¾-in. plate. The plates are ASTM A572 Grade 50 and have been properly sized. Use 70-ksi electrodes. Note that the plates would normally be specified as ASTM A36, but $F_y = 50$ ksi plate has been used here to demonstrate the requirements for long welds.

Verify the welds for the loads shown.

Solution:

From Chapter 2 of ASCE/SEI 7, the required strength is:

\[
\begin{align*}
P_{u, LRFD} &= 1.2(33.0 \text{ kips}) + 1.6(100 \text{ kips}) \\
&= 200 \text{ kips} \\

P_{u, ASD} &= 33.0 \text{ kips} + 100 \text{ kips} \\
&= 133 \text{ kips}
\end{align*}
\]

Maximum and Minimum Weld Size

Because the thickness of the overlapping plate is ¼ in., the maximum fillet weld size that can be used without special notation per AISC Specification Section J2.2b, is a ¾-in.-fillet weld. A ¾-in.-fillet weld can be deposited in the flat or horizontal position in a single pass (true up to ¾-in.).

From AISC Specification Table J2.4, the minimum size of fillet weld, based on a material thickness of ¼ in. is ⅛ in.

Length of Weld Required

The nominal weld strength per inch of ¾-in. weld, determined from AISC Specification Section J2.4(a) is:

\[
R_n = F_{we} A_{we} = (0.60 F_{wx})(A_{we}) = 0.60(70 \text{ ksi})(\frac{3}{16} \text{ in.}/\sqrt{2}) = 5.57 \text{ kips/in.}
\]
From AISC Specification Section J2.2b, for longitudinal fillet welds used alone in end connections of flat-bar tension members, the length of each fillet weld shall be not less than the perpendicular distance between them.

24 in. ≥ 18 in.  o.k.

From AISC Specification Section J2.2b, check the weld length to weld size ratio, because this is an end loaded fillet weld.

\[
\frac{L}{w} = \frac{24 \text{ in.}}{\frac{1}{6} \text{ in.}} = 128 > 100. \text{ therefore, AISC Specification Equation J2-1 must be applied, and the length of weld increased, because the resulting } \beta \text{ will reduce the available strength below the required strength.}
\]

Try a weld length of 27 in.

The new length to weld size ratio is:

\[
\frac{27.0 \text{ in.}}{\frac{1}{6} \text{ in.}} = 144
\]

For this ratio:

\[
\beta = 1.2 - 0.002(\frac{l}{w}) \leq 1.0 \quad \text{(Spec. Eq. J2-1)}
\]

\[
= 1.2 - 0.002(144)
\]

\[
= 0.912
\]

Recheck the weld at its reduced strength.

\[
\phi R_u = (0.912)(0.75)(5.57 \text{ kips/in.})(54.0 \text{ in.}) = 206 \text{ kips} \quad \text{o.k.}
\]

Therefore, use 27 in. of weld on each side.
EXAMPLE J.2 FILLET WELD LOADED AT AN ANGLE

Given:

Design a fillet weld at the edge of a gusset plate to carry a force of 50.0 kips due to dead load and 150 kips due to live load, at an angle of 60° relative to the weld. Assume the beam and the gusset plate thickness and length have been properly sized. Use a 70-ksi electrode.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

\[
P_u = 1.2(50.0 \text{ kips}) + 1.6(150 \text{ kips}) = 300 \text{ kips}
\]

\[
P_a = 50.0 \text{ kips} + 150 \text{ kips} = 200 \text{ kips}
\]

Assume a 3/8-in. fillet weld is used on each side of the plate.

Note that from AISC Specification Table J2.4, the minimum size of fillet weld, based on a material thickness of \( \frac{3}{8} \) in. is \( \frac{3}{4} \) in.

Available Shear Strength of the Fillet Weld Per Inch of Length

From AISC Specification Section J2.4(a), the nominal strength of the fillet weld is determined as follows:

\[
A_{we} = \frac{\gamma_{tu}}{\sqrt{2}} = 0.221 \text{ in.}
\]

\[
F_{uw} = 0.60 F_{yxx} \left( 1.0 + 0.5 \sin^{1.5} \theta \right)
\]

\[
= 0.60(70 \text{ ksi}) \left( 1.0 + 0.5 \sin^{1.5} 60° \right)
\]

\[
= 58.9 \text{ ksi}
\]

\[
R_e = F_{uw} A_{we}
\]

\[
= 58.9 \text{ ksi}(0.221 \text{ in.})
\]

\[
= 13.0 \text{ kip/\text{in.}}
\]
From AISC *Specification* Section J2.4(a), the available shear strength per inch of weld length is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td></td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_u = 0.75(13.0 \text{ kip/in.})$</td>
<td>$R_u = \frac{13.0 \text{ kip/in.}}{2.00}$</td>
<td>$\Omega$ = 2.00</td>
</tr>
<tr>
<td></td>
<td>$= 9.75 \text{ kip/in.}$</td>
<td>$= 6.50 \text{ kip/in.}$</td>
</tr>
<tr>
<td>For 2 sides:</td>
<td></td>
<td>For 2 sides:</td>
</tr>
<tr>
<td>$\phi R_u = 2(0.75)(13.0 \text{ kip/in.})$</td>
<td>$R_u = \frac{2(13.0 \text{ kip/in.})}{2.00}$</td>
<td>$\Omega$ = 2.00</td>
</tr>
<tr>
<td></td>
<td>$= 19.5 \text{ kip/in.}$</td>
<td>$= 13.0 \text{ kip/in.}$</td>
</tr>
</tbody>
</table>

**Required Length of Weld**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l = \frac{300 \text{ kips}}{19.5 \text{ kip/in.}} = 15.4 \text{ in.}$</td>
<td>$l = \frac{200 \text{ kips}}{13.0 \text{ kip/in.}} = 15.4 \text{ in.}$</td>
<td></td>
</tr>
</tbody>
</table>

Use 16 in. on each side of the plate.
EXAMPLE J.3 COMBINED TENSION AND SHEAR IN BEARING TYPE CONNECTIONS

Given:

A ¾-in.-diameter ASTM A325-N bolt is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load. Check the combined stresses according to AISC Specification Equations J3-3a and J3-3b.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile and shear strengths are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tension</strong>:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_u$</td>
<td>$1.2(3.50 \text{ kips}) + 1.6(12.0 \text{ kips})$</td>
<td>$3.50 \text{ kips} + 12.0 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 23.4 \text{ kips}$</td>
<td>$= 15.5 \text{ kips}$</td>
</tr>
<tr>
<td><strong>Shear</strong>:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_u$</td>
<td>$1.2(1.33 \text{ kips}) + 1.6(4.00 \text{ kips})$</td>
<td>$1.33 \text{ kips} + 4.00 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 8.00 \text{ kips}$</td>
<td>$= 5.33 \text{ kips}$</td>
</tr>
</tbody>
</table>

Available Tensile Strength

When a bolt is subjected to combined tension and shear, the available tensile strength is determined according to the limit states of tension and shear rupture, from AISC Specification Section J3.7 as follows.

From AISC Specification Table J3.2,

$F_{tu} = 90 \text{ ksi}, F_{nv} = 54 \text{ ksi}$

From AISC Manual Table 7-1, for a ¾-in.-diameter bolt,

$A_b = 0.442 \text{ in.}^2$

The available shear stress is determined as follows and must equal or exceed the required shear stress.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi = 0.75$</td>
<td></td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi F_{nv} = 0.75(54 \text{ ksi})$</td>
<td>$\frac{F_{nv}}{\Omega} = \frac{54 \text{ ksi}}{2.00} = 27.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 40.5 \text{ ksi}$</td>
<td></td>
</tr>
<tr>
<td>$f_{nv}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= \frac{V_u}{A_b}$</td>
<td>$f_{nv} = \frac{V_u}{A_b}$</td>
<td>$= \frac{5.33 \text{ kips}}{0.442 \text{ in.}^2} = 12.1 \text{ ksi} \leq 27.0 \text{ ksi}$</td>
</tr>
<tr>
<td></td>
<td>$= 8.00 \text{ kips}$</td>
<td>$= 18.1 \text{ ksi} \leq 40.5 \text{ ksi}$</td>
</tr>
<tr>
<td></td>
<td>$= 0.442 \text{ in.}^2$</td>
<td></td>
</tr>
</tbody>
</table>

The available tensile strength of a bolt subject to combined tension and shear is as follows:
### LRFD

\[
F'_{ut} = 1.3 \frac{F_{ut}}{\phi_{F_{ut}}} - f_{rc} \leq F_{ut} \quad (Spec. \ Eq. \ J3.3a)
\]

\[
= 1.3\left(90 \text{ ksi}\right) - \frac{90 \text{ ksi}}{0.75(54 \text{ ksi})} (18.1 \text{ ksi})
\]

\[
= 76.8 \text{ ksi} \leq 90 \text{ ksi}
\]

\[
R_s = F'_{ut} A_b \quad (Spec. \ Eq. \ J3-2)
\]

\[
= 76.8 \text{ ksi} \left(0.442 \text{ in.}^2\right)
\]

\[
= 33.9 \text{ kips}
\]

For combined tension and shear,
\[\phi = 0.75 \text{ from AISC Specification Section J3.7}\]

### Design tensile strength:

\[
\phi R_s = 0.75(33.9 \text{ kips})
\]

\[
= 25.4 \text{ kips} > 23.4 \text{ kips} \quad \text{o.k.}
\]

### ASD

\[
F'_{ut} = 1.3 \frac{\Omega F_{ut}}{F_{cw}} - f_{rc} \leq F_{ut} \quad (Spec. \ Eq. \ J3.3b)
\]

\[
= 1.3(90 \text{ ksi}) - \frac{2.00(90 \text{ ksi})}{54 \text{ ksi}} (12.1 \text{ ksi})
\]

\[
= 76.7 \text{ ksi} \leq 90 \text{ ksi}
\]

\[
R_s = F'_{ut} A_b \quad (Spec. \ Eq. \ J3-2)
\]

\[
= 76.7 \text{ ksi} \left(0.442 \text{ in.}^2\right)
\]

\[
= 33.9 \text{ kips}
\]

For combined tension and shear,
\[\Omega = 2.00 \text{ from AISC Specification Section J3.7}\]

### Allowable tensile strength:

\[
\frac{R_s}{\Omega} = \frac{33.9 \text{ kips}}{2.00}
\]

\[
= 17.0 \text{ kips} > 15.5 \text{ kips} \quad \text{o.k.}
\]
EXAMPLE J.4A  SLIP-CRITICAL CONNECTION WITH SHORT-SLOTTED HOLES

Slip-critical connections shall be designed to prevent slip and for the limit states of bearing-type connections.

**Given:**

Select the number of ¾-in.-diameter ASTM A325 slip-critical bolts with a Class A faying surface that are required to support the loads shown when the connection plates have short slots transverse to the load and no fillers are provided. Select the number of bolts required for slip resistance only.

**Solution:**

From Chapter 2 of ASCE/SEI 7, the required strength is:

\[
P_u = 1.2(17.0 \text{ kips}) + 1.6(51.0 \text{ kips}) = 102 \text{ kips}
\]

\[
P_a = 17.0 \text{ kips} + 51.0 \text{ kips} = 68.0 \text{ kips}
\]

From AISC Specification Section J3.8(a), the available slip resistance for the limit state of slip for standard size and short-slotted holes perpendicular to the direction of the load is determined as follows:

\[
\phi = 1.00 \quad \Omega = 1.50
\]

\[
\mu = 0.30 \text{ for Class A surface}
\]

\[
D_a = 1.13
\]

\[
h_f = 1.0 \text{, factor for fillers, assuming no more than one filler}
\]

\[
T_b = 28 \text{ kips, from AISC Specification Table J3.1}
\]

\[
n_s = 2 \text{, number of slip planes}
\]

\[
R_n = \mu D_a h_f T_b n_s
\]

\[
= 0.30(1.13)(1.0)(28 \text{ kips})(2)
\]

\[
= 19.0 \text{ kips/bolt}
\]

The available slip resistance is:

\[
\phi R_n = 1.00(19.0 \text{ kips/bolt}) = 19.0 \text{ kips/bolt}
\]

\[
\frac{R_n}{\Omega} = \frac{19.0 \text{ kips/bolt}}{1.50} = 12.7 \text{ kips/bolt}
\]
Required Number of Bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_b = \frac{P_u}{\phi R_s}$</td>
<td>$n_b = \frac{P_u}{(\frac{R_u}{\Omega})}$</td>
</tr>
<tr>
<td>$= \frac{102 \text{ kips}}{19.0 \text{ kips/bolt}}$</td>
<td>$= \frac{68.0 \text{ kips}}{(12.7 \text{ kips/bolt})}$</td>
</tr>
<tr>
<td>$= 5.37 \text{ bolts}$</td>
<td>$= 5.37 \text{ bolts}$</td>
</tr>
</tbody>
</table>

Use 6 bolts

Use 6 bolts

Note: To complete the design of this connection, the limit states of bolt shear, bolt bearing, tensile yielding, tensile rupture, and block shear rupture must be determined.
EXAMPLE J.4B  SLIP-CRITICAL CONNECTION WITH LONG-SLOTTED HOLES

Given:

Repeat Example J.4A with the same loads, but assuming that the connected pieces have long-slotted holes in the direction of the load.

Solution:

The required strength from Example J.4A is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 102 \text{ kips}$</td>
<td>$P_a = 68.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

From AISC Specification Section J3.8(c), the available slip resistance for the limit state of slip for long-slotted holes is determined as follows:

$$\phi = 0.70 \quad \Omega = 2.14$$

$$\mu = 0.30 \text{ for Class A surface}$$

$$D_a = 1.13$$

$$h_f = 1.0, \text{ factor for fillers, assuming no more than one filler}$$

$$T_b = 28 \text{ kips, from AISC Specification Table J3.1}$$

$$n_s = 2, \text{ number of slip planes}$$

$$R_n = \mu D_a h_f T_b n_s$$  \hspace{1cm} (Spec. Eq. J3-4)

$$= 0.30 \times 1.13 \times 1.0 \times 28 \text{ kips} \times 2$$

$$= 19.0 \text{ kips/bolt}$$

The available slip resistance is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 0.70 \times 19.0 \text{ kips/bolt}$</td>
<td>$\frac{R_n}{\Omega} = \frac{19.0 \text{ kips/bolt}}{2.14} = 8.88 \text{ kips/bolt}$</td>
</tr>
<tr>
<td>$= 13.3 \text{ kips/bolt}$</td>
<td></td>
</tr>
</tbody>
</table>
### Required Number of Bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_b = \frac{P_u}{\phi R_u} )</td>
<td>( n_b = \frac{P_u}{\left(\frac{R_u}{\Omega}\right)} )</td>
</tr>
<tr>
<td>( \frac{102 \text{ kips}}{13.3 \text{ kips/bolt}} )</td>
<td>( \frac{68.0 \text{ kips}}{8.88 \text{ kips/bolt}} )</td>
</tr>
<tr>
<td>= 7.67 bolts</td>
<td>= 7.66 bolts</td>
</tr>
</tbody>
</table>

Use 8 bolts

Note: To complete the design of this connection, the limit states of bolt shear, bolt bearing, tensile yielding, tensile rupture, and block shear rupture must be determined.
EXAMPLE J.5  COMBINED TENSION AND SHEAR IN A SLIP-CRITICAL CONNECTION

Because the pretension of a bolt in a slip-critical connection is used to create the clamping force that produces the shear strength of the connection, the available shear strength must be reduced for any load that produces tension in the connection.

Given:

The slip-critical bolt group shown as follows is subjected to tension and shear. Use ¼-in.-diameter ASTM A325 slip-critical Class A bolts in standard holes. This example shows the design for bolt slip resistance only, and assumes that the beams and plates are adequate to transmit the loads. Determine if the bolts are adequate.

Solution:

\[ \mu = 0.30 \text{ for Class A surface} \]
\[ D_s = 1.13 \]
\[ n_b = 8, \text{ number of bolts carrying the applied tension} \]
\[ h_f = 1.0, \text{ factor for fillers, assuming no more than one filler} \]
\[ T_b = 28 \text{ kips, from AISC Specification Table J3.1} \]
\[ n_s = 1, \text{ number of slip planes} \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_u = 1.2(15.0 \text{ kips}) + 1.6(45.0 \text{ kips}) ] = 90.0 kips</td>
<td>[ P_u = 15.0 \text{ kips} + 45.0 \text{ kips} ] = 60.0 kips</td>
</tr>
<tr>
<td>By geometry,</td>
<td>By geometry,</td>
</tr>
<tr>
<td>[ T_u = \frac{4}{5} \text{(90.0 kips)} = 72.0 \text{ kips} ]</td>
<td>[ T_u = \frac{4}{5} \text{(60.0 kips)} = 48.0 \text{ kips} ]</td>
</tr>
<tr>
<td>[ V_u = \frac{3}{5} \text{(90.0 kips)} = 54.0 \text{ kips} ]</td>
<td>[ V_u = \frac{3}{5} \text{(60.0 kips)} = 36.0 \text{ kips} ]</td>
</tr>
</tbody>
</table>

Available Bolt Tensile Strength

The available tensile strength is determined from AISC Specification Section J3.6.
From AISC Specification Table J3.2 for Group A bolts, the nominal tensile strength in ksi is, $F_{nt} = 90$ ksi. From AISC Manual Table 7-1, $A_b = 0.442$ in.²

$$A_b = \frac{\pi (\frac{3}{4} \text{ in.})^2}{4} = 0.442 \text{ in.}^2$$

The nominal tensile strength in kips is,

$$R_n = F_{nt} A_b \quad \text{(from Spec. Eq. J3-1)}$$

$$= 90 \text{ ksi}(0.442 \text{ in.}^2)$$

$$= 39.8 \text{ kips}$$

The available tensile strength is,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 0.75 \left( \frac{39.8 \text{ kips}}{\text{bolt}} \right) &gt; \frac{72.0 \text{ kips}}{8 \text{ bolts}}$</td>
<td>$\Omega \left( \frac{39.8 \text{ kips/bolt}}{2.00} \right) &gt; \frac{48.0 \text{ kips}}{8 \text{ bolts}}$</td>
</tr>
<tr>
<td>$= 29.9 \text{ kips/bolt} &gt; 9.00 \text{ kips/bolt}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Available Slip Resistance Per Bolt**

The available slip resistance of one bolt is determined using AISC Specification Equation J3-4 and Section J3.8.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the available slip resistance $(T_a = 0)$ of a bolt.</td>
<td>Determine the available slip resistance $(T_a = 0)$ of a bolt.</td>
</tr>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = \phi \mu D_h T_s n_s$</td>
<td>$R_n = \frac{\mu D_h T_s n_s}{\Omega}$</td>
</tr>
<tr>
<td>$= 1.00(0.30)(1.13)(1.0)(28 \text{ kips})(1)$</td>
<td>$= 0.30(1.13)(1.0)(28 \text{ kips})(1)$</td>
</tr>
<tr>
<td>$= 9.49 \text{ kips/bolt}$</td>
<td>$= 6.33 \text{ kips/bolt}$</td>
</tr>
</tbody>
</table>

**Available Slip Resistance of the Connection**

Because the clip-critical connection is subject to combined tension and shear, the available slip resistance is multiplied by a reduction factor provided in AISC Specification Section J3.9.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slip-critical combined tension and shear coefficient: $k_{sc} = 1 - \frac{T_u}{D_h T_s n_s}$</td>
<td>Slip-critical combined tension and shear coefficient: $k_{sc} = 1 - \frac{1.5T_u}{D_h T_s n_s}$</td>
</tr>
<tr>
<td>$(Spec. \text{ Eq. J3-5a})$</td>
<td>$(Spec. \text{ Eq. J3-5b})$</td>
</tr>
<tr>
<td>$= 1 - \frac{72.0 \text{ kips}}{1.13(28 \text{ kips})(8)}$</td>
<td>$= 1 - \frac{1.5(48.0 \text{ kips})}{1.13(28 \text{ kips})(8)}$</td>
</tr>
<tr>
<td>$= 0.716$</td>
<td>$= 0.716$</td>
</tr>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
</tbody>
</table>
\[ \phi R_s = \phi R_s k, n_b \]

\[ = 9.49 \text{kips/bolt}(0.716)(8 \text{ bolts}) \]

\[ = 54.4 \text{kips} > 54.0 \text{kips} \quad \text{o.k.} \]

\[ \frac{R_s}{\Omega} = \frac{R_s}{\Omega} k, n_b \]

\[ = 6.33 \text{kips/bolt}(0.716)(8 \text{ bolts}) \]

\[ = 36.3 \text{kips} > 36.0 \text{kips} \quad \text{o.k.} \]

Note: The bolt group must still be checked for all applicable strength limit states for a bearing-type connection.
EXAMPLE J.6    BEARING STRENGTH OF A PIN IN A DRILLED HOLE

Given:

A 1-in.-diameter pin is placed in a drilled hole in a 1 1/2-in. ASTM A36 plate. Determine the available bearing strength of the pinned connection, assuming the pin is stronger than the plate.

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

ASTM A36

\[ F_y = 36 \text{ ksi} \]

\[ F_u = 58 \text{ ksi} \]

The available bearing strength is determined from AISC Specification Section J7, as follows:

The projected bearing area is,

\[ A_{pb} = d t_p \]

\[ = 1.00 \text{ in.} \times 1.50 \text{ in.} \]

\[ = 1.50 \text{ in.}^2 \]

The nominal bearing strength is,

\[ R_n = 1.8 F_y A_{pb} \]

\[ = 1.8(36 \text{ ksi})(1.50 \text{ in.}^2) \]

\[ = 97.2 \text{ kips} \]

The available bearing strength of the plate is:

\[ \Phi R_n \]

\[ = 0.75(97.2 \text{ kips}) \]

\[ = 72.9 \text{ kips} \]

\[ \Omega = 2.00 \]

\[ R_n = 97.2 \text{ kips} \]

\[ \Omega = 2.00 \]

\[ = 48.6 \text{ kips} \]
EXAMPLE J.7  BASE PLATE BEARING ON CONCRETE

Given:

An ASTM A992 W12×96 column bears on a 24-in. × 24-in. concrete pedestal with \( f'_c = 3 \) ksi. The space between the base plate and the concrete pedestal has grout with \( f'_c = 4 \) ksi. Design the ASTM A36 base plate to support the following loads in axial compression:

\[
\begin{align*}
P_D &= 115 \text{ kips} \\
P_L &= 345 \text{ kips}
\end{align*}
\]

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Column
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Base Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

Column
W12×96
\( d = 12.7 \text{ in.} \)
\( b_f = 12.2 \text{ in.} \)
\( t_f = 0.900 \text{ in.} \)
\( t_w = 0.550 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:
Base Plate Dimensions

Determine the required base plate area from AISC Specification Section J8 assuming bearing on the full area of the concrete support.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(115 \text{ kips}) + 1.6(345 \text{ kips}) )</td>
<td>( P_u = 115 \text{ kips} + 345 \text{ kips} )</td>
</tr>
<tr>
<td>( = 690 \text{ kips} )</td>
<td>( = 460 \text{ kips} )</td>
</tr>
</tbody>
</table>

\[
\phi_c = 0.65 \\
A_{l(eq)} = \frac{P_u}{\phi_c 0.85 f'_c} \\
\quad = \frac{690 \text{ kips}}{0.65(0.85)(3 \text{ ksi})} \\
\quad = 416 \text{ in.}^2
\]

\[
\Omega_c = 2.31 \\
A_{l(eq)} = \frac{\Omega_c P_u}{0.85 f'_c} \\
\quad = \frac{2.31(460 \text{ kips})}{0.85(3 \text{ ksi})} \\
\quad = 417 \text{ in.}^2
\]

Note: The strength of the grout has conservatively been neglected, as its strength is greater than that of the concrete pedestal.

Try a 22.0 in. \( \times \) 22.0 in. base plate.

Verify \( N \geq d + 2(3.00 \text{ in.}) \) and \( B \geq b_y + 2(3.00 \text{ in.}) \) for anchor rod pattern shown in diagram:

\[
d + 2(3.00 \text{ in.}) = 12.7 \text{ in.} + 2(3.00 \text{ in.}) \]
\[
= 18.7 \text{ in.} < 22 \text{ in.} \quad \text{o.k.}
\]

\[
b_y + 2(3.00 \text{ in.}) = 12.2 \text{ in.} + 2(3.00 \text{ in.}) \]
\[
= 18.2 \text{ in.} < 22 \text{ in.} \quad \text{o.k.}
\]

Base plate area:

\[
A_{t} = NB \\
\quad = 22.0 \text{ in.} \times 22.0 \text{ in.} \]
\[
= 484 \text{ in.}^2 > 417 \text{ in.}^2 \quad \text{o.k.}
\]

Note: A square base plate with a square anchor rod pattern will be used to minimize the chance for field and shop problems.

Concrete Bearing Strength

Use AISC Specification Equation J8-2 because the base plate covers less than the full area of the concrete support.

Because the pedestal is square and the base plate is a concentrically located square, the full pedestal area is also the geometrically similar area. Therefore,

\[
A_2 = 24.0 \text{ in.} \times 24.0 \text{ in.} \\
\quad = 576 \text{ in.}^2
\]

The available bearing strength is,
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c = 0.65$</td>
<td>$\Omega_c = 2.31$</td>
</tr>
<tr>
<td>$\phi_c P_r = \phi_c 0.85 f'c A_i \sqrt{\frac{A_c}{A_i}} \leq \phi_c 1.7 f'c A_i$</td>
<td>$P_r = 0.85 f'c A_i \sqrt{\frac{A_c}{A_i}} \leq 1.7 f'c A_i$</td>
</tr>
<tr>
<td>$= 0.65 (0.85)(3 \text{ksi})(484 \text{in.}^2) \sqrt{\frac{576 \text{in.}^2}{484 \text{in.}^2}}$</td>
<td>$= 0.85(3 \text{ksi})(484 \text{in.}^2) \sqrt{\frac{576 \text{in.}^2}{484 \text{in.}^2}}$</td>
</tr>
<tr>
<td>$\leq 0.65(1.7)(3 \text{ksi})(484 \text{in.}^2)$</td>
<td>$\leq 1.7 (3 \text{ksi})(484 \text{in.}^2)$</td>
</tr>
<tr>
<td>= 875 kips $\leq$ 1,600 kips, use 875 kips</td>
<td>$= 583$ kips $\leq$ 1,070 kips, use 583 kips</td>
</tr>
</tbody>
</table>

875 kips > 690 kips o.k. 583 kips > 460 kips o.k.

Notes:
1. $A_2/A_1 \leq 4$; therefore, the upper limit in AISC Specification Equation J8-2 does not control.
2. As the area of the base plate approaches the area of concrete, the modifying ratio, $A_c/A$, approaches unity and AISC Specification Equation J8-2 converges to AISC Specification Equation J8-1.

Required Base Plate Thickness

The base plate thickness is determined in accordance with AISC Manual Part 14.

\[ m = \frac{N - 0.95d}{2} \]  \hspace{1cm} (Manual Eq. 14-2)

\[ = \frac{22.0 \text{in.} - 0.95(12.7 \text{in.})}{2} \]
\[ = 4.97 \text{in.} \]

\[ n = \frac{B - 0.8b_f}{2} \]  \hspace{1cm} (Manual Eq. 14-3)

\[ = \frac{22.0 \text{in.} - 0.8(12.2 \text{in.})}{2} \]
\[ = 6.12 \text{in.} \]

\[ n' = \frac{\sqrt{db_f}}{4} \]  \hspace{1cm} (Manual Eq. 14-4)

\[ = \frac{\sqrt{12.7 \text{in.}(12.2 \text{in.})}}{4} \]
\[ = 3.11 \text{in.} \]
\[
X = \left[ \frac{4db}{(d + b)^2} \right] \frac{P_s}{P_f} \quad \text{(Manual Eq. 14-6a)}
\]
\[= \left[ \frac{4(12.7 \text{ in.})(12.2 \text{ in.})}{(12.7 \text{ in.} + 12.2 \text{ in.})^2} \right] \frac{690 \text{ kips}}{875 \text{ kips}} = 0.788\]

\[
\lambda = \frac{2\sqrt{X}}{1 + \sqrt{1 - X}} \leq 1
\]
\[= \frac{2\sqrt{0.788}}{1 + \sqrt{1 - 0.788}} = 1.22 > 1, \text{ use } \lambda = 1 \quad \text{(Manual Eq. 14-5)}
\]

Note: \(\lambda\) can always be conservatively taken equal to 1.

\[
\lambda \nu' = (1)(3.11 \text{ in.}) = 3.11 \text{ in.}
\]

\[l = \max \left( m, n, \lambda \nu' \right)
\]
\[= \max (4.97 \text{ in., } 6.12 \text{ in., } 3.11 \text{ in.}) = 6.12 \text{ in.}\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_{pu} = \frac{P_s}{BN}) &amp; (f_{pu} = \frac{P_u}{BN})</td>
<td></td>
</tr>
<tr>
<td>(= \frac{690 \text{ kips}}{22.0 \text{ in.}(22.0 \text{ in.})} = 1.43 \text{ ksi}) &amp; (= \frac{460 \text{ kips}}{22.0 \text{ in.}(22.0 \text{ in.})} = 0.950 \text{ ksi})</td>
<td></td>
</tr>
</tbody>
</table>

From AISC Manual Equation 14-7a:
\[
I_{min} = l \sqrt{\frac{2f_{pu}}{0.9F_y}} = 6.12 \text{ in.} \sqrt{\frac{2(1.43 \text{ ksi})}{0.9(36 \text{ ksi})}} = 1.82 \text{ in.}
\]

From AISC Manual Equation 14-7b:
\[
I_{min} = l \sqrt{\frac{3.33f_{pu}}{F_y}} = 6.12 \text{ in.} \sqrt{\frac{3.33(0.950 \text{ ksi})}{36 \text{ ksi}}} = 1.81 \text{ in.}
\]

Use a 2.00-in.-thick base plate.
Chapter K
Design of HSS and Box Member Connections

Examples K.1 through K.6 illustrate common beam to column shear connections that have been adapted for use with HSS columns. Example K.7 illustrates a through-plate shear connection, which is unique to HSS columns. Calculations for transverse and longitudinal forces applied to HSS are illustrated in Examples K.8 and K.9. An example of an HSS truss connection is given in Example K.10. Examples of HSS cap plate, base plate and end plate connections are given in Examples K.11 through K.13.
EXAMPLE K.1  WELDED/BOLTED WIDE TEE CONNECTION TO AN HSS COLUMN

Given:

Design a connection between an ASTM A992 W16×50 beam and an ASTM A500 Grade B HSS8×8×¾ column using an ASTM A992 WT5×24.5. Use ¾-in.-diameter ASTM A325-N bolts in standard holes with a bolt spacing, s, of 3 in., vertical edge distance Lev of 1¾ in. and 3 in. from the weld line to the bolt line. Design as a flexible connection for the following vertical shear loads:

\[ P_D = 6.20 \text{ kips} \]
\[ P_L = 18.5 \text{ kips} \]

Note: A tee with a flange width wider than 8 in. was selected to provide sufficient surface for flare bevel groove welds on both sides of the column, because the tee will be slightly offset from the column centerline.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Tee
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Column
ASTM A500 Grade B
\[ F_y = 46 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]
From AISC *Manual* Tables 1-1, 1-8 and 1-12, the geometric properties are as follows:

W16×50
- \( t_w = 0.380 \text{ in.} \)
- \( d = 16.3 \text{ in.} \)
- \( t_f = 0.630 \text{ in.} \)
- \( T = 13\% \text{ in.} \)

WT5×24.5
- \( t_s = t_w = 0.340 \text{ in.} \)
- \( d = 4.99 \text{ in.} \)
- \( t_f = 0.560 \text{ in.} \)
- \( b_f = 10.0 \text{ in.} \)
- \( k_1 = \frac{1}{60} \text{ in.} \)

HSS8×8×\frac{3}{4}
- \( t = 0.233 \text{ in.} \)
- \( B = 8.00 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u )</td>
<td>1.2(6.20 kips) + 1.6(18.5 kips)</td>
<td>6.20 kips + 18.5 kips</td>
</tr>
<tr>
<td>( = 37.0 \text{ kips} )</td>
<td>( = 24.7 \text{ kips} )</td>
<td></td>
</tr>
</tbody>
</table>

Calculate the available strength assuming the connection is flexible.

**Required Number of Bolts**

The required number of bolts will ultimately be determined using the coefficient, \( C \), from AISC *Manual* Table 7-6. First, the available strength per bolt must be determined.

Determine the available shear strength of a single bolt.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi r_a )</td>
<td>17.9 kips from AISC <em>Manual</em> Table 7-1</td>
<td>( r_a ) = 11.9 kips from AISC <em>Manual</em> Table 7-1</td>
</tr>
</tbody>
</table>

Determine single bolt bearing strength based on edge distance.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{e,v} )</td>
<td>1¼ in. ( \geq ) 1 in. from AISC <em>Specification</em> Table J3.4</td>
<td>1¼ in. ( \geq ) 1 in. from AISC <em>Specification</em> Table J3.4</td>
</tr>
<tr>
<td>( \phi r_a )</td>
<td>49.4 kips/in.(0.340 in.) ( = 16.8 \text{ kips} )</td>
<td>( r_a ) = 32.9 kips/in.(0.340 in.) ( = 11.2 \text{ kips} )</td>
</tr>
</tbody>
</table>

Determine single bolt bearing capacity based on spacing and AISC *Specification* Section J3.3.
Bolt bearing strength based on edge distance controls over available shear strength of the bolt.

Determine the coefficient for the eccentrically loaded bolt group.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 3.00$ in. $\geq \frac{2}{3}(\frac{2}{3}$ in.)</td>
<td>$s = 3.00$ in. $\geq \frac{2}{3}(\frac{2}{3}$ in.)</td>
</tr>
<tr>
<td>$= 2.00$ in.</td>
<td>$= 2.00$ in.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-4,</td>
<td>From AISC Manual Table 7-4,</td>
</tr>
<tr>
<td>$\phi r_n = 87.8$ kips/in.(0.340 in.)</td>
<td>$\frac{r_n}{\Omega} = 58.5$ kips/in.(0.340 in.)</td>
</tr>
<tr>
<td>$= 29.9$ kips</td>
<td>$= 19.9$ kips</td>
</tr>
</tbody>
</table>

Using $e = 3.00$ in. and $s = 3.00$ in., determine $C$ from AISC Manual Table 7-6.

Try four rows of bolts, $C = 2.81 > 2.20$ \textbf{o.k.}

WT Stem Thickness and Length

AISC Manual Part 9 stipulates a maximum tee stem thickness that should be provided for rotational ductility as follows:

$$t_{s, \text{max}} = \frac{d_{s, \text{max}}}{2} + \frac{1}{16} \text{ in.}$$

$= (\frac{1}{4} \text{ in.}) + \frac{1}{16} \text{ in.}$

$= 0.438 \text{ in.} > 0.340 \text{ in.}$ \textbf{o.k.}

Note: The beam web thickness is greater than the WT stem thickness. If the beam web were thinner than the WT stem, this check could be satisfied by checking the thickness of the beam web.

Determine the length of the WT required as follows:

A W16×50 has a $T$-dimension of 13\% in.

$$L_{\text{min}} = T/2$$

From AISC Manual Part 10

$= (13\% \text{ in.})/2$

$= 6.81 \text{ in.}$

Determine WT length required for bolt spacing and edge distances.
\[ L = 3(3.00 \text{ in.}) + 2(1\frac{1}{4} \text{ in.}) = 11.5 \text{ in.} < T = 13 \frac{3}{8} \text{ in.} \quad \text{o.k.} \]

Try \( L = 11.5 \text{ in.} \).

**Stem Shear Yielding Strength**

Determine the available shear strength of the tee stem based on the limit state of shear yielding from AISC Specification Section J4.2.

\[
R_s = 0.6F_vA_{sv} \quad \text{(Spec. Eq. J4-3)}
\]

\[
= 0.6(50 \text{ ksi})(11.5 \text{ in.})(0.340 \text{ in.}) = 117 \text{ kips}
\]

<table>
<thead>
<tr>
<th>( LRFD )</th>
<th>( ASD )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_s = 1.00(117 \text{ kips}) = 117 \text{ kips} )</td>
<td>( R_n = 117 \text{ kips} )</td>
</tr>
</tbody>
</table>

117 kips > 37.0 kips \text{o.k.}

117 kips = 78.0 kips > 24.7 kips \text{o.k.}

Because of the geometry of the WT and because the WT flange is thicker than the stem and carries only half of the beam reaction, flexural yielding and shear yielding of the flange are not critical limit states.

**Stem Shear Rupture Strength**

Determine the available shear strength of the tee stem based on the limit state of shear rupture from AISC Specification Section J4.2.

\[
R_s = 0.6F_vA_{sv} \quad \text{(Spec. Eq. J4-4)}
\]

\[
= 0.6(65 \text{ ksi})[11.5 \text{ in.} - 4(1\frac{3}{8} \text{ in.})(0.340 \text{ in.})] = 106 \text{ kips}
\]

<table>
<thead>
<tr>
<th>( LRFD )</th>
<th>( ASD )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_s = 0.75(106 \text{ kips}) = 79.5 \text{ kips} )</td>
<td>( R_n = 106 \text{ kips} )</td>
</tr>
</tbody>
</table>

79.5 kips > 37.0 kips \text{o.k.}

53.0 kips > 24.7 kips \text{o.k.}

**Stem Block Shear Rupture Strength**

Determine the available strength for the limit state of block shear rupture from AISC Specification Section J4.3.

For this case \( U_{bs} = 1.0 \).

Use AISC Manual Tables 9-3a, 9-3b and 9-3c. Assume \( L_{eh} = 1.99 \text{ in.} \approx 2.00 \text{ in.} \).
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi F_A_w = \frac{76.2 \text{kips/}in.}{t} ]</td>
<td>( F_A_w = \frac{50.8 \text{kips/}in.}{t\Omega} )</td>
</tr>
<tr>
<td>[ \phi 0.60 F_y A_{pv} = \frac{231 \text{kips/}in.}{t} ]</td>
<td>( 0.60 F_y A_{pv} = \frac{154 \text{kips/}in.}{t\Omega} )</td>
</tr>
<tr>
<td>[ \phi 0.60 F_y A_{nv} = \frac{210 \text{kips/}in.}{t} ]</td>
<td>( 0.60 F_y A_{nv} = \frac{140 \text{kips/}in.}{t\Omega} )</td>
</tr>
<tr>
<td>[ \phi R_n = \phi 0.60 F_y A_{pv} + \phi U_{nv} F_y A_{nv} \leq \phi 0.60 F_y A_{pv} + \phi U_{nv} F_y A_{nv} ] ((\text{from Spec. Eq. J4-5}))</td>
<td>( R_n = \frac{0.60 F_y A_{pv} + U_{nv} F_y A_{nv}}{\Omega} \leq \frac{0.60 F_y A_{pv} + U_{nv} F_y A_{nv}}{\Omega} ) ((\text{from Spec. Eq. J4-5}))</td>
</tr>
<tr>
<td>( \phi R_n = 0.340 \text{in.}(210 \text{kips/in.} + 76.2 \text{kips/in.}) \leq 0.340 \text{in.}(231 \text{kips/in.} + 76.2 \text{kips/in.}) ) ( = 97.3 \text{kips} \leq 104 \text{kips} )</td>
<td>( \phi R_n = 0.340 \text{in.}(140 \text{kips/in.} + 50.8 \text{kips/in.}) \leq 0.340 \text{in.}(154 \text{kips/in.} + 50.8 \text{kips/in.}) ) ( = 64.9 \text{kips} \leq 69.6 \text{kips} )</td>
</tr>
<tr>
<td>97.3 kips &gt; 37.0 kips ( \text{o.k.} )</td>
<td>64.9 kips &gt; 24.7 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

**Stem Flexural Strength**

The required flexural strength for the tee stem is,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = P_u e ) ( = 37.0 \text{kips}(3.00 \text{in.}) ) ( = 111 \text{kip-in.} )</td>
<td>( M_u = P_u e ) ( = 24.7 \text{kips}(3.00 \text{in.}) ) ( = 74.1 \text{kip-in.} )</td>
</tr>
</tbody>
</table>

The tee stem available flexural strength due to yielding is determined as follows, from AISC Specification Section F11.1. The stem, in this case, is treated as a rectangular bar.

\[
Z = \frac{t_d^2}{4} = \frac{0.340 \text{in.}(11.5 \text{in.})^2}{4} = 11.2 \text{ in.}^3
\]

\[
S = \frac{t_d^2}{6} = \frac{0.340 \text{in.}(11.5 \text{in.})^2}{6} = 7.49 \text{ in.}^3
\]

\[
M_u = M_p = F_y Z \leq 1.6 M_y \quad (\text{Spec. Eq. F11-1})
\]

\[
= 50 \text{ksi}(11.2 \text{ in.}^3) \leq 1.6(50 \text{ksi})(7.49 \text{ in.}^3)
\]

\[
= 560 \text{kip-in.} \leq 599 \text{kip-in.}
\]
Therefore, use \( M_n = 560 \text{ kip-in.} \).

Note: The 1.6 limit will never control for a plate, because the shape factor for a plate is 1.5.

The tee stem available flexural yielding strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi M_n = 0.90(560 \text{ kip-in.}) )</td>
<td>( M_n = 560 \text{ kip-in.} )</td>
</tr>
<tr>
<td>= 504 kip-in. &gt; 111 kip-in. o.k.</td>
<td>( \Omega = 1.67 )</td>
</tr>
</tbody>
</table>

The tee stem available flexural strength due to lateral-torsional buckling is determined from Section F11.2.

\[
\frac{L_d d}{t_s^2} = \frac{(3.00 \text{ in.})(11.5 \text{ in.})}{(0.340 \text{ in.})^2} = 298
\]

\[
\frac{0.08E}{F_y} = \frac{0.08(29,000 \text{ ksi})}{50 \text{ ksi}} = 46.4
\]

\[
\frac{1.9E}{F_y} = \frac{1.9(29,000 \text{ ksi})}{50 \text{ ksi}} = 1,102
\]

Because 46.4 < 298 < 1,102, Equation F11.2 is applicable.

\[
M_n = C_b \left[ 1.52 - 0.274 \left( \frac{L_d d}{t_s^2} \right) \frac{F_y}{E} \right] M_y \leq M_p
\]

\[
= 1.00 \left[ 1.52 - 0.274(298) \frac{50 \text{ ksi}}{29,000 \text{ ksi}} \right] (50 \text{ ksi})(7.49 \text{ ksi}) \leq (50 \text{ ksi})(11.2 \text{ in.}^3)
\]

= 517 kip-in. ≤ 560 kip-in.

The tee stem available flexural rupture strength is determined from Part 9 of the AISC Manual as follows:

\[
Z_{net} = \frac{d^2}{4} - 2r(d_h + \frac{\sqrt{6}}{4} \text{ in.})(1.5 \text{ in.} + 4.5 \text{ in.})
\]

\[
= \frac{0.340 \text{ in.}(11.5 \text{ in.})^2}{4} - 2(0.340 \text{ in.})(\frac{1.5}{16} \text{ in.} + \frac{\sqrt{6}}{4} \text{ in.})(1.5 \text{ in.} + 4.5 \text{ in.})
\]

= 7.67 in.³
\[ M_n = F_u Z_{net} \]
\[ = 65 \text{ksi}(7.67 \text{ in.}^3) \]
\[ = 499 \text{kip-in.} \]

\[ \phi M_n = 0.75(499 \text{ kip-in.}) \]
\[ = 374 \text{ kip-in.} > 111 \text{ kip-in.} \text{ o.k.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ \frac{M_n}{\Omega} = 499 \text{ kip-in.} \]
| \[ \Omega = 2.00 \]
| \[ M_n = 250 \text{ kip-in.} > 74.1 \text{ kip-in.} \text{ o.k.} \] |

**Beam Web Bearing**

\[ t_w > t_s \]
\[ 0.380 \text{ in.} > 0.340 \text{ in.} \]

Beam web is satisfactory for bearing by comparison with the WT.

**Weld Size**

Because the flange width of the WT is larger than the width of the HSS, a flare bevel groove weld is required. Taking the outside radius as \[ 2(0.233 \text{ in.}) = 0.466 \text{ in.} \] and using AISC Specification Table J2.2, the effective throat thickness of the flare bevel weld is \[ E = \frac{\phi}{\xi}(0.466 \text{ in.}) = 0.146 \text{ in.} \]

Using AISC Specification Table J2.3, the minimum effective throat thickness of the flare bevel weld, based on the 0.233 in. thickness of the HSS column, is \( \frac{1}{8} \) in.

\[ E = 0.146 \text{ in.} > \frac{1}{8} \text{ in.} \]

The equivalent fillet weld that provides the same throat dimension is:

\[ \left( \frac{D}{16} \right) \left( \frac{1}{\sqrt{2}} \right) = 0.146 \]
\[ D = 16\sqrt{2}(0.146) \]
\[ = 3.30 \text{ sixteenths of an inch} \]

The equivalent fillet weld size is used in the following calculations.

**Weld Ductility**

Let \[ b_f = B = 8.00 \text{ in.} \]
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

\[ b = \frac{b_f - 2k_i}{2} \]
\[ = \frac{8.00 \text{ in.} - 2(\frac{3}{32} \text{ in.})}{2} \]
\[ = 3.19 \text{ in.} \]

\[ w_{\text{min}} = 0.0155 \frac{F_y t_f^2}{b} \left( \frac{b^2}{L} + 2 \right) \leq \% t_s \]  
\[ (\text{Manual Eq. 9-36}) \]
\[ = 0.0155 \frac{50 \text{ ksi}(0.560 \text{ in.})^2}{3.19 \text{ in.}} \left[ \frac{(3.19 \text{ in.})^2}{(11.5 \text{ in.})^2} + 2 \right] \leq \% (0.340 \text{ in.}) \]
\[ = 0.158 \text{ in.} \leq 0.213 \text{ in.} \]

0.158 in. = 2.53 sixteenths of an inch

\[ D_{\text{min}} = 2.53 < 3.30 \text{ sixteenths of an inch} \quad \text{o.k.} \]

**Nominal Weld Shear Strength**

The load is assumed to act concentrically with the weld group (flexible connection).

\[ a = 0, \text{ therefore, } C = 3.71 \text{ from AISC Manual Table 8-4} \]

\[ R_n = CC_Dl \]
\[ = 3.71(1.00)(3.30 \text{ sixteenths of an inch})(11.5 \text{ in.}) \]
\[ = 141 \text{ kips} \]

**Shear Rupture of the HSS at the Weld**

\[ t_{\text{min}} = \frac{3.09D}{F_y} \]
\[ = \frac{3.09(3.30 \text{ sixteenths})}{58 \text{ ksi}} \]
\[ = 0.176 \text{ in.} < 0.233 \text{ in.} \]

By inspection, shear rupture of the WT flange at the welds will not control.

Therefore, the weld controls.

From AISC Specification Section J2.4, the available weld strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(141 \text{ kips}) )</td>
<td>( R_n = 141 \text{ kips} )</td>
</tr>
<tr>
<td>( = 106 \text{ kips} &gt; 37.0 \text{ kips} \quad \text{o.k.} )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( = 70.5 \text{ kips} &gt; 24.7 \text{ kips} \quad \text{o.k.} )</td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE K.2  WELDED/BOLTED NARROW TEE CONNECTION TO AN HSS COLUMN

Given:

Design a connection for an ASTM A992 W16×50 beam to an ASTM A500 Grade B HSS8×8×¾ column using a tee with fillet welds against the flat width of the HSS. Use ¾-in.-diameter A325-N bolts in standard holes with a bolt spacing, $s$, of 3.00 in., vertical edge distance $L_{ev}$ of 1¼ in. and 3.00 in. from the weld line to the center of the bolt line. Use 70-ksi electrodes. Assume that, for architectural purposes, the flanges of the WT from the previous example have been stripped down to a width of 5 in. Design as a flexible connection for the following vertical shear loads:

\[
P_D = 6.20 \text{ kips} \\
P_L = 18.5 \text{ kips}
\]

Note: This is the same problem as Example K.1 with the exception that a narrow tee will be selected which will permit fillet welds on the flat of the column. The beam will still be centered on the column centerline; therefore, the tee will be slightly offset.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Tee
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Column
ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi
From AISC Manual Tables 1-1, 1-8 and 1-12, the geometric properties are as follows:

- **W16×50**
  - \( t_w = 0.380 \text{ in.} \)
  - \( d = 16.3 \text{ in.} \)
  - \( t_f = 0.630 \text{ in.} \)

- **HSS8×8×\( \frac{3}{4} \)**
  - \( t = 0.233 \text{ in.} \)
  - \( B = 8.00 \text{ in.} \)

- **WT5×24.5**
  - \( t_s = t_w = 0.340 \text{ in.} \)
  - \( d = 4.99 \text{ in.} \)
  - \( t_f = 0.560 \text{ in.} \)
  - \( k_1 = \frac{3}{16} \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u )</td>
<td>1.2(6.20 kips) + 1.6(18.5 kips) = 37.0 kips</td>
<td>( P_u = 6.20 \text{ kips} + 18.5 \text{ kips} = 24.7 \text{ kips} )</td>
</tr>
</tbody>
</table>

The WT stem thickness, WT length, WT stem strength and beam web bearing strength are verified in Example K.1. The required number of bolts is also determined in Example K.1.

**Maximum WT Flange Width**

Assume \( \frac{3}{8} \)-in. welds and HSS corner radius equal to 2.25 times the nominal thickness \( 2.25(\frac{3}{8} \text{ in.}) = \frac{9}{4} \text{ in.} \).

The recommended minimum shelf dimension for \( \frac{3}{8} \)-in. fillet welds from AISC Manual Figure 8-11 is \( \frac{1}{2} \text{ in.} \).

Connection offset:

\[
\frac{0.380}{2} + \frac{0.340}{2} = 0.360 \text{ in.}
\]

\( b_f \leq 8.00 \text{ in.} - 2(\frac{9}{4} \text{ in.}) - 2(\frac{1}{2} \text{ in.}) - 2(0.360 \text{ in.}) \)

5.00 in. \( \leq 5.16 \text{ in.} \quad \text{o.k.} \)

**Minimum Fillet Weld Size**

From AISC Specification Table J2.4, the minimum fillet weld size = \( \frac{1}{6} \text{ in.} \) (\( D = 2 \)) for welding to 0.233-in. material.

**Weld Ductility**

As defined in Figure 9-5 of the AISC Manual Part 9,

\[
b = \frac{b_f - 2k_1}{2}
\]
\[
\frac{D - \frac{3}{16} \text{ in.}}{2} = 1.69 \text{ in.}
\]

\[
w_{\text{min}} = 0.0155 \frac{F_y t_f^2}{b} \left( \frac{b^2}{L^2} + 2 \right) \leq \frac{(\%)}{t_i} \quad (\text{Manual Eq. 9-36})
\]

\[
= 0.0155 \frac{50 \text{ ksi} (0.560 \text{ in.})^2}{1.69 \text{ in.}} \left[ \frac{(1.69 \text{ in.})^2}{(11.5 \text{ in.})^2} + 2 \right] \leq \frac{(\%)}{(0.340 \text{ in.)}}
\]

\[
= 0.291 \text{ in.} \leq 0.213 \text{ in.}, \text{ use 0.213 in.}
\]

\[
D_{\text{min}} = 0.213 \text{ in.} (16)
\]

\[
= 3.41 \text{ sixteenths of an inch.}
\]

Try a \(\frac{1}{4}\)-in. fillet weld as a practical minimum, which is less than the maximum permitted weld size of \(t_f - \frac{3}{16} \text{ in.} = 0.560 \text{ in.} - \frac{3}{16} \text{ in.} = 0.498 \text{ in.}
\)

Provide \(\frac{1}{2}\)-in. return welds at the top of the WT to meet the criteria listed in AISC Specification Section J2.2b.

**Minimum HSS Wall Thickness to Match Weld Strength**

\[
t_{\text{min}} = \frac{3.09D}{F_y} \quad (\text{Manual Eq. 9-2})
\]

\[
= \frac{3.09(4)}{58 \text{ ksi}}
\]

\[
= 0.213 \text{ in.} < 0.233 \text{ in.}
\]

By inspection, shear rupture of the flange of the WT at the welds will not control.

Therefore, the weld controls.

**Available Weld Shear Strength**

The load is assumed to act concentrically with the weld group (flexible connection).

\(a = 0\), therefore, \(C = 3.71\) from AISC Manual Table 8-4

\[
R_n = CC_i D_l

= 3.71(1.00)(4 \text{ sixteenths of an inch})(11.5 \text{ in.})

= 171 \text{ kips}
\]

From AISC Specification Section J2.4, the available fillet weld shear strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.75)</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>(\phi R_n = 0.75(171 \text{ kips}) = 128 \text{ kips})</td>
<td>(R_n = 171 \text{ kips})</td>
</tr>
<tr>
<td>128 kips &gt; 37.0 kips</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>85.5 kips &gt; 24.7 kips</td>
<td>(= 85.5 \text{ kips})</td>
</tr>
</tbody>
</table>
EXAMPLE K.3 DOUBLE ANGLE CONNECTION TO AN HSS COLUMN

Given:

Use AISC Manual Tables 10-1 and 10-2 to design a double-angle connection for an ASTM A992 W36×231 beam to an ASTM A500 Grade B HSS14×14×½ column. Use ⅝-in.-diameter ASTM A325-N bolts in standard holes. The angles are ASTM A36 material. Use 70-ksi electrodes. The bottom flange cope is required for erection. Use the following vertical shear loads:

\[ P_D = 37.5 \text{ kips} \]
\[ P_L = 113 \text{ kips} \]

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Column
ASTM A500 Grade B
\[ F_y = 46 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

Angles
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]
From AISC Manual Tables 1-1 and 1-12, the geometric properties are as follows:

\[
\begin{align*}
W36 \times 231 \\
t_w &= 0.760 \text{ in.} \\
T &= 31\frac{3}{8} \text{ in.}
\end{align*}
\]

\[
\begin{align*}
\text{HSS}14 \times 14 \times \frac{3}{8} \\
t &= 0.465 \text{ in.} \\
B &= 14.0 \text{ in.}
\end{align*}
\]

From Chapter 2 of ASCE/SEI 7, the required strength is:

\[
\begin{align*}
\text{LRFD } Ru &= 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips}) \\
&= 226 \text{ kips} \\
\text{LRFD } Ra &= 37.5 \text{ kips} + 113 \text{ kips} \\
&= 151 \text{ kips}
\end{align*}
\]

**Bolt and Weld Design**

Try 8 rows of bolts and 3/16-in. welds.

Obtain the bolt group and angle available strength from AISC Manual Table 10-1.

\[
\begin{align*}
\text{LRFD } \phi R_u &= 286 \text{ kips} > 226 \text{ kips} & \text{o.k.} \\
\text{ASD } R_u &= 191 \text{ kips} > 151 \text{ kips} & \text{o.k.}
\end{align*}
\]

Obtain the available weld strength from AISC Manual Table 10-2 (welds B).

\[
\begin{align*}
\text{LRFD } \phi R_u &= 279 \text{ kips} > 226 \text{ kips} & \text{o.k.} \\
\text{ASD } R_u &= 186 \text{ kips} > 151 \text{ kips} & \text{o.k.}
\end{align*}
\]

**Minimum Support Thickness**

The minimum required support thickness using Table 10-2 is determined as follows for $F_u = 58 \text{ ksi}$ material.

\[
0.238 \text{ in.} \left( \frac{65 \text{ ksi}}{58 \text{ ksi}} \right) = 0.267 \text{ in.} < 0.465 \text{ in.} & \text{o.k.}
\]

**Minimum Angle Thickness**

\[
t_{\text{min}} = w + \frac{3}{16} \text{ in., from AISC Specification Section J2.2b} \\
= \frac{3}{16} \text{ in.} + \frac{3}{16} \text{ in.} \\
= \frac{3}{8} \text{ in.}
\]

Use 3/8-in. angle thickness to accommodate the welded legs of the double angle connection.

Use 2L4 × 32 × 3/8 × 1’-11½”.

**Minimum Angle Length**

\[
L = 23.5 \text{ in.} > T/2
\]
> 31¼ in./2
> 15.7 in.  o.k.

**Minimum Column Width**

The workable flat for the HSS column is 14.0 in. – 2(2.25)(½ in.) = 11.8 in.

The recommended minimum shelf dimension for ½-in. fillet welds from AISC Manual Figure 8-11 is ¾ in.

The minimum acceptable width to accommodate the connection is:

2(4.00 in.) + 0.760 in. + 2(¾ in.) = 9.89 in. < 11.8 in.  o.k.

**Available Beam Web Strength**

The available beam web strength, from AISC Manual Table 10-1 for an uncoped beam with $L_{eh}=1\frac{3}{4}$ in., is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 702$ kips/in.(0.760 in.)</td>
<td>534 kips</td>
<td>$R_\Omega = 468$ kips/in.(0.760 in.)</td>
</tr>
<tr>
<td>534 kips &gt; 226 kips</td>
<td>o.k.</td>
<td>356 kips</td>
</tr>
<tr>
<td>356 kips &gt; 151 kips</td>
<td>o.k.</td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE K.4  UNSTIFFENED SEATED CONNECTION TO AN HSS COLUMN

Given:

Use AISC Manual Table 10-6 to design an unstiffened seated connection for an ASTM A992 W21×62 beam to an ASTM A500 Grade B HSS12×12×½ column. The angles are ASTM A36 material. Use 70-ksi electrodes. Use the following vertical shear loads:

\[ P_D = 9.00 \text{ kips} \]
\[ P_L = 27.0 \text{ kips} \]

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Column
ASTM A500 Grade B
\[ F_y = 46 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

Angles
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]
From AISC Manual Tables 1-1 and 1-12, the geometric properties are as follows:

\[ W_{21 \times 62} \]
\[ t_w = 0.400 \text{ in.} \]
\[ d = 21.0 \text{ in.} \]
\[ k_{des} = 1.12 \text{ in.} \]

\[ HSS_{12 \times 12 \times \frac{3}{4}} \]
\[ t = 0.465 \text{ in.} \]
\[ B = 12.0 \text{ in.} \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[ R_u = 1.2(9.00 \text{ kips}) + 1.6(27.0 \text{ kips}) ]</td>
<td>[ R_u = 9.00 \text{ kips} + 27.0 \text{ kips} ]</td>
</tr>
<tr>
<td></td>
<td>[ = 54.0 \text{ kips} ]</td>
<td>[ = 36.0 \text{ kips} ]</td>
</tr>
</tbody>
</table>

**Seat Angle and Weld Design**

Check local web yielding of the \[ W_{21 \times 62} \] using AISC Manual Table 9-4 and Part 10.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From AISC Manual Equation 9-45a and Table 9-4,</td>
<td>From AISC Manual Equation 9-45b and Table 9-4,</td>
</tr>
<tr>
<td></td>
<td>[ l_{b, \text{min}} = \frac{R_u - \phi R_i}{\phi R_i} \geq k_{des} ]</td>
<td>[ l_{b, \text{min}} = \frac{R_u - R_i / \Omega}{R_i / \Omega} \geq k_{des} ]</td>
</tr>
<tr>
<td></td>
<td>[ = \frac{54.0 \text{ kips} - 56.0 \text{ kips}}{20.0 \text{ kips/in.}} \geq 1.12 \text{ in.} ]</td>
<td>[ = \frac{36.0 \text{ kips} - 37.3 \text{ kips}}{13.3 \text{ kips/in.}} \geq 1.12 \text{ in.} ]</td>
</tr>
<tr>
<td></td>
<td>Use [ l_{b, \text{min}} = 1.12 \text{ in.} ]</td>
<td>Use [ l_{b, \text{min}} = 1.12 \text{ in.} ]</td>
</tr>
<tr>
<td></td>
<td>Check web crippling when [ l_b/d \leq 0.2 ]</td>
<td>Check web crippling when [ l_b/d \leq 0.2 ]</td>
</tr>
<tr>
<td></td>
<td>From AISC Manual Equation 9-47a,</td>
<td>From AISC Manual Equation 9-47b,</td>
</tr>
<tr>
<td></td>
<td>[ l_{b, \text{min}} = \frac{R_u - \phi R_i}{\phi R_i} ]</td>
<td>[ l_{b, \text{min}} = \frac{R_u - R_i / \Omega}{R_i / \Omega} ]</td>
</tr>
<tr>
<td></td>
<td>[ = \frac{54.0 \text{ kips} - 71.7 \text{ kips}}{5.37 \text{ kips/in.}} ]</td>
<td>[ = \frac{36.0 \text{ kips} - 47.8 \text{ kips}}{3.58 \text{ kips/in.}} ]</td>
</tr>
<tr>
<td></td>
<td>which results in a negative quantity.</td>
<td>which results in a negative quantity.</td>
</tr>
<tr>
<td></td>
<td>Check web crippling when [ l_b/d &gt; 0.2 ]</td>
<td>Check web crippling when [ l_b/d &gt; 0.2 ]</td>
</tr>
<tr>
<td></td>
<td>From AISC Manual Equation 9-48a,</td>
<td>From AISC Manual Equation 9-48b,</td>
</tr>
<tr>
<td></td>
<td>[ l_{b, \text{min}} = \frac{R_u - \phi R_i}{\phi R_i} ]</td>
<td>[ l_{b, \text{min}} = \frac{R_u - R_i / \Omega}{R_i / \Omega} ]</td>
</tr>
<tr>
<td></td>
<td>[ = \frac{54.0 \text{ kips} - 64.2 \text{ kips}}{7.16 \text{ kips/in.}} ]</td>
<td>[ = \frac{36.0 \text{ kips} - 42.8 \text{ kips}}{4.77 \text{ kips/in.}} ]</td>
</tr>
<tr>
<td></td>
<td>which results in a negative quantity.</td>
<td>which results in a negative quantity.</td>
</tr>
</tbody>
</table>
Note: Generally, the value of $l_b/d$ is not initially known and the larger value determined from the web crippling equations in the preceding text can be used conservatively to determine the bearing length required for web crippling.

For this beam and end reaction, the beam web strength exceeds the required strength (hence the negative bearing lengths) and the lower-bound bearing length controls ($l_{b, re\text{q}} = k_{des} = 1.12$ in.). Thus, $l_{b, min} = 1.12$ in.

Try an $L8\times4\times\frac{3}{8}$ seat with $\frac{3}{16}$-in. fillet welds.

**Outstanding Angle Leg Available Strength**

From AISC Manual Table 10-6 for an 8-in. angle length and $l_{b, re\text{q}} = 1.12$ in. $\approx 1\frac{1}{8}$ in., the outstanding angle leg available strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_w = 81.0$ kips $&gt; 54.0$ kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{R}{\Omega} = 53.9$ kips $&gt; 36.0$ kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Available Weld Strength**

From AISC Manual Table 10-6, for an 8 in. x 4 in. angle and $\frac{3}{16}$-in. weld size, the available weld strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_w = 66.7$ kips $&gt; 54.0$ kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{R}{\Omega} = 44.5$ kips $&gt; 36.0$ kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Minimum HSS Wall Thickness to Match Weld Strength**

$$t_{min} = \frac{3.09D}{F_v}$$  \hspace{1cm}(Manual Eq. 9-2)

$$= \frac{3.09(5)}{58 \text{ ksi}}$$

$$= 0.266 \text{ in.} < 0.465 \text{ in.}$$

Because $t$ of the HSS is greater than $t_{min}$ for the $\frac{3}{16}$-in. weld, no reduction in the weld strength is required to account for the shear in the HSS.

**Connection to Beam and Top Angle (AISC Manual Part 10)**

Use a $L4\times4\times\frac{3}{4}$ angle. Use a $\frac{3}{16}$-in. fillet weld across the toe of the angle for attachment to the HSS. Attach both the seat and top angles to the beam flanges with two $\frac{3}{4}$-in.-diameter ASTM A325-N bolts.
EXAMPLE K.5  STIFFENED SEATED CONNECTION TO AN HSS COLUMN

Given:

Use AISC Manual Tables 10-8 and 10-15 to design a stiffened seated connection for an ASTM A992 W21×68 beam to an ASTM A500 Grade B HSS14×14×½ column. Use the following vertical shear loads:

\[ P_D = 20.0 \text{ kips} \]
\[ P_L = 60.0 \text{ kips} \]

Use ¾-in.-diameter ASTM A325-N bolts in standard holes to connect the beam to the seat plate. Use 70-ksi electrode welds to connect the stiffener, seat plate and top angle to the HSS. The angle and plate material is ASTM A36.
Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

**Beam**
- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

**Column**
- ASTM A500 Grade B
  - $F_y = 46$ ksi
  - $F_u = 58$ ksi

**Angles and Plates**
- ASTM A36
  - $F_y = 36$ ksi
  - $F_u = 58$ ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

**W21×68**
- $t_w = 0.430$ in.
- $d = 21.1$ in.
- $k_{des} = 1.19$ in.

**HSS14×14×\frac{3}{2}**
- $t = 0.465$ in.
- $B = 14.0$ in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 1.2(20.0 \text{ kips}) + 1.6(60.0 \text{ kips})$</td>
<td>$P_u = 20.0 \text{ kips} + 60.0 \text{ kips}$</td>
<td>$P_u = 80.0 \text{ kips}$</td>
</tr>
<tr>
<td>= 120 kips</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Limits of Applicability for AISC Specification Section K1.3**

AISC *Specification* Table K1.2A gives the limits of applicability for plate-to-rectangular HSS connections. The limits that are applicable here are:

- **Strength:** $F_y = 46$ ksi $\leq$ 52 ksi \text{ o.k.}
- **Ductility:** $\frac{F_y}{F_u} = \frac{46 \text{ ksi}}{58 \text{ ksi}} = 0.793 \leq 0.8 \text{ o.k.}$

**Stiffener Width, W, Required for Web Local Crippling and Web Local Yielding**

The stiffener width is determined based on web local crippling and web local yielding of the beam.

For web local crippling, assume $l_e/d > 0.2$ and use constants $R_5$ and $R_6$ from AISC *Manual* Table 9-4. Assume a $\frac{1}{4}$-in. setback for the beam end.
For web local yielding, use constants $R_1$ and $R_2$ from AISC Manual Table 9-4. Assume a $\frac{3}{4}$-in. setback for the beam end.

The minimum stiffener width, $W_{min}$, for web local crippling controls. Use $W = 7.00$ in.

Check the assumption that $l_i/d > 0.2$.

$$l_i = 7.00 \text{ in.} - \frac{3}{4} \text{ in.}$$
$$l_i = 6.25 \text{ in.}$$
$$d = 21.1 \text{ in.}$$
$$d = 0.296 > 0.2, \text{ as assumed}$$

**Weld Strength Requirements for the Seat Plate**

Try a stiffener of length, $L = 24$ in. with $\frac{3}{16}$-in. fillet welds. Enter AISC Manual Table 10-8 using $W = 7$ in. as determined in the preceding text.

From AISC Manual Part 10, Figure 10-10(b), the minimum length of the seat-plate-to-HSS weld on each side of the stiffener is $0.2L = 4.8$ in. This establishes the minimum weld between the seat plate and stiffener; use 5 in. of $\frac{3}{16}$-in. weld on each side of the stiffener.

**Minimum HSS Wall Thickness to Match Weld Strength**

The minimum HSS wall thickness required to match the shear rupture strength of the base metal to that of the weld is:
Because \( t \) of the HSS is greater than \( t_{\text{min}} \) for the \( \frac{1}{8}\) in. fillet weld, no reduction in the weld strength to account for shear in the HSS is required.

**Stiffener Plate Thickness**

From AISC *Manual* Part 10, Design Table 10-8 discussion, to develop the stiffener-to-seat-plate welds, the minimum stiffener thickness is:

\[
t_{p\text{, min}} = 2w
\]

\[
= 2\left(\frac{1}{8}\right) \text{ in.}
\]

\[
= \frac{1}{4} \text{ in.}
\]

For a stiffener with \( F_y = 36 \text{ ksi} \) and a beam with \( F_y = 50 \text{ ksi} \), the minimum stiffener thickness is:

\[
t_{p\text{, min}} = \left( \frac{F_y\text{beam}}{F_y\text{stiffener}} \right) t_w
\]

\[
= \left( \frac{50\text{ksi}}{36\text{ksi}} \right) (0.430 \text{ in.})
\]

\[
= 0.597 \text{ in.}
\]

For a stiffener with \( F_y = 36 \text{ ksi} \) and a column with \( F_u = 58 \text{ ksi} \), the maximum stiffener thickness is determined from AISC *Specification* Table K1.2 as follows:

\[
t_{p\text{, max}} = \frac{F_t}{F_y}
\]

\[
= \frac{58 \text{ ksi}(0.465 \text{ in.})}{36 \text{ ksi}}
\]

\[
= 0.749 \text{ in.}
\]

Use stiffener thickness of \( \frac{3}{8} \) in.

Determine the stiffener length using AISC *Manual* Table 10-15.
To satisfy the minimum, select a stiffener $L = 24$ in. from AISC Manual Table 10-15.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{R \cdot W}{t^2}$</td>
<td>$3,910$ kips/in. $&gt; 3,880$ kips/in.</td>
<td>o.k.</td>
</tr>
<tr>
<td></td>
<td>$2,600$ kips/in. $&gt; 2,590$ kips/in.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Use PL% in.×7 in.×2 ft-0 in. for the stiffener.

**HSS Width Check**

The minimum width is $0.4L + t_p + 2(2.25t)$

$$B = 14.0 \text{ in.} > 0.4(24.0 \text{ in.}) + \frac{3}{8} \text{ in.} + 2(2.25)(0.465 \text{ in.})$$

$$= 12.3 \text{ in.}$$

**Seat Plate Dimensions**

To accommodate two $\frac{3}{8}$-in.-diameter ASTM A325-N bolts on a $5\frac{1}{2}$-in. gage connecting the beam flange to the seat plate, a width of 8 in. is required. To accommodate the seat-plate-to-HSS weld, the required width is:

$$2(5.00 \text{ in.}) + \frac{3}{8} \text{ in.} = 10.6 \text{ in.}$$

Note: To allow room to start and stop welds, an 11.5 in. width is used.

Use PL% in.×7 in.×0 ft-11$\frac{1}{2}$ in. for the seat plate.

**Top Angle, Bolts and Welds (AISC Manual Part 10)**

The minimum weld size for the HSS thickness according to AISC Specification Table J2.4 is $\frac{3}{8}$ in. The angle thickness should be $\frac{1}{8}$ in. larger.

Use L4×4×$\frac{3}{8}$ with $\frac{3}{16}$-in. fillet welds along the toes of the angle to the beam flange and HSS. Alternatively, two $\frac{3}{4}$-in.-diameter ASTM A325-N bolts may be used to connect the leg of the angle to the beam flange.
EXAMPLE K.6  SINGLE-PLATE CONNECTION TO A RECTANGULAR HSS COLUMN

Given:

Use AISC Manual Table 10-10a to design a single-plate connection for an ASTM A992 W18×35 beam framing into an ASTM A500 Grade B HSS6×6×¾ column. Use 3⁄8-in.-diameter ASTM A325-N bolts in standard holes and 70-ksi weld electrodes. The plate material is ASTM A36. Use the following vertical shear loads:

\[ P_D = 6.50 \text{ kips} \]
\[ P_L = 19.5 \text{ kips} \]

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

- **Beam**  
  ASTM A992  
  \( F_y = 50 \text{ ksi} \)  
  \( F_u = 65 \text{ ksi} \)

- **Column**  
  ASTM A500 Grade B  
  \( F_y = 46 \text{ ksi} \)  
  \( F_u = 58 \text{ ksi} \)

- **Plate**  
  ASTM A36  
  \( F_y = 36 \text{ ksi} \)  
  \( F_u = 58 \text{ ksi} \)

From AISC Manual Tables 1-1 and 1-12, the geometric properties are as follows:

- **W18×35**  
  \( d = 17.7 \text{ in.} \)  
  \( t_w = 0.300 \text{ in.} \)  
  \( T = 15\frac{1}{2} \text{ in.} \)
HSS \(6\times 6\times 0.349\) in.

\[ B = H = 6.00 \text{ in.} \]
\[ t = 0.349 \text{ in.} \]
\[ b/t = 14.2 \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u )</td>
<td>[ 1.2 \times (6.50 \text{ kips}) + 1.6 \times (19.5 \text{ kips}) ]</td>
<td>[ 6.50 \text{ kips} + 19.5 \text{ kips} ]</td>
</tr>
<tr>
<td></td>
<td>[ = 39.0 \text{ kips} ]</td>
<td>[ = 26.0 \text{ kips} ]</td>
</tr>
</tbody>
</table>

### Limits of Applicability of AISC Specification Section K1.3

AISC Specification Table K1.2A gives the following limits of applicability for plate-to-rectangular HSS connections. The limits applicable here are:

**HSS wall slenderness:**

\[
\frac{(B - 3t)}{t} \leq 1.40 \sqrt{\frac{E}{F_y}}
\]

\[
\frac{[6.00 \text{ in.} - 3(0.349 \text{in.})]}{0.349 \text{in.}} \leq 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}}
\]

14.2 < 35.2 \( \text{o.k.} \)

**Material Strength:**

\[ F_y = 46 \text{ ksi} \leq 52 \text{ ksi} \] \( \text{o.k.} \)

**Ductility:**

\[
\frac{F_y}{F_y} = \frac{46 \text{ ksi}}{58 \text{ ksi}}
\]

0.793 \( \leq 0.8 \) \( \text{o.k.} \)

Using AISC Manual Part 10, determine if a single-plate connection is suitable (the HSS wall is not slender).

**Maximum Single-Plate Thickness**

From AISC Specification Table K1.2, the maximum single-plate thickness is:

\[
t_p \leq \frac{F_y}{F_{yw}}
\]

\[
= \frac{58 \text{ ksi}}{36 \text{ ksi}} \times (0.349 \text{ in.})
\]

= 0.562 in.

Note: Limiting the single-plate thickness precludes a shear yielding failure of the HSS wall.

**Single-Plate Connection**

Try 3 bolts and a \( \frac{3}{16} \)-in. plate thickness with \( \frac{1}{4} \)-in. fillet welds.

\[
t_p = \frac{3}{16} \text{ in.} < 0.562 \text{ in.} \] \( \text{o.k.} \)
Note: From AISC Manual Table 10-9, either the plate or the beam web must satisfy:

\[ t = \frac{d}{6} \leq d/2 + \frac{d}{10} \leq \frac{d}{4} + \frac{d}{10} \leq 0.438 \text{ in.} \quad \text{o.k.} \]

Obtain the available single-plate connection strength from AISC Manual Table 10-10a.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_a = 43.4 \text{ kips} &gt; 39.0 \text{ kips} )</td>
<td>( R_\phi \Omega = 28.8 \text{ kips} &gt; 26.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Use a PL\( 5\frac{3}{6} \times 4\frac{3}{8} \times 0' - 8'' \).

**HSS Shear Rupture at Welds**

The minimum HSS wall thickness required to match the shear rupture strength of the HSS wall to that of the weld is:

\[ t_{\text{min}} = \frac{3.09D}{F_s} \]

\[ = \frac{3.09(4)}{58 \text{ ksi}} \]

\[ = 0.213 \text{ in.} < t = 0.349 \text{ in.} \quad \text{o.k.} \]

**Available Beam Web Bearing Strength (AISC Manual Table 10-1)**

For three \( \frac{3}{8} \)-in.-diameter bolts and \( L_{eb} = 1\frac{1}{2} \) in., the bottom of AISC Manual Table 10-1 gives the uncoped beam web available bearing strength per inch of thickness. The available beam web bearing strength can then be calculated as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_b = 263 \text{ kips/in.}(0.300 \text{ in.}) )</td>
<td>( R_\phi \Omega = 176 \text{ kips/in.}(0.300 \text{ in.}) )</td>
</tr>
<tr>
<td>( = 78.9 \text{ kips} &gt; 39.0 \text{ kips} )</td>
<td>( = 52.8 \text{ kips} &gt; 26.0 \text{ kips} )</td>
</tr>
</tbody>
</table>
EXAMPLE K.7 THROUGH-PLATE CONNECTION

Given:

Use AISC Manual Table 10-10a to check a through-plate connection between an ASTM A992 W18×35 beam and an ASTM A500 Grade B HSS6×4×3/8 with the connection to one of the 6 in. faces, as shown in the figure. A thin-walled column is used to illustrate the design of a through-plate connection. Use ⅜-in.-diameter ASTM A325-N bolts in standard holes and 70-ksi weld electrodes. The plate is ASTM A36 material. Use the following vertical shear loads:

\[ P_D = 3.30 \text{ kips} \]
\[ P_L = 9.90 \text{ kips} \]

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Column
ASTM A500 Grade B
\[ F_y = 46 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

Plate
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC Manual Tables 1-1 and 1-11, the geometric properties are as follows:

W18×35
\[ d = 17.7 \text{ in.} \]
\[ t_w = 0.300 \text{ in.} \]
\[ T = 15\frac{1}{2} \text{ in.} \]
HSS6×4×½
B = 4.00 in.
H = 6.00 in.
t = 0.116 in.
h/t = 48.7

Limits of Applicability of AISC Specification Section K1.3

AISC Specification Table K1.2A gives the following limits of applicability for plate-to-rectangular HSS connections. The limits applicable here follow.

HSS wall slenderness: Check if a single-plate connection is allowed.

\[
\frac{(H - 3t)}{t} \leq 1.40 \sqrt{\frac{E}{F_y}}
\]
\[
\frac{6.00 - 3(0.116 \text{ in.})}{0.116 \text{ in.}} \leq 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}}
\]

48.7 > 35.2 n.g.

Because the HSS6×4×½ is slender, a through-plate connection should be used instead of a single-plate connection. Through-plate connections are typically very expensive. When a single-plate connection is not adequate, another type of connection, such as a double-angle connection may be preferable to a through-plate connection.

AISC Specification Chapter K does not contain provisions for the design of through-plate shear connections. The following procedure treats the connection of the through-plate to the beam as a single-plate connection.

From Chapter 2 of ASCE/SEI 7, the required strength is:

\[
R_u = 1.2(3.30 \text{ kips}) + 1.6(9.90 \text{ kips}) = 19.8 \text{ kips}
\]

\[
R_y = 3.30 \text{ kips} + 9.90 \text{ kips} = 13.2 \text{ kips}
\]

Portion of the Through-Plate Connection that Resembles a Single-Plate

Try three rows of bolts (L = 8½ in.) and a ¼-in. plate thickness with ½-in. fillet welds.

\[
L = 8\frac{1}{2} \text{ in.} \geq T/2 \\
\geq (15\frac{1}{2} \text{ in.})/2 \\
\geq 7.75 \text{ in.} \text{ o.k.}
\]

Note: From AISC Manual Table 10-9, either the plate or the beam web must satisfy:

\[
t = \frac{3}{16} \text{ in.} \leq d_t/2 + \frac{1}{16} \text{ in.} \\
\leq \frac{3}{4} \text{ in.} / 2 + \frac{1}{16} \text{ in.} \\
\leq 0.438 \text{ in.} \text{ o.k.}
\]
Obtain the available single-plate connection strength from AISC Manual Table 10-10a.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_v = 38.3 \text{ kips} &gt; 19.8 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$R_{\Omega} = 25.6 \text{ kips} &gt; 13.2 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Required Weld Strength**

The available strength for the welds in this connection is checked at the location of the maximum reaction, which is along the weld line closest to the bolt line. The reaction at this weld line is determined by taking a moment about the weld line farthest from the bolt line.

$$a = 3.00 \text{ in.} \quad \text{(distance from bolt line to nearest weld line)}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{fs} = \frac{R_v (B + a)}{B} = \frac{19.8 \text{ kips} (4.00 \text{ in.} + 3.00 \text{ in.})}{4.00 \text{ in.}} = 34.7 \text{ kips}$</td>
<td>$V_{fs} = \frac{R_v (B + a)}{B} = \frac{13.2 \text{ kips} (4.00 \text{ in.} + 3.00 \text{ in.})}{4.00 \text{ in.}} = 23.1 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Available Weld Strength**

The minimum required weld size is determined using AISC Manual Part 8.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{req} = \frac{\phi R_v}{1.392 I} = \frac{34.7 \text{ kips}}{1.392 (8.50 \text{ in.}) (2)} = 1.47 \text{ sixteenths} &lt; 3 \text{ sixteenths}$</td>
<td>$D_{req} = \frac{R_{\Omega}}{0.928 I} = \frac{23.1 \text{ kips}}{0.928 (8.50 \text{ in.}) (2)} = 1.46 \text{ sixteenths} &lt; 3 \text{ sixteenths}$</td>
</tr>
</tbody>
</table>

**HSS Shear Yielding and Rupture Strength**

The available shear strength of the HSS due to shear yielding and shear rupture is determined from AISC Specification Section J4.2.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For shear yielding of HSS at the connection, $\phi = 1.00$ $\phi R_v = \phi 0.60 F_s A_y$ (from Spec. Eq. J4-3) $= 1.00 (0.60)(46 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)$ $= 54.4 \text{ kips}$</td>
<td>For shear yielding of HSS at the connection, $\Omega = 1.50$ $\frac{R_{\Omega}}{\Omega} = \frac{0.60 F_s A_y}{\Omega}$ (from Spec. Eq. J4-3) $= \frac{(0.60)(46 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)}{1.50}$ $= 36.3 \text{ kips}$</td>
</tr>
<tr>
<td>54.4 kips &gt; 34.7 kips</td>
<td>36.3 kips &gt; 23.1 kips</td>
</tr>
<tr>
<td>For shear rupture of HSS at the connection, $\phi = 0.75$</td>
<td>For shear rupture of HSS at the connection, $\Omega = 2.00$</td>
</tr>
</tbody>
</table>
Available Beam Web Bearing Strength

The available beam web bearing strength is determined from the bottom portion of Table 10-1, for three 3/8-in.-diameter bolts and an uncoped beam. The table provides the available strength in kips/in. and the available beam web bearing strength is:

\[
\phi R_w = 0.60F_a A_w \quad \text{(from Spec. Eq. J4-4)}
\]

\[
\begin{align*}
\phi R_w &= 0.60(58 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2) \\
&= 51.5 \text{ kips} \\
51.5 \text{ kips} &> 34.7 \text{ kips} \quad \text{o.k.}
\end{align*}
\]

\[
R_w = \frac{0.60F_a A_w}{\Omega} \quad \text{(from Spec. Eq. J4-4)}
\]

\[
\begin{align*}
\frac{R_w}{\Omega} &= \frac{(0.60)(58 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)}{2.00} \\
&= 34.3 \text{ kips} \\
34.3 \text{ kips} &> 23.1 \text{ kips} \quad \text{o.k.}
\end{align*}
\]
EXAMPLE K.8  TRANSVERSE PLATE LOADED PERPENDICULAR TO THE HSS AXIS ON A RECTANGULAR HSS

Given:
Verify the local strength of the ASTM A500 Grade B HSS column subject to the transverse loads given as follows, applied through a 5½-in.-wide ASTM A36 plate. The HSS8×8×½ is in compression with nominal axial loads of $P_D_{column} = 54.0$ kips and $P_L_{column} = 162$ kips. The HSS has negligible required flexural strength.

Solution:
From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
<th>Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ASTM A500 Grade B</td>
<td>ASTM A36</td>
</tr>
<tr>
<td>$F_y$</td>
<td>46 ksi</td>
<td>36 ksi</td>
</tr>
<tr>
<td>$F_u$</td>
<td>58 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

From AISC Manual Table 1-12, the geometric properties are as follows:

<table>
<thead>
<tr>
<th></th>
<th>HSS8×8×½</th>
<th>Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>8.00 in.</td>
<td>5½ in.</td>
</tr>
<tr>
<td>$t$</td>
<td>0.465 in.</td>
<td>½ in.</td>
</tr>
</tbody>
</table>

Limits of Applicability of AISC Specification Section K1.3

AISC Specification Table K1.2A provides the limits of applicability for plate-to-rectangular HSS connections. The following limits are applicable in this example.

HSS wall slenderness:

$B/t = 14.2 \leq 35$  o.k.
Width ratio:
\[ B_p/B = \frac{5\frac{1}{2} \text{ in.}}{8.00 \text{ in.}} = 0.688 \]
0.25 ≤ \( B_p/B \) ≤ 1.0  o.k.

Material strength:
\[ F_y = 46 \text{ ksi} \leq 52 \text{ ksi} \text{ for HSS} \quad \text{o.k.} \]

Ductility:
\[ \frac{F_y}{F_u} = \frac{46 \text{ ksi}}{58 \text{ ksi}} = 0.793 \leq 0.8 \text{ for HSS} \quad \text{o.k.} \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse force from the plate:</td>
<td>[ P_a = 1.2(10.0 \text{ kips}) + 1.6(30.0 \text{ kips}) = 60.0 \text{ kips} ]</td>
<td>[ P_a = 10.0 \text{ kips} + 30.0 \text{ kips} = 40.0 \text{ kips} ]</td>
</tr>
</tbody>
</table>
| Column axial force: | \[ P_r = P_a \text{ column} \]
  \[ = 1.2(54.0 \text{ kips}) + 1.6(162 \text{ kips}) = 324 \text{ kips} \] | \[ P_r = P_a \text{ column} \]
  \[ = 54.0 \text{ kips} + 162 \text{ kips} = 216 \text{ kips} \] |

Available Local Yielding Strength of Plate from AISC Specification Table K1.2

The available local yielding strength of the plate is determined from AISC Specification Table K1.2.

\[ R_a = \frac{10}{B/t} F_y B_p \leq F_y t_p B_p \]
\[ = \frac{10}{8.00 \text{ in.}/0.465 \text{ in.}} \leq \frac{36 \text{ ksi}(0.465 \text{ in.})(5\frac{1}{2} \text{ in.})}{36 \text{ ksi}(0.465 \text{ in.})(5\frac{1}{2} \text{ in.})} \]
\[ = 68.4 \text{ kips} \leq 99.0 \text{ kips} \quad \text{o.k.} \]

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi ) = 0.95</td>
<td></td>
<td>( \Omega = 1.58 )</td>
</tr>
</tbody>
</table>
| \( \phi R_a = 0.95(68.4 \text{ kips}) \) | \[ R = 68.4 \text{ kips} \]
  \[ \frac{R}{\Omega} = 1.58 \] | \[ R = 43.3 \text{ kips} \]
  \[ \frac{R}{\Omega} = 1.58 \] |
| \[ = 65.0 \text{ kips} > 60.0 \text{ kips} \] | \quad \text{o.k.}            | \[ = 43.3 \text{ kips} > 40.0 \text{ kips} \] \quad \text{o.k.} |

HSS Shear Yielding (Punching)

The available shear yielding (punching) strength of the HSS is determined from AISC Specification Table K1.2.

This limit state need not be checked when \( B_p > B - 2t \), nor when \( B_p < 0.85B \).

\[ B - 2t = 8.00 \text{ in.} - 2(0.465 \text{ in.}) = 7.07 \text{ in.} \]
\[ 0.85B = 0.85(8.00 \text{ in.}) = 6.80 \text{ in.} \]
Therefore, because $B_p < 6.80$ in., this limit state does not control.

**Other Limit States**

The other limit states listed in AISC Specification Table K1.2 apply only when $\beta = 1.0$. Because $B_p / B < 1.0$, these limit states do not apply.
EXAMPLE K.9 LONGITUDINAL PLATE LOADED PERPENDICULAR TO THE HSS AXIS ON A ROUND HSS

Given:

Verify the local strength of the ASTM A500 Grade B HSS6.000×0.375 tension chord subject to transverse loads, \( P_D = 4.00 \) kips and \( P_L = 12.0 \) kips, applied through a 4 in. wide ASTM A36 plate.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Chord
- ASTM A500 Grade B
- \( F_y = 42 \) ksi
- \( F_u = 58 \) ksi

Plate
- ASTM A36
- \( F_{yp} = 36 \) ksi
- \( F_u = 58 \) ksi

From AISC Manual Table 1-13, the geometric properties are as follows:

- HSS6.000×0.375
- \( D = 6.00 \) in.
- \( t = 0.349 \) in.

Limits of Applicability of AISC Specification Section K1.2, Table K1.1A

AISC Specification Table K1.1A provides the limits of applicability for plate-to-round connections. The applicable limits for this example are:

- HSS wall slenderness:
  - \( D/t = 6.00 \) in./0.349 in.
  - \( = 17.2 \leq 50 \) for T-connections o.k.

- Material strength:
  - \( F_y = 42 \) ksi \leq 52 ksi for HSS o.k.

Ductility:
\[ \frac{F_t}{F_u} = \frac{42 \text{ ksi}}{58 \text{ ksi}} = 0.724 \leq 0.8 \text{ for HSS} \quad \text{o.k.} \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u )</td>
<td>1.2(4.00 kips) + 1.6(12.0 kips)</td>
<td>4.00 kips + 12.0 kips</td>
</tr>
<tr>
<td></td>
<td>= 24.0 kips</td>
<td>= 16.0 kips</td>
</tr>
</tbody>
</table>

**HSS Plastification Limit State**

The limit state of HSS plastification applies and is determined from AISC Specification Table K1.1.

\[
R_s = 5.5F_t r^2 \left( 1 + 0.25 \frac{f}{D} \right) Q_t 
\]

\[(\text{Spec. Eq. K1-2)}\]

From the AISC Specification Table K1.1 Functions listed at the bottom of the table, for an HSS connecting surface in tension, \( Q_t = 1.0 \).

\[
R_s = 5.5(42 \text{ ksi})(0.349 \text{ in.})^2 \left( 1 + 0.25 \frac{4.00 \text{ in.}}{6.00 \text{ in.}} \right)(1.0) 
\]

\[ = 32.8 \text{ kips} \]

The available strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_a )</td>
<td>0.90(32.8 kips)</td>
<td>( R_a = \frac{32.8 \text{ kips}}{1.67} )</td>
</tr>
<tr>
<td></td>
<td>= 29.5 kips &gt; 24.0 kips</td>
<td>( \frac{R_a}{\Omega} )</td>
</tr>
</tbody>
</table>
EXAMPLE K.10  HSS BRACE CONNECTION TO A W-SHAPE COLUMN

Given:

Verify the strength of an ASTM A500 Grade B HSS3½×3½×¾ brace for required axial forces of 80.0 kips (LRFD) and 52.0 kips (ASD). The axial force may be either tension or compression. The length of the brace is 6 ft. Design the connection of the HSS brace to the gusset plate. Allow ⅛ in. for fit of the slot over the gusset plate.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Brace
ASTM A500 Grade B

\[ F_y = 46 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

Plate
ASTM A36

\[ F_{yp} = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-12, the geometric properties are as follows:

HSS3½×3½×¾

\[ A = 2.91 \text{ in.}^2 \]
\[ r = 1.32 \text{ in.} \]
\[ t = 0.233 \text{ in.} \]

Available Compressive Strength of Brace

Obtain the available axial compressive strength of the brace from AISC Manual Table 4-4.

\[ K = 1.0 \]
\[ L_b = 6.00 \text{ ft} \]
Available Tensile Yielding Strength of Brace

Obtain the available tensile yielding strength of the brace from AISC Manual Table 5-5.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \frac{P_n}{\Omega_c} = 98.6 \text{ kips} &gt; 80.0 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{P_n}{\Omega_c} = 65.6 \text{ kips} &gt; 52.0 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Available Tensile Rupture Strength of the Brace

Due to plate geometry, $\frac{8}{3}$ in. of overlap occurs. Try four $\frac{3}{6}$-in. fillet welds, each 7-in. long. Based on AISC Specification Table J2.4 and the HSS thickness of $\frac{1}{4}$ in., the minimum weld size is an $\frac{1}{6}$-in. fillet weld.

Determine the available tensile strength of the brace from AISC Specification Section D2.

$$A_e = A_n U$$  
(Spec. Eq. D3-1)

where

$$A_n = A_g - 2(t) \left( \text{gusset plate thickness} + \frac{3}{8} \text{ in.} \right)$$

$$= 2.91 \text{ in.}^2 - 2(0.233 \text{ in.}) \left( \frac{3}{8} \text{ in.} + \frac{3}{8} \text{ in.} \right)$$

$$= 2.71 \text{ in.}^2$$

Because $l = 7 \text{ in.} > H = 3\frac{1}{2} \text{ in.}$, from AISC Specification Table D3.1, Case 6,

$$U = 1 - \frac{\bar{x}}{l}$$

$$\bar{x} = \frac{B^2 + 2BH}{4(B + H)} \text{ from AISC Specification Table D3.1}$$

$$= \frac{(3\frac{1}{2} \text{ in.})^2 + 2(3\frac{1}{2} \text{ in.})(3\frac{1}{2} \text{ in.})}{4(3\frac{1}{2} \text{ in.} + 3\frac{1}{2} \text{ in.})}$$

$$= 1.31 \text{ in.}$$

Therefore,

$$U = 1 - \frac{\bar{x}}{l}$$

$$= 1 - \frac{1.31 \text{ in.}}{7.00 \text{ in.}}$$

$$= 0.813$$

And,

$$A_e = A_n U$$  
(Spec. Eq. D3-1)

$$= 2.71 \text{ in.}^2(0.813)$$

$$= 2.20 \text{ in.}^2$$

$$P_n = F_w A_e$$  
(Spec. Eq. D2-2)

$$= 58 \text{ ksi}(2.20 \text{ in.}^2)$$
Using AISC Specification Section D2, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_t = 0.75 )</td>
<td>( \Omega_t = 2.00 )</td>
</tr>
<tr>
<td>( \phi_tP_n = 0.75(128 \text{ kips}) )</td>
<td>( \frac{P_n}{\Omega_t} = \frac{128 \text{ kips}}{2.00} )</td>
</tr>
<tr>
<td>( = 96.0 \text{ kips} &gt; 80.0 \text{ kips} )</td>
<td>( = 64.0 \text{ kips} &gt; 52.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Available Strength of \( \frac{3}{16} \)-in. Weld of Plate to HSS

From AISC Manual Part 8, the available strength of a \( \frac{3}{16} \)-in. fillet weld is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_R = 4(1.392Dl) ) (from Manual Eq. 8-2a)</td>
<td>( R = 4(0.928Dl) ) (from Manual Eq. 8-2b)</td>
</tr>
<tr>
<td>( = 4(1.392)(3 \text{ sixteenths})(7.00 \text{ in.}) )</td>
<td>( = 4(0.928)(3 \text{ sixteenths})(7.00 \text{ in.}) )</td>
</tr>
<tr>
<td>( = 117 \text{ kips} &gt; 80.0 \text{ kips} )</td>
<td>( = 78.0 \text{ kips} &gt; 52.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

HSS Shear Rupture Strength at Welds (weld on one side)

\[
 t_{min} = \frac{3.09D}{F_u} = \frac{3.09(3)}{58} = 0.160 \text{ in.} < 0.233 \text{ in.} \quad \text{o.k.}
\]

Gusset Plate Shear Rupture Strength at Welds (weld on two sides)

\[
 t_{min} = \frac{6.19D}{F_u} = \frac{6.19(3)}{58} = 0.320 \text{ in.} < \frac{3}{8} \text{ in.} \quad \text{o.k.}
\]

A complete check of the connection would also require consideration of the limit states of the other connection elements, such as:

- Whitmore buckling
- Local capacity of column web yielding and crippling
- Yielding of gusset plate at gusset-to-column intersection
EXAMPLE K.11 RECTANGULAR HSS COLUMN WITH A CAP PLATE, SUPPORTING A CONTINUOUS BEAM

Given:

Verify the local strength of the ASTM A500 Grade B HSS8×8×¾ column subject to the given ASTM A992 W18×35 beam reactions through the ASTM A36 cap plate. Out of plane stability of the column top is provided by the beam web stiffeners; however, the stiffeners will be neglected in the column strength calculations. The column axial forces are $R_D = 24$ kips and $R_L = 30$ kips.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

**Beam**
- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

**Column**
- ASTM A500 Grade B
  - $F_y = 46$ ksi
  - $F_u = 58$ ksi

**Cap Plate**
- ASTM A36
  - $F_{yp} = 36$ ksi
  - $F_u = 58$ ksi

From AISC Manual Tables 1-1 and 1-12, the geometric properties are as follows:

**W18×35**
- $d = 17.7$ in.
- $b_f = 6.00$ in.
- $t_w = 0.300$ in.
- $t_r = 0.425$ in.
- $k_1 = ¾$ in.

**HSS8×8×¾**
- $t = 0.233$ in.
From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$</td>
<td>$1.2(24.0 \text{ kips}) + 1.6(30.0 \text{ kips})$</td>
<td>$24.0 \text{ kips} + 30.0 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 76.8 \text{ kips}$</td>
<td>$= 54.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

Assume the vertical beam reaction is transmitted to the HSS through bearing of the cap plate at the two column faces perpendicular to the beam.

**Bearing Length, $l_b$ at Bottom of W18×35**

Assume the dispersed load width, $l_b = 5t_p + 2k_1$. With $t_p = t_f$.

$$l_b = 5t_f + 2k_1$$

$$= 5(0.425 \text{ in.}) + 2(\frac{3}{4} \text{ in.})$$

$$= 3.63 \text{ in.}$$

**Available Strength: Local Yielding of HSS Sidewalls**

Determine the applicable equation from AISC Specification Table K1.2.

$$5t_p + l_b = 5(\frac{1}{2} \text{ in.}) + 3.63 \text{ in.}$$

$$= 6.13 \text{ in.} < 8.00 \text{ in.}$$

Therefore, only two walls contribute and AISC Specification Equation K1-14a applies.

$$R_u = 2F\bar{t}\left(5t_p + l_b \right)$$

(Spec. Eq. K1-14a)

$$= 2(46 \text{ ksi})(0.233 \text{ in.})(6.13 \text{ in.})$$

$$= 131 \text{ kips}$$

The available wall local yielding strength of the HSS is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$\phi R_u$</td>
<td>1.00(131 kips)</td>
<td>o.k.</td>
</tr>
<tr>
<td></td>
<td>$= 131 \text{ kips} &gt; 76.8 \text{ kips}$</td>
<td></td>
</tr>
</tbody>
</table>

**Available Strength: Local Crippling of HSS Sidewalls**

From AISC Specification Table K1.2, the available wall local crippling strength of the HSS is determined as follows:

$$R_u = 1.6t^2 \left[1 + \frac{6l_b}{B} \left(\frac{t}{t_p}\right)^{1.5} \right] \sqrt{\frac{EF}{t}}$$

(Spec. Eq. K1-15)

$$= 1.6(0.233 \text{ in.})^2 \left[1 + \left(\frac{6(6.13 \text{ in.})}{8.00 \text{ in.}}\right)\left(\frac{0.233 \text{ in.}}{\frac{1}{2} \text{ in.}}\right)^{1.5} \right] \sqrt{\frac{29,000 \text{ ksi}(46 \text{ ksi})}{\frac{1}{2} \text{ in.} / 0.233 \text{ in.}}}$$

$$= 362 \text{ kips}$$
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R = 0.75(362 \text{ kips})$</td>
<td>$R = \frac{362 \text{ kips}}{2.00}$</td>
</tr>
<tr>
<td>$= 272 \text{ kips} &gt; 76.8 \text{ kips}$</td>
<td>$= 181 \text{ kips} &gt; 54.0 \text{ kips}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note: This example illustrates the application of the relevant provisions of Chapter K of the AISC Specification. Other limit states should also be checked to complete the design.
EXAMPLE K.12 RECTANGULAR HSS COLUMN BASE PLATE

Given:

An ASTM A500 Grade B HSS6×6×½ column is supporting loads of 40.0 kips of dead load and 120 kips of live load. The column is supported by a 7 ft-6 in. × 7 ft-6 in. concrete spread footing with $f'_c = 3,000$ psi. Size an ASTM A36 base plate for this column.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>$F_y = 46$ ksi, $F_u = 58$ ksi</td>
</tr>
<tr>
<td>Base Plate</td>
<td>$F_y = 36$ ksi, $F_u = 58$ ksi</td>
</tr>
</tbody>
</table>

From AISC Manual Table 1-12, the geometric properties are as follows:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSS6×6×½</td>
<td>$B = H = 6.00$ in.</td>
</tr>
</tbody>
</table>

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>Type</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>$1.2(40.0$ kips) + 1.6(120 kips)</td>
<td>$40.0$ kips + 120 kips</td>
</tr>
<tr>
<td></td>
<td>= 240 kips</td>
<td>= 160 kips</td>
</tr>
</tbody>
</table>

Note: The procedure illustrated here is similar to that presented in AISC Design Guide 1, Base Plate and Anchor Rod Design (Fisher and Kloiber, 2006), and Part 14 of the AISC Manual.

Try a base plate which extends 3½ in. from each face of the HSS column, or 13 in. × 13 in.
**Available Strength for the Limit State of Concrete Crushing**

On less than the full area of a concrete support,

\[ P_p = 0.85 f'c A \sqrt{\frac{A_2}{A_1}} \leq 1.7 f'c A_1 \]  

(Spec. Eq. J8-2)

\[ A_1 = BN = 13.0 \text{ in.} (13.0 \text{ in.}) = 169 \text{ in}^2 \]

\[ A_2 = 90.0 \text{ in.} (90.0 \text{ in.}) = 8,100 \text{ in}^2 \]

\[ P_p = 0.85 (3 \text{ ksi})(169 \text{ in}^2) \sqrt{\frac{8,100 \text{ in}^2}{169 \text{ in}^2}} \leq 1.7 (3 \text{ ksi})(169 \text{ in}^2) \]

\[ = 2,980 \text{ kips} \leq 862 \text{ kips} \]

Use \( P_p = 862 \text{ kips} \).

Note: The limit on the right side of AISC Specification Equation J8-2 will control when \( A_2/A_1 \) exceeds 4.0.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.65 ) from AISC Specification Section J8</td>
<td>( \Omega_c = 2.31 ) from AISC Specification Section J8</td>
</tr>
<tr>
<td>( \phi_c P_p = 0.65(862 \text{ kips}) = 560 \text{ kips} &gt; 240 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>( P_p = 862 \text{ kips} )</td>
<td>( \Omega_c = 2.31 )</td>
</tr>
<tr>
<td></td>
<td>( = 373 \text{ kips} &gt; 160 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Pressure Under Bearing Plate and Required Thickness**

For a rectangular HSS, the distance \( m \) or \( n \) is determined using 0.95 times the depth and width of the HSS.

\[ m = n = \frac{N - 0.95(\text{outside dimension})}{2} \]

\[ = \frac{13.0 \text{ in.} - 0.95(6.00 \text{ in.})}{2} \]

\[ = 3.65 \text{ in.} \]

The critical bending moment is the cantilever moment outside the HSS perimeter. Therefore, \( m = n = l \).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{pu} = \frac{P_p}{A_1} )</td>
<td>( f_{pu} = \frac{P_p}{A_1} )</td>
</tr>
<tr>
<td>( = \frac{240 \text{ kips}}{169 \text{ in}^2} )</td>
<td>( = \frac{160 \text{ kips}}{169 \text{ in}^2} )</td>
</tr>
<tr>
<td>( = 1.42 \text{ ksi} )</td>
<td>( = 0.947 \text{ ksi} )</td>
</tr>
<tr>
<td>( M_a = \frac{f_{pu} l^2}{2} )</td>
<td>( M_a = \frac{f_{pu} l^2}{2} )</td>
</tr>
<tr>
<td>( Z = \frac{t^2}{4} )</td>
<td>( Z = \frac{t^2}{4} )</td>
</tr>
</tbody>
</table>
\[
\phi_b = 0.90 \\
M_u = M_p = F_y Z \quad \text{(Spec. Eq. F11-1)}
\]

Equating:

\[
M_u = \phi_b M_u \quad \text{and solving for } t_p \text{ gives:}
\]

\[
t_{p,(req)} = \frac{2 f_y l^2}{\phi_b F_y} = \frac{2(1.42 \text{ ksi})(3.65 \text{ in.})^2}{0.90(36 \text{ ksi})} = 1.08 \text{ in.}
\]

Or use AISC Manual Equation 14-7a:

\[
t_{p,(req)} = \sqrt{\frac{2P_u}{0.9F_y BN}} = 3.65 \sqrt{\frac{2(240 \text{ kips})}{0.9(36 \text{ ksi})(13.0 \text{ in.})(13.0 \text{ in.})}} = 1.08 \text{ in.}
\]

Therefore, use a 1\(\frac{1}{4}\) in.-thick base plate.
EXAMPLE K.13  RECTANGULAR HSS STRUT END PLATE

Given:

Determine the weld leg size, end plate thickness, and the size of ASTM A325 bolts required to resist forces of 16 kips from dead load and 50 kips from live load on an ASTM A500 Grade B HSS4×4×¼ section. The end plate is ASTM A36. Use 70-ksi weld electrodes.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Strut
ASTM A500 Grade B
\( F_y = 46 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

End Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-12, the geometric properties are as follows:

HSS4×4×¼
\( t = 0.233 \text{ in.} \)
\( A = 3.37 \text{ in.}^2 \)

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_d = 1.2(16.0 \text{ kips}) + 1.6(50.0 \text{ kips}) )</td>
<td>( P_d = 16.0 \text{ kips} + 50.0 \text{ kips} )</td>
</tr>
<tr>
<td>= 99.2 kips</td>
<td>= 66.0 kips</td>
</tr>
</tbody>
</table>
**Preliminary Size of the (4) ASTM A325 Bolts**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{ed}$</td>
<td>$\frac{P}{n}$</td>
<td>$\frac{P}{n}$</td>
</tr>
<tr>
<td></td>
<td>$= 99.2 \text{ kips}$</td>
<td>$= 66.0 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 24.8 \text{ kips}$</td>
<td>$= 16.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

Using AISC *Manual* Table 7-2, try $\frac{3}{16}$-in.-diameter ASTM A325 bolts.

$\phi r_e = 29.8 \text{ kips}$

End-Plate Thickness with Consideration of Prying Action (AISC Manual Part 9)

\[
a' = \left( a + \frac{d_b}{2} \right) \leq \left( 1.25b + \frac{d_b}{2} \right) \quad (\text{Manual Eq. 9-27})
\]

\[
= 1.50 \text{ in.} + \frac{3/4 \text{ in.}}{2} \leq 1.25(1.50 \text{ in.}) + \frac{3/4 \text{ in.}}{2}
\]

\[
= 1.88 \text{ in.} \leq 2.25 \text{ in.} \quad \text{o.k.}
\]

\[
b' = b - \frac{d_b}{2}\quad (\text{Manual Eq. 9-21})
\]

\[
= 1.50 \text{ in.} - \frac{3/4 \text{ in.}}{2}
\]

\[
= 1.13 \text{ in.}
\]

\[
\rho = \frac{b'}{a'}\quad (\text{Manual Eq. 9-26})
\]

\[
= 1.13\frac{\text{in.}}{1.88 = 0.601}
\]

\[
d' = \frac{3}{16} \text{ in.}
\]

The tributary length per bolt,

\[
p = \frac{\text{full plate width}}{\text{(number of bolts per side)}}
\]

\[
= \frac{10.0 \text{ in.}}{1}
\]

\[
= 10.0 \text{ in.}
\]

\[
\delta = 1 - \frac{d'}{p}\quad (\text{Manual Eq. 9-24})
\]

\[
= 1 - \frac{3/16 \text{ in.}}{10.0 \text{ in.}}
\]

\[
= 0.919
\]
Use Equation 9-23 for $t_{req}$ in Chapter 9 of the AISC Manual, except that $F_u$ is replaced by $F_y$ per recommendation of Willibald, Packer and Puthli (2003) and Packer et al. (2010).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = \frac{1}{\rho} \left( \frac{F_{w,u}}{F_{u}} - 1 \right)$</td>
<td>(Manual Eq. 9-25)</td>
</tr>
<tr>
<td>$= \frac{1}{0.601} \left( \frac{29.8 \text{ kips}}{24.8 \text{ kips}} - 1 \right)$</td>
<td>$= \frac{1}{0.601} \left( \frac{19.9 \text{ kips}}{16.5 \text{ kips}} - 1 \right)$</td>
</tr>
<tr>
<td>$= 0.335$</td>
<td>$= 0.343$</td>
</tr>
</tbody>
</table>

| $\alpha' = \frac{\beta}{\delta (1 - \beta)}$ | |
| $= \frac{1}{0.919} \left( \frac{0.335}{1 - 0.335} \right)$ | $= \frac{1}{0.919} \left( \frac{0.343}{1 - 0.343} \right)$ |
| $= 0.548 \leq 1.0$ | $= 0.568 \leq 1.0$ |

Use a ½-in. end plate, $t_e > 0.480$ in., further bolt check for prying not required.

Use (4) ¼-in.-diameter A325 bolts.

**Required Weld Size**

\[ R_{w} = F_{w,u} A_{w,e} \]  \hspace{1cm} (Spec. Eq. J2-4)

\[ F_{w,u} = 0.60 F_{xx} \left( 1.0 + 0.50 \sin^{1.5} \theta \right) \]  \hspace{1cm} (Spec. Eq. J2-5)

\[ = 0.60 \left( 70 \text{ ksi} \right) \left( 1.0 + 0.50 \sin^{1.5} 0^\circ \right) \]

\[ = 63.0 \text{ ksi} \]

\[ l = 4 \left( 4.00 \text{ in.} \right) \]

\[ = 16.0 \text{ in.} \]

Note: This weld length is approximate. A more accurate length could be determined by taking into account the curved corners of the HSS.

From AISC Specification Table J2.5:
For shear load on fillet welds $\phi = 0.75$

$$w \geq \frac{P_v}{\phi F_{sw} (0.707) l}$$

from AISC Manual Part 8

$$\geq \frac{99.2 \text{ kips}}{0.75 \times (63.0 \text{ ksi}) (0.707) (16.0 \text{ in.})}$$

$$\geq 0.186 \text{ in.}$$

For shear load on fillet welds $\Omega = 2.00$

$$w \geq \frac{\Omega P_v}{F_{sw} (0.707) l}$$

from AISC Manual Part 8

$$\geq \frac{2.00 \times (66.0 \text{ kips})}{(63.0 \text{ ksi}) (0.707) (16.0 \text{ in.})}$$

$$\geq 0.185 \text{ in.}$$

Try $w = \frac{3}{16} \text{ in.} > 0.186 \text{ in.}$

**Minimum Weld Size Requirements**

For $t = \frac{1}{4} \text{ in.}$, the minimum weld size = $\frac{1}{6} \text{ in.}$ from AISC Specification Table J2.4.

**Results:**

Use $\frac{3}{16}$-in. weld with $\frac{1}{2}$-in. end plate and (4) $\frac{3}{4}$-in.-diameter ASTM A325 bolts, as required for strength in the previous calculation.
CHAPTER K DESIGN EXAMPLE REFERENCES


APPENDIX 6

STABILITY BRACING FOR COLUMNS AND BEAMS

This Appendix contains provisions for evaluating column and beam braces.

The governing limit states for column and beam design may include flexural, torsional and flexural-torsional buckling for columns and lateral-torsional buckling for beams. In the absence of other intermediate bracing, column unbraced lengths are defined between points of obviously adequate lateral restraint, such as floor and roof diaphragms which are part of the building’s lateral force resisting systems. Similarly, beams are often braced against lateral-torsional buckling by relatively strong and stiff bracing elements such as a continuously connected floor slab or roof diaphragm. However, at times, unbraced lengths are bounded by elements that may or may not possess adequate strength and stiffness to provide sufficient bracing. AISC Specification Appendix 6 provides equations for determining the required strength and stiffness of braces that have not been included in the second-order analysis of the structural system. It is not intended that the provisions of Appendix 6 apply to bracing that is part of the lateral force resisting system.

Background for the provisions can be found in references cited in the Commentary including “Fundamentals of Beam Bracing” (Yura, 2001) and the Guide to Stability Design Criteria for Metal Structures (Ziemian, 2010). AISC Manual Part 2 also provides information on member stability bracing.

6.1 GENERAL PROVISIONS

Lateral column and beam bracing may be either relative or nodal while torsional beam bracing may be nodal or continuous. The User Note in AISC Specification Appendix 6, Section 6.1 states “A relative brace controls the movement of the brace point with respect to adjacent braced points. A nodal brace controls the movement at the braced point without direct interaction with adjacent braced points. A continuous bracing system consists of bracing that is attached along the entire member length;…..” Relative and nodal bracing systems are discussed further in AISC Specification Commentary Appendix 6, Section 6.1. Examples of each are shown in the Commentary Figure C-A-6.1.

A rigorous second-order analysis, including initial out-of-straightness, may be used in lieu of the requirements of this appendix.

6.2 COLUMN BRACING

The requirements in this section apply to bracing associated with the limit state of flexural buckling. For columns that could experience torsional or flexural-torsional buckling, the designer must ensure that sufficient bracing to resist the torsional component of buckling is provided. See Helwig and Yura (1999).

Column braces may be relative or nodal. The type of bracing must be determined before the requirements for strength and stiffness can be determined. The requirements are derived for an infinite number of braces along the column and are thus conservative for most columns as explained in the Commentary. Provision is made in this section for reducing the required brace stiffness for nodal bracing when the column required strength is less than the available strength of the member. The Commentary also provides an approach to reduce the requirements when a finite number of nodal braces are provided.

6.3 BEAM BRACING
The requirements in this section apply to bracing associated with the limit state of lateral-torsional buckling. Bracing to resist lateral-torsional buckling may be accomplished by a lateral brace, or a torsional brace, or a combination of the two to prevent twist of the section. Lateral bracing should normally be connected near the compression flange. The exception is for cantilevers and near inflection points. Torsional bracing may be connected anywhere on the cross section in a manner to prevent twist of the section.

According to AISC Specification Section F1(2), the design of members for flexure is based on the assumption that points of support are restrained against rotation about their longitudinal axis. The bracing requirements in Appendix 6 are for intermediate braces in addition to those at the support.

In members subject to double curvature, inflection points are not to be considered as braced points unless bracing is provided at that location. In addition, the bracing nearest the inflection point must be attached to prevent twist, either as a torsional brace or as lateral braces attached to both flanges as described in AISC Specification Appendix 6, Section 6.3.1(2).

6.3.1 Lateral Bracing. As with column bracing, beam bracing may be relative or nodal. In addition, it is permissible to provide torsional bracing. This section provides requirements for determining the required lateral brace strength and stiffness for relative and nodal braces.

For nodal braces, provision is made in this section to reduce the required brace stiffness when the actual unbraced length is less than the maximum unbraced length for the required flexural strength.

The Commentary provides alternative equations for required brace strength and stiffness that may result in a significantly smaller required strength and stiffness due to the conservative nature of the requirements of this section.

6.3.2 Torsional Bracing. This section provides requirements for determining the required bracing flexural strength and stiffness for nodal and continuous torsional bracing. Torsional bracing can be connected to the section at any location. However, if the beam has inadequate distortional (out-of-plane) bending stiffness, torsional bracing will be ineffective. Web stiffeners can be provided when necessary, to increase the web distortional stiffness for nodal torsional braces.

As is the case for columns and for lateral beam nodal braces, it is possible to reduce the required brace stiffness when the required strength of the member is less than the available strength for the provided location of bracing.

Provisions for continuous torsional bracing are also provided. A slab connected to the top flange of a beam in double curvature may provide sufficient continuous torsional bracing as discussed in the Commentary. For this condition there is no unbraced length between braces so the unbraced length used in the strength and stiffness equations is the maximum unbraced length permitted to provide the required strength in the beam. In addition, for continuous torsional bracing, stiffeners are not permitted to be used to increase web distortional stiffness.

6.4 BEAM-COLUMN BRACING

For bracing of beam-columns, the required strength and stiffness are to be determined for the column and beam independently as specified in AISC Specification Appendix 6, Sections 6.2 and 6.3. These values are then to be combined, depending on the type of bracing provided.
APPENDIX 6 REFERENCES


EXAMPLE A-6.1 NODAL STABILITY BRACING OF A COLUMN

Given:

An ASTM A992 W12×72 column carries a dead load of 105 kips and a live load of 315 kips. The column is 36 ft long and is braced laterally and torsionally at its ends. Intermediate lateral braces for the x- and y-axis are provided at the one-third points as shown. Thus, the unbraced length for the limit state of flexural-torsional buckling is 36 ft and the unbraced length for flexural buckling is 12 ft. The column has sufficient strength to support the applied loads with this bracing. Find the strength and the stiffness requirements for the intermediate nodal braces, such that the unbraced length for the column can be taken as 12 ft.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Column
ASTM A992
$F_y = 50$ ksi

Required Compressive Strength

The required strengths from the governing load combinations are:

<table>
<thead>
<tr>
<th>LRFD Load Combination 2 from ASCE/SEI 7 Section 2.3.2</th>
<th>ASD Load Combination 2 from ASCE/SEI 7 Section 2.4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_a = 1.2(105 \text{ kips}) + 1.6(315 \text{ kips})$</td>
<td>$P_a = 105 \text{ kips} + 315 \text{ kips}$</td>
</tr>
<tr>
<td>$= 630 \text{ kips}$</td>
<td>$= 420 \text{ kips}$</td>
</tr>
</tbody>
</table>

From AISC Manual Table 4-1 at $KL_y = 12$ ft, the available strength of the W12×72 is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_a = 806 \text{ kips}$</td>
<td>$\frac{P_a}{\Omega_e} = 536 \text{ kips}$</td>
</tr>
</tbody>
</table>
| $806 \text{ kips} > 630 \text{ kips}$      | $536 \text{ kips} > 420 \text{kips}$      | o.k.}
Calculate the required nodal brace strength

From AISC Specification Equation A-6-3, the required nodal brace strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_{rb} = 0.01P_u )</td>
<td>( P_{rb} = 0.01P_u )</td>
</tr>
<tr>
<td></td>
<td>= 0.01 (630 kips)</td>
<td>= 0.01 (420 kips)</td>
</tr>
<tr>
<td></td>
<td>= 6.30 kips</td>
<td>= 4.20 kips</td>
</tr>
</tbody>
</table>

Calculate the required nodal brace stiffness

From AISC Specification Equation A-6-4, the required nodal brace stiffness is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi ) = 0.75</td>
<td>( \Omega = 2.00 )</td>
<td></td>
</tr>
<tr>
<td>( \beta_{br} = \frac{1}{\phi} \left( \frac{8P_u}{L_u} \right) )</td>
<td>( \beta_{br} = \Omega \left( \frac{8P_u}{L_u} \right) )</td>
<td></td>
</tr>
<tr>
<td>= \frac{1}{0.75} \left( \frac{8(630 \text{ kips})}{(12 \text{ ft})(12 \text{ in./ft})} \right)</td>
<td>= 2.00 \left( \frac{8(420 \text{ kips})}{(12 \text{ ft})(12 \text{ in./ft})} \right)</td>
<td></td>
</tr>
<tr>
<td>= 46.7 \text{ kip/in.}</td>
<td>= 46.7 \text{ kip/in.}</td>
<td></td>
</tr>
</tbody>
</table>

Determine the maximum permitted unbraced length for the required strength

From AISC Manual Table 4-1:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( KL = 18.9 \text{ ft for } P_u = 632 \text{ kips} )</td>
<td>( KL = 18.9 \text{ ft for } P_u = 421 \text{ kips} )</td>
<td></td>
</tr>
</tbody>
</table>

Calculate the required nodal brace stiffness for this increased unbraced length

It is permissible to size the braces to provide the lower stiffness determined using the maximum unbraced length permitted to carry the required strength according to AISC Specification Appendix 6, Section 6.2.2. From AISC Specification Equation A-6-4:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 ) (LRFD)</td>
<td>( \Omega = 2.00 ) (ASD)</td>
<td></td>
</tr>
<tr>
<td>( \beta_{br} = \frac{1}{\phi} \left( \frac{8P_u}{L_u} \right) )</td>
<td>( \beta_{br} = \Omega \left( \frac{8P_u}{L_u} \right) )</td>
<td></td>
</tr>
<tr>
<td>= \frac{1}{0.75} \left( \frac{8(630 \text{ kips})}{(18.9 \text{ ft})(12 \text{ in./ft})} \right)</td>
<td>= 2.00 \left( \frac{8(420 \text{ kips})}{(18.9 \text{ ft})(12 \text{ in./ft})} \right)</td>
<td></td>
</tr>
<tr>
<td>= 29.6 \text{ kip/in.}</td>
<td>= 29.6 \text{ kip/in.}</td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE A-6.2  NODAL STABILITY BRACING OF A COLUMN

Given:

An ASTM A992 WT7×34 column carries a dead load of 25 kips and a live load of 75 kips. The column is 30 ft long as shown. The unbraced length for this column is 7.5 ft. Bracing about the y-axis is provided by the axial resistance of a W-shape connected to the flange of the WT, while bracing about the x-axis is provided by the flexural resistance of the same W-shape loaded at the midpoint of a 12-ft simple span. Find the strength and stiffness requirements for the nodal braces and select an appropriate W-shape, based on x-axis flexural buckling of the WT. Assume that the axial strength and stiffness of the W-shape are adequate to brace the y-axis of the WT. Also, assume the column is braced laterally and torsionally at its ends and torsionally at each brace point.

Solution:

This column is braced at each end by the supports and at the one-quarter points by a W-shape as shown.

From AISC Manual Table 2-4, the material properties are as follows:

Column and brace
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Required Compressive Strength

The required strength is determined from the following governing load combinations:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD Load Combination 2 from ASCE/SEI 7 Section 2.3.2</td>
<td>ASD Load Combination 2 from ASCE/SEI 7 Section 2.4.1</td>
</tr>
<tr>
<td>$P_u = 1.2(25\text{ kips}) + 1.6(75\text{ kips})$</td>
<td>$P_a = 25\text{ kips} + 75\text{ kips}$</td>
</tr>
<tr>
<td>$= 150\text{ kips}$</td>
<td>$= 100\text{ kips}$</td>
</tr>
</tbody>
</table>

Available Compressive Strength

From AISC Manual Table 4-7 at $KL_x = 7.5$ ft, the available axial compressive strength of the WT7×34 is:
Required Nodal Brace Size

The required nodal brace strength from AISC Specification Equation A-6-3 is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{rb} = 0.01P_u$</td>
<td>$P_{rb} = 0.01P_u$</td>
</tr>
<tr>
<td>= 0.01(150 kips)</td>
<td>= 0.01(100 kips)</td>
</tr>
<tr>
<td>= 1.50 kips</td>
<td>= 1.00 kips</td>
</tr>
</tbody>
</table>
Note that because the live-to-dead load ratio is 3, the LRFD and ASD results are identical.

The required stiffness can be reduced if the maximum permitted unbraced length is used as described in AISC Specification Appendix 6, Section 6.2, and also if the actual number of braces are considered, as discussed in the commentary. The following demonstrates how this affects the design.

From AISC Manual Table 4-7, the maximum permitted unbraced length for the required strength is as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( KL = 18.6 \text{ ft for } P_u = 150 \text{ kips} )</td>
<td>( KL = 18.6 \text{ ft for } P_u = 100 \text{ kips} )</td>
</tr>
</tbody>
</table>

From AISC Specification Commentary Equation C-A-6-4, determine the reduction factor for three intermediate braces:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2n-1}{2n} = \frac{2(3)-1}{2(3)} = 0.833 )</td>
<td>( \frac{2n-1}{2n} = \frac{2(3)-1}{2(3)} = 0.833 )</td>
</tr>
</tbody>
</table>

Determine the required nodal brace stiffness for the increased unbraced length and number of braces using AISC Specification Equation A-6-4.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \beta_\phi = 0.833 \left[ \frac{1}{\phi} \left( \frac{8P_u}{L_o} \right) \right] )</td>
<td>( \beta_\phi = 0.833 \left[ \frac{\Omega}{L_o} \left( \frac{8P_u}{L_o} \right) \right] )</td>
</tr>
<tr>
<td>( = 0.833 \left[ \frac{1}{0.75} \left( \frac{8(150 \text{ kips})}{18.6 \text{ ft}(12 \text{ in./ft})} \right) \right] = 5.97 \text{ kip/in.} )</td>
<td>( = 0.833 \left[ 2.00 \left( \frac{8(100 \text{ kips})}{18.6 \text{ ft}(12 \text{ in./ft})} \right) \right] = 5.97 \text{ kip/in.} )</td>
</tr>
</tbody>
</table>

Determine the required brace size based on this new stiffness requirement.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Based on brace stiffness, the minimum required moment of inertia of the beam is:</td>
<td>Based on brace stiffness, the minimum required moment of inertia of the beam is:</td>
</tr>
</tbody>
</table>
Based on the unchanged flexural strength for a compact laterally supported beam, the minimum required plastic section modulus is:

\[ I_{pl} \geq \frac{\beta_{pl} E L}{48} \]

\[ \geq \frac{(5.97 \text{ kip/in.})(12 \text{ ft})^3 (12 \text{ in./ft}^3)}{48(29,000 \text{ ksi})} \]

\[ \geq 12.8 \text{ in.}^4 \]

Select a W6\times8.5 noncompact member with \( I_x = 14.9 \text{ in.}^4 \) and \( Z_x = 5.73 \text{ in.}^4 \).
EXAMPLE A-6.3  NODAL STABILITY BRACING OF A BEAM—CASE I

Given:

A walkway in an industrial facility has a span of 28 ft as shown in the given plan view. The walkway has a deck of grating which is not sufficient to brace the beams. The ASTM A992 W12×22 beams along walkway edges are braced against twist at the ends as required by AISC Specification Section F1(2) and are connected by an L3×3×⅜ strut at midspan. The two diagonal ASTM A36 L5×5×⅜ braces are connected to the top flange of the beams at the supports and at the strut at the middle. The strut and the brace connections are welded; therefore bolt slippage does not need to be accounted for in the stiffness calculation. The dead load on each beam is 0.0500 kip/ft and the live load is 0.125 kip/ft. Determine if the diagonal braces are strong enough and stiff enough to brace this walkway.

Solution:

Because the diagonal braces are connected directly to an unyielding support that is independent of the midspan brace point, they are designed as nodal braces. The strut will be assumed to be sufficiently strong and stiff to force the two beams to buckle together.

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Diagonal braces
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC Manual Tables 1-1 and 1-7, the geometric properties are as follows:

Beam
W12×22

$ho = 11.9$ in.

Diagonal braces
L5×5×⅜

$A = 3.07$ in.$^2$

Required Strength

Determine the required strength for each beam using the governing load combination.
Determine the required flexural strength for a uniformly loaded simply supported beam.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From LRFD Load Combination 2 of ASCE/SEI 7 Section 2.3.2</td>
<td>From ASD Load Combination 2 of ASCE/SEI 7 Section 2.4.1</td>
</tr>
<tr>
<td>$w_u = 1.2(0.0500 \text{ kip/ft}) +1.6(0.125 \text{ kip/ft})$</td>
<td>$w_u = 0.0500 \text{ kip/ft} + 0.125 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$= 0.260 \text{ kip/ft}$</td>
<td>$= 0.175 \text{ kip/ft}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u = (0.260 \text{ kip/ft})(28.0 \text{ ft})^2/8$</td>
<td>$M_u = (0.175 \text{ kip/ft})(28.0 \text{ ft})^2/8$</td>
</tr>
<tr>
<td>$= 25.5 \text{ kip-ft}$</td>
<td>$= 17.2 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

It can be shown that the W12×22 beams are adequate with the unbraced length of 14.0 ft. Both beams need bracing in the same direction simultaneously.

**Required Brace Strength and Stiffness**

From AISC Specification Appendix 6, Section 6.3, and Equation A-6-7, determine the required nodal brace strength for each beam as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{rb} = 0.02M_uC_d/h_o$</td>
<td>$P_{rb} = 0.02M_uC_d/h_o$</td>
</tr>
<tr>
<td>$C_d = 1.0$ for bending in single curvature</td>
<td>$C_d = 1.0$ for bending in single curvature</td>
</tr>
<tr>
<td>$P_{rb} = 0.02(25.5 \text{ kip-ft})(12 \text{ in./ft})(1.0)/(11.9 \text{ in.})$</td>
<td>$P_{rb} = 0.02(17.2 \text{ kip-ft})(12 \text{ in./ft})(1.0)/(11.9 \text{ in.})$</td>
</tr>
<tr>
<td>$= 0.514 \text{ kips}$</td>
<td>$= 0.347 \text{ kips}$</td>
</tr>
</tbody>
</table>

Because there are two beams to be braced, the total required brace strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{rb} = 2(0.514 \text{ kips})$</td>
<td>$P_{rb} = 2(0.347 \text{ kips})$</td>
</tr>
<tr>
<td>$= 1.03 \text{ kips}$</td>
<td>$= 0.694 \text{ kips}$</td>
</tr>
</tbody>
</table>

There are two beams to brace and two braces to share the load. The worst case for design of the braces will be when they are in compression.

By geometry, the diagonal bracing length is $l = \sqrt{(14.0 \text{ ft})^2 + (5.00 \text{ ft})^2} = 14.9 \text{ ft}$, and the required brace strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_b \cos \theta = F_b (5.00 \text{ ft}/14.9 \text{ ft})$</td>
<td>$F_b \cos \theta = F_b (5.00 \text{ ft}/14.9 \text{ ft})$</td>
</tr>
<tr>
<td>$= 1.03 \text{ kips}$</td>
<td>$= 0.694 \text{ kips}$</td>
</tr>
</tbody>
</table>

Because there are two braces, the required brace strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_b = \frac{1.03 \text{ kips}}{2(5.00 \text{ ft}/14.9 \text{ ft})}$</td>
<td>$F_b = \frac{0.694 \text{ kips}}{2(5.00 \text{ ft}/14.9 \text{ ft})}$</td>
</tr>
<tr>
<td>$= 1.53 \text{ kips}$</td>
<td>$= 1.03$</td>
</tr>
</tbody>
</table>
The required nodal brace stiffness is determined from AISC Specification Equation A-6-8 as follows:

$$\phi = 0.75$$

$$\beta_{br} = \frac{1}{\phi} \left( \frac{10M_uC_d}{L_uh_u} \right)$$

$$= \frac{1}{0.75} \left[ \frac{10(25.5 \text{ kip-ft})(12 \text{ in./ft})(1.0)}{(14.0 \text{ ft})(12 \text{ in./ft})(11.9 \text{ in.})} \right]$$

$$= 2.04 \text{ kip/in.}$$

$$\Omega = 2.00$$

$$\beta_{br} = \Omega \left( \frac{10M_uC_d}{L_uh_u} \right)$$

$$= 2.00 \left[ \frac{10(17.2 \text{ kip-ft})(12 \text{ in./ft})(1.0)}{(14.0 \text{ ft})(12 \text{ in./ft})(11.9 \text{ in.})} \right]$$

$$= 2.06 \text{ kip/in.}$$

Because there are two beams to be braced, the total required nodal brace stiffness is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$\beta_{br} = 2(2.04 \text{ kip/in.})$$</td>
<td>$$\beta_{br} = 2(2.06 \text{ kip/in.})$$</td>
</tr>
<tr>
<td>$$= 4.08 \text{ kip/in.}$$</td>
<td>$$= 4.12 \text{ kip/in.}$$</td>
</tr>
</tbody>
</table>

The beams require bracing in order to have sufficient strength to carry the given load. However, locating that brace at the midspan provides flexural strength greater than the required strength. The maximum unbraced length permitted for the required flexural strength is 17.8 ft from AISC Specification Section F2. Thus, according to AISC Specification Appendix 6, Section 6.3.1b, this length could be used in place of 14.0 ft to determine the required stiffness. However, because the required stiffness is so small, the 14.0 ft length will be used here.

For a single brace located as shown previously, the stiffness is:

$$\beta = \frac{AE \cos^2 \theta}{L}$$

$$= \frac{(3.07 \text{ in.}^2)(29,000 \text{ ksi})(5.00 \text{ ft}/14.9 \text{ ft})^2}{(14.9 \text{ ft})(12 \text{ in./ft})}$$

$$= 56.1 \text{ kip/in.}$$

Because there are two braces, the system stiffness is twice this. Thus,

$$\beta = 2(56.1 \text{ kip/in.})$$

$$= 112 \text{ kip/in.}$$

Determine if the braces provide sufficient stiffness.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$\beta = 112 \text{ kip/in.} &gt; 4.08 \text{ kip/in.}$$</td>
<td>$$\beta = 112 \text{ kip/in.} &gt; 4.12 \text{ kip/in.}$$</td>
</tr>
</tbody>
</table>

Available Strength of Braces

The braces may be called upon to act in either tension or compression, depending on which transverse direction the system tries to buckle. Brace compression buckling will control over tension yielding. Therefore, determine the compressive strength of the braces assuming they are eccentrically loaded using AISC Manual Table 4-12.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
For $KL = 14.9$ ft,  

$\phi P_n = 17.2$ kips

$1.53$ kips < $17.2$ kips \textbf{o.k.}

Therefore the L5x5x5% has sufficient strength.

For $KL = 14.9$ ft,  

$\frac{P_n}{\Omega} = 11.2$ kips

$1.03$ kips < $11.2$ kips \textbf{o.k.}

Therefore the L5x5x5% has sufficient strength.

The L5x5x5% braces have sufficient strength and stiffness to act as the nodal braces for this system.
EXAMPLE A-6.4  NODAL STABILITY BRACING OF A BEAM—CASE II

Given:

A walkway in an industrial facility has a span of 28 ft as shown. The walkway has a deck of grating which is not sufficient to brace the beams. The ASTM A992 W12×22 beams are braced against twist at the ends, and they are connected by a strut connected at midspan. At that same point they are braced to an adjacent ASTM A500 Grade B HSS 8×8×¾ column by the attachment of a 5-ft-long ASTM A36 2L3×3×¾. The brace connections are all welded; therefore, bolt slippage does not need to be accounted for in the stiffness calculation. The adjacent column is not braced at the walkway level, but is adequately braced 12 ft below and 12 ft above the walkway level. The dead load on each beam is 0.050 kip/ft and the live load is 0.125 kip/ft. Determine if the bracing system has adequate strength and stiffness to brace this walkway.

Solution:

Because the bracing system does not interact directly with any other braced point on the beam, the double angle and column constitute a nodal brace system. The strut will be assumed to be sufficiently strong and stiff to force the two beams to buckle together.

From AISC Manual Table 2-4, the material properties are as follows:

- Beam
  ASTM A992
  $F_y = 50$ ksi
  $F_u = 65$ ksi

- HSS column
  ASTM A500 Grade B
  $F_y = 46$ ksi
  $F_u = 58$ ksi

- Double-angle brace
  ASTM A36
  $F_y = 36$ ksi
  $F_u = 58$ ksi

From AISC Manual Tables 1-1 and 1-7, the geometric properties are as follows:

- Beam
  W12×22
  $h_o = 11.9$ in.

- HSS column
  HSS8×8×¾
Required Strength

Determine the required strength in kip/ft for each beam using the governing load combination.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From LRFD Load Combination 2 of ASCE/SEI 7 Section 2.3.2</td>
<td>From ASD Load Combination 2 of ASCE/SEI 7 Section 2.4.1</td>
</tr>
<tr>
<td>$w_u = 1.2(0.0500 \text{kip/ft}) + 1.6(0.125 \text{kip/ft})$</td>
<td>$w_u = 0.0500 \text{kip/ft} + 0.125 \text{kip/ft}$</td>
</tr>
<tr>
<td>$= 0.260 \text{kip/ft}$</td>
<td>$= 0.175 \text{kip/ft}$</td>
</tr>
</tbody>
</table>

Determine the required flexural strength for a uniformly distributed load on the simply supported beam as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u = (0.260 \text{kip/ft})(28 \text{ ft})^2/8$</td>
<td>$M_u = (0.175 \text{ kips/ft})(28 \text{ ft})^2/8$</td>
</tr>
<tr>
<td>$= 25.5 \text{kip-ft}$</td>
<td>$= 17.2 \text{kip-ft}$</td>
</tr>
</tbody>
</table>

It can be shown that the W12x22 beams are adequate with this unbraced length of 14.0 ft. Both beams need bracing in the same direction simultaneously.

Required Brace Strength and Stiffness

The required brace force for each beam is determined from AISC Specification Equation A-6-7 as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{rb} = 0.02M_uC_d/h_o$</td>
<td>$P_{rb} = 0.02M_uC_d/h_o$</td>
</tr>
<tr>
<td>$C_d = 1.0$ for bending in single curvature.</td>
<td>$C_d = 1.0$ for bending in single curvature.</td>
</tr>
<tr>
<td>$P_{rb} = 0.02(25.5 \text{kip-ft})(12 \text{in./ft})(1.0)/(11.9 \text{ in.})$</td>
<td>$P_{rb} = 0.02(17.2 \text{kip-ft})(12 \text{in./ft})(1.0)/(11.9 \text{ in.})$</td>
</tr>
<tr>
<td>$= 0.514 \text{kips}$</td>
<td>$= 0.347 \text{kips}$</td>
</tr>
</tbody>
</table>

Because there are two beams, the total required brace force is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{rb} = 2(0.514 \text{kips})$</td>
<td>$P_{rb} = 2(0.347 \text{kips})$</td>
</tr>
<tr>
<td>$= 1.03 \text{kips}$</td>
<td>$= 0.694 \text{kips}$</td>
</tr>
</tbody>
</table>

By inspection, the 2L3x3x$\frac{3}{4}$ can carry the required bracing force. The HSS column can also carry the bracing force through bending on a 24-ft-long span. It will be shown that the change in length of the 2L3x3x$\frac{3}{4}$ is negligible, so the available brace stiffness will come from the flexural stiffness of the column only.

Determine the required brace stiffness using AISC Specification Equation A-6-8.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$ (LRFD)</td>
<td>$\Omega = 2.00$ (ASD)</td>
</tr>
</tbody>
</table>
The beams require one brace in order to have sufficient strength to carry the given load. However, locating that brace at midspan provides flexural strength greater than the required strength. The maximum unbraced length permitted for the required flexural strength is 17.8 ft from AISC Specification Section F2. Thus, according to AISC Specification Appendix 6, Section 6.3.1b, this length could be used in place of 14.0 ft to determine the required stiffness.

**Available Stiffness**

Because the brace stiffness comes from the combination of the axial stiffness of the double-angle member and the flexural stiffness of the column loaded at its midheight, the individual element stiffness will be determined and then combined.

The axial stiffness of the double angle is:

\[
\beta = \frac{AE}{L} = \frac{2.88 \text{ in.}^2 (29,000 \text{ ksi})}{(5.00 \text{ ft})(12 \text{ in./ft})} = 1,390 \text{ kip/in.}
\]

The available flexural stiffness of the HSS column with a point load at midspan is:

\[
\beta = \frac{48EI}{L} = \frac{48(29,000 \text{ ksi})(70.7 \text{ in.}^3)}{(24.0 \text{ ft})^3(12 \text{ in./ft})^3} = 4.12 \text{ kip/in.}
\]

The combined stiffness is:

\[
\frac{1}{\beta} = \frac{1}{\beta_{\text{angles}}} + \frac{1}{\beta_{\text{column}}} = \frac{1}{1,390 \text{ kip/in.}} + \frac{1}{4.12 \text{ kip/in.}} = 0.243 \text{ in./kip}
\]

Thus, the system stiffness is:

\[
\beta = 4.12 \text{ kip/in.}
\]

The stiffness of the double-angle member could have reasonably been ignored.

Because the double-angle brace is ultimately bracing two beams, the required stiffness is multiplied by 2:
The HSS8×8×$\frac{3}{4}$ column is an adequate brace for the beams. However, if the column also carries an axial force, it must be checked for combined forces.
EXAMPLE A-6.5  NODAL STABILITY BRACING OF A BEAM WITH REVERSE CURVATURE BENDING

Given:

A roof system is composed of 26K8 steel joists spaced at 5-ft intervals and supported on ASTM A992 W21×50 girders as shown in Figure A-6.5-1. The roof dead load is 33 psf and the roof live load is 25 psf. Determine the required strength and stiffness of the braces needed to brace the girder at the support and near the inflection point. Determine the size of single-angle kickers connected to the bottom flange of the girder and the top chord of the joist, as shown, where the brace force will be taken by a connected rigid diaphragm.

(a) Plan

(b) Section B-B: Beam with bracing and moment diagram
Solution:

Since the braces will transfer their force to a rigid roof diaphragm, they will be treated as nodal braces.

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi
Single-angle brace
ASTM A36
$F_y = 36 \text{ ksi}$
$F_u = 58 \text{ ksi}$

From the Steel Joist Institute:

Joist
K-Series
$F_y = 50 \text{ ksi}$

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W21×50
$h_o = 20.3 \text{ in.}$

Required Flexural Strength

The required flexural strength of the beam is determined as follows.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From LRFD Load Combination 2 of ASCE/SEI 7 Section 2.3.2</td>
<td>From ASD Load Combination 2 of ASCE/SEI 7 Section 2.4.1</td>
</tr>
<tr>
<td>$w_u = 1.2(33.0 \text{ psf}) + 1.6(25.0 \text{ psf})$</td>
<td>$w_u = 33.0 \text{ psf} + 25.0 \text{ psf}$</td>
</tr>
<tr>
<td>$= 79.6 \text{ psf}$</td>
<td>$= 58.0 \text{ psf}$</td>
</tr>
<tr>
<td>$w_a = 79.6 \text{ psf} / (40.0 \text{ ft}) / (1,000 \text{ lb/kip})$</td>
<td>$w_a = 58.0 \text{ psf} / (40.0 \text{ ft}) / (1,000 \text{ lb/kip})$</td>
</tr>
<tr>
<td>$= 3.18 \text{ kip/ft}$</td>
<td>$= 2.32 \text{ kip/ft}$</td>
</tr>
<tr>
<td>From Figure A-6.5-1(d), $M_{ab} = 88.8 \text{ (3.18 kip/ft)}$</td>
<td>From Figure A-6.5-1(d), $M_{ab} = 88.8 \text{ (2.32 kip/ft)}$</td>
</tr>
<tr>
<td>$= 282 \text{ kip-ft}$</td>
<td>$= 206 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Required Brace Strength and Stiffness

Determine the required force to brace the bottom flange of the girder with a nodal brace. The braces at points B and C will be determined based on the moment at B. However, because the brace at C is the closest to the inflection point, its strength and stiffness requirements are greater since they are influenced by the variable $C_d$ which will be equal to 2.0.

The required brace force is determined from AISC Specification Equation A-6-7 as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{rb} = 0.02M_{bc}C_d/h_o$</td>
<td>$P_{rb} = 0.02M_{bc}C_d/h_o$</td>
</tr>
<tr>
<td>$= 0.02(282 \text{ kip-ft})(12 \text{ in./ft})(2.0)/(20.3 \text{ in.})$</td>
<td>$= 0.02(206 \text{ kip-ft})(12 \text{ in./ft})(2.0)/(20.3 \text{ in.})$</td>
</tr>
<tr>
<td>$= 6.67 \text{ kips}$</td>
<td>$= 4.87 \text{ kips}$</td>
</tr>
</tbody>
</table>

Determine the required stiffness of the nodal brace at point C. The required brace stiffness is a function of the unbraced length. It is permitted to use the maximum unbraced length permitted for the beam based upon the required flexural strength. Thus, determine the maximum unbraced length permitted.
Based on AISC Specification Section F1, Equation F1-1, and the moment diagram shown in Figure A-6-6(d), for the beam between points B and C, the lateral-torsional buckling modification factor, $C_b$, is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_b = \frac{12.5M_{ax}}{2.5M_{ax} + 3M_{d} + 4M_{a} + 3M_{c}}$</td>
<td>$C_b = \frac{12.5M_{ax}}{2.5M_{ax} + 3M_{d} + 4M_{a} + 3M_{c}}$</td>
</tr>
<tr>
<td>$= \frac{12.5[-88.8w]}{2.5[-88.8w] + 3[-42.1w] + 4[-1.5w] + 3[32.9w]}$</td>
<td>$= \frac{12.5[-88.8w]}{2.5[-88.8w] + 3[-42.1w] + 4[-1.5w] + 3[32.9w]}$</td>
</tr>
<tr>
<td>$= 2.45$</td>
<td>$= 2.45$</td>
</tr>
</tbody>
</table>

The maximum unbraced length for the required flexural strength can be determined by setting the available flexural strength based on AISC Specification Equation F2-3 (lateral-torsional buckling) equal to the required strength and solving for $L_b$ (this is assuming that $L_b > L_r$).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For a required flexural strength, $M_u = 282 \text{ kip-ft}$, with $C_b = 2.45$, the unbraced length may be taken as $L_b = 22.0 \text{ ft}$</td>
<td>For a required flexural strength, $M_u = 206 \text{ kip-ft}$, with $C_b = 2.45$, the unbraced length may be taken as $L_b = 20.6 \text{ ft}$</td>
</tr>
</tbody>
</table>

From AISC Specification Appendix 6, Section 6.3.1b, Equation A-6-8:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\beta_{br} = \left( \frac{10M_{c}}{L_b h_o} \right)$</td>
<td>$\beta_{br} = \left( \frac{10M_{c}}{L_b h_o} \right)$</td>
</tr>
<tr>
<td>$= \left( \frac{10(282 \text{ kip-ft})(12 \text{ in./ft})(2.0)}{22.0 \text{ ft}(12 \text{ in./ft})(20.3 \text{ in.})} \right)$</td>
<td>$= \left( \frac{10(206 \text{ kip-ft})(12 \text{ in./ft})(2.0)}{20.6 \text{ ft}(12 \text{ in./ft})(20.3 \text{ in.})} \right)$</td>
</tr>
<tr>
<td>$= 16.8 \text{ kip/in.}$</td>
<td>$= 19.7 \text{ kip/in.}$</td>
</tr>
</tbody>
</table>

Because no deformation will be considered in the connections, only the brace itself will be used to provide the required stiffness. The brace is oriented with the geometry as shown in Figure A-6.5-1(e). Thus, the force in the brace is $F_{br} = P_{br}/(\cos \theta)$ and the stiffness of the brace is $AE(\cos^2 \theta)/L$. There are two braces at each brace point. One would be in tension and one in compression, depending on the direction that the girder attempts to buckle. For simplicity in design, a single brace will be selected that will be assumed to be in tension. Only the limit state of yielding will be considered.

Select a single angle to meet the requirements of strength and stiffness, with a length of:

$\sqrt{(48.0 \text{ in.})^2 + (20.0 \text{ in.})^2} = 52 \text{ in.}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required brace force</td>
<td>Required brace force</td>
</tr>
<tr>
<td>$F_{br} = P_{br}/(\cos \theta)$</td>
<td>$F_{br} = P_{br}/(\cos \theta)$</td>
</tr>
<tr>
<td>$= (6.67 \text{ kips})/(48.0 \text{ ft}/52.0 \text{ ft})$</td>
<td>$= (4.87 \text{ kips})/(48.0 \text{ ft}/52.0 \text{ ft})$</td>
</tr>
<tr>
<td>$= 7.23 \text{ kips}$</td>
<td>$= 5.28 \text{ kips}$</td>
</tr>
</tbody>
</table>
From AISC Specification Section D2(a), the required area based on available tensile strength is determined as follows:

\[ A_g = \frac{F_{br}}{\phi F_y} \]
\[ = 7.23 \text{ kips}/[0.90(36 \text{ ksi})] \]
\[ = 0.223 \text{ in.}^2 \]

The required area based on stiffness is:

\[ A_g = \frac{\beta_{br} L}{E \cos^2 \theta} \]
\[ = \frac{(16.8 \text{ kip/in.})(52.0 \text{ in.})}{(29,000 \text{ ksi})(48.0 \text{ in./52.0 ft})^2} \]
\[ = 0.0354 \text{ in.}^2 \]

The strength requirement controls, therefore select L2×2×1/8 with \( A = 0.491 \text{ in.}^2 \)

From AISC Specification Section D2(a), the required area based on available tensile strength is determined as follows:

\[ A_g = \frac{\Omega F_{br}}{F_y} \]
\[ = 1.67(5.28 \text{ kips})/(36 \text{ ksi}) \]
\[ = 0.245 \text{ in.}^2 \]

The required area based on stiffness is:

\[ A_g = \frac{\beta_{br} L}{E \cos^2 \theta} \]
\[ = \frac{(19.7 \text{ kip/in.})(52.0 \text{ in.})}{(29,000 \text{ ksi})(48.0 \text{ in./52.0 ft})^2} \]
\[ = 0.0415 \text{ in.}^2 \]

The strength requirement controls, therefore select L2×2×1/8 with \( A = 0.491 \text{ in.}^2 \)

At the column at point B, the required strength would be one-half of that at point C, because \( C_d = 1.0 \) at point B instead of 2.0. However, since the smallest angle available has been selected for the brace, there is no reason to check further at the column and the same angle will be used there.
EXAMPLE A-6.6  NODAL TORSIONAL STABILITY BRACING OF A BEAM

Given:

A roof system is composed of W12×40 intermediate beams spaced 5 ft on center supporting a connected panel roof system that cannot be used as a diaphragm. The beams span 30 ft and are supported on W30×90 girders spanning 60 ft. This is an isolated roof structure with no connections to other structures that could provide lateral support to the girder compression flanges. Thus, the flexural resistance of the attached beams must be used to provide torsional stability bracing of the girders. The roof dead load is 40 psf and the roof live load is 24 psf. Determine if the beams are sufficient to provide nodal torsional stability bracing.

Solution:

Because the bracing beams are not connected in a way that would permit them to transfer an axial bracing force, they must behave as nodal torsional braces if they are to effectively brace the girders.

From AISC Manual Table 2-4, the material properties are as follows:

Beam and girder
ASTM A992
\(F_y = 50\) ksi
From AISC *Manual* Table 1-1, the geometric properties are as follows:

**Beam**

*W12×40*

- $t_w = 0.295$ in.
- $I_x = 307$ in.$^4$

**Girder**

*W30×90*

- $t_w = 0.470$ in.
- $h_o = 28.9$ in.
- $I_y = 115$ in.$^4$

### Required Flexural Strength

The required flexural strength of the girder is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From LRFD Load Combination 3 of ASCE/SEI 7 Section 2.3.2</td>
<td>From ASD Load Combination 3 of ASCE/SEI 7 Section 2.4.1</td>
</tr>
<tr>
<td>$w_u = 1.2(40 \text{ psf}) + 1.6(24 \text{ psf}) = 86.4 \text{ psf}$</td>
<td>$w_a = 40 \text{ psf} + 24 \text{ psf} = 64.0 \text{ psf}$</td>
</tr>
<tr>
<td>$w_u = 86.4 \text{ psf} (15 \text{ ft})/(1,000 \text{ lb/kip}) = 1.30 \text{ kip/ft}$</td>
<td>$w_u = 64.0 \text{ psf} (15 \text{ ft})/(1,000 \text{ lb/kip}) = 0.960 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$M_u = (1.30 \text{ kip/ft})(60.0 \text{ ft})^2/8 = 585 \text{ kip-ft}$</td>
<td>$M_a = (0.960 \text{ kip/ft})(60.0 \text{ ft})^2/8 = 432 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

With $C_b = 1.0$, from AISC *Manual* Table 3-10, the maximum unbraced length permitted for the *W30×90* based upon required flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For $M_{ub} = 585 \text{ kip-ft}, L_b = 22.0$ ft</td>
<td>For $M_{ub} = 432 \text{ kip-ft}, L_b = 20.7$ ft</td>
</tr>
</tbody>
</table>

### Nodal Torsional Brace Design

The required flexural strength for a nodal torsional brace for the girder is determined from AISC *Specification* Appendix 6, Section 6.3.2a, Equation A-6-9, with braces every 5 ft, $n = 11$, and assuming $C_b = 1$.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{rb} = 0.024M_{JL}/nC_bL_b$</td>
<td>$M_{rb} = 0.024M_{JL}/nC_bL_b$</td>
</tr>
<tr>
<td>$= 0.024(585 \text{ kip-ft})(60.0 \text{ ft})/11(1.0)(22.0 \text{ ft}) = 3.48 \text{ kip-ft}$</td>
<td>$= 0.024(432 \text{ kip-ft})(60.0 \text{ ft})/11(1.0)(20.7 \text{ ft}) = 2.73 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

The required overall nodal torsional brace stiffness is determined from AISC *Specification* Appendix 6, Section 6.3.2a, Equation A-6-11, as follows:
The distortional buckling stiffness of the girder web is a function of the web slenderness and the presence of any stiffeners, using AISC Specification Equation A-6-12. The web distortional stiffness is:

\[ \beta_{web} = \frac{3.3E}{h_b} \left( \frac{1.5h_b t_w^2}{12} + \frac{t_w h_w^3}{12} \right) \]  

\( (\text{Spec. Eq. A-6-12}) \)

The distortional stiffness of the girder web alone is:

\[ \beta_{web} = \frac{3.3E}{h_b} \left( \frac{1.5h_b t_w^2}{12} \right) \]

\[ = \frac{3.3E}{h_b} \left( \frac{1.5h_b t_w^2}{12} \right) = \frac{3.3(29,000 \text{ ksi})}{28.9 \text{ in.}} \left( \frac{1.5(28.9 \text{ in.})(0.470 \text{ in.})^3}{12} \right) \]

\[ = 1,240 \text{ kip-in./rad} \]

For AISC Specification Equation A-6-10 to give a nonnegative result, the web distortional stiffness given by Equation A-6-12 must be greater than the required nodal torsional stiffness given by Equation A-6-11. Because the web distortional stiffness of the girder is less than the required nodal torsional stiffness for both LRFD and ASD, web stiffeners will be required.

Determine the torsional stiffness contributed by the beams. Both girders will buckle in the same direction forcing the beams to bend in reverse curvature. Thus, the flexural stiffness of the beam is:

\[ \beta_{tb} = \frac{6EI}{L} \]

\[ = \frac{6(29,000 \text{ ksi})(307 \text{ in}^4)}{(30.0 \text{ ft})(12 \text{ in./ft})} \]

\[ = 148,000 \text{ kip-in./rad} \]

Determining the required distortional stiffness of the girder will permit determination of the required stiffener size. The total stiffness is determined by summing the inverse of the distortional and flexural stiffnesses. Thus:

\[ \frac{1}{\beta_T} = \frac{1}{\beta_{tb}} + \frac{1}{\beta_{web}} \]

Determine the minimum web distortional stiffness required to provide bracing for the girder.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 3.00$</td>
</tr>
<tr>
<td>$\beta_T = \frac{1}{\phi} \left( \frac{2.4LM_w^2}{nEI/C_w} \right)$</td>
<td>$\beta_T = \Omega \left( \frac{2.4LM_w^2}{nEI/C_w} \right)$</td>
</tr>
<tr>
<td>$= \frac{1}{0.75} \left[ \frac{2.4(60.0 \text{ ft})(585 \text{ kip-ft})^2}{11(29,000 \text{ ksi})(115 \text{ in.}^4)(1.0)^2} \right]$</td>
<td>$= 3.00 \left( \frac{2.4(60.0 \text{ ft})(432 \text{ kip-ft})^2}{11(29,000 \text{ ksi})(115 \text{ in.}^4)(1.0)^2} \right)$</td>
</tr>
<tr>
<td>$= 3,100 \text{ kip-in./rad}$</td>
<td>$= 3,800 \text{ kip-in./rad}$</td>
</tr>
</tbody>
</table>
Using AISC Specification Equation A-6-12, determine the required width, $b_s$, of ⅜-in.-thick stiffeners.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{sec} = 3,170 \text{ kip-in./rad}$</td>
<td>$\beta_{sec} = 3,900 \text{ kip-in./rad}$</td>
</tr>
<tr>
<td>3,170 kip-in./rad = 1,240 kip-in./rad</td>
<td>3,900 kip-in./rad = 1,240 kip-in./rad</td>
</tr>
<tr>
<td>$Z = \left( \frac{0.295 \text{ in.} \cdot (9.00 \text{ in.})^2}{4} \right)$</td>
<td>$Z = \left( \frac{0.295 \text{ in.} \cdot (9.00 \text{ in.})^2}{4} \right)$</td>
</tr>
<tr>
<td>$= 5.97 \text{ in.}^3$</td>
<td>$= 5.97 \text{ in.}^3$</td>
</tr>
<tr>
<td>$M_n = \left( 50 \text{ ksi} \cdot (5.97 \text{ in.})^3 \right) / 12$</td>
<td>$M_n = \left( 50 \text{ ksi} \cdot (5.97 \text{ in.})^3 \right) / 12$</td>
</tr>
<tr>
<td>$= 24.9 \text{ kip-ft}$</td>
<td>$= 24.9 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$\phi M_n = 0.90(24.9 \text{ kip-ft})$</td>
<td>$M_n \leq 1.67$</td>
</tr>
<tr>
<td>$= 22.4 \text{ kip-ft} &gt; 3.48 \text{ kip-ft}$ o.k.</td>
<td>$\phi M_n = 24.9 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Available Flexural Strength

Each beam is connected to a girder web stiffener. Thus, each beam will be coped at the top and bottom as shown in Figure A-6.6-1(b) with a depth at the coped section of 9 in. The available flexural strength of the coped beam is determined using the provisions of AISC Specification Sections J4.5 and F11.

From AISC Specification Equation F11-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_n = M_p = F_j Z \leq 1.6M_y$</td>
<td>$M_n = M_p = F_j Z \leq 1.6M_y$</td>
</tr>
<tr>
<td>For a rectangle, $Z &lt; 1.6S$. Therefore strength will be controlled by $F_j Z$ and</td>
<td>For a rectangle, $Z \leq 1.6S$. Therefore strength will be controlled by $F_j Z$ and</td>
</tr>
<tr>
<td>$Z = \left( \frac{0.295 \text{ in.} \cdot (9.00 \text{ in.})^2}{4} \right)$</td>
<td>$Z = \left( \frac{0.295 \text{ in.} \cdot (9.00 \text{ in.})^2}{4} \right)$</td>
</tr>
<tr>
<td>$= 5.97 \text{ in.}^3$</td>
<td>$= 5.97 \text{ in.}^3$</td>
</tr>
<tr>
<td>$M_n = \left( 50 \text{ ksi} \cdot (5.97 \text{ in.})^3 \right) / 12$</td>
<td>$M_n = \left( 50 \text{ ksi} \cdot (5.97 \text{ in.})^3 \right) / 12$</td>
</tr>
<tr>
<td>$= 24.9 \text{ kip-ft}$</td>
<td>$= 24.9 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$\phi M_n = 0.90(24.9 \text{ kip-ft})$</td>
<td>$M_n \leq 1.67$</td>
</tr>
<tr>
<td>$= 22.4 \text{ kip-ft} &gt; 3.48 \text{ kip-ft}$ o.k.</td>
<td>$= 14.9 \text{ kip-ft} &gt; 2.73 \text{ kip-ft}$ o.k.</td>
</tr>
</tbody>
</table>
Neglecting any rotation due to the bolts moving in the holes or any influence of the end moments on the strength of the beams, this system has sufficient strength and stiffness to provide nodal torsional bracing to the girders.

Additional connection design limit states may also need to be checked.
Part II
Examples Based on the AISC Steel Construction Manual

This part contains design examples demonstrating design aids and concepts provided in the AISC Steel Construction Manual.
Chapter IIA
Simple Shear Connections

The design of simple shear connections is covered in Part 10 of the AISC Steel Construction Manual.
EXAMPLE II.A-1  ALL-BOLTED DOUBLE-ANGLE CONNECTION

Given:

Select an all-bolted double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

\[ R_D = 37.5 \text{ kips} \]
\[ R_L = 113 \text{ kips} \]

Use \( \frac{3}{8} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Column
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Angles

*This dimension (see sketch, Section A) is determined as one-half of the decimal web thickness rounded to the next higher \( \frac{1}{64} \) in. Example: 0.760/2 = 0.380; use \( \frac{3}{8} \) in. This will produce spacing of holes in the supporting beam slightly larger than detailed in the angles to permit spreading of angles (angles can be spread but not closed) at time of erection to supporting member. Alternatively, consider using horizontal short slots in the support legs of the angles.
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W36\(\times\)231
\[ t_w = 0.760 \text{ in.} \]

Column
W14\(\times\)90
\[ t_f = 0.710 \text{ in.} \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

\[
\begin{align*}
\text{LRFD} & \quad \text{ASD} \\
R_u &= 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips}) & R_u &= 37.5 \text{ kips} + 113 \text{ kips} \\
&= 226 \text{ kips} & \theta &= 151 \text{ kips}
\end{align*}
\]

Connection Design

AISC Manual Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

Try 8 rows of bolts and 2L5\(\times\)3\(\frac{3}{8}\)\(\times\)\(\frac{3}{8}\) (SLBB).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_a = 247 \text{ kips} &gt; 226 \text{ kips} )</td>
<td>(\frac{R_a}{\Omega} = 165 \text{ kips} &gt; 151 \text{ kips} )</td>
</tr>
<tr>
<td>Beam web strength from AISC Manual Table 10-1: Uncoped, (L_{eh} = 1\frac{1}{4} \text{ in.} )</td>
<td>Beam web strength from AISC Manual Table 10-1: Uncoped, (L_{eh} = 1\frac{1}{4} \text{ in.} )</td>
</tr>
<tr>
<td>(\phi R_a = 702 \text{ kips/in.}(0.760 \text{ in.}) )</td>
<td>(\frac{R_a}{\Omega} = 468 \text{ kips/in.}(0.760 \text{ in.}) )</td>
</tr>
<tr>
<td>(= 534 \text{ kips} &gt; 226 \text{ kips} )</td>
<td>(= 356 \text{ kips} &gt; 151 \text{ kips} )</td>
</tr>
<tr>
<td>Bolt bearing on column flange from AISC Manual Table 10-1:</td>
<td>Bolt bearing on column flange from AISC Manual Table 10-1:</td>
</tr>
<tr>
<td>(\phi R_a = 1,400 \text{ kips/in.}(0.710 \text{ in.}) )</td>
<td>(\frac{R_a}{\Omega} = 936 \text{ kips/in.}(0.710 \text{ in.}) )</td>
</tr>
<tr>
<td>(= 994 \text{ kips} &gt; 226 \text{ kips} )</td>
<td>(= 665 \text{ kips} &gt; 151 \text{ kips} )</td>
</tr>
</tbody>
</table>
EXAMPLE II.A-2  BOLTED/WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1 using AISC Manual Table 10-2 to substitute welds for bolts in the support legs of the double-angle connection (welds B). Use 70-ksi electrodes.

Note: Bottom flange cope for erection.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Column
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Angles
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W36×231
\( t_w = 0.760 \text{ in.} \)
Column

W14×90
t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips}) = 226 \text{ kips})</td>
<td>(R_u = 37.5 \text{ kips} + 113 \text{ kips} = 151 \text{ kips})</td>
</tr>
</tbody>
</table>

Weld Design using AISC Manual Table 10-2 (welds B)

Try \(\frac{3}{16}\)-in. weld size, \(L = 23 \frac{1}{2} \text{ in.}\).

\(t_f \text{ min} = 0.238 \text{ in.} < 0.710 \text{ in.} \quad \text{o.k.}\)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_u = 279 \text{ kips} &gt; 226 \text{ kips})</td>
<td>(R_u \Omega = 186 \text{ kips} &gt; 151 \text{ kips} \quad \text{o.k.})</td>
</tr>
</tbody>
</table>

Angle Thickness

The minimum angle thickness for a fillet weld from AISC Specification Section J2.2b is:

\[ t_{\text{min}} = w + \frac{1}{16} \text{ in.} \]
\[ = \frac{3}{16} \text{ in.} + \frac{1}{16} \text{ in.} \]
\[ = \frac{1}{8} \text{ in.} \]

Try 2L4×3\(\frac{3}{4}\)×\(\frac{3}{8}\) (SLBB).

Angle and Bolt Design

AISC Manual Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

Check 8 rows of bolts and \(\frac{3}{8}\)-in. angle thickness.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_u = 286 \text{ kips} &gt; 226 \text{ kips})</td>
<td>(R_u \Omega = 191 \text{ kips} &gt; 151 \text{ kips} \quad \text{o.k.})</td>
</tr>
<tr>
<td>Beam web strength:</td>
<td>Beam web strength:</td>
</tr>
<tr>
<td>Uncoped, (L_{\text{ch}} = 1\frac{1}{4} \text{ in.})</td>
<td>Uncoped, (L_{\text{ch}} = 1\frac{1}{8} \text{ in.})</td>
</tr>
<tr>
<td>(\phi R_u = 702 \text{ kips/in.}(0.760 \text{ in.}) = 534 \text{ kips} &gt; 226 \text{ kips})</td>
<td>(R_u \Omega = 468 \text{ kips/in.}(0.760 \text{ in.}) = 356 \text{ kips} &gt; 151 \text{ kips} \quad \text{o.k.})</td>
</tr>
</tbody>
</table>

Note: In this example, because of the relative size of the cope to the overall beam size, the coped section does not control. When this cannot be determined by inspection, see AISC Manual Part 9 for the design of the coped section.
EXAMPLE II.A-3  ALL-WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1 using AISC Manual Table 10-3 to design an all-welded double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange. Use 70-ksi electrodes and ASTM A36 angles.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Column
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Angles
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W36×231
$tw = 0.760$ in.
From Chapter 2 of ASCE/SEI 7, the required strength is:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips}) & R_u = 37.5 \text{ kips} + 113 \text{ kips} \\
= 226 \text{ kips} & = 151 \text{ kips} \\
\hline
\end{array}
\]

**Design of Weld Between Beam Web and Angle (welds A)**

Try \(\frac{3}{16}\)-in. weld size, \(L = 24 \text{ in.}\)

\(t_{w \min} = 0.286 \text{ in.} < 0.760 \text{ in.} \quad \text{o.k.}\)

From AISC *Manual* Table 10-3:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi R_u = 257 \text{ kips} > 226 \text{ kips} & \frac{R_n}{\Omega} = 171 \text{ kips} > 151 \text{ kips} \quad \text{o.k.} \\
\hline
\end{array}
\]

**Design of Weld Between Column Flange and Angle (welds B)**

Try \(\frac{3}{8}\)-in. weld size, \(L = 24 \text{ in.}\)

\(t_{f \min} = 0.190 \text{ in.} < 0.710 \text{ in.} \quad \text{o.k.}\)

From AISC *Manual* Table 10-3:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi R_u = 229 \text{ kips} > 226 \text{ kips} & \frac{R_n}{\Omega} = 153 \text{ kips} > 151 \text{ kips} \quad \text{o.k.} \\
\hline
\end{array}
\]

**Angle Thickness**

Minimum angle thickness for weld from AISC *Specification* Section J2.2b:

\[
t_{\text{min}} = w + \frac{1}{8} \text{ in.} \\
= \frac{3}{8} \text{ in.} + \frac{1}{16} \text{ in.} \\
= \frac{5}{16} \text{ in.}
\]

Try \(2L4\times3\times\frac{5}{16}\) (SLBB).

**Shear Yielding of Angles (AISC Specification Section J4.2)**

\[
A_{gy} = 2(24.0 \text{ in.})\left(\frac{3}{8} \text{ in.}\right) \\
= 15.0 \text{ in.}^2
\]

\[
R_n = 0.60 F_y A_{gy} \quad (\text{Spec. Eq. J4-3}) \\
= 0.60(36 \text{ ksi})(15.0 \text{ in.}^2) \\
= 324 \text{ kips}
\]
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = 1.00(324 \text{ kips})$</td>
<td>$R_n = \frac{324 \text{ kips}}{1.50}$</td>
</tr>
<tr>
<td>= 324 kips &gt; 226 kips</td>
<td>= 216 kips &gt; 151 kips</td>
</tr>
</tbody>
</table>

o.k.
EXAMPLE II.A-4  ALL-BOLTED DOUBLE-ANGLE CONNECTION IN A COPED BEAM

Given:

Use AISC Manual Table 10-1 to select an all-bolted double-angle connection between an ASTM A992 W18×50 beam and an ASTM A992 W21×62 girder web to support the following beam end reactions:

\[ R_D = 10 \text{ kips} \]
\[ R_L = 30 \text{ kips} \]

The beam top flange is coped 2 in. deep by 4 in. long, \( L_{ew} = 1\frac{1}{4} \text{ in.} \), \( L_{eh} = 1\frac{3}{4} \text{ in.} \). Use \( \frac{3}{8} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
W18×50
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Girder
W21×62
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Angles
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)
From AISC Manual Tables 1-1 and 9-2 and AISC Manual Figure 9-2, the geometric properties are as follows:

**Beam**

W18×50  
\(d = 18.0 \text{ in.}\)  
\(t_w = 0.355 \text{ in.}\)  
\(S_{net} = 23.4 \text{ in.}^3\)  
\(c = 4.00 \text{ in.}\)  
\(d_c = 2.00 \text{ in.}\)  
\(e = 4.00 \text{ in.} + 0.500 \text{ in.} = 4.50 \text{ in.}\)  
\(h_o = 16.0 \text{ in.}\)

**Girder**

W21×62  
\(t_w = 0.400 \text{ in.}\)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips}) = 60.0 \text{ kips})</td>
<td>(R_a = 10 \text{ kips} + 30 \text{ kips} = 40.0 \text{ kips})</td>
</tr>
</tbody>
</table>

**Connection Design**

AISC Manual Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

Try 3 rows of bolts and 2L4×3½×¾ (SLBB).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_u = 76.4 \text{ kips} &gt; 60.0 \text{ kips})</td>
<td>(R_u = 50.9 \text{ kips} &gt; 40.0 \text{ kips})</td>
</tr>
</tbody>
</table>
| Beam web strength from AISC Manual Table 10-1: Top flange coped, \(L_{ev} = 1\frac{1}{4} \text{ in.}, L_{eh} = 1\frac{1}{4} \text{ in.}\):  
\(\phi R_u = 200 \text{ kips/in.}(0.355 \text{ in.}) = 71.0 \text{ kips} > 60.0 \text{ kips}\) | \(R_u = 133 \text{ kips/in.}(0.355 \text{ in.}) = 47.2 \text{ kips} > 40.0 \text{ kips}\) |
| Bolt bearing on girder web from AISC Manual Table 10-1:  
\(\phi R_u = 526 \text{ kips/in.}(0.400 \text{ in.}) = 210 \text{ kips} > 60.0 \text{ kips}\) | \(R_u = 351 \text{ kips/in.}(0.400 \text{ in.}) = 140 \text{ kips} > 40.0 \text{ kips}\) |

Note: The middle portion of AISC Manual Table 10-1 includes checks of the limit-state of bolt bearing on the beam web and the limit-state of block shear rupture on coped beams. AISC Manual Tables 9-3a, 9-3b and 9-3c may be used to determine the available block shear strength for values of \(L_{ev}\) and \(L_{eh}\) beyond the limits of AISC Manual Table 10-1. For coped members, the limit states of flexural yielding and local buckling must be checked independently per AISC Manual Part 9.
Coped Beam Strength (AISC Manual Part 9)

Flexural Local Web Buckling

Verify \( c \leq 2d \) and \( d_c \leq \frac{d}{2} \).

\[
c = 4.00 \text{ in.} \leq 2(18.0 \text{ in.}) = 36.0 \text{ in.} \quad \text{o.k.}
\]

\[
d_c = 2.00 \text{ in.} \leq \frac{18.0 \text{ in.}}{2} = 9.00 \text{ in.} \quad \text{o.k.}
\]

\[
c = 4.00 \text{ in.} \\
\frac{c}{d} = \frac{4.00 \text{ in.}}{18.0 \text{ in.}} = 0.222
\]

\[
c = 4.00 \text{ in.} \\
\frac{c}{h_o} = \frac{4.00 \text{ in.}}{16.0 \text{ in.}} = 0.250
\]

Since \( \frac{c}{d} \leq 1.0 \),

\[
f = \frac{2c}{d} = 2(0.222) = 0.444 \quad \text{(Manual Eq. 9-8)}
\]

Since \( \frac{c}{h_o} \leq 1.0 \),

\[
k = 2.2\left(\frac{h_o}{c}\right)^{1.65} = 2.2\left(\frac{16.0 \text{ in.}}{4.00 \text{ in.}}\right)^{1.65} = 21.7 \quad \text{(Manual Eq. 9-10)}
\]

\[
F_{cr} = 26,210\left(\frac{f_k}{h_o}\right)^2 f_k
\]

\[
= 26,210\left(\frac{0.355 \text{ in.}}{16.0 \text{ in.}}\right)^2 (0.444)(21.7)
\]

\[
= 124 \text{ ksi} \leq 50 \text{ ksi}
\]

Use \( F_{cr} = 50 \text{ ksi} \).

\[
R_n = \frac{F_{cr}S_{net}}{c} \quad \text{from AISC Manual Equation 9-6}
\]

\[
= \frac{50 \text{ ksi}(23.4 \text{ in.}^3)}{4.50 \text{ in.}}
\]

\[
= 260 \text{ kips}
\]
### Shear Yielding of Beam Web (AISC Specification Section J4.2)

\[
R_n = 0.60F_y A_{gv} \\
= 0.60(50 \text{ ksi})(0.355 \text{ in.})(16.0 \text{ in.}) \\
= 170 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(260 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = 260 \text{ kips} )</td>
</tr>
<tr>
<td>( = 234 \text{ kips} &gt; 60.0 \text{ kips} )</td>
<td>( = 156 \text{ kips} &gt; 40.0 \text{ kips} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

### Shear Rupture of Beam Web (AISC Specification Section J4.2)

\[
A_{nv} = t_w \left[ h_o - 3\left(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}\right)\right] \\
= 0.355 \text{ in.}(16.0 \text{ in.} - 2.63 \text{ in.}) \\
= 4.75 \text{ in.}^2
\]

\[
R_n = 0.60F_y A_{nv} \\
= 0.60(65.0 \text{ ksi})(4.75 \text{ in.}^2) \\
= 185 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(170 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = 170 \text{ kips} )</td>
</tr>
<tr>
<td>( = 170 \text{ kips} &gt; 60.0 \text{ kips} )</td>
<td>( = 113 \text{ kips} &gt; 40.0 \text{ kips} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Because the cope is not greater than the length of the connection angle, it is assumed that other flexural limit states of rupture and lateral-torsional buckling do not control.
EXAMPLE II.A-5  WELDED/BOLTED DOUBLE-ANGLE CONNECTION IN A COPED BEAM

Given:

Repeat Example II.A-4 using AISC Manual Table 10-2 to substitute welds for bolts in the supported-beam-web legs of the double-angle connection (welds A). Use 70-ksi electrodes and ¼-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
W18×50
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Girder
W21×62
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Angles
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Tables 1-1 and 9-2 and AISC Manual Figure 9-2, the geometric properties are as follows:

Beam
W18×50
\( d = 18.0 \text{ in.} \)
\( t_w = 0.355 \text{ in.} \)
\( S_{net} = 23.4 \text{ in.}^3 \)
\( c = 4.00 \text{ in.} \)
\( d_c = 2.00 \text{ in.} \)
\( e = 4.00 \text{ in.} + 0.500 \text{ in.} \)
= 4.50 in.  
\( h_w = 16.0 \) in.  

Girder  
W21×62  
\( t_w = 0.400 \) in.  

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips}) )</td>
<td>( R_u = 10 \text{ kips} + 30 \text{ kips} )</td>
</tr>
<tr>
<td>= 60.0 kips</td>
<td>=40.0 kips</td>
</tr>
</tbody>
</table>

Weld Design (welds A)

Try \( \frac{1}{16} \)-in. weld size, \( L = 8\frac{1}{2} \) in from AISC Manual Table 10-2.

\( t_{w.min} = 0.286 \) in. < 0.355 in. \ o.k.  

From AISC Manual Table 10-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 110 \text{ kips} &gt; 60.0 \text{ kips} )</td>
<td>( \frac{R_u}{\Omega} ) = 73.5 kips &gt; 40.0 kips \ o.k.</td>
</tr>
</tbody>
</table>

Minimum Angle Thickness for Weld

\( w = \) weld size

\( t_{min} = w + \frac{1}{16} \) in. from AISC Specification Section J2.2b

\( = \frac{1}{16} \) in. + \( \frac{1}{16} \) in.

\( = \frac{1}{4} \) in.  

Bolt Bearing on Supporting Member Web

From AISC Manual Table 10-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 526 \text{ kips/in.}(0.400\text{in.}) )</td>
<td>( \frac{R_n}{\Omega} ) = 351 kips/in.(0.400 in.)</td>
</tr>
<tr>
<td>= 210 kips &gt; 60.0 kips</td>
<td>= 140 kips &gt; 40.0 kips \ o.k.</td>
</tr>
</tbody>
</table>

Bearing, Shear and Block Shear for Bolts and Angles

From AISC Manual Table 10-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 76.4 \text{ kips} &gt; 60.0 \text{ kips} )</td>
<td>( \phi R_n = 50.9 \text{ kips} &gt; 40.0 \text{ kips} ) \ o.k.</td>
</tr>
</tbody>
</table>

Note: The middle portion of AISC Manual Table 10-1 includes checks of the limit state of bolt bearing on the beam web and the limit state of block shear rupture on the beam web. AISC Manual Tables 9-3a, 9-3b and 9-3c may be used to determine the available block shear strength for values of \( L_{e,y} \) and \( L_{e,b} \) beyond the limits of AISC Manual Table 10-1. For coped members, the limit states of flexural yielding and local buckling must be checked independently per AISC Manual Part 9.
Coped Beam Strength (AISC Manual Part 9)

The coped beam strength is verified in Example II.A-4.

Shear Yielding of Beam Web

\[ R_n = 0.60F_yA_{yw} \]  

\[ = 0.60(50.0 \text{ ksi})(0.355 \text{ in.})(16.0 \text{ in.}) \]  

\[ = 170 \text{ kips} \]  

From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(170 \text{ kips}) )</td>
<td>o.k.</td>
</tr>
<tr>
<td>( = 170 \text{ kips} &gt; 60.0 \text{ kips} )</td>
<td>( \frac{R_n}{\Omega} = 170 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{1.50}{\Omega} = 113 \text{ kips} &gt; 40.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Shear Rupture of Beam Web

\[ R_n = 0.60F_uA_{yw} \]  

\[ = 0.60(65.0 \text{ ksi})(0.355 \text{ in.})(16.0 \text{ in.}) \]  

\[ = 222 \text{ kips} \]  

From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(222 \text{ kips}) )</td>
<td>o.k.</td>
</tr>
<tr>
<td>( = 167 \text{ kips} &gt; 60.0 \text{ kips} )</td>
<td>( \frac{R_n}{\Omega} = 222 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{2.00}{\Omega} = 111 \text{ kips} &gt; 40.0 \text{ kips} )</td>
</tr>
</tbody>
</table>
EXAMPLE II.A-6  BEAM END COPED AT THE TOP FLANGE ONLY

Given:

For an ASTM A992 W21×62 coped 8 in. deep by 9 in. long at the top flange only, assuming e = 9½ in. and using an ASTM A36 plate:

A. Calculate the available strength of the beam end, considering the limit states of flexural yielding, local buckling, shear yielding and shear rupture.

B. Choose an alternate ASTM A992 W21 shape to eliminate the need for stiffening for an end reaction of \( R_D = 16.5 \) kips and \( R_L = 47 \) kips.

C. Determine the size of doubler plate needed to stiffen the W21×62 for the given end reaction in Solution B.

D. Determine the size of longitudinal stiffeners needed to stiffen the W2 for the given end reaction in Solution B.

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W21×62
ASTM A992
\( F_y = 50 \) ksi
\( F_u = 65 \) ksi

Plate
ASTM A36
\( F_y = 36 \) ksi
\( F_u = 58 \) ksi
From AISC Manual Tables 1-1 and 9-2 and AISC Manual Figure 9-2, the geometric properties are as follows:

Beam

W21×62

d = 21.0 in.

tw = 0.400 in.

bf = 8.24 in.

tw = 0.615 in.

Snelt = 17.8 in.3

c = 9.00 in.

dc = 8.00 in.

e = 9.50 in.

ho = 13.0 in.

Solution A:

Flexural Yielding and Local Web Buckling (AISC Manual Part 9)

Verify parameters.

c ≤ 2d

9.00 in. ≤ 2(21.0 in.)

≤ 42.0 in.  o.k.

dc ≤ d/2

8.00 in. ≤ 21.0 in.

≤ 10.5 in.  o.k.

\[ \frac{c}{d} = \frac{9.00 \text{ in.}}{21.0 \text{ in.}} = 0.429 \]

\[ \frac{c}{ho} = \frac{9.00 \text{ in.}}{13.0 \text{ in.}} = 0.692 \]

Because \( \frac{c}{d} \leq 1.0 \),

\[ f = 2 \left( \frac{c}{d} \right) \quad (\text{Manual Eq. 9-8}) \]

\[ = 2(0.429) = 0.858 \]

Because \( \frac{c}{ho} \leq 1.0 \),

\[ k = 2.2 \left( \frac{ho}{c} \right)^{1.65} \quad (\text{Manual Eq. 9-10}) \]

\[ = 2.2 \left( \frac{13.0 \text{ in.}}{9.00 \text{ in.}} \right)^{1.65} = 4.04 \]
For a top cope only, the critical buckling stress is:

\[
F_{cr} = 26,210 \left( \frac{t_w}{h_o} \right)^2 f_k \leq F_y
\]

\[
= 26,210 \left( \frac{0.400 \text{ in.}}{13.0 \text{ in.}} \right)^2 (0.858)(4.04) \leq F_y
\]

\[
= 86.0 \text{ ksi} \leq F_y
\]

Use \( F_{cr} = F_y = 50 \text{ ksi} \)

\[
R_n = \frac{F_{cr}S_{net}}{e} \text{ from AISC Manual Equation 9-6}
\]

\[
= \frac{50 \text{ ksi}(17.8 \text{ in.}^3)}{9.50 \text{ in.}}
\]

\[
= 93.7 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(93.7 \text{ kips}) )</td>
<td>( R_n ) = 93.7 kips</td>
</tr>
<tr>
<td>( = 84.3 \text{ kips} )</td>
<td>( \Omega ) = 1.67</td>
</tr>
<tr>
<td></td>
<td>( R_n \Omega = 56.1 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Shear Yielding of Beam Web**

\[
R_n = 0.60F_xA_{gy}
\]

\[
= 0.60(50 \text{ ksi})(0.400 \text{ in.})(13.0 \text{ in.})
\]

\[
= 156 \text{ kips}
\]

From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(156 \text{ kips}) )</td>
<td>( R_n ) = 156 kips</td>
</tr>
<tr>
<td>( = 156 \text{ kips} )</td>
<td>( \Omega ) = 1.50</td>
</tr>
<tr>
<td></td>
<td>( R_n \Omega = 104 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Shear Rupture of Beam Web**

\[
R_n = 0.60F_xA_{nv}
\]

\[
= 0.60(65 \text{ ksi})(0.400 \text{ in.})(13.0 \text{ in.})
\]

\[
= 203 \text{ kips}
\]
From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$ = 0.75</td>
<td></td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75(203 \text{ kips})$</td>
<td>$R_n = 203 \text{ kips}$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td></td>
<td>$= 152 \text{ kips}$</td>
<td>$= 102 \text{ kips}$</td>
</tr>
</tbody>
</table>

Thus, the available strength is controlled by local buckling.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 84.3 \text{ kips}$</td>
<td>$R_n = 56.1 \text{ kips}$</td>
<td></td>
</tr>
</tbody>
</table>

**Solution B:**

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(16.5 \text{ kips}) + 1.6(47 \text{ kips})$</td>
<td>$R_u = 16.5 \text{ kips} + 47 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 95.0 \text{ kips}$</td>
<td>$= 63.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

As determined in Solution A, the available critical stress due to local buckling for a W21×62 with an 8-in.-deep cope is limited to the yield stress.

*Required Section Modulus Based on Local Buckling*

From AISC Manual Equations 9-5 and 9-6:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{req} = \frac{R_u e}{\phi F_y}$</td>
<td>$S_{req} = \frac{R_u e \Omega}{F_y}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 95.0 \text{ kips}(9.50 \text{ in.})$</td>
<td>$= 63.5 \text{ kips}(9.50 \text{ in.})(1.67)$</td>
</tr>
<tr>
<td></td>
<td>$= 0.90(50.0 \text{ ksi})$</td>
<td>$= 50.0 \text{ ksi}$</td>
</tr>
<tr>
<td></td>
<td>$= 20.1 \text{ in.}^3$</td>
<td>$= 20.1 \text{ in.}^3$</td>
</tr>
</tbody>
</table>

Try a W21×73.

From AISC Manual Table 9-2:

$S_{net} = 21.0 \text{ in.}^3 > 20.1 \text{ in.}^3 \quad \text{o.k.}$

Note: By comparison to a W21×62, a W21×73 has sufficient shear strength.
Solution C:

**Doubler Plate Design (AISC Manual Part 9)**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doubler plate must provide a required strength of:</td>
<td>Doubler plate must provide a required strength of:</td>
</tr>
<tr>
<td>$95.0 \text{ kips} - 84.3 \text{ kips} = 10.7 \text{ kips}$</td>
<td>$63.5 \text{ kips} - 56.1 \text{ kips} = 7.40 \text{ kips}$</td>
</tr>
</tbody>
</table>

$$S_{req} = \frac{(R_u - \phi R_{a,\text{beam}})e}{\phi F_y} = \frac{(95.0 \text{ kips} - 84.3 \text{ kips})(9.50 \text{ in.})}{0.90(50 \text{ ksi})} = 2.26 \text{ in.}^3$$

For an 8-in.-deep plate,

$$t_{req} = \frac{6S_{req}}{d^2} = \frac{6(2.26 \text{ in.}^3)}{(8.00 \text{ in.})^2} = 0.212 \text{ in.}$$

Note: ASTM A572 Grade 50 plate is recommended in order to match the beam yield strength.

Thus, since the doubler plate must extend at least $d_c$ beyond the cope, use a PL $\frac{3}{4}$ in.$\times$8 in.$\times$1 ft in. with $\frac{3}{16}$-in. welds top and bottom.

Solution D:

**Longitudinal Stiffener Design**

Try PL $\frac{3}{4}$ in.$\times$4 in. slotted to fit over the beam web with $F_y = 50 \text{ ksi}$.

From section property calculations for the neutral axis and moment of inertia, conservatively ignoring the beam fillets, the neutral axis is located 4.39 in. from the bottom flange (8.86 in. from the top of the stiffener).

<table>
<thead>
<tr>
<th></th>
<th>$I_o$ (in.$^4$)</th>
<th>$Ad^2$ (in.$^4$)</th>
<th>$I_o + Ad^2$ (in.$^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffener</td>
<td>0.00521</td>
<td>76.3</td>
<td>76.3</td>
</tr>
<tr>
<td>W21×62 web</td>
<td>63.3</td>
<td>28.9</td>
<td>92.2</td>
</tr>
<tr>
<td>W21×62 bottom flange</td>
<td>0.160</td>
<td>84.5</td>
<td>84.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Sigma = I_o + Ad^2 = 253 \text{ in.}^4$</td>
<td></td>
</tr>
</tbody>
</table>

**Slenderness of the Longitudinal Stiffener**

$$\lambda_c = 0.95\sqrt{\frac{k_c}{E \cdot F_t}}$$ from AISC Specification Table B4.1b Case 11
\[ k_c = \frac{4}{\sqrt{h/t_w}} \text{ where } 0.35 \leq k_c \leq 0.76 \]
\[ = \frac{4}{\sqrt{11.9 \text{ in.}/0.400 \text{ in.}}} \]
\[ = 0.733 \]

use \( k_c = 0.733 \)

\[ S_{xc} = \frac{I_x}{c} \]
\[ = \frac{253 \text{ in.}^4}{8.86 \text{ in.}} \]
\[ = 28.6 \text{ in.}^3 \]

\[ S_{xt} = \frac{253 \text{ in.}^4}{4.39 \text{ in.}} \]
\[ = 57.6 \text{ in.}^3 \]

\[ S_{xt} = \frac{57.6 \text{ in.}^3}{28.6 \text{ in.}^3} \]
\[ = 2.01 \geq 0.7, \text{ therefore,} \]

\[ F_e = 0.7F_y \]
\[ = 0.7(50 \text{ ksi}) \]
\[ = 35.0 \text{ ksi} \]

\[ \lambda_e = 0.95\sqrt{0.733(29,000 \text{ ksi})/35.0 \text{ ksi}} \]
\[ = 23.4 \]

\[ b = 4.00 \text{ in.} \]
\[ t = 2(\frac{3}{4} \text{ in.}) \]
\[ = 8.00 < 23.4, \text{ therefore, the stiffener is not slender} \]

\[ S_{net} = S_{xc} \]

The nominal strength of the reinforced section using AISC Manual Equation 9-6 is:

\[ R_n = \frac{F_e S_{net}}{e} \]
\[ = \frac{50 \text{ ksi}(28.6 \text{ in.}^3)}{9.50 \text{ in.}} \]
\[ = 151 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(151 \text{ kips}) )</td>
<td></td>
</tr>
</tbody>
</table>
= 136 kips > 95.0 kips\hline
\( R_n \)
\hline
\( \Omega \)
\hline
\( 151 \text{ kips} \)
\hline
\( 1.67 \)
\hline
\( = 90.4 \text{ kips} > 63.5 \text{ kips} \)
\hline
\( \text{o.k.} \)
\hline
\( \text{Note: ASTM A572 Grade 50 plate is recommended in order to match the beam yield strength.} \)

\textit{Plate Dimensions}

Since the longitudinal stiffening must extend at least \( d_c \) beyond the cope, use PL\( \frac{1}{4} \text{ in.} \times 4 \text{ in.} \times 1 \text{ ft 5 in.} \) with \( \frac{1}{4} \text{-in.} \) welds.
EXAMPLE II.A-7  BEAM END COPED AT THE TOP AND BOTTOM FLANGES

Given:

For an ASTM A992 W16×40 coped 3½ in. deep by 9½ in. wide at the top flange and 2 in. deep by 11½ in. wide at the bottom flange calculate the available strength of the beam end, considering the limit states of flexural yielding and local buckling. Assume a ½-in. setback from the face of the support to the end of the beam.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
\( F_y = 50 \) ksi
\( F_u = 65 \) ksi

From AISC Manual Table 1-1 and AISC Manual Figure 9-3, the geometric properties are as follows:

\[
\begin{align*}
  d & = 16.0 \text{ in.} \\
  t_w & = 0.305 \text{ in.} \\
  t_f & = 0.505 \text{ in.} \\
  b_f & = 7.00 \text{ in.} \\
  c_t & = 9.50 \text{ in.} \\
  d_{ct} & = 3.50 \text{ in.} \\
  c_b & = 11.5 \text{ in.} \\
  d_{cb} & = 2.00 \text{ in.} \\
  e_b & = 11.5 \text{ in.} + 0.50 \text{ in.} \\
    & = 12.0 \text{ in.} \\
  e_t & = 9.50 \text{ in.} + 0.50 \text{ in.} \\
    & = 10.0 \text{ in.} \\
  h_o & = 16.0 \text{ in.} - 2.00 \text{ in.} - 3.50 \text{ in.} \\
    & = 10.5 \text{ in.}
\end{align*}
\]

Local Buckling at the Compression (Top) Flange Cope

Because the bottom cope (tension) is longer than the top cope (compression) and \( d_c > 0.2d \), the available buckling stress is calculated using AISC Manual Equation 9-14.
\[ \lambda = \frac{h_o \sqrt{F_y}}{10 h w \sqrt{475 + 280 \left( \frac{h_o}{c_t} \right)^2}} \]  
\[ = \frac{10.5 \text{ in.} \sqrt{50 \text{ ksi}}}{10(0.305 \text{ in.}) \sqrt{475 + 280 \left( \frac{10.5 \text{ in.}}{9.50 \text{ in.}} \right)^2}} \]  
\[ = 0.852 \]  

Because, 0.7 < \lambda ≤ 1.41:

\[ Q = 1.34 - 0.486 \lambda \]  
\[ = 1.34 - 0.486(0.852) \]  
\[ = 0.926 \]  

Available Buckling Stress

\[ F_{cr} = F_y Q \]  
\[ = 50 \text{ ksi}(0.926) \]  
\[ = 46.3 \text{ ksi} < 50 \text{ ksi} \]  
(buckling controls)

Determine the net elastic section modulus:

\[ S_{net} = \frac{t_w h_o^2}{6} \]  
\[ = \frac{(0.305 \text{ in.})(10.5 \text{ in.}^2)^2}{6} \]  
\[ = 5.60 \text{ in.}^3 \]  

The strength based on flexural local buckling is determined as follows:

\[ M_n = F_{cr} S_{net} \]  
\[ = 46.3 \text{ ksi}(5.60 \text{ in.}^3) \]  
\[ = 259 \text{ kip-in.} \]  

\[ R_n = \frac{M_n}{c_t} \]  
\[ = 259 \text{ kip-in.} \]  
\[ = 10.0 \text{ in.} \]  
\[ = 25.9 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b R_n = 0.90(25.9 \text{ kips}) )</td>
<td>( R_n = 25.9 \text{ kips} )</td>
</tr>
<tr>
<td>= 23.3 kips</td>
<td>( \frac{R_n}{\Omega_b} = \frac{25.9 \text{ kips}}{1.67} )</td>
</tr>
</tbody>
</table>
| = 15.5 kips |}

Check flexural yielding of the tension (bottom) flange cope.
From AISC *Manual* Table 9-2 the elastic section modulus of the remaining section is $S_{net} = 15.6\text{ in.}^3$.

The strength based on flexural yielding is determined as follows:

$$M_n = F_y S_{net}$$
$$= 50 \text{ ksi}(15.6\text{ in.}^3)$$
$$= 780 \text{ kip-in.}$$

$$R_a = \frac{M_n}{e_b}$$
$$= \frac{780 \text{ kip-in.}}{12.0 \text{ in.}}$$
$$= 65.0 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_b R_a = 0.90(65.0 \text{ kips})$</td>
<td>$\frac{R_a}{\Omega_b} = \frac{65.0 \text{ kips}}{1.67} = 38.9 \text{ kips}$</td>
</tr>
</tbody>
</table>

Thus, the available strength is controlled by local buckling in the top (compression) cope of the beam.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_b R_a = 23.3 \text{ kips}$</td>
<td>$\frac{R_a}{\Omega_b} = \frac{15.5 \text{ kips}}{1.67} = 9.3 \text{ kips}$</td>
</tr>
</tbody>
</table>
EXAMPLE IIA-8  ALL-BOLTED DOUBLE-ANGLE CONNECTIONS (BEAMS-TO-GIRDER WEB)

Given:

Design the all-bolted double-angle connections between the ASTM A992 W12×40 beam (A) and ASTM A992 W21×50 beam (B) and the ASTM A992 W30×99 girder-web to support the following beam end reactions:

<table>
<thead>
<tr>
<th>Beam A</th>
<th>Beam B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{DA} = 4.17$ kips</td>
<td>$R_{DB} = 18.3$ kips</td>
</tr>
<tr>
<td>$R_{LA} = 12.5$ kips</td>
<td>$R_{LB} = 55.0$ kips</td>
</tr>
</tbody>
</table>

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and assume $e = 5.50$ in. Use ASTM A36 angles.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam A
- W12×40
- ASTM A992
- $F_y = 50$ ksi
- $F_u = 65$ ksi

Beam B
- W21×50
- ASTM A992
- $F_y = 50$ ksi
- $F_u = 65$ ksi
Girder
W30×99
ASTM A992
\(F_y = 50\) ksi
\(F_u = 65\) ksi

Angle
ASTM A36
\(F_y = 36\) ksi
\(F_u = 58\) ksi

From AISC Manual Tables 1-1 and 9-2, the geometric properties are as follows:

**Beam A**
W12×40
\(t_w = 0.295\) in.
\(d = 11.9\) in.
\(h_o = 9.90\) in.
\(S_{net} = 8.03\) in.\(^3\)
\(d_c = 2.00\) in.
\(c = 5.00\) in.
\(e = 5.50\) in.

**Beam B**
W21×50
\(t_w = 0.380\) in.
\(d = 20.8\) in.
\(h_o = 18.8\) in.
\(S_{net} = 32.5\) in.\(^3\)
\(d_c = 2.00\) in.
\(c = 5.00\) in.
\(e = 5.50\) in.

Girder
W30×99
\(t_w = 0.520\) in.
\(d = 29.7\) in.

**Beam A:**

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_{da} = 1.2(4.17) kips (+ 1.6(12.5) kips) (= 25.0) kips</td>
<td>(R_{da} = 4.17) kips (+ 12.5) kips (= 16.7) kips</td>
</tr>
</tbody>
</table>
Bolt Shear and Bolt Bearing, Shear Yielding, Shear Rupture and Block Shear Rupture of Angles

From AISC Manual Table 10-1, for two rows of bolts and ¼-in. angle thickness:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n ) = 48.9 kips &gt; 25.0 kips</td>
<td>o.k.</td>
<td>( \frac{R_e}{\Omega} ) = 32.6 kips &gt; 16.7 kips</td>
</tr>
</tbody>
</table>

Bolt Bearing and Block Shear Rupture of Beam Web

From AISC Manual Table 10-1, for two rows of bolts and \( L_{cv} = 1\frac{1}{4} \text{ in.} \) and \( L_{ch} = 1\frac{1}{2} \text{ in.} \):

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n ) = 126 kips/in.(0.295 in.) = 37.2 kips &gt; 25.0 kips</td>
<td>o.k.</td>
<td>( \frac{R_e}{\Omega} ) = 83.7 kips/in.(0.295 in.) = 24.7 kips &gt; 16.7 kips</td>
</tr>
</tbody>
</table>

Coped Beam Strength (AISC Manual Part 9)

Flexural Yielding and Local Web Buckling

Verify parameters.

\[
c \leq 2d \\
5.00 \text{ in.} \leq 2(11.9 \text{ in.}) \\
\leq 23.8 \text{ in.} \quad \text{ o.k.}
\]

\[
d_c \leq \frac{d}{2} \\
2.00 \text{ in.} \leq \frac{11.9 \text{ in.}}{2} \\
\leq 5.95 \text{ in.} \quad \text{ o.k.}
\]

\[
c = \frac{5.00 \text{ in.}}{11.9 \text{ in.}} = 0.420 \leq 1.0
\]

\[
c = \frac{5.00 \text{ in.}}{9.90 \text{ in.}} = 0.505 \leq 1.0
\]

Because \( \frac{c}{d} \leq 1.0 \), the plate buckling model adjustment factor:

\[
f = 2 \left( \frac{c}{d} \right) \\
= 2(0.420) \\
= 0.840
\]

Because \( \frac{c}{h_0} \leq 1.0 \), the plate buckling coefficient is:
\[ k = 2.2 \left( \frac{h_o}{c} \right)^{1.65} \]  
\[ = 2.2 \left( \frac{9.90 \text{ in.}}{5.00 \text{ in.}} \right)^{1.65} \]
\[ = 6.79 \]

For top cope only, the critical buckling stress is:
\[ F_{cr} = 26,210 \left( \frac{t_w}{h_o} \right)^2 f k \leq F_y \]  
\[ = 26,210 \left( \frac{0.295 \text{ in.}}{9.90 \text{ in.}} \right)^2 (0.840)(6.79) \]
\[ = 133 \text{ ksi} \leq F_y \]

Use \( F_{cr} = F_y = 50 \text{ ksi.} \)

From AISC Manual Equation 9-6:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \phi R_n = \frac{\phi F_{cr} S_{net}}{e} )</td>
<td>( R_n = \frac{F_{cr} S_{net}}{\Omega e} )</td>
</tr>
<tr>
<td></td>
<td>( = 0.90(50 \text{ ksi})(8.03 \text{ in.}^3) )</td>
<td>( = 50 \text{ ksi}(8.03 \text{ in.}^3) )</td>
</tr>
<tr>
<td></td>
<td>( = 65.7 \text{ kips} &gt; 25.0 \text{ kips} )</td>
<td>( = 43.7 \text{ kips} &gt; 16.7 \text{ kips} )</td>
</tr>
</tbody>
</table>

Shear Yielding of Beam Web
\[ R_n = 0.60 F_y A_{gv} \]  
\[ = 0.60(50 \text{ ksi})(0.295 \text{ in.})(9.90 \text{ in.}) \]
\[ = 87.6 \text{ kips} \]

From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \phi R_n = \frac{\phi F_{cr} S_{net}}{e} )</td>
<td>( R_n = \frac{F_{cr} S_{net}}{\Omega e} )</td>
</tr>
<tr>
<td></td>
<td>( = 87.6 \text{ kips} )</td>
<td>( = 87.6 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( = 87.6 \text{ kips} &gt; 25.0 \text{ kips} )</td>
<td>( = 58.4 \text{ kips} &gt; 16.7 \text{ kips} )</td>
</tr>
</tbody>
</table>

Shear Rupture of Beam Web
\[ A_{nv} = t_w \left[ h_o - 2(\frac{1}{12} \text{ in.} + \frac{1}{16} \text{ in.}) \right] \]
\[ = 0.295 \text{ in.}(9.90 \text{ in.} - 1.75 \text{ in.}) \]
\[ = 2.40 \text{ in.}^2 \]
\[ R_n = 0.60 F_{cr} A_{nv} \]  
\[ = 0.60(65 \text{ ksi})(2.40 \text{ in.}^2) \]
From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_e = 0.75(93.6 \text{ kips})$</td>
<td>$R_e = 93.6 \text{ kips} \over \Omega = 2.00$</td>
</tr>
<tr>
<td>= 70.2 kips &gt; 25.0 kips</td>
<td>= 46.8 kips &gt; 16.7 kips</td>
</tr>
<tr>
<td><strong>o.k.</strong></td>
<td><strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Beam B:**

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{Bu} = 1.2(18.3 \text{ kips}) + 1.6(55.0 \text{ kips})$</td>
<td>$R_{Bu} = 18.3 \text{ kips} + 55.0 \text{ kips}$</td>
</tr>
<tr>
<td>= 110 kips</td>
<td>= 73.3 kips</td>
</tr>
<tr>
<td><strong>o.k.</strong></td>
<td><strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Bolt Shear and Bolt Bearing, Shear Yielding, Shear Rupture and Block Shear Rupture of Angles**

From AISC Manual Table 10-1, for five rows of bolts and $\frac{1}{4}$-in. angle thickness:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_e = 125 \text{ kips} &gt; 110 \text{ kips}$</td>
<td>$R_e = 83.3 \text{ kips} &gt; 73.3 \text{ kips}$</td>
</tr>
<tr>
<td><strong>o.k.</strong></td>
<td><strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Bolt Bearing and Block Shear Rupture of Beam Web**

From AISC Manual Table 10-1, for five rows of bolts and $L_v = 1\frac{1}{4}$ in. and $L_{vb} = 1\frac{1}{2}$ in.:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_e = 312 \text{ kips/in.}(0.380 \text{ in.})$</td>
<td>$R_e = 208 \text{ kips/in.}(0.380 \text{ in.})$</td>
</tr>
<tr>
<td>= 119 kips &gt; 110 kips</td>
<td>= 79.0 kips &gt; 73.3 kips</td>
</tr>
<tr>
<td><strong>o.k.</strong></td>
<td><strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Coped Beam Strength (AISC Manual Part 9)**

Flexural Yielding and Local Web Buckling

Verify parameters.

\[ c \leq 2d \]
5.00 in. \leq 2(20.8 in.)
\leq 41.6 in.  \text{ o.k.}

\[ d_c \leq \frac{d}{2} \]
2.00 in. \leq \frac{20.8 \text{ in.}}{2}
\leq 10.4 in.  \text{ o.k.}
\[
c = \frac{5.00 \text{ in.}}{20.8 \text{ in.}} = 0.240 \leq 1.0
\]

\[
c = \frac{5.00 \text{ in.}}{18.8 \text{ in.}} = 0.266 \leq 1.0
\]

Because \( \frac{c}{d} \leq 1.0 \), the plate buckling model adjustment factor is:

\[
f = \frac{2c}{d} = 2(0.240) = 0.480
\]

Because \( \frac{c}{h_o} \leq 1.0 \), the plate buckling coefficient is:

\[
k = 2.2\left(\frac{h_o}{c}\right)^{1.65} = 2.2\left(\frac{18.8 \text{ in.}}{5.00 \text{ in.}}\right)^{1.65} = 19.6
\]

\[
F_{cr} = 26,210\left(\frac{t_w}{h_o}\right)^2 f k \leq F_y
\]

\[
= 26,210\left(\frac{0.380 \text{ in.}}{18.8 \text{ in.}}\right)^2 (0.480)(19.6) = 101 \text{ ksi} \leq F_y
\]

Use \( F_{cr} = F_y = 50 \text{ ksi} \).

From AISC Manual Equation 9-6:

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>1.67</td>
</tr>
</tbody>
</table>

\[
\phi R_n = \frac{\phi F_{cr} S_{net}}{e} = \frac{0.90(50 \text{ ksi})(32.5 \text{ in.}^3)}{5.50 \text{ in.}} = 266 \text{ kips} > 110 \text{ kips}
\]

\( \text{o.k.} \)

\[
\frac{R_n}{\Omega} = \frac{F_{cr} S_{net}}{\Omega e} = \frac{50 \text{ ksi}(32.5 \text{ in.}^3)}{1.67(5.50 \text{ in.})} = 177 \text{ kips} > 73.3 \text{ kips}
\]

\( \text{o.k.} \)

**Shear Yielding of Beam Web**
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

\[
R_n = 0.60F_u A_{sv}
\]
\[
= 0.60(50 \text{ ksi})(0.380 \text{ in.})(18.8 \text{ in.})
\]
\[
= 214 \text{ kips}
\]

From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 1.00)</td>
<td>(\Omega = 1.50)</td>
</tr>
<tr>
<td>(\phi R_n = 1.00(214 \text{ kips}))</td>
<td>(\frac{R_n}{\Omega} = 143 \text{ kips} &gt; 73.3 \text{ kips} \quad \text{o.k.})</td>
</tr>
<tr>
<td>= 214 kips &gt; 110 kips o.k.</td>
<td></td>
</tr>
</tbody>
</table>

Shear Rupture of Beam Web

\[
A_{sv} = t_w [h_o - (5)(\frac{1}{36} \text{ in.} + \frac{1}{60} \text{ in.})]
\]
\[
= 0.380 \text{ in.}(18.8 \text{ in.} - 4.38 \text{ in.})
\]
\[
= 5.48 \text{ in.}^2
\]

\[
R_n = 0.60F_u A_{sv}
\]
\[
= 0.60(65 \text{ ksi})(5.48 \text{ in.}^2)
\]
\[
= 214 \text{ kips}
\]

From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.75)</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>(\phi R_n = 0.75(214 \text{ kips}))</td>
<td>(\frac{R_n}{\Omega} = 107 \text{ kips} &gt; 73.3 \text{ kips} \quad \text{o.k.})</td>
</tr>
<tr>
<td>= 161 kips &gt; 110 kips o.k.</td>
<td></td>
</tr>
</tbody>
</table>

Supporting Girder

Supporting Girder Web

The required bearing strength per bolt is greatest for the bolts that are loaded by both connections. Thus, for the design of these four critical bolts, the required strength is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Beam A, each bolt must support one-fourth of 25.0 kips or 6.25 kips/bolt.</td>
<td>From Beam A, each bolt must support one-fourth of 16.7 kips or 4.18 kips/bolt.</td>
</tr>
</tbody>
</table>

From Beam B, each bolt must support one-tenth of 110 kips or 11.0 kips/bolt.  

Thus,

\[
R_n = 6.25 \text{ kips/bolt} + 11.0 \text{ kips/bolt}
\]
\[
= 17.3 \text{ kips/bolt}
\]

From AISC Manual Table 7-4, the allowable bearing strength per bolt is:
The tabulated values may be verified by hand calculations, as follows:

From AISC *Specification* Equation J3-6a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_u = \phi 1.2 l_c F_u \leq \phi 2.4 d t F_u )</td>
<td>( r_n = \frac{1.2 l_c F_u}{\Omega} \leq \frac{2.4 d t F_u}{\Omega} )</td>
</tr>
<tr>
<td>( l_c = 3.00 \text{ in.} - \frac{1}{16} \text{ in.} )</td>
<td>( l_c = 3.00 \text{ in.} - \frac{1}{16} \text{ in.} )</td>
</tr>
<tr>
<td>( = 2.19 \text{ in.} )</td>
<td>( = 2.19 \text{ in.} )</td>
</tr>
<tr>
<td>( \phi 1.2 l_c F_u = 0.75 (1.2) (2.19 \text{ in.}) (0.520 \text{ in.}) (65 \text{ ksi}) )</td>
<td>( \frac{1.2 l_c F_u}{\Omega} = \frac{1.2 (2.19 \text{ in.}) (0.520 \text{ in.}) (65 \text{ ksi})}{2.00} )</td>
</tr>
<tr>
<td>( = 66.6 \text{ kips} )</td>
<td>( = 44.4 \text{ kips} )</td>
</tr>
<tr>
<td>( \phi (2.4 d t F_u) = 0.75 (2.4) (0.750 \text{ in.}) (0.520 \text{ in.}) (65 \text{ ksi}) )</td>
<td>( \frac{2.4 d t F_u}{\Omega} = \frac{2.4 (0.750 \text{ in.}) (0.520 \text{ in.}) (65 \text{ ksi})}{2.00} )</td>
</tr>
<tr>
<td>( = 45.6 \text{ kips} &lt; 66.6 \text{ kips} )</td>
<td>( = 30.4 \text{ kips} &lt; 44.4 \text{ kips} )</td>
</tr>
<tr>
<td>( \phi r_u = 45.6 \text{ kips/bolt} &gt; 17.3 \text{ kips/bolt} )</td>
<td>( r_n = \frac{30.4 \text{ kips/bolt}}{\Omega} &gt; 11.5 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>

\( \frac{r_n}{\Omega} = 58.5 \text{ kips/in.} (0.520 \text{ in.}) \)
\( = 30.4 \text{ kips/bolt} > 11.5 \text{ kips/bolt} \) o.k.
EXAMPLE II.A-9 OFFSET ALL-BOLTED DOUBLE-ANGLE CONNECTIONS (BEAMS-TO-GIRDER WEB)

Given:

Two all-bolted double-angle connections are made back-to-back with offset beams. Design the connections to accommodate an offset of 6 in. Use an ASTM A992 beam, and ASTM A992 beam and ASTM A36 angles.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Girder
W18×50
ASTM A992

F_y = 50 ksi
F_u = 65 ksi
Beam
W16×45
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Angles
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Girder
W18×50
$tw = 0.355$ in.
$d = 18.0$ in.

Beam
W16×45
$tw = 0.345$ in.
$d = 16.1$ in.

Modify the 2L4×3½×¼ SLBB connection designed in Example II.A-4 to work in the configuration shown in the preceding figure. The offset dimension (6 in.) is approximately equal to the gage on the support from the previous example (6½ in.) and, therefore, is not recalculated.

Thus, the bearing strength of the middle vertical row of bolts (through both connections), which carries a portion of the reaction for both connections, must be verified for this new configuration.

For each beam,

$R_D = 10$ kips
$R_L = 30$ kips

From Chapter 2 of ASCE 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(10 \text{kips}) + 1.6(30 \text{kips})$</td>
<td>$R_u = 10 \text{kips} + 30 \text{kips}$</td>
</tr>
<tr>
<td>60.0 kips</td>
<td>40.0 kips</td>
</tr>
</tbody>
</table>

*Bolt Shear*

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_u = 60.0 \text{kips}$</td>
<td>$r_u = 40.0 \text{kips}$</td>
</tr>
<tr>
<td>$6 \text{ bolts}$</td>
<td>$6 \text{ bolts}$</td>
</tr>
<tr>
<td>$10.0 \text{kips/bolt}$</td>
<td>$6.67 \text{kips/bolt}$</td>
</tr>
</tbody>
</table>

From AISC *Manual* Table 7-1, the available shear strength of a single bolt in double shear is:

17.9 kips/bolt > 10.0 kips/bolt  **o.k.**

11.9 kips/bolt > 6.67 kips/bolt  **o.k.**
**Supporting Girder Web**

At the middle vertical row of bolts, the required bearing strength for one bolt is the sum of the required shear strength per bolt for each connection. The available bearing strength per bolt is determined from AISC Manual Table 7-4.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_u = 2(10.0 \text{ kips/bolt})$</td>
<td>$r_a = 2(6.67 \text{ kips/bolt})$</td>
<td>o.k.</td>
</tr>
<tr>
<td>= 20.0 kips/bolt</td>
<td>= 13.3 kips/bolt</td>
<td></td>
</tr>
<tr>
<td>$\phi r_n = 87.8 \text{ kips/in}(0.355 \text{ in.})$</td>
<td>$\frac{r_n}{\Omega} = 58.5 \text{ kips/in.}(0.355 \text{ in.})$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$= 31.2 \text{ kips/bolt} &gt; 20.0 \text{ kips/bolt}$</td>
<td>$= 20.8 \text{ kips/bolt} &gt; 13.3 \text{ kips/bolt}$</td>
<td></td>
</tr>
</tbody>
</table>

Note: If the bolts are not spaced equally from the supported beam web, the force in each column of bolts should be determined by using a simple beam analogy between the bolts, and applying the laws of statics.
EXAMPLE II.A-10  SKEWED DOUBLE BENT-PLATE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Design the skewed double bent-plate connection between an ASTM A992 W16×77 beam and ASTM A992 W27×94 girder-web to support the following beam end reactions:

\[ R_D = 13.3 \text{ kips} \]
\[ R_L = 40.0 \text{ kips} \]

Use 3⁄8-in.-diameter ASTM A325-N or F1852-N bolts in standard holes through the support and ASTM A36 plates. Use 70-ksi electrode welds to the supported beam.

Solution:

Fig. II.A-10. Skewed Double Bent-Plate Connection (Beam-to-Girder Web)
From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×77
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Girder
W27×94
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W16×77
\( t_w = 0.455 \text{ in.} \)
\( d = 16.5 \text{ in.} \)

Girder
W27×94
\( t_w = 0.490 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2 (13.3 \text{ kips}) + 1.6 (40.0 \text{ kips}) ) = 80.0 kips</td>
<td>( R_u = 13.3 \text{ kips} + 40.0 \text{ kips} ) = 53.3 kips</td>
</tr>
</tbody>
</table>

Using figure (c) of the connection, assign load to each vertical row of bolts by assuming a simple beam analogy between bolts and applying the laws of statics.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| Required strength for bent plate A:
\( R_u = \frac{80.0 \text{ kips (2\frac{1}{4} \text{ in.})}}{6.00 \text{ in.}} \) = 30.0 kips | Required strength for bent plate A:
\( R_u = \frac{53.3 \text{ kips (2\frac{1}{4} \text{ in.})}}{6.00 \text{ in.}} \) = 20.0 kips |
| Required strength for bent plate B:
\( R_u = 80.0 \text{ kips} - 30.0 \text{ kips} \) = 50.0 kips | Required strength for bent plate B:
\( R_u = 53.3 \text{ kips} - 20.0 \text{ kips} \) = 33.3 kips |

Assume that the welds across the top and bottom of the plates will be 2½ in. long, and that the load acts at the intersection of the beam centerline and the support face.
While the welds do not coincide on opposite faces of the beam web and the weld groups are offset, the locations of the weld groups will be averaged and considered identical. See figure (d).

**Weld Design**

Assume a plate length of 8½ in.

\[ k = \frac{kl}{l} = \frac{2\frac{1}{2} \text{ in.}}{8\frac{1}{2} \text{ in.}} = 0.294 \]

\[ xl = \frac{2\frac{1}{2} \text{ in.}(1\frac{1}{4} \text{ in.})(2)}{2\frac{1}{2} \text{ in.}(2) + 8\frac{1}{2} \text{ in.}} = 0.463 \text{ in.} \]

\[ a = \frac{(al + xl) - xl}{l} = \frac{3\frac{3}{4} \text{ in} - 0.463 \text{ in.}}{8.50 \text{ in.}} = 0.373 \]

Interpolating from AISC Manual Table 8-8, with \( \theta = 0^\circ, a = 0.373, \) and \( k = 0.294, \)

\[ C = 2.52 \]

The required weld size for two such welds using AISC Manual Equation 8-13 is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( D_{req} = \frac{R_u}{\phi CC/l} )</td>
<td>( D_{req} = \frac{\Omega R_u}{CC/l} )</td>
</tr>
<tr>
<td>( = \frac{50.0 \text{ kips}}{0.75(2.52)(1.0)(8\frac{1}{2} \text{ in.})} = 3.11 \rightarrow 4 \text{ sixteenths} )</td>
<td>( = \frac{2.00(33.3 \text{ kips})}{2.52(1.0)(8\frac{1}{2} \text{ in.})} = 3.11 \rightarrow 4 \text{ sixteenths} )</td>
</tr>
</tbody>
</table>

Use ¼-in. fillet welds and at least ⅛-in.-thick bent plates to allow for the welds.

**Beam Web Thickness**

According to Part 9 of the AISC Manual, with \( F_{EXX} = 70 \text{ ksi} \) on both sides of the connection, the minimum thickness required to match the available shear rupture strength of the connection element to the available shear rupture strength of the base metal is:

\[ t_{min} = \frac{6.19D}{F_u} \]

\[ = \frac{6.19(3.11 \text{ sixteenths})}{65 \text{ ksi}} = 0.296 \text{ in.} < 0.455 \text{ in.} \text{ o.k.} \]
Bolt Shear

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum shear to one bent plate = 50.0 kips</td>
<td>Maximum shear to one bent plate = 33.3 kips</td>
<td>Try 3 rows of ¾-in.-diameter ASTM A325-N bolts.</td>
</tr>
<tr>
<td></td>
<td>Try 3 rows of ¾-in.-diameter ASTM A325-N bolts.</td>
<td>From AISC Manual Table 7-1:</td>
</tr>
<tr>
<td></td>
<td>From AISC Manual Table 7-1:</td>
<td>[ \phi R_n = n \left( \phi r_n \right) ]</td>
</tr>
<tr>
<td></td>
<td>[ \phi R_n = n \left( \phi r_n \right) ]</td>
<td>[ R_n = n \left( \frac{r_n}{\Omega} \right) ]</td>
</tr>
<tr>
<td></td>
<td>= 3 bolts(24.3 kips/bolt)</td>
<td>= 3 bolts(16.2 kips/bolt)</td>
</tr>
<tr>
<td></td>
<td>= 72.9 kips &gt; 50.0 kips</td>
<td>= 48.6 kips &gt; 33.3 kips</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Bearing on Support

From AISC Manual Table 7-4 with 3-in. spacing in standard holes:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi r_{ni} = 102 \text{kips/in.}(0.490 \text{ in.})(3 \text{ bolts}) ]</td>
<td>[ R_n = \frac{68.3 \text{kips/in.}(0.490 \text{ in.})(3 \text{ bolts})}{\Omega} ]</td>
<td>o.k.</td>
</tr>
<tr>
<td>= 150 kips &gt; 50.0 kips</td>
<td>= 100 kips &gt; 33.3 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Bent Plate Design

Try a ¼ in plate.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing on plate from AISC Manual Tables 7-4 and 7-5:</td>
<td>Bearing on plate from AISC Manual Tables 7-4 and 7-5:</td>
<td>[ r_{si} = 60.9 \text{kips/in.} ]</td>
</tr>
<tr>
<td>[ \phi r_{si} = 91.4 \text{kips/in.} ]</td>
<td>[ r_{so} = 27.2 \text{kips/in.} ]</td>
<td>[ R_n = \frac{60.9 \text{kips/in.}(0.490 \text{ in.})(2 \text{ bolts})}{\Omega} + \frac{27.2 \text{kips/in.}(1 \text{ bolt})}{\Omega} ]</td>
</tr>
<tr>
<td>[ \phi r_{so} = 40.8 \text{kips/in.} ]</td>
<td>= 69.9 kips &gt; 50.0 kips</td>
<td>= 46.6 kips &gt; 33.3 kips</td>
</tr>
<tr>
<td>Shear yielding of plate using AISC Specification Equation J4-3:</td>
<td>o.k.</td>
<td>o.k.</td>
</tr>
<tr>
<td>[ \phi = 1.00 ]</td>
<td>[ \phi = 1.50 ]</td>
<td>[ \Omega = 1.50 ]</td>
</tr>
<tr>
<td>[ \phi R_n = \phi 0.60 F_y A_{ev} ]</td>
<td>[ R_n = \frac{0.60 F_y A_{ev}}{\Omega} ]</td>
<td>= [ 0.60(36 \text{ ksi})(\frac{8}{32} \text{ in.})(\frac{1}{4} \text{ in.}) ]</td>
</tr>
<tr>
<td>= 1.00(0.60)(36 ksi)(8/32 in.)(1/4 in.)</td>
<td>= 38.3 kips &gt; 33.3 kips</td>
<td>= 38.3 kips &gt; 33.3 kips</td>
</tr>
<tr>
<td>= 57.4 kips &gt; 50.0 kips</td>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
### LRFD

Shear rupture of plate using AISC *Specification* Equation J4-4:

\[ A_{nv} = \left[ 8\frac{1}{2} \text{ in.} - 3(1.00 \text{ in.}) \right] (\% \text{ in.}) \]
\[ = 1.72 \text{ in.}^2 \]

\[ \phi = 0.75 \]

\[ \phi R_n = \phi 0.60 F_u A_{nv} \]
\[ = 0.75(0.60)(58 \text{ ksi})(1.72 \text{ in.}^2) \]
\[ = 44.9 \text{ kips} < 50.0 \text{ kips} \]

**n.g.**

Increase the plate thickness to \( \frac{3}{8} \) in.

\[ A_{nv} = \left[ 8\frac{1}{2} \text{ in.} - 3(1.00 \text{ in.}) \right] (\% \text{ in.}) \]
\[ = 2.06 \text{ in.}^2 \]

\[ \phi = 0.75 \]

\[ \phi R_n = 0.75(0.60)(58 \text{ ksi})(2.06 \text{ in.}^2) \]
\[ = 53.8 \text{ kips} > 50.0 \text{ kips} \]

**o.k.**

Block shear rupture of plate using AISC *Specification* Equation J4-5 with \( n = 3, L_{ev} = L_{eh} = \frac{1}{4} \text{ in.}, U_{bs} = 1 \):

\[ \phi R_n = \phi U_{bs} F_u A_{nt} + \min \left( \phi 0.60 F_y A_{gyv}, \phi 0.60 F_u A_{nv} \right) \]

Tension rupture component from AISC *Manual* Table 9-3a:

\[ \phi U_{bs} F_u A_{nt} = (1.0)(32.6 \text{ kips/in.}) \left( \frac{3}{8} \text{ in.} \right) \]

Shear yielding component from AISC *Manual* Table 9-3b:

\[ \phi 0.60 F_y A_{gyv} = 117 \text{ kips/in.} \left( \frac{3}{8} \text{ in.} \right) \]

Shear rupture component from AISC *Manual* Table 9-3c:

\[ \phi 0.60 F_u A_{nv} = 124 \text{ kips/in.} \left( \frac{3}{8} \text{ in.} \right) \]

### ASD

Shear rupture of plate using AISC *Specification* Equation J4-4:

\[ A_{nv} = \left[ 8\frac{1}{2} \text{ in.} - 3(1.00 \text{ in.}) \right] (\% \text{ in.}) \]
\[ = 1.72 \text{ in.}^2 \]

\[ \Omega = 2.00 \]

\[ R_n = \frac{0.60 F_u A_{nv}}{\Omega} \]
\[ = \frac{0.60(58 \text{ ksi})(1.72 \text{ in.}^2)}{2.00} \]
\[ = 29.9 \text{ kips} < 33.3 \text{ kips} \]

**n.g.**

Increase the plate thickness to \( \frac{3}{8} \) in.

\[ A_{nv} = \left[ 8\frac{1}{2} \text{ in.} - 3(1.00 \text{ in.}) \right] (\% \text{ in.}) \]
\[ = 2.06 \text{ in.}^2 \]

\[ \Omega = 2.00 \]

\[ R_n = \frac{0.60(58 \text{ ksi})(2.06 \text{ in.}^2)}{2.00} \]
\[ = 35.8 \text{ kips} > 33.3 \text{ kips} \]

**o.k.**

Block shear rupture of plate using AISC *Specification* Equation J4-5 with \( n = 3, L_{ev} = L_{eh} = \frac{1}{4} \text{ in.}, U_{bs} = 1 \):

\[ \frac{R_n}{\Omega} = \frac{U_{bs} F_u A_{nt}}{\Omega} + \min \left( \frac{0.60 F_y A_{gyv}}{\Omega}, \frac{0.60 F_u A_{nv}}{\Omega} \right) \]

Tension rupture component from AISC *Manual* Table 9-3a:

\[ \frac{U_{bs} F_u A_{nt}}{\Omega} = (1.0)(21.8 \text{ kips/in.}) \left( \frac{3}{8} \text{ in.} \right) \]

Shear yielding component from AISC *Manual* Table 9-3b:

\[ \frac{0.60 F_y A_{gyv}}{\Omega} = 78.3 \text{ kips/in.} \left( \frac{3}{8} \text{ in.} \right) \]

Shear rupture component from AISC *Manual* Table 9-3c:

\[ \frac{0.60 F_u A_{nv}}{\Omega} = 82.6 \text{ kips/in.} \left( \frac{3}{8} \text{ in.} \right) \]
\[
\phi R_n = (32.6 \text{ kips/in.} + 117 \text{ kips/in.})(\frac{3}{8} \text{ in.}) \\
= 56.1 \text{ kips} > 50.0 \text{ kips} \\
\text{o.k.}
\]
\[
\frac{R_n}{\Omega} = (21.8 \text{ kips/in.} + 78.3 \text{ kips/in.})(\frac{3}{8} \text{ in.}) \\
= 37.5 \text{ kips} > 33.3 \text{ kips} \\
\text{o.k.}
\]

Thus, the configuration shown in Figure II.A-10 can be supported using \(\frac{3}{8}\)-in. bent plates, and \(\frac{1}{4}\)-in. fillet welds.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_n) = (32.6 kips/in. + 117 kips/in.)((\frac{3}{8}) in.)</td>
<td>o.k.</td>
</tr>
<tr>
<td>(\frac{R_n}{\Omega}) = (21.8 kips/in. + 78.3 kips/in.)((\frac{3}{8}) in.)</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
**EXAMPLE II.A-11  SHEAR END-PLATE CONNECTION (BEAM TO GIRDER WEB)**

**Given:**

Design a shear end-plate connection to connect an ASTM A992 W18×50 beam to an ASTM A992 W21×62 girder web, to support the following beam end reactions:

- \( R_D = 10 \text{ kips} \)
- \( R_L = 30 \text{ kips} \)

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes, 70-ksi electrodes and ASTM A36 plates.

**Solution:**

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

**Beam**

ASTM A992

- \( F_y = 50 \text{ ksi} \)
- \( F_u = 65 \text{ ksi} \)

**Girder**

ASTM A992

- \( F_y = 50 \text{ ksi} \)
- \( F_u = 65 \text{ ksi} \)

**Plate**

ASTM A36

- \( F_y = 36 \text{ ksi} \)
- \( F_u = 58 \text{ ksi} \)

From AISC Manual Tables 1-1 and 9-2 and AISC Manual Figure 9-2, the geometric properties are as follows:

**Beam**

- \( W18\times50 \)
- \( d = 18.0 \text{ in.} \)
- \( t_w = 0.355 \text{ in.} \)
- \( S_{net} = 23.4 \text{ in}^3 \)
- \( c = 4\frac{1}{4} \text{ in.} \)
- \( d_c = 2 \text{ in.} \)
- \( e = 4\frac{1}{2} \text{ in.} \)
$h_w = 16.0$ in.

Girder

W21×62

$t_w = 0.400$ in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2 \times (10 \text{ kips}) + 1.6 \times (30 \text{ kips})$</td>
<td>$R_u = 10 \text{ kips} + 30 \text{ kips}$</td>
</tr>
<tr>
<td>$= 60.0 \text{ kips}$</td>
<td>$= 40.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

Bolt Shear and Bolt Bearing, Shear Yielding, Shear Rupture, and Block Shear Rupture of End-Plate

From AISC Manual Table 10-4, for 3 rows of bolts and $\frac{1}{4}$-in. plate thickness:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = 76.4 \text{ kips} &gt; 60.0 \text{ kips}$</td>
<td>$\dfrac{R_n}{\Omega} = 50.9 \text{ kips} &gt; 40.0 \text{ kips}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Weld Shear and Beam Web Shear Rupture

Try $\frac{1}{10}$-in. weld. From AISC Manual Table 10-4, the minimum beam web thickness is,

$t_{w,\text{min}} = 0.286$ in. $< 0.355$ in. o.k.

From AISC Manual Table 10-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = 67.9 \text{ kips} &gt; 60.0 \text{ kips}$</td>
<td>$\dfrac{R_n}{\Omega} = 45.2 \text{ kips} &gt; 40.0 \text{ kips}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Bolt Bearing on Girder Web

From AISC Manual Table 10-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = 526 \text{ kips/in.(0.400 in.)}$</td>
<td>$\dfrac{R_n}{\Omega} = 351 \text{ kips/in.(0.400 in.)}$</td>
</tr>
<tr>
<td>$= 210 \text{ kips} &gt; 60.0 \text{ kips}$</td>
<td>$= 140 \text{ kips} &gt; 40.0 \text{ kips}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Coped Beam Strength

As was shown in Example II.A-4, the coped section does not control the design. o.k.

Beam Web Shear

As was shown in Example II.A-4, beam web shear does not control the design. o.k.
EXAMPLE II.A-12  ALL-BOLTED UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Design an all-bolted unstiffened seated connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column web to support the following end reactions:

\[ R_D = 9.0 \text{ kips} \]
\[ R_L = 27.5 \text{ kips} \]

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles.

Note: For calculation purposes, assume setback is equal to ¾ in. to account for possible beam underrun.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
W16×50
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Column
W14×90
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Angles
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W16×50
\[ t_{ew} = 0.380 \text{ in.} \]
\[ d = 16.3 \text{ in.} \]
\[ b_f = 7.07 \text{ in.} \]
\[ t_f = 0.630 \text{ in.} \]
\[ k_{des} = 1.03 \text{ in.} \]

Column
W14x90
\[ t_w = 0.440 \text{ in.} \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ R_u = 1.2 (9.0 \text{ kips}) + 1.6 (27.5 \text{ kips}) = 54.8 \text{ kips} ]</td>
<td>[ R_a = 9.0 \text{ kips} + 27.5 \text{ kips} = 36.5 \text{ kips} ]</td>
</tr>
</tbody>
</table>

**Web Local Yielding Bearing Length (AISC Specification Section J10.2):**

\( l_{b \text{ min}} \) is the length of bearing required for the limit states of web local yielding and web local crippling on the beam, but not less than \( k_{des} \).

From AISC Manual Table 9-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ l_{b \text{ min}} = \frac{R_a - \phi R_3}{\phi R_2} \geq k_{des} \quad \text{(from Manual Eq. 9-45a)} ]</td>
<td>[ l_{b \text{ min}} = \frac{R_a - R_3}{R_2} / \Omega \geq k_{des} \quad \text{(from Manual Eq. 9-45b)} ]</td>
</tr>
<tr>
<td>[ \frac{54.8 \text{ kips} - 48.9 \text{ kips}}{19.0 \text{ kips/in.}} \geq 1.03 \text{ in.} ]</td>
<td>[ \frac{36.5 \text{ kips} - 32.6 \text{ kips}}{12.7 \text{ kips/in.}} \geq 1.03 \text{ in.} ]</td>
</tr>
</tbody>
</table>
| Use \( l_{b \text{ min}} = 1.03 \text{ in.} \) | Use \( l_{b \text{ min}} = 1.03 \text{ in.} \)

**Web Local Crippling Bearing Length (AISC Specification Section J10.3):**

\[ \left( \frac{l_{b}}{d} \right)_{\text{max}} = \frac{3.25 \text{ in.}}{16.3 \text{ in.}} = 0.199 < 0.2 \]

From AISC Manual Table 9-4, when \( \frac{l_{b}}{d} \leq 0.2 \),

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ l_{b \text{ min}} = \frac{R_u - \phi R_3}{\phi R_4} \quad \text{(from Manual Eq. 9-47a)} ]</td>
<td>[ l_{b \text{ min}} = \frac{R_a - R_3}{R_4} / \Omega \quad \text{(from Manual Eq. 9-47b)} ]</td>
</tr>
<tr>
<td>[ \frac{54.8 \text{ kips} - 67.2 \text{ kips}}{5.79 \text{ kips/in.}} ]</td>
<td>[ \frac{36.5 \text{ kips} - 44.8 \text{ kips}}{3.86 \text{ kips/in.}} ]</td>
</tr>
<tr>
<td>which results in a negative quantity.</td>
<td>which results in a negative quantity.</td>
</tr>
</tbody>
</table>
| Therefore, \( l_{b \text{ min}} = k_{des} = 1.03 \text{ in.} \) | Therefore, \( l_{b \text{ min}} = k_{des} = 1.03 \text{ in.} \)

**Shear Yielding and Flexural Yielding of Angle and Local Yielding and Crippling of Beam Web**

Try an 8-in. angle length with a \( \frac{3}{8} \) in. thickness, a 3\( \frac{1}{2} \)-in. minimum outstanding leg and \( l_{b \text{ req}} = 1.03 \text{ in.} \).
Conservatively, use \( t_h = 1\frac{\pi}{6} \) in.

From AISC Manual Table 10-5:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 90.0 ) kips ( &gt; ) 54.8 kips</td>
<td>o.k. ( \frac{R_n}{\Omega} = 59.9 ) kips ( &gt; ) 36.5 kips</td>
</tr>
</tbody>
</table>

Try \( L6\times4\times\frac{3}{8} \) (4-in. OSL), 8-in. long with 5½-in. bolt gage, connection type B (four bolts).

From AISC Manual Table 10-5, for \( \frac{3}{8} \)-in. diameter ASTM A325-N bolts:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 71.6 ) kips ( &gt; ) 54.8 kips</td>
<td>o.k. ( \frac{R_n}{\Omega} = 47.7 ) kips ( &gt; ) 36.5 kips</td>
</tr>
</tbody>
</table>

**Bolt Bearing on the Angle**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required bearing strength:</td>
<td>Required bearing strength:</td>
</tr>
<tr>
<td>( r_u = \frac{54.8 \text{ kips}}{4 \text{ bolts}} ) = 13.7 kips/bolt</td>
<td>( r_a = \frac{36.5 \text{ kips}}{4 \text{ bolts}} ) = 9.13 kips/bolt</td>
</tr>
</tbody>
</table>

By inspection, tear-out does not control; therefore, only the limit on AISC Specification Equation J3-6a need be checked.

From AISC Specification Equation J3-6a:

\[
\phi R_e = \phi 2.4 dt F_u = 0.75(2.4)(\frac{3}{8} \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) = 48.9 \text{ kips} > 13.7 \text{ kips} \quad \text{O.K.}
\]

\[
\frac{R_n}{\Omega} = 2.4(\frac{3}{8} \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) = 32.6 \text{ kips} > 9.13 \text{ kips} \quad \text{O.K.}
\]

**Bolt Bearing on the Column**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_e = \phi 2.4 dt F_u ) = 0.75(2.4)(\frac{3}{8} \text{ in.})(0.440 \text{ in.})(65 \text{ ksi}) = 38.6 \text{ kips} &gt; 13.7 \text{ kips}</td>
<td>( \frac{R_n}{\Omega} = 2.4(\frac{3}{8} \text{ in.})(0.440 \text{ in.})(65 \text{ ksi}) ) = 25.7 kips &gt; 9.13 kips</td>
</tr>
</tbody>
</table>

**Top Angle and Bolts**

Use an \( L4\times4\times\frac{3}{8} \) with two \( \frac{3}{8} \)-in.-diameter ASTM A325-N or F1852-N bolts through each leg.
EXAMPLE IIA-13  BOLTED/WELDED UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design an unstiffened seated connection between an ASTM A992 W21×62 beam and an ASTM A992 W14×61 column flange to support the following beam end reactions:

\[ R_D = 9.0 \text{ kips} \]
\[ R_L = 27.5 \text{ kips} \]

Use ¼-in.-diameter ASTM A325-N or F1852-N bolts in standard holes to connect the supported beam to the seat and top angles. Use 70-ksi electrode welds to connect the seat and top angles to the column flange and ASTM A36 angles.

Note: For calculation purposes, assume setback is equal to ¾ in. to account for possible beam underrun.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
W21×62
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Column
W14×61
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]
Angles
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

**Beam**
W21×62
- \( t_w = 0.400 \text{ in.} \)
- \( d = 21.0 \text{ in.} \)
- \( b_f = 8.24 \text{ in.} \)
- \( t_f = 0.615 \text{ in.} \)
- \( k_{des} = 1.12 \text{ in.} \)

**Column**
W14×61
\( t_f = 0.645 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2 (9.0 \text{ kips}) + 1.6 (27.5 \text{ kips}) )</td>
<td>( R_u = 9.0 \text{ kips} + 27.5 \text{ kips} )</td>
</tr>
<tr>
<td>= 54.8 kips</td>
<td>= 36.5 kips</td>
</tr>
</tbody>
</table>

**Web Local Yielding Bearing Length (AISC Specification Section J10.2):**

\( l_{b\ min} \) is the length of bearing required for the limit states of web local yielding and web local crippling of the beam, but not less than \( k_{des} \).

From AISC Manual Table 9-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_{b\ min} = \frac{R_u - \phi R_l}{\phi R_2} \geq k_{des} ) (from Manual Eq. 9-45a)</td>
<td>( l_{b\ min} = \frac{R_u - R_l}{R_2 / \Omega} \geq k_{des} ) (from Manual Eq. 9-45b)</td>
</tr>
<tr>
<td>= \frac{54.8 \text{ kips} - 56.0 \text{ kips}}{20.0 \text{ kips/in.}} \geq 1.12 \text{ in.}</td>
<td>= \frac{36.5 \text{ kips} - 37.3 \text{ kips}}{13.3 \text{ kips/in.}} \geq 1.12 \text{ in.}</td>
</tr>
</tbody>
</table>

which results in a negative quantity.

Therefore, \( l_{b\ min} = k_{des} = 1.12 \text{ in.} \)

**Web Local Crippling Bearing Length (AISC Specification Section J10.3):**

\( \left( \frac{l_b}{d} \right)_{\text{max}} = \frac{3/4 \text{ in.}}{21.0 \text{ in.}} \)

\( = 0.155 < 0.2 \)
From AISC Manual Table 9-4, when $\frac{l_b}{d} \leq 0.2$,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{b, min} = \frac{R_u - \phi R_3}{\phi R_4}$ (from Manual Eq. 9-47a)</td>
<td>$l_{b, min} = \frac{R_u - R_3 / \Omega}{R_4 / \Omega}$ (from Manual Eq. 9-47b)</td>
</tr>
<tr>
<td>$= \frac{54.8 \text{ kips} - 71.7 \text{ kips}}{5.37 \text{ kips/in.}}$</td>
<td>$= \frac{36.5 \text{ kips} - 47.8 \text{ kips}}{3.58 \text{ kips/in.}}$</td>
</tr>
<tr>
<td>which results in a negative quantity.</td>
<td>which results in a negative quantity.</td>
</tr>
<tr>
<td>Therefore, $l_{b, min} = k_{det} = 1.12 \text{ in.}$</td>
<td>Therefore, $l_{b, min} = k_{det} = 1.12 \text{ in.}$</td>
</tr>
</tbody>
</table>

**Shear Yielding and Flexural Yielding of Angle and Local Yielding and Crippling of Beam Web**

Try an 8-in. angle length with a $\frac{5}{8}$-in. thickness and a $3\frac{1}{2}$-in. minimum outstanding leg.

Conservatively, use $l_b = 1\frac{3}{8}$ in.

From AISC Manual Table 10-6:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_4 = 81.0 \text{ kips} &gt; 54.8 \text{ kips}$</td>
<td>$R_\Omega = 53.9 \text{ kips} &gt; 36.5 \text{ kips}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Try an L8×4×5 (4 in. OSL), 8 in. long with $\frac{5}{8}$-in. fillet welds.

From AISC Manual Table 10-6:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_4 = 66.7 \text{ kips} &gt; 54.8 \text{ kips}$</td>
<td>$R_\Omega = 44.5 \text{ kips} &gt; 36.5 \text{ kips}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Use two $\frac{3}{4}$-in.-diameter ASTM A325-N bolts to connect the beam to the seat angle.

The strength of the bolts, welds and angles must be verified if horizontal forces are added to the connection.

**Top Angle, Bolts and Welds**

Use an L4×4×4 with two $\frac{3}{4}$-in.-diameter ASTM A325-N or F1852-N bolts through the supported beam leg of the angle. Use a $\frac{3}{16}$-in. fillet weld along the toe of the angle to the column flange. See the discussion in AISC Manual Part 10.
EXAMPLE II.A-14 STIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a stiffened seated connection between an ASTM A992 W21×68 beam and an ASTM A992 W14×90 column flange, to support the following end reactions:

\[ R_D = 21 \text{ kips} \]
\[ R_L = 62.5 \text{ kips} \]

Use \( \frac{3}{4} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes to connect the supported beam to the seat plate and top angle. Use 70-ksi electrode welds to connect the stiffener and top angle to the column flange and ASTM A36 plates and angles.

Note: For calculation purposes, assume setback is equal to \( \frac{3}{4} \) in. to account for possible beam underrun.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W21×68
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Column
W14×90
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)
Angles and plates
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

**Beam**

W21×68
\( t_w = 0.430 \text{ in.} \)
\( d = 21.1 \text{ in.} \)
\( b_f = 8.27 \text{ in.} \)
\( t_f = 0.685 \text{ in.} \)
\( k_{des} = 1.19 \text{ in.} \)

**Column**

W14×90
\( t_f = 0.710 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u )</td>
<td>1.2 (21 kips) + 1.6 (62.5 kips)</td>
<td>( R_a = 21 \text{ kips} + 62.5 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>= 125 kips</td>
<td>= 83.5 kips</td>
</tr>
</tbody>
</table>

**Required Stiffener Width, W**

For web local crippling, assume \( l_b/d > 0.2 \).

From AISC Manual Table 9-4, \( W_{min} \) for local crippling is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{min} )</td>
<td>( R_u - \phi R_0 - \phi R_3 ) + setback ( \phi R_0 ) ( \phi R_3 ) ( R_0 ) ( \Omega ) ( \Omega ) ( \text{+ ( \frac{\pi}{4} ) in.} ) ( \text{+ ( \frac{\pi}{4} ) in.} )</td>
<td>( W_{min} = \frac{R_u - R_3}{R_0} / \Omega ) + setback ( \frac{R_3}{R_0} ) ( \frac{R_0}{\Omega} ) ( \text{+ ( \frac{\pi}{4} ) in.} ) ( \text{+ ( \frac{\pi}{4} ) in.} )</td>
</tr>
<tr>
<td></td>
<td>= 125 kips - 75.9 kips / 7.95 kips / in.</td>
<td>= 83.5 kips - 50.6 kips / 5.30 kips / in.</td>
</tr>
<tr>
<td></td>
<td>= 6.93 in.</td>
<td>= 6.96 in.</td>
</tr>
</tbody>
</table>

From AISC Manual Table 9-4, \( W_{min} \) for web local yielding is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{min} )</td>
<td>( R_u - \phi R_1 ) + setback ( \phi R_2 ) ( \phi R_1 ) ( R_2 ) ( R_1 ) ( \Omega ) ( \Omega )</td>
<td>( W_{min} = \frac{R_u - R_1}{R_2} / \Omega ) + setback ( \frac{R_1}{R_2} ) ( \frac{R_2}{\Omega} )</td>
</tr>
<tr>
<td></td>
<td>= 125 kips - 64.0 kips / 21.5 kips / in.</td>
<td>= 83.5 kips - 42.6 kips / 14.3 kips / in.</td>
</tr>
<tr>
<td></td>
<td>= 3.59 in. &lt; 6.93 in.</td>
<td>= 3.61 in. &lt; 6.96 in.</td>
</tr>
</tbody>
</table>

Use \( W = 7 \text{ in.} \).

Check assumption:

\[
\frac{l_b}{d} = \frac{7.00 \text{ in.} - \frac{\pi}{4} \text{ in.}}{21.1 \text{ in.}}
\]
= 0.296 > 0.2  

**Stiffener Length, L, and Stiffener to Column Flange Weld Size**

Try a stiffener with $L = 15$ in. and $\frac{1}{16}$-in. fillet welds.

From AISC Manual Table 10-8:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_n = 139$ kips $&gt; 125$ kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{R_n}{\Omega} = 93.0$ kips $&gt; 83.5$ kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Seat Plate Welds (AISC Manual Part 10)**

Use $\frac{1}{16}$-in. fillet welds on each side of the stiffener. Minimum length of seat plate to column flange weld is $0.2(L) = 3$ in. The weld between the seat plate and stiffener plate is required to have a strength equal to or greater than the weld between the seat plate and the column flange, use $\frac{1}{16}$-in. fillet welds on each side of the stiffener to the seat plate; length of weld = 6 in.

**Seat Plate Dimensions (AISC Manual Part 10)**

A width of 9 in. is adequate to accommodate two $\frac{3}{4}$-in.-diameter ASTM A325-N bolts on a $5\frac{1}{2}$ in. gage connecting the beam flange to the seat plate.

Use a PL $\frac{3}{8}$ in.$\times$7 in.$\times$9 in. for the seat.

**Stiffener Plate Thickness (AISC Manual Part 10)**

Determine the minimum plate thickness to develop the stiffener to the seat plate weld.

$$t_{min} = 2w$$
$$= 2(\frac{1}{16} 	ext{ in.})$$
$$= \frac{1}{8} 	ext{ in.}$$

Determine the minimum plate thickness for a stiffener with $F_y = 36$ ksi and a beam with $F_y = 50$ ksi.

$$t_{min} = \frac{50 	ext{ ksi}}{36 	ext{ ksi}} t_w$$
$$= \frac{50 	ext{ ksi}}{36 	ext{ ksi}} (0.430 \text{ in.})$$
$$= 0.597 \text{ in.} < \frac{1}{8} \text{ in.}$$

Use a PL $\frac{3}{8}$ in.$\times$7 in.$\times$1 ft 3 in.

**Top Angle, Bolts and Welds**

Use an L4$\times$4$\times$$\frac{3}{4}$ with two $\frac{3}{4}$-in.-diameter A325-N or F1852-N bolts through the supported beam leg of the angle. Use a $\frac{1}{16}$-in. fillet weld along the toe of the angle to the column flange.
EXAMPLE II.A-15 STIFFENED SEATED CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Design a stiffened seated connection between an ASTM A992 W21×68 beam and an ASTM A992 W14×90 column web to support the following beam end reactions:

\[ R_D = 21 \text{ kips} \]
\[ R_L = 62.5 \text{ kips} \]

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes to connect the supported beam to the seat plate and top angle. Use 70-ksi electrode welds to connect the stiffener and top angle to the column web. Use ASTM A36 angles and plates.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W21×68
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Column
W14×90
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)
Angles and Plates
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W21×68
$tw = 0.430$ in.
d = 21.1 in.
b_f = 8.27 in.
t_f = 0.685 in.
k_de = 1.19 in.

Column
W14×90
$tw = 0.440$ in.
T = 10 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ra = 1.2 (21 \text{ kips}) + 1.6 (62.5 \text{ kips})$</td>
<td>$Ra = 21 \text{ kips} + 62.5 \text{ kips}$</td>
</tr>
<tr>
<td>= 125 kips</td>
<td>= 83.5 kips</td>
</tr>
</tbody>
</table>

Required Stiffener Width, W

As calculated in Example II.A-14, use $W = 7$ in.

Stiffener Length, L, and Stiffener to Column Web Weld Size

As calculated in Example II.A-14, use $L = 15$ in. and $\frac{3}{16}$-in. fillet welds.

Seat Plate Welds (AISC Manual Part 10)

As calculated in Example II.A-14, use 3 in. of $\frac{3}{16}$-in. weld on both sides of the seat plate for the seat plate to column web welds and for the seat plate to stiffener welds.

Seat Plate Dimensions (AISC Manual Part 10)

For a column web support, the maximum distance from the face of the support to the line of the bolts between the beam flange and seat plate is $3\frac{1}{2}$ in. The $PL 5\frac{3}{8}$ in.$\times 7$ in.$\times 9$ in. selected in Example II.A-14 will accommodate these bolts.

Stiffener Plate Thickness (AISC Manual Part 10)

As calculated in Example II.A-14, use a $PL 5\frac{3}{8}$ in.$\times 7$ in.$\times 1$ ft 3 in.

Top Angle, Bolts and Welds

Use an L4×4×$\frac{3}{8}$ with two $\frac{3}{4}$-in.-diameter ASTM A325-N bolts through the supported beam leg of the angle. Use a $\frac{3}{16}$-in. fillet weld along the toe of the angle to the column web.

Column Web
If only one side of the column web has a stiffened seated connection, then,

\[
t_{w\text{min}} = \frac{3.09D}{F_u}
\]

\[
= \frac{65 \text{ ksi}}{65 \text{ ksi}} = 0.238 \text{ in.}
\]

If both sides of the column web have a stiffened seated connection, then,

\[
t_{w\text{min}} = \frac{6.19D}{F_u}
\]

\[
= \frac{6.19 (5\text{ sixteenths})}{65 \text{ ksi}} = 0.476 \text{ in.}
\]

Column \( t_w = 0.440 \text{ in.} \), which is sufficient for the one-sided stiffened seated connection shown.

Note: Additional detailing considerations for stiffened seated connections are given in Part 10 of the AISC Manual.
EXAMPLE II.A-16 OFFSET UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Determine the seat angle and weld size required for the unstiffened seated connection between an ASTM A992 W14×48 beam and an ASTM A992 W12×65 column flange connection with an offset of 5½ in., to support the following beam end reactions:

\[ R_D = 5.0 \text{ kips} \]
\[ R_L = 15 \text{ kips} \]

Use 70-ksi electrode welds to connect the seat angle to the column flange and an ASTM A36 angle.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

**Beam**

W14×48
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

**Column**

W12×65
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

**Angle**

Note A: End return is omitted because the AWS Code does not permit weld returns to be carried around the corner formed by the column flange toe and seat angle heel.

Note B: Beam and top angle not shown for clarity.

Note C: The nominal setback of the beam from the face of the flange is ½ in. A setback of ¾ in. is used in the calculations to accommodate potential beam underrun.
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC *Manual* Table 1-1, the geometric properties are as follows:

**Beam**
W14×48
- \( t_w = 0.340 \text{ in.} \)
- \( d = 13.8 \text{ in.} \)
- \( b_f = 8.03 \text{ in.} \)
- \( t_f = 0.595 \text{ in.} \)
- \( k_{des} = 1.19 \text{ in.} \)

**Column**
W12×65
- \( b_f = 12.0 \text{ in.} \)
- \( t_f = 0.605 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_u = 1.2 \times (5.0 \text{ kips}) + 1.6 \times (15 \text{ kips}) )</td>
<td>( R_u = 5.0 \text{ kips} + 15 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>= 30.0 kips</td>
<td>= 20.0 kips</td>
</tr>
</tbody>
</table>

**Web Local Yielding Bearing Length (AISC Specification Section J10.2):**

\( l_{b,\text{min}} \) is the length of bearing required for the limit states of web local yielding and web local crippling, but not less than \( k_{des} \).

From AISC *Manual* Table 9-4:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( l_{b,\text{min}} = \frac{R_u - \phi R_3}{\phi R_2} \geq k_{des} ) (from Manual Eq. 9-45a)</td>
<td>( l_{b,\text{min}} = \frac{R_u - (R_1 / \Omega)}{(R_2 / \Omega)} \geq k_{des} ) (from Manual Eq. 9-45b)</td>
</tr>
<tr>
<td></td>
<td>= ( \frac{30.0 \text{ kips} - 50.6 \text{ kips}}{17.0 \text{ kips} / \text{in.}} \geq 1.19 \text{ in.} )</td>
<td>= ( \frac{20.0 \text{ kips} - 33.7 \text{ kips}}{11.3 \text{ kips} / \text{in.}} \geq 1.19 \text{ in.} )</td>
</tr>
<tr>
<td></td>
<td>which results in a negative quantity.</td>
<td>which results in a negative quantity.</td>
</tr>
<tr>
<td></td>
<td>Therefore, ( l_{b,\text{min}} = k_{des} = 1.19 \text{ in.} )</td>
<td>Therefore, ( l_{b,\text{min}} = k_{des} = 1.19 \text{ in.} )</td>
</tr>
</tbody>
</table>

**Web Local Crippling Bearing Length (AISC Specification Section J10.3):**

From AISC *Manual* Table 9-4, when \( \frac{l_b}{d} \leq 0.2 \),

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( l_{b,\text{min}} = \frac{R_u - \phi R_3}{\phi R_4} ) (from Manual Eq. 9-47a)</td>
<td>( l_{b,\text{min}} = \frac{R_u - (R_3 / \Omega)}{(R_4 / \Omega)} ) (from Manual Eq. 9-47b)</td>
</tr>
<tr>
<td></td>
<td>= ( \frac{30.0 \text{ kips} - 55.2 \text{ kips}}{5.19 \text{ kips/in.}} )</td>
<td>= ( \frac{20.0 \text{ kips} - 36.8 \text{ kips}}{3.46 \text{ kips/in.}} )</td>
</tr>
<tr>
<td></td>
<td>which results in a negative quantity.</td>
<td>which results in a negative quantity.</td>
</tr>
</tbody>
</table>
which results in a negative quantity. 
Therefore, \( l_{h,req} = k_{des} = 1.19 \text{ in.} \)

**Seat Angle and Welds**

The required strength for the righthand weld can be determined by summing moments about the lefthand weld.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{uR} )</td>
<td>( \frac{30.0 \text{ kips}(3.00 \text{ in.})}{3.50 \text{ in.}} = 25.7 \text{ kips} )</td>
<td>( \frac{20.0 \text{ kips}(3.00 \text{ in.})}{3.50 \text{ in.}} = 17.1 \text{ kips} )</td>
</tr>
</tbody>
</table>

Conservatively design the seat for twice the force in the more highly loaded weld. Therefore design the seat for the following:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u )</td>
<td>2(25.7 kips)</td>
<td>2(17.1 kips)</td>
</tr>
<tr>
<td></td>
<td>= 51.4 kips</td>
<td>= 34.2 kips</td>
</tr>
</tbody>
</table>

Try a 6-in. angle length with a \( \frac{1}{8} \)-in. thickness.

From AISC Manual Table 10-6, with \( l_{h,req} = 1\frac{3}{16} \) in.:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n )</td>
<td>55.2 kips &gt; 51.4 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>( \frac{R_n}{\Omega} )</td>
<td></td>
<td>36.7 kips &gt; 34.2 kips</td>
</tr>
</tbody>
</table>

For an \( L7 \times 4 \) angle with \( \frac{1}{8} \)-in. fillet welds, the weld strength from AISC Manual Table 10-6 is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n )</td>
<td>53.4 kips &gt; 51.4 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>( \frac{R_n}{\Omega} )</td>
<td></td>
<td>35.6 kips &gt; 34.2 kips</td>
</tr>
</tbody>
</table>

Use \( L7 \times 4 \times \frac{3}{8} \times 6 \) in. for the seat angle. Use two \( \frac{3}{8} \)-in.-diameter ASTM A325-N or F1852-N bolts to connect the beam to the seat angle. Weld the angle to the column with \( \frac{1}{8} \)-in. fillet welds.

**Top Angle, Bolts and Welds**

Use an \( L4 \times 4 \times \frac{1}{4} \) with two \( \frac{3}{8} \)-in.-diameter ASTM A325-N or F1852-N bolts through the outstanding leg of the angle.

Use a \( \frac{1}{8} \)-in. fillet weld along the toe of the angle to the column flange (maximum size permitted by AISC Specification Section J2.2b).
EXAMPLE II.A-17 SINGLE-PLATE CONNECTION (CONVENTIONAL—BEAM-TO-COLUMN FLANGE)

Given:

Design a single-plate connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

\[ R_D = 8.0 \text{ kips} \]
\[ R_L = 25 \text{ kips} \]

Use \( \frac{3}{4} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes, 70-ksi electrode welds and an ASTM A36 plate.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×50
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Column
W14×90
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)
From AISC Manual Table 1-1, the geometric properties are as follows:

**Beam**

W16×50

t_w = 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

**Column**

W14×90

t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2(8.0 \text{ kips}) + 1.6(25 \text{ kips}) )</td>
<td>( R_u = 8.0 \text{ kips} + 25 \text{ kips} )</td>
</tr>
<tr>
<td>= 49.6 kips</td>
<td>= 33.0 kips</td>
</tr>
</tbody>
</table>

**Bolt Shear, Weld Shear, and Bolt Bearing, Shear Yielding, Shear Rupture, and Block Shear Rupture of the Plate**

Try four rows of bolts, \( \frac{1}{8} \)-in. plate thickness, and \( \frac{5}{16} \)-in. fillet weld size.

From AISC Manual Table 10-10a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 52.2 \text{ kips} &gt; 49.6 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>( \frac{R_u}{\Omega} = 34.8 \text{ kips} &gt; 33.0 \text{ kips} )</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Bolt Bearing for Beam Web**

Block shear rupture, shear yielding and shear rupture will not control for an uncoped section.

From AISC Manual Table 10-1, for an uncoped section, the beam web available strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 351 \text{ kips/in.}(0.380 \text{ in.}) )</td>
<td>o.k.</td>
</tr>
<tr>
<td>= 133 kips &gt; 49.6 kips</td>
<td>( \frac{R_u}{\Omega} = 234 \text{ kips/in.}(0.380 \text{ in.}) )</td>
</tr>
<tr>
<td>= 88.9 kips &gt; 33.0 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note: To provide for stability during erection, it is recommended that the minimum plate length be one-half the T-dimension of the beam to be supported. AISC Manual Table 10-1 may be used as a reference to determine the recommended maximum and minimum connection lengths for a supported beam.
EXAMPLE II.A-18 SINGLE-PLATE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Design a single-plate connection between an ASTM A992 W18×35 beam and an ASTM A992 W21×62 girder web to support the following beam end reactions:

\[ R_D = 6.5 \text{ kips} \]
\[ R_L = 20 \text{ kips} \]

The top flange is coped 2 in. deep by 4 in. long, \( L_{cc} = 1\frac{1}{2} \text{ in.} \). Use \( \frac{3}{4} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes, 70-ksi electrode welds and an ASTM A36 plate.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×35
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Girder
W21×62
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)
From AISC Manual Table 1-1 and Figure 9-2, the geometric properties are as follows:

**Beam**

W18×35  
\( t_w = 0.300 \text{ in.} \)  
\( d = 17.7 \text{ in.} \)  
\( t_f = 0.425 \text{ in.} \)  
\( c = 4.00 \text{ in.} \)  
\( d_c = 2.00 \text{ in.} \)  
\( h_o = 15.7 \text{ in.} \)

**Girder**

W21×62  
\( t_w = 0.400 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

\[
\begin{align*}
LRFD & \\
R_u &= 1.2(6.5 \text{ kips}) + 1.6(20 \text{ kips}) \\
&= 39.8 \text{ kips} \\
\hline
ASD & \\
R_d &= 6.5 \text{ kips} + 20 \text{ kips} \\
&= 26.5 \text{ kips}
\end{align*}
\]

*Bolt Shear, Weld Shear, and Bolt Bearing, Shear Yielding, Shear Rupture, and Block Shear Rupture of the Plate*

Try four rows of bolts, \( \frac{3}{8} \)-in. plate thickness, and \( \frac{3}{16} \)-in. fillet weld size.

From AISC Manual Table 10-10a:

\[
\begin{align*}
LRFD & \\
\phi R_u &= 52.2 \text{ kips} > 39.8 \text{ kips} & \text{o.k.} \\
\hline
ASD & \\
\frac{R_u}{\Omega} &= 34.8 \text{ kips} > 26.5 \text{ kips} & \text{o.k.}
\end{align*}
\]

*Bolt Bearing and Block Shear Rupture for Beam Web*

From AISC Manual Table 10-1, for a coped section with \( n = 4, L_{cv} = 1\frac{1}{2} \text{ in.}, \) and \( L_{eh} > 1\frac{1}{8} \text{ in.}:

\[
\begin{align*}
LRFD & \\
\phi R_u &= 269 \text{ kips/in.}(0.300 \text{ in.}) \\
&= 80.7 \text{ kips} > 39.8 \text{ kips} & \text{o.k.} \\
\hline
ASD & \\
\frac{R_u}{\Omega} &= 180 \text{ kips/in.}(0.300 \text{ in.}) \\
&= 54.0 \text{ kips} > 26.5 \text{ kips} & \text{o.k.}
\end{align*}
\]

*Shear Rupture of the Girder Web at the Weld*

\[
t_{min} = \frac{3.09D}{F_u} \\
= \frac{3.09(3\text{sixteenths})}{65 \text{ ksi}} \\
= 0.143 \text{ in.} < 0.400 \text{ in.} \quad \text{o.k.}
\]

Note: For coped beam sections, the limit states of flexural yielding and local buckling should be checked independently per AISC Manual Part 9. The supported beam web should also be checked for shear yielding and shear rupture per AISC Specification Section J4.2. However, for the shallow cope in this example, these limit states do not govern. For an illustration of these checks, see Example II.A-4.
EXAMPLE IIA-19  EXTENDED SINGLE-PLATE CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Design the connection between an ASTM A992 W16×36 beam and the web of an ASTM A992 W14×90 column, to support the following beam end reactions:

\[ R_D = 6.0 \text{ kips} \]
\[ R_L = 18 \text{ kips} \]

Use \( \frac{3}{4} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and an ASTM A36 plate. The beam is braced by the floor diaphragm. The plate is assumed to be thermally cut.

Note: All dimensional limitations are satisfied.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×36
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Column
W14×90
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:
Column
W14×90
\( t_w = 0.440 \text{ in.} \)
\( b_f = 14.5 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_u = 1.2(6.0 \text{ kips}) + 1.6(18 \text{ kips}) )</td>
<td>( R_a = 6.0 \text{ kips} + 18 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>= 36.0 kips</td>
<td>= 24.0 kips</td>
</tr>
</tbody>
</table>

Determine the distance from the support to the first line of bolts and the distance to the center of gravity of the bolt group.

\[ a = 9.00 \text{ in.} \]
\[ e = 9.00 \text{ in.} + 1.50 \text{ in.} = 10.5 \text{ in.} \]

*Bearing Strength of One Bolt on the Beam Web*

Tear out does not control by inspection.

From AISC *Manual* Table 7-4, determine the bearing strength (right side of AISC *Specification* Equation J3-6a):

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \phi r_n = 87.8 \text{ kips/in.}(0.295 \text{ in.}) )</td>
<td>( r_n = 58.5 \text{ kips/in.}(0.295 \text{ in.}) )</td>
</tr>
<tr>
<td></td>
<td>= 25.9 kips</td>
<td>( \Omega = 17.3 \text{ kips} )</td>
</tr>
</tbody>
</table>

*Shear Strength of One Bolt*

From AISC *Manual* Table 7-1:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \phi r_n = 17.9 \text{ kips} )</td>
<td>( r_n = 11.9 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( \Omega )</td>
<td>( \Omega )</td>
</tr>
</tbody>
</table>

Therefore, shear controls over bearing.

*Strength of the Bolt Group*

By interpolating AISC *Manual* Table 7-7, with \( e = 10.5 \text{ in.} \):

\[ C = 2.33 \]
Maximum Plate Thickness

Determine the maximum plate thickness, \( t_{\text{max}} \), that will result in the plate yielding before the bolts shear.

\[ F_{nv} = 54 \text{ ksi from AISC Specification Table J3.2} \]

\[ C' = 26.0 \text{ in. from AISC Manual Table 7-7 for the moment-only case} \]

\[ M_{\text{max}} = \frac{F_{nv}}{0.90} (A_b C') \quad (\text{Manual Eq. 10-3}) \]

\[ = \frac{54 \text{ ksi}}{0.90} (0.442 \text{ in.}^2)(26.0 \text{ in.}) \]

\[ = 690 \text{ kip-in.} \]

\[ t_{\text{max}} = \frac{6M_{\text{max}}}{F_d d^2} \quad (\text{Manual Eq. 10-2}) \]

\[ = \frac{6(690 \text{ kip-in.})}{36 \text{ ksi}(12.0 \text{ in.})^2} \]

\[ = 0.799 \text{ in.} \]

Try a plate thickness of \( \frac{1}{2} \text{ in.} \).

Bolt Bearing on Plate

\[ l_c = 1.50 \text{ in.} - \frac{13.56 \text{ in.}}{2} \]

\[ = 1.09 \text{ in.} \]

\[ R_n = 1.2l_c t F_u \leq 2.4 dt F_u \quad (\text{Spec. Eq. J3-6a}) \]

\[ 1.2(1.09 \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) \leq 2.4(\frac{1}{2} \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) \]

\[ 37.9 \text{ kips/bolt} \leq 52.2 \text{ kips/bolt} \]

Therefore, bolt shear controls.

Shear Yielding of Plate

Using AISC Specification Equation J4-3:
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = \phi 0.60 F_y A_{gv}$</td>
<td>$R_n = 0.60 F_y A_{gv}$</td>
</tr>
<tr>
<td>$= 1.00(0.60)(36\text{ ksi})(12.0\text{ in.})(\frac{1}{2}\text{ in.})$</td>
<td>$\frac{\Omega}{\Omega}$ $= 0.60(36\text{ ksi})(12.0\text{ in.})(\frac{1}{2}\text{ in.})$</td>
</tr>
<tr>
<td>$= 130\text{ kips} &gt; 36.0\text{ kips}$</td>
<td>$= 1.50$</td>
</tr>
<tr>
<td>o.k.</td>
<td>$= 86.4\text{ kips} &gt; 24.0\text{ kips}$</td>
</tr>
</tbody>
</table>

**Shear Rupture of Plate**

$$A_{nv} = t_p \left[ d - n(d_b + \frac{1}{8}\text{ in.}) \right]$$

$$= \frac{1}{2} \text{ in.} \left[ 12.0\text{ in.} - 4(\frac{1}{32}\text{ in.} + \frac{1}{16}\text{ in.}) \right]$$

$$= 4.25\text{ in.}^2$$

Using AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = \phi 0.60 F_y A_{nv}$</td>
<td>$R_n = 0.60 F_y A_{nv}$</td>
</tr>
<tr>
<td>$= 0.75(0.60)(58\text{ ksi})(4.25\text{ in.}^2)$</td>
<td>$\frac{\Omega}{\Omega}$ $= 0.60(58\text{ ksi})(4.25\text{ in.}^2)$</td>
</tr>
<tr>
<td>$= 111\text{ kips} &gt; 36.0\text{ kips}$</td>
<td>$= 2.00$</td>
</tr>
<tr>
<td>o.k.</td>
<td>$= 74.0\text{ kips} &gt; 24.0\text{ kips}$</td>
</tr>
</tbody>
</table>

**Block Shear Rupture of Plate**

$n = 4, L_{nv} = 1\frac{1}{2}\text{ in.}, L_{ch} = 4\frac{1}{4}\text{ in.}$

Using AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = \phi U_{bs} F_y A_{nt} + \min(\phi 0.60 F_y A_{gv}, \phi 0.60 F_y A_{nv})$</td>
<td>$R_n = \frac{U_{bs} F_y A_{nt}}{\Omega} + \min \left( \frac{0.60 F_y A_{gv}}{\Omega}, \frac{0.60 F_y A_{nv}}{\Omega} \right)$</td>
</tr>
<tr>
<td>Tension rupture component:</td>
<td>Tension rupture component:</td>
</tr>
<tr>
<td>$U_{bs} = 0.5$ from AISC Specification Section J4.3</td>
<td>$U_{bs} = 0.5$ from AISC Specification Section J4.3</td>
</tr>
<tr>
<td>$A_{nt} = \frac{1}{2} \text{ in.} \left[ 4\frac{1}{4}\text{ in.} - 1.5(\frac{1}{32}\text{ in.} + \frac{1}{16}\text{ in.}) \right]$</td>
<td>$A_{nt} = \frac{1}{2} \text{ in.} \left[ 4\frac{1}{4}\text{ in.} - 1.5(\frac{1}{32}\text{ in.} + \frac{1}{16}\text{ in.}) \right]$</td>
</tr>
<tr>
<td>$= 1.47\text{ in.}^2$</td>
<td>$= 1.47\text{ in.}^2$</td>
</tr>
<tr>
<td>$\phi U_{bs} F_y A_{nt} = 0.75(0.5)(58\text{ ksi})(1.47\text{ in.}^2)$</td>
<td>$U_{bs} F_y A_{nt} = 0.5(58\text{ ksi})(1.47\text{ in.}^2)$</td>
</tr>
<tr>
<td>$= 32.0\text{ kips}$</td>
<td>$= 21.3\text{ kips}$</td>
</tr>
</tbody>
</table>
Shear Yielding, Shear Buckling and Flexural Yielding of Plate

Check local buckling of plate

This check is analogous to the local buckling check for doubly coped beams as illustrated in AISC Manual Part 9, where \( c = 9 \) in. and \( h_o = 12 \) in.

\[
\lambda = \frac{h_o \sqrt{F_y}}{10 \sqrt{475 + 280 \left( \frac{h_o}{c} \right)^2}}
\]

\[
= \frac{(12.0 \text{ in.}) \sqrt{36 \text{ ksi}}}{10(\frac{1}{2} \text{ in.}) \sqrt{475 + 280 \left( \frac{12.0 \text{ in.}}{9.00 \text{ in.}} \right)^2}}
\]

\[
= 0.462
\]

\( \lambda \leq 0.7 \), therefore, \( Q = 1.0 \)

\( QF_y = F_y \)

Therefore, plate buckling is not a controlling limit state.

From AISC Manual Equation 10-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>\left( \frac{V_u}{\phi V_n} \right)^2 + \left( \frac{M_u}{\phi_b M_n} \right)^2 \leq 1.0</td>
<td>\left( \frac{V_a}{V_n / \Omega_y} \right)^2 + \left( \frac{M_a}{M_n / \Omega_b} \right)^2 \leq 1.0</td>
</tr>
</tbody>
</table>

From preceding calculations:

\( V_u = 36.0 \) kips

\( V_a = 24.0 \) kips
Design Examples V14.1

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<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi V_n = 130 \text{ kips} )</td>
<td>( V_n = 86.4 \text{ kips} )</td>
</tr>
<tr>
<td>( M_n = V_n e )</td>
<td>( M_n = V_n e )</td>
</tr>
<tr>
<td>( = 36.0 \text{ kips}(9.00 \text{ in.}) )</td>
<td>( = 24.0 \text{ kips}(9.00 \text{ in.}) )</td>
</tr>
<tr>
<td>( = 324 \text{ kip-in.} )</td>
<td>( = 216 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi_{bM_n} = \phi_b QF Z_{pl} )</td>
<td>( \frac{M_n}{\Omega_b} = \frac{QF Z_{pl}}{\Omega_b} )</td>
</tr>
<tr>
<td>( \phi = 0.90(1.0)(36 \text{ ksi}) \left( \frac{\frac{1}{2} \text{ in.}(12.0 \text{ in.})^2}{4} \right) )</td>
<td>( \phi = 1.0(36 \text{ ksi}) \left( \frac{\frac{1}{2} \text{ in.}(12.0 \text{ in.})^2}{1.67} \right) )</td>
</tr>
<tr>
<td>( = 583 \text{ kip-in.} )</td>
<td>( = 388 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( \left( \frac{36.0 \text{ kips}}{130 \text{ kips}} \right)^2 + \left( \frac{324 \text{ kip-in.}}{583 \text{ kip-in.}} \right)^2 = 0.386 \leq 1.0 \text{ o.k.} )</td>
<td>( \left( \frac{24.0 \text{ kips}}{86.4 \text{ kips}} \right)^2 + \left( \frac{216 \text{ kip-in.}}{388 \text{ kip-in.}} \right)^2 = 0.387 \leq 1.0 \text{ o.k.} )</td>
</tr>
</tbody>
</table>

**Flexural Rupture of Plate**

\( Z_{net} = 12.8 \text{ in.}^3 \) from AISC Manual Table 15-3

From AISC Manual Equation 9-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi_{bM_n} = \phi F_u Z_{net} )</td>
<td>( M_n = \frac{F_u Z_{net}}{\Omega} )</td>
</tr>
<tr>
<td>( = 0.75(58 \text{ ksi})(12.8 \text{ in.}^3) )</td>
<td>( = \frac{58 \text{ ksi}(12.8 \text{ in.}^3)}{2.00} )</td>
</tr>
<tr>
<td>( = 557 \text{ kip-in.} &gt; 324 \text{ kip-in.} \text{ o.k.} )</td>
<td>( = 371 \text{ kip-in.} &gt; 216 \text{ kip-in.} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

**Weld Between Plate and Column Web (AISC Manual Part 10)**

\( w = \frac{3}{8} t_p \)

\( = \frac{1}{3}(\frac{1}{2} \text{ in.}) \)

\( = 0.313 \text{ in.}, \text{ therefore, use a } \frac{3}{8} \text{-in. fillet weld on both sides of the plate.} \)

**Strength of Column Web at Weld**

\( t_{min} = \frac{3.09D}{F_u} \)

\( = \frac{3.09(5 \text{ sixteenths})}{65 \text{ ksi}} \)

\( = 0.238 \text{ in.} < 0.440 \text{ in.} \text{ o.k.} \)
EXAMPLE II.A-20  ALL-BOLTED SINGLE-PLATE SHEAR SPLICE

Given:

Design an all-bolted single-plate shear splice between an ASTM A992 W24×55 beam and an ASTM A992 W24×68 beam.

\( R_D = 10 \text{ kips} \)
\( R_L = 30 \text{ kips} \)

Use \( \frac{3}{8}-\text{in.-diameter} \) ASTM A325-N or F1852-N bolts in standard holes with 5 in. between vertical bolt rows and an ASTM A36 plate.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W24×55
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Beam
W24×68
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W24×55
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

**Beam**

- W24×68
- $t_w = 0.415$ in.

**Bolt Group Design**

Note: When the splice is symmetrical, the eccentricity of the shear to the center of gravity of either bolt group is equal to half the distance between the centroids of the bolt groups. Therefore, each bolt group can be designed for the shear, $R_u$ or $R_a$, and one-half the eccentric moment, $R_{ue}$ or $R_{ae}$.

Using a symmetrical splice, each bolt group will carry one-half the eccentric moment. Thus, the eccentricity on each bolt group, $e/2 = 2\frac{1}{2}$ in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$</td>
<td>$1.2(10 \text{ kips}) + 1.6(30 \text{ kips})$</td>
<td>$R_a = 10 \text{ kips} + 30 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 60.0 \text{ kips}$</td>
<td>$= 40.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Bolt Shear**

From AISC Manual Table 7-1:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi r_u$</td>
<td>24.3 kips/bolt</td>
<td>$r_n = 16.2$ kips/bolt</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi r_n$</td>
<td>0.75(26.9 kips)</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r_n = 26.9$ kips</td>
</tr>
</tbody>
</table>

**Bolt Bearing on 4½-in. Plate**

Note: The available bearing strength based on edge distance will conservatively be used for all of the bolts.

$$l_c = 1.50\text{ in.} - \frac{15\frac{1}{8}\text{ in.}}{2} = 1.03\text{ in.}$$

$$r_u = 1.2l_c t_F u \leq 2.4dt_F u\quad (Spec.\ Eq.\ J3-6a)$$

$$= 1.2(1.03 \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) \leq 2.4(\frac{3}{8} \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi})$$

$$= 26.9\text{ kips/bolt} \leq 45.7\text{ kips/bolt}$$

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td></td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi r_n$</td>
<td>0.75(26.9 kips)</td>
<td>$r_n = 26.9$ kips</td>
</tr>
<tr>
<td></td>
<td>$= 20.2$ kips/bolt</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r_n = 26.9$ kips</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 13.5$ kips/bolt</td>
</tr>
</tbody>
</table>

Note: By inspection, bearing on the webs of the W24 beams will not govern.
Since bearing is more critical,

$$C_{\text{min}} = \frac{R_u}{\phi r_n} = \frac{60.0 \text{ kips}}{20.2 \text{ kips/bolt}} = 2.97$$

By interpolating AISC Manual Table 7-6, with $n = 4$, $\theta = 0^\circ$ and $e_y = 2\frac{1}{2}$ in.:

$$C = 3.07 > 2.97 \quad \text{o.k.}$$

Flexural Yielding of Plate

Try PL3\% in. $\times$ 8 in. $\times$ 1'-0".

The required flexural strength is:

$$M_a = \frac{R_u e}{2} = \frac{60.0 \text{ kips}(5.00 \text{ in.})}{2} = 150 \text{ kip-in.}$$

$$\phi = 0.90$$

$$\phi M_a = \phi F_y Z_x$$

$$= 0.90(36 \text{ ksi}) \left[ \frac{3\text{ in.}(12.0 \text{ in.})^2}{4} \right]$$

$$= 437 \text{ kip-in.} > 150 \text{ kip-in.} \quad \text{o.k.}$$

Flexural Rupture of Plate

$$Z_{\text{net}} = 9.00 \text{ in.}^3$$ from AISC Manual Table 15-3

From AISC Manual Equation 9-4:

$$M_a = \frac{R_u e}{2} = \frac{40.0 \text{ kips}(5.00 \text{ in.})}{2} = 100 \text{ kip-in.}$$

$$\Omega = 1.67$$

$$\frac{M_a}{\Omega} = \frac{F_y Z_x}{\Omega}$$

$$= \frac{36 \text{ ksi}(3\text{ in.}(12.0 \text{ in.})^2)}{1.67}$$

$$= 291 \text{ kip-in.} > 100 \text{ kip-in.} \quad \text{o.k.}$$
Shear Yielding of Plate

From AISC Specification Equation J4-3:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ</td>
<td>1.00</td>
<td>Ω = 1.50</td>
</tr>
<tr>
<td>φRn = φ0.60FyAgy</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 1.00(0.60)(36 ksi)(12.0 in.)(⅜ in.)</td>
<td>= 0.60(36 ksi)(12.0 in.)(⅜ in.)</td>
</tr>
<tr>
<td></td>
<td>= 97.2 kips &gt; 60.0 kips</td>
<td>= 64.8 kips &gt; 40.0 kips</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Shear Rupture of Plate

\[ A_{sv} = \frac{3}{8}\text{ in.}[12.0\text{ in.} - 4(15/16\text{ in.} + 1/16\text{ in.})] = 3.00 \text{ in.}^2 \]

From AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ</td>
<td>0.75</td>
<td>Ω = 2.00</td>
</tr>
<tr>
<td>φRn = φ0.60FyAgy</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 0.75(0.60)(58 ksi)(3.00 in.²)</td>
<td>= 0.60(58 ksi)(3.00 in.²)</td>
</tr>
<tr>
<td></td>
<td>= 78.3 kips &gt; 60.0 kips</td>
<td>= 52.2 kips &gt; 40.0 kips</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Block Shear Rupture of Plate

\[ L_{sh} = L_{sv} = 1\frac{1}{2}\text{ in.} \]

From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>φRn = φU_{bs}FuAm + min(φ0.60FyAgy, φ0.60FuAgy)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U_{bs} = 1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tension rupture component from AISC Manual Table 9-3a:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>φU_{bs}FuAm = 1.0(43.5 kips/in.)(⅞ in.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tension rupture component from AISC Manual Table 9-3a:

\[ U_{bs}FuAm = 1.0(29.0 \text{ kips/in.})(\frac{5}{8} \text{ in.}) \]
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear yielding component from AISC Manual Table 9-3b:</td>
<td>Shear yielding component from AISC Manual Table 9-3b:</td>
</tr>
<tr>
<td>$\phi 0.60 F_y A_{gv} = 170 \text{kips/in.}(% \text{ in.})$</td>
<td>$\frac{0.60 F_y A_{gv}}{\Omega} = 113 \text{kips/in.}(% \text{ in.})$</td>
</tr>
<tr>
<td>Shear rupture component from AISC Manual Table 9-3c:</td>
<td>Shear rupture component from AISC Manual Table 9-3c:</td>
</tr>
<tr>
<td>$\phi 0.60 F_u A_{mu} = 183 \text{kips/in.}(% \text{ in.})$</td>
<td>$\frac{0.60 F_u A_{mu}}{\Omega} = 122 \text{kips/in.}(% \text{ in.})$</td>
</tr>
<tr>
<td>$\phi R_y = (43.5 \text{kips/in.} + 170 \text{kips/in.})(% \text{ in.})$</td>
<td></td>
</tr>
<tr>
<td>$= 80.1 \text{kips} &gt; 60.0 \text{kips}$</td>
<td>$R_y = (29.0 \text{kips/in.} + 113 \text{kips/in.})(% \text{ in.})$</td>
</tr>
<tr>
<td></td>
<td>$= 53.3 \text{kips} &gt; 40.0 \text{kips}$</td>
</tr>
</tbody>
</table>

Use PL\% in. × 8 in. × 1 ft 0 in.
EXAMPLE II.A-21  BOLTED/WELDED SINGLE-PLATE SHEAR SPLICE

Given:

Design a single-plate shear splice between an ASTM A992 W16×31 beam and an ASTM A992 W16×50 beam to support the following beam end reactions:

\[
R_D = 8.0 \text{ kips} \\
R_L = 24.0 \text{ kips}
\]

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts through the web of the W16×50 and 70-ksi electrode welds to the web of the W16×31. Use an ASTM A36 plate.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×31
ASTM A992
\(F_y = 50 \text{ ksi} \)
\(F_u = 65 \text{ ksi} \)

Beam
W16×50
ASTM A992
\(F_y = 50 \text{ ksi} \)
\(F_u = 65 \text{ ksi} \)

Plate
ASTM A36
\(F_y = 36 \text{ ksi} \)
\(F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W16×31
\(t_w = 0.275 \text{ in.} \)
Beam
W16×50
\( t_w = 0.380 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_u = 1.2(8.0 \text{ kips}) + 1.6(24 \text{ kips}) = 48.0 \text{ kips} )</td>
<td>( R_u = 8.0 \text{ kips} + 24 \text{ kips} = 32.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Weld Design**

Since the splice is unsymmetrical and the weld group is more rigid, it will be designed for the full moment from the eccentric shear.

Assume \( PL \frac{3}{8} \text{ in.} \times 8 \text{ in.} \times 1 \text{ ft 0 in.} \). This plate size meets the dimensional and other limitations of a single-plate connection with a conventional configuration from AISC *Manual* Part 10.

Use AISC *Manual* Table 8-8 to determine the weld size.

\[
k = \frac{kl}{l} = \frac{3\frac{1}{2} \text{ in.}}{12.0 \text{ in.}} = 0.292
\]

\[
xl = \frac{(kl)^2}{2(kl)+l} = \frac{(3\frac{1}{2} \text{ in.})^2}{2(3\frac{1}{2} \text{ in.})+12.0 \text{ in.}} = 0.645 \text{ in.}
\]

\[
a = \frac{xl}{l} = \frac{6.50 \text{ in.} - 0.645 \text{ in.}}{12.0 \text{ in.}} = 0.586 \text{ in.}
\]

\[a = \frac{al}{l} = \frac{5.86 \text{ in.}}{12.0 \text{ in.}} = 0.488
\]

By interpolating AISC *Manual* Table 8-8, with \( \theta = 0^\circ \),

\[C = 2.15
\]

The required weld size is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{req} )</td>
<td>( P_a ) ( \frac{\Omega}{C C l} )</td>
<td>( D_{req} = \frac{P_a \Omega}{C C l} )</td>
</tr>
</tbody>
</table>
The minimum weld size from AISC Specification Table J2.4 is $\frac{3}{32}$ in.

Use a $\frac{3}{32}$-in. fillet weld.

**Shear Rupture of W16×31 Beam Web at Weld**

For fillet welds with $F_{EY} = 70$ ksi on one side of the connection, the minimum thickness required to match the available shear rupture strength of the connection element to the available shear rupture strength of the base metal is:

$$t_{\min} = \frac{3.09D}{F_u}$$

$$= \frac{3.09(2.48 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.118 < 0.275 \text{ in.} \quad \text{o.k.}$$

**Bolt Group Design**

Since the weld group was designed for the full eccentric moment, the bolt group will be designed for shear only.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolt shear strength from AISC Manual Table 7-1:</td>
<td>Bolt shear strength from AISC Manual Table 7-1:</td>
</tr>
<tr>
<td>$\phi_{n} = 17.9 \text{ kips/bolt}$</td>
<td>$\frac{r_n}{\Omega} = 11.9 \text{ kips/bolt}$</td>
</tr>
<tr>
<td>For bearing on the $\frac{3}{8}$-in.-thick single plate, conservatively use the design values provided for $L_e = 1\frac{1}{4}$ in.</td>
<td>For bearing on the $\frac{3}{8}$-in.-thick single plate, conservatively use the design values provided for $L_e = 1\frac{1}{4}$ in.</td>
</tr>
<tr>
<td>Note: By inspection, bearing on the web of the W16×50 beam will not govern.</td>
<td>Note: By inspection, bearing on the web of the W16×50 beam will not govern.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-5:</td>
<td>From AISC Manual Table 7-5:</td>
</tr>
<tr>
<td>$\phi_{n} = 44.0 \text{ kips/in./bolt}(\frac{3}{8} \text{ in.})$</td>
<td>$\frac{r_n}{\Omega} = 29.4 \text{ kips/in./bolt}(\frac{3}{8} \text{ in.})$</td>
</tr>
<tr>
<td>= 16.5 kips/bolt</td>
<td>= 11.0 kips/bolt</td>
</tr>
<tr>
<td>Since bolt bearing is more critical than bolt shear,</td>
<td>Since bolt bearing is more critical than bolt shear,</td>
</tr>
<tr>
<td>$n_{\min} = \frac{R_u}{\phi_{n}}$</td>
<td>$n_{\min} = \frac{R_u}{\phi_{n}/\Omega}$</td>
</tr>
<tr>
<td>$= \frac{48.0 \text{ kips}}{16.5 \text{ kips/bolt}}$</td>
<td>$= \frac{32.0 \text{ kips}}{11.0 \text{ kips/bolt}}$</td>
</tr>
<tr>
<td>= 2.91 bolts &lt; 4 bolts</td>
<td>= 2.91 bolts &lt; 4 bolts</td>
</tr>
</tbody>
</table>
**Flexural Yielding of Plate**

As before, try a PL\% in. × 8 in. × 1 ft 0 in.

The required flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u = R_e$</td>
<td>$M_a = R_e$</td>
<td>$M_a = R_e$</td>
</tr>
<tr>
<td>$= 48.0 \text{kips}(5.86 \text{ in.})$</td>
<td>$= 32.0 \text{kips}(5.86 \text{ in.})$</td>
<td>$= 32.0 \text{kips}(5.86 \text{ in.})$</td>
</tr>
<tr>
<td>$= 281 \text{ kip-in.}$</td>
<td>$= 188 \text{ kip-in.}$</td>
<td>$= 188 \text{ kip-in.}$</td>
</tr>
<tr>
<td>$\phi = 0.90$</td>
<td></td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi M_a = \phi F_y Z_x$</td>
<td>$\frac{M_a}{\Omega} = \frac{F_y Z_x}{\Omega}$</td>
<td>$\frac{M_a}{\Omega} = \frac{F_y Z_x}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.9(36 \text{ ksi}) \left[ \frac{% \text{ in.}(12.0 \text{ in.})^2}{4} \right]$</td>
<td>$= 36 \text{ ksi} \left[ \frac{% \text{ in.}(12.0 \text{ in.})^2}{1.67} \right] \frac{1}{4}$</td>
<td>$= 36 \text{ ksi} \left[ \frac{% \text{ in.}(12.0 \text{ in.})^2}{1.67} \right] \frac{1}{4}$</td>
</tr>
<tr>
<td>$= 437 \text{ kip-in.} &gt; 281 \text{ kip-in.}$ o.k.</td>
<td>$= 291 \text{ kip-in.} &gt; 188 \text{ kip-in.}$ o.k.</td>
<td>$= 291 \text{ kip-in.} &gt; 188 \text{ kip-in.}$ o.k.</td>
</tr>
</tbody>
</table>

**Shear Yielding of Plate**

From AISC Specification Equation J4-3:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td></td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = \phi F_y A_g v$</td>
<td>$\frac{R_n}{\Omega} = \frac{0.60 F_y A_g v}{\Omega}$</td>
<td>$\frac{R_n}{\Omega} = \frac{0.60 F_y A_g v}{\Omega}$</td>
</tr>
<tr>
<td>$= 1.00(0.60)(36 \text{ ksi})(12.0 \text{ in.})(% \text{ in.})$</td>
<td>$= 0.60(36 \text{ ksi})(12.0 \text{ in.})(% \text{ in.})$</td>
<td>$= 0.60(36 \text{ ksi})(12.0 \text{ in.})(% \text{ in.})$</td>
</tr>
<tr>
<td>$= 97.2 \text{ kips} &gt; 48.0 \text{ kips}$ o.k.</td>
<td>$= 64.8 \text{ kips} &gt; 32.0 \text{ kips}$ o.k.</td>
<td>$= 64.8 \text{ kips} &gt; 32.0 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

**Shear Rupture of Plate**

$$A_{nv} = \frac{3\% \text{ in.} \cdot 12.0 \text{ in.} - 4(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})}{2} = 3.19 \text{ in.}^2$$

From AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td></td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = \phi F_y A_{nv}$</td>
<td>$\frac{R_n}{\Omega} = \frac{0.60 F_y A_{nv}}{\Omega}$</td>
<td>$\frac{R_n}{\Omega} = \frac{0.60 F_y A_{nv}}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.75(0.60)(58 \text{ ksi})(3.19 \text{ in.}^2)$</td>
<td>$= 0.60(58 \text{ ksi})(3.19 \text{ in.}^2)$</td>
<td>$= 0.60(58 \text{ ksi})(3.19 \text{ in.}^2)$</td>
</tr>
<tr>
<td>$= 83.3 \text{ kips} &gt; 48.0 \text{ kip}$ o.k.</td>
<td>$= 55.5 \text{ kips} &gt; 32.0 \text{ kips}$ o.k.</td>
<td>$= 55.5 \text{ kips} &gt; 32.0 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>
Block Shear Rupture of Plate

$L_{eh} = L_{rv} = 1\frac{1}{2}$ in.

From AISC Specification Equation J4-5:

$$U_{bs} = 1.0$$

Tension rupture component from AISC Manual Table 9-3a:

$$\phi U_{bs} F_u A_{nt} = 1.0(46.2 \text{ kips/in.})(\% \text{ in.})$$

Shear yielding component from AISC Manual Table 9-3b:

$$\phi 0.60 F_y A_{gy} = 170 \text{ kips/in.}(\% \text{ in.})$$

Shear rupture component from AISC Manual Table 9-3c:

$$\phi 0.60 F_u A_{nv} = 194 \text{ kips/in.}(\% \text{ in.})$$

$$\phi R_n = (46.2 \text{ kips/in.} + 170 \text{ kips/in.})(\% \text{ in.})$$

$$= 81.1 \text{ kips} > 48.0 \text{ kips} \text{ o.k.}$$

Tension rupture component from AISC Manual Table 9-3a:

$$U_{bs} = 1.0$$

Shear yielding component from AISC Manual Table 9-3b:

$$0.60 F_y A_{gy} = 113 \text{ kips/in.}(\% \text{ in.})$$

Shear rupture component from AISC Manual Table 9-3c:

$$0.60 F_u A_{nv} = 129 \text{ kips/in.}(\% \text{ in.})$$

$$R_n = (30.8 \text{ kips/in.} + 113 \text{ kips/in.})(\% \text{ in.})$$

$$= 53.9 \text{ kips} > 32.0 \text{ kips} \text{ o.k.}$$

Use PL\% in. × 8 in. × 1 ft 0 in.
EXAMPLE II.A-22  BOLTED BRACKET PLATE DESIGN

Given:

Design a bracket plate to support the following loads:

\[ P_D = 6 \text{ kips} \]
\[ P_L = 18 \text{ kips} \]

Use \( \frac{3}{8} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and an ASTM A36 plate. Assume the column has sufficient available strength for the connection.
Solution:

For discussion of the design of a bracket plate, see AISC Manual Part 15.

From AISC Manual Table 2-5, the material properties are as follows:

- Plate
  - ASTM A36
  - $F_y = 36$ ksi
  - $F_u = 58$ ksi

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$</td>
<td>$1.2(6\text{ kips}) + 1.6(18\text{ kips})$</td>
<td>$6\text{ kips} + 18\text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$36.0\text{ kips}$</td>
<td>$24.0\text{ kips}$</td>
</tr>
</tbody>
</table>

**Bolt Design**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolt shear from AISC Manual Table 7-1:</td>
<td>Bolt shear from AISC Manual Table 7-1:</td>
<td></td>
</tr>
<tr>
<td>$\phi r_n$</td>
<td>$17.9\text{ kips}$</td>
<td>$r_n/\Omega$</td>
</tr>
<tr>
<td>$= 17.9\text{ kips}$</td>
<td>$= 11.9\text{ kips}$</td>
<td></td>
</tr>
</tbody>
</table>

For bearing on the bracket plate:

- Try PL½ in. × 20 in., $L_e \geq 2$ in.
- From AISC Manual Table 7-5:
  - $\phi r_n = 78.3\text{ kips/bolt/in.}(\%\text{ in.})$
  - $= 29.4\text{ kips/bolt}$
- Bolt shear controls.

By interpolating AISC Manual Table 7-8 with $\theta = 0^\circ$, a 5½ in. gage with $s = 3$ in., $e_v = 12.0$ in., $n = 6$ and $C = 4.53$:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{min}} = \frac{R_u}{\phi r_n}$</td>
<td>$C_{\text{min}} = \frac{\Omega R_u}{r_n}$</td>
<td></td>
</tr>
<tr>
<td>$= \frac{36.0\text{ kips}}{17.9\text{ kips/bolt}}$</td>
<td>$= \frac{24.0\text{ kips}}{11.9\text{ kips/bolt}}$</td>
<td></td>
</tr>
<tr>
<td>$= 2.01$</td>
<td>$= 2.02$</td>
<td></td>
</tr>
<tr>
<td>$C = 4.53 &gt; 2.01$</td>
<td>o.k.</td>
<td>$C = 4.53 &gt; 2.02$</td>
</tr>
</tbody>
</table>

**Flexural Yielding of Bracket Plate on Line K**

The required strength is:
\[
M_a = P_a e \quad \text{(Manual Eq. 15-1a)} \quad M_a = P_a e \quad \text{(Manual Eq. 15-1b)}
\]
\[
= (36.0 \text{ kips})(12.0 \text{ in.} - 2\frac{3}{4} \text{ in.}) \quad = (24.0 \text{ kips})(12.0 \text{ in.} - 2\frac{3}{4} \text{ in.})
\]
\[
= 333 \text{ kip-in.} \quad = 222 \text{ kip-in.}
\]

From AISC Manual Equation 15-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_a = F_y Z)</td>
<td>(M_a = F_y Z)</td>
</tr>
<tr>
<td>(\phi = 0.90)</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(\phi M_n = \phi F_y Z)</td>
<td>(M_n = F_y Z)</td>
</tr>
<tr>
<td>(= 0.90(36 \text{ ksi}) \left{ \frac{\frac{3}{8} \text{ in.}(20.0 \text{ in.})^2}{4} \right})</td>
<td>(= \frac{36 \text{ ksi} \left(\frac{\frac{3}{8} \text{ in.}(20.0 \text{ in.})^2}{4}\right)}{\Omega})</td>
</tr>
<tr>
<td>(= 1,220 \text{ kip-in.} &gt; 333 \text{ kip-in.})</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(= 808 \text{ kip-in.} &gt; 222 \text{ kip-in.})</td>
<td>(= 1,670 \text{ kip-in.} &gt; 222 \text{ kip-in.})</td>
</tr>
</tbody>
</table>

**Flexural Rupture of Bracket Plate on Line K**

From Table 15-3, for a \(\frac{3}{8}\)-in.-thick bracket plate, with \(\frac{3}{4}\)-in. bolts and six bolts in a row, \(Z_{net} = 21.5 \text{ in.}^3\).

From AISC Manual Equation 15-3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_a = F_y Z)</td>
<td>(M_a = F_y Z)</td>
</tr>
<tr>
<td>(\phi = 0.75)</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>(\phi M_n = \phi F_y Z_{net})</td>
<td>(M_n = F_y Z_{net})</td>
</tr>
<tr>
<td>(= 0.75(58 \text{ ksi})(21.5 \text{ in.}^3))</td>
<td>(\Omega = \frac{58 \text{ ksi}(21.5 \text{ in.}^3)}{2.00})</td>
</tr>
<tr>
<td>(= 935 \text{ kip-in.} &gt; 333 \text{ kip-in.})</td>
<td>(= 624 \text{ kip-in.} &gt; 222 \text{ kip-in.})</td>
</tr>
</tbody>
</table>

**Shear Yielding of Bracket Plate on Line J**

\[
\tan \theta = \frac{b}{a} = \frac{15\frac{3}{4} \text{ in.}}{20.0 \text{ in.}} = 0.75 \quad \theta = 37.3^\circ
\]

\[
b' = a \sin \theta = 20.0 \text{ in.} \times (\sin 37.3^\circ) = 12.1 \text{ in.}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_r = V_a = P_a \sin \theta)</td>
<td>(V_r = V_a = P_a \sin \theta)</td>
</tr>
<tr>
<td>(= 36.0 \text{ kips}(\sin 37.3^\circ))</td>
<td>(= 24.0 \text{ kips}(\sin 37.3^\circ))</td>
</tr>
<tr>
<td>(= 21.8 \text{ kips})</td>
<td>(= 14.5 \text{ kips})</td>
</tr>
<tr>
<td>(V_n = 0.6 F_y b')</td>
<td>(V_n = 0.6 F_y b')</td>
</tr>
<tr>
<td>(= 0.6(36 \text{ ksi})(\frac{3}{8} \text{ in.})(12.1 \text{ in.}))</td>
<td>(= 0.6(36 \text{ ksi})(\frac{3}{8} \text{ in.})(12.1 \text{ in.}))</td>
</tr>
<tr>
<td>(= 98.0 \text{ kips})</td>
<td>(= 98.0 \text{ kips})</td>
</tr>
</tbody>
</table>
Local Yielding and Local Buckling of Bracket Plate on Line J

For local yielding:

\[ F_{cy} = F_y \]
\[ = 36 \text{ ksi} \]  

(Manual Eq. 15-13)

For local buckling:

\[ F_{cr} = QF_y \]  

(Manual Eq. 15-14)

where

\[ a' = \frac{a}{\cos \theta} \]
\[ = \frac{20.0 \text{ in.}}{\cos 37.3^\circ} \]
\[ = 25.1 \text{ in.} \]  

(Manual Eq. 15-18)

\[ \lambda = \frac{\left( \frac{b'}{t} \right) \sqrt{F_y}}{5 \sqrt{475 + 1,120 \left( \frac{b'}{a'} \right)^2}} \]
\[ = \frac{\left( 12.1 \text{ in.} \right)}{\left( \frac{3/4 \text{ in.}}{25.1 \text{ in.}} \right)^2} \sqrt{36 \text{ ksi}} \]
\[ = 1.43 \]  

(Manual Eq. 15-17)

Because 1.43 < \lambda,

\[ Q = \frac{1.30}{\lambda^2} \]
\[ = \frac{1.30}{\left(1.43\right)^2} \]
\[ = 0.636 \]  

(Manual Eq. 15-16)

\[ F_{cr} = QF_y \]
\[ = 0.636(36 \text{ ksi}) \]
\[ = 22.9 \text{ ksi} \]  

(Manual Eq. 15-14)

Local buckling controls over local yielding.

Interaction of Normal and Flexural Strengths

Check that Manual Equation 15-10 is satisfied:
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

### LRFD

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_r = N_a = P_a \cos \theta )</td>
<td>(Manual Eq. 15-9a)</td>
</tr>
<tr>
<td>[ N_a = F_{crt}(b') ]</td>
<td>(Manual Eq. 15-11)</td>
</tr>
<tr>
<td>( M_r = M_a = P_a e - N_a (b'/2) )</td>
<td>(Manual Eq. 15-8a)</td>
</tr>
<tr>
<td>( M_n = F_{crt}(b') )</td>
<td>(Manual Eq. 15-12)</td>
</tr>
<tr>
<td>( N_c = \phi N_n = 0.90(104 \text{ kips}) )</td>
<td></td>
</tr>
<tr>
<td>( M_c = \phi M_n = 0.90(314 \text{ kip-in.}) )</td>
<td></td>
</tr>
<tr>
<td>[ \frac{N_r + M_r}{N_c + M_c} \leq 1.0 ]</td>
<td>(Manual Eq. 15-10)</td>
</tr>
</tbody>
</table>

### ASD

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_r = N_a = P_a \cos \theta )</td>
<td>(Manual Eq. 15-9b)</td>
</tr>
<tr>
<td>( N_n = F_{crt}(b') )</td>
<td>(Manual Eq. 15-11)</td>
</tr>
<tr>
<td>( M_r = M_a = P_a e - N_a (b'/2) )</td>
<td>(Manual Eq. 15-8b)</td>
</tr>
<tr>
<td>( M_n = F_{crt}(b') )</td>
<td>(Manual Eq. 15-12)</td>
</tr>
<tr>
<td>( \Omega = 1.67 )</td>
<td></td>
</tr>
<tr>
<td>[ \frac{N_r}{\Omega} \leq 1.0 ]</td>
<td></td>
</tr>
</tbody>
</table>

**Shear Yielding of Bracket Plate on Line K (using AISC Specification Equation J4-3)**

\[
R_n = 0.60 F_{crt} A_{gv} \\
= 0.6(36 \text{ ksi})(20.0 \text{ in.})(1/8 \text{ in.}) \\
= 162 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(162 \text{ kips}) )</td>
<td>( R_n = 162 \text{ kips} )</td>
</tr>
<tr>
<td>( = 162 \text{ kips} &gt; 36.0 \text{ kips} )</td>
<td>( \frac{R_n}{\Omega} = 162 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Shear Rupture of Bracket Plate on Line K (using AISC Specification Equation J4-4)**

\[
A_{nv} = \left[ 20.0 \text{ in.} - 6 \left(1/16 \text{ in.} + 1/16 \text{ in.} \right) \right] (1/8 \text{ in.}) \\
= 5.53 \text{ in.}^2
\]

---

\[ \]
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = \phi 0.60 F_u A_{nv}$</td>
<td>$R_n = \frac{0.60 F_u A_{nv}}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.75(0.60)(58 \text{ ksi})(5.53 \text{ in.}^2)$</td>
<td>$= \frac{0.60(58 \text{ ksi})(5.53 \text{ in.}^2)}{2.00}$</td>
</tr>
<tr>
<td>$= 144 \text{ kips} &gt; 36.0 \text{ kips}$</td>
<td>$= 96.2 \text{ kips} &gt; 24.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

**o.k.**
EXAMPLE II.A-23  WELDED BRACKET PLATE DESIGN

Given:

Design a welded bracket plate, using 70-ksi electrodes, to support the following loads:

\[ P_D = 9 \text{ kips} \]
\[ P_L = 27 \text{ kips} \]

Assume the column has sufficient available strength for the connection. Use an ASTM A36 plate.

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

Plate
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From Chapter 2 of ASCE/SEI 7, the required strength is:
Try PL½ in. × 18 in.

Try a C-shaped weld with kl = 3 in. and l = 18 in.

\[
k = \frac{kl}{l} = \frac{3.00 \text{ in.}}{18.0 \text{ in.}} = 0.167
\]

\[
xl = \frac{(kl)^2}{2(kl) + l} = \frac{(3.00 \text{ in.})^2}{2(3.00 \text{ in.}) + 18.0 \text{ in.}} = 0.375 \text{ in.}
\]

\[
al = 11.0 \text{ in.} - 0.375 \text{ in.} = 10.6 \text{ in.}
\]

\[
al = \frac{al}{l} = \frac{10.6 \text{ in.}}{18.0 \text{ in.}} = 0.589
\]

Interpolate AISC Manual Table 8-8 using \( \theta = 0^\circ \), \( k = 0.167 \), and \( a = 0.589 \).

\[C = 1.49\]

From AISC Manual Table 8-3:

\[C_1 = 1.0 \text{ for E70 electrode}\]

From AISC Manual Equation 8-13:

\[
\phi = 0.75
\]

\[
D_{\text{min}} = \frac{P_u}{\phi CC_l} = \frac{54.0 \text{ kips}}{0.75(1.49)(1.0)(18.0 \text{ in.})} = 2.68 \rightarrow 3 \text{ sixteenths}
\]

\[
\Omega = 2.00
\]

\[
D_{\text{min}} = \frac{\Omega P_u}{CC_l} = \frac{2.00(36.0 \text{ kips})}{1.49(1.0)(18.0 \text{ in.})} = 2.68 \rightarrow 3 \text{ sixteenths}
\]
\[ w_{\text{max}} = \frac{1}{2} \text{ in.} - \frac{1}{16} \text{ in.} = \frac{7}{16} \text{ in.} \geq \frac{3}{16} \text{ in.} \quad \text{o.k.} \]

From AISC Specification Table J2.4:

\[ w_{\text{min}} = \frac{3}{16} \text{ in.} \]

Use a \( \frac{3}{16} \)-in. fillet weld.

**Flexural Yielding of Bracket Plate**

Conservatively taking the required moment strength of the plate as equal to the moment strength of the weld group,

\[
M_a = P_e (a f) = 54.0 \text{ kips}(10.6 \text{ in.}) = 572 \text{ kip-in.}
\]

\[
M_a = F_y z = 36.0 \text{ kips}(10.6 \text{ in.}) = 382 \text{ kip-in.}
\]

\[
M_a = F_y Z = \frac{\frac{1}{2} \text{ in.}(18.0 \text{ in.})^2}{4} = 1,460 \text{ kip-in.}
\]

\[
\phi = 0.90, \quad \phi M_a = 0.90(1,460 \text{ kip-in.}) = 1,310 \text{ kip-in.}
\]

\[ 1,310 \text{ kip-in.} > 572 \text{ kip-in.} \quad \text{o.k.} \]

\[
\phi = 0.90, \quad \phi M_a = 0.90(1,460 \text{ kip-in.}) = 1,310 \text{ kip-in.}
\]

\[
\Omega = 1.67, \quad M_a = \frac{1,460 \text{ kip-in.}}{1.67} = 874 \text{ kip-in.}
\]

\[ 874 \text{ kip-in.} > 382 \text{ kip-in.} \quad \text{o.k.} \]

**Shear Yielding of Bracket Plate on Line J**

\[
\tan \theta = \frac{b}{a}
\]

\[ 11 \frac{3}{4} \text{ in.} \quad 18 \text{ in.} \]

\[ \theta = 33.1^\circ \]

\[ b' = a \sin \theta = 18.0 \text{ in.} \sin 33.1^\circ = 9.83 \text{ in.} \]

\[
V_r = V_a = P_a \sin \theta = 54.0 \text{ kips}(\sin 33.1^\circ) = 29.5 \text{ kips}
\]

\[
V_r = V_a = P_a \sin \theta = 36.0 \text{ kips}(\sin 33.1^\circ) = 19.7 \text{ kips}
\]

\[
V_n = 0.6F_y b'_t = 0.6(36 \text{ ksi})(\frac{1}{2} \text{ in.})(9.83 \text{ in.}) = 106 \text{ kips}
\]

\[
V_n = 0.6F_y b'_t = 0.6(36 \text{ ksi})(\frac{1}{2} \text{ in.})(9.83 \text{ in.}) = 106 \text{ kips}
\]
Local Yielding and Local Buckling of Bracket Plate on Line J

For local yielding:

\[ F_{cr} = F_y \]
\[ = 36 \text{ ksi} \]  

(Manual Eq. 15-13)

For local buckling:

\[ F_{cr} = QF_y \]

(Manual Eq. 15-14)

where

\[ a' = \frac{a}{\cos \theta} \]
\[ = \frac{18.0 \text{ in.}}{\cos 33.1^\circ} \]
\[ = 21.5 \text{ in.} \]

(Manual Eq. 15-18)

\[ \lambda = \frac{\left( \frac{b'}{t} \right) \sqrt{F_y}}{5 \sqrt{475 + 1.120 \left( \frac{b'}{a'} \right)^2}} \]
\[ = \frac{9.83 \text{ in.}}{0.5 \text{ in.}} \sqrt{36 \text{ ksi}} \]
\[ = 0.886 \]

Because \( 0.70 < \lambda \leq 1.41 \),

\[ Q = 1.34 - 0.486\lambda \]
\[ = 1.34 - 0.486(0.886) \]
\[ = 0.909 \]

(Manual Eq. 15-15)

\[ F_{cr} = QF_y \]
\[ = 0.909(36 \text{ ksi}) \]
\[ = 32.7 \text{ ksi} \]

(Manual Eq. 15-14)

Local buckling controls over local yielding. Therefore, the required and available normal and flexural strengths are determined as follows:
\[
N_r = N_a = P_a \cos \theta \\
= 54.0 \text{kips}(\cos 33.1^\circ) \\
= 45.2 \text{kips} \\
\]
\[
N_a = F_c tb' \\
= 32.7 \text{ksi}(\frac{1}{2} \text{ in.})(9.83 \text{ in.}) \\
= 161 \text{kips} \\
\]
\[
\phi = 0.90 \\
N_c = \phi N_a = 0.90(161 \text{kips}) \\
= 145 \text{kips} \\
\]
\[
M_r = M_a = P_a e - N_a (b'/2) \\
= 54.0 \text{kips}(8.00 \text{ in.}) - 45.2 \text{kips}(9.83 \text{ in.}/2) \\
= 210 \text{ kip-in.} \\
\]
\[
M_n = \frac{F_{cr} (b')^2}{4} \\
= \frac{32.7 \text{ksi}(\frac{1}{2} \text{ in.})(9.83 \text{ in.})^2}{4} \\
= 395 \text{ kip-in.} \\
\]
\[
M_c = \phi M_n = 0.90(395 \text{ kip-in.}) \\
= 356 \text{ kip-in.} \\
\]
\[
\frac{N_r + M_r}{N_c} \leq 1.0 \\
\leq 0.902 \leq 1.0 \\
\text{o.k.} \\
\]

**Shear Yielding of Bracket Plate on Line K**

\[
R_n = 0.60 F_y A_{gy} \\
= 0.60(36 \text{ksi})(18.0 \text{ in.})(\frac{1}{2} \text{ in.}) \\
= 194 \text{kips} \\
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_r = N_a = P_a \cos \theta) (Manual Eq. 15-9a)</td>
<td>(N_r = N_a = P_a \cos \theta) (Manual Eq. 15-9b)</td>
</tr>
<tr>
<td>(= 36.0 \text{kips}(\cos 33.1^\circ))</td>
<td>(= 30.2 \text{kips})</td>
</tr>
<tr>
<td>(N_a = F_c tb') (Manual Eq. 15-11)</td>
<td>(N_a = F_c tb') (Manual Eq. 15-11)</td>
</tr>
<tr>
<td>(= 32.7 \text{ksi}(\frac{1}{2} \text{ in.})(9.83 \text{ in.}))</td>
<td>(= 161 \text{kips})</td>
</tr>
<tr>
<td>(\phi = 0.90)</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(N_c = \phi N_a = 0.90(161 \text{kips}))</td>
<td>(N_c = \frac{N_a}{\Omega} = 161 \text{kips} \cdot \frac{1}{1.67} = 96.4 \text{kips})</td>
</tr>
<tr>
<td>(= 145 \text{kips})</td>
<td>(= 96.4 \text{kips})</td>
</tr>
<tr>
<td>(M_r = M_a = P_a e - N_a (b'/2)) (Manual Eq. 15-8a)</td>
<td>(M_r = M_a = P_a e - N_a (b'/2)) (Manual Eq. 15-8b)</td>
</tr>
<tr>
<td>(= 54.0 \text{kips}(8.00 \text{ in.}) - 45.2 \text{kips}(9.83 \text{ in.}/2))</td>
<td>(= 36.0 \text{kips}(8.00 \text{ in.}) - 30.1 \text{kips}(9.83 \text{ in.}/2))</td>
</tr>
<tr>
<td>(= 210 \text{ kip-in.})</td>
<td>(= 140 \text{ kip-in.})</td>
</tr>
<tr>
<td>(M_n = \frac{F_{cr} (b')^2}{4}) (Manual Eq. 15-12)</td>
<td>(M_n = \frac{F_{cr} (b')^2}{4}) (Manual Eq. 15-12)</td>
</tr>
<tr>
<td>(= \frac{32.7 \text{ksi}(\frac{1}{2} \text{ in.})(9.83 \text{ in.})^2}{4})</td>
<td>(= \frac{32.7 \text{ksi}(\frac{1}{2} \text{ in.})(9.83 \text{ in.})^2}{4})</td>
</tr>
<tr>
<td>(= 395 \text{ kip-in.})</td>
<td>(= 395 \text{ kip-in.})</td>
</tr>
<tr>
<td>(M_c = \phi M_n = 0.90(395 \text{ kip-in.}))</td>
<td>(M_c = \frac{M_n}{\Omega} = \frac{395 \text{ kip-in.}}{1.67} = 237 \text{ kip-in.})</td>
</tr>
<tr>
<td>(= 356 \text{ kip-in.})</td>
<td>(= 237 \text{ kip-in.})</td>
</tr>
<tr>
<td>(\frac{N_r + M_r}{N_c} \leq 1.0) (Manual Eq. 15-10)</td>
<td>(\frac{N_r + M_r}{N_c} \leq 1.0) (Manual Eq. 15-10)</td>
</tr>
<tr>
<td>(\leq 0.902 \leq 1.0)</td>
<td>(\leq 0.902 \leq 1.0)</td>
</tr>
<tr>
<td>\text{o.k.}</td>
<td>\text{o.k.}</td>
</tr>
</tbody>
</table>

\[
\phi = 1.00 \\
\phi R_n = 1.00(194 \text{kips}) \\
= 194 \text{kips} > 54.0 \text{kips} \\
\text{o.k.} \\
\]

\[
\Omega = 1.50 \\
\frac{R_n}{\Omega} = \frac{194 \text{kips}}{1.50} = 129 \text{kips} > 36.0 \text{kips} \\
\text{o.k.} \\
\]

**Design Examples V14.1**  
**AMERICAN INSTITUTE OF STEEL CONSTRUCTION**
EXAMPLE II.A-24  ECCENTRICALLY LOADED BOLT GROUP (IC METHOD)

Given:

Determine the largest eccentric force, acting vertically and at a 15° angle, which can be supported by the available shear strength of the bolts using the instantaneous center of rotation method. Use 3/8-in.-diameter ASTM A325-N or F1852-N bolts in standard holes. Assume that bolt shear controls over bearing. Use AISC Manual Table 7-8.

Solution A (θ = 0°):

Assume the load is vertical (θ = 0°) as shown:

From AISC Manual Table 7-8, with θ = 0°, s = 3.00 in., ε = 16.0 in. and n = 6:

\[ C = 3.55 \]

From AISC Manual Table 7-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ ΦP_n = 24.3 \text{ kips} ]</td>
<td>[ \frac{r_n}{Ω} = 16.2 \text{ kips} ]</td>
</tr>
<tr>
<td>[ ΦR_n = CΦP_n ]</td>
<td>[ \frac{R_n}{Ω} = C \frac{r_n}{Ω} ]</td>
</tr>
<tr>
<td>[ = 3.55(24.3 \text{ kips}) ]</td>
<td>[ = 3.55(16.2 \text{ kips}) ]</td>
</tr>
<tr>
<td>[ = 86.3 \text{ kips} ]</td>
<td>[ = 57.5 \text{ kips} ]</td>
</tr>
</tbody>
</table>

Thus, \( P_n \) must be less than or equal to 86.3 kips. Thus, \( P_n \) must be less than or equal to 57.5 kips.

Note: The eccentricity of the load significantly reduces the shear strength of the bolt group.
Solution B \((\theta = 15^\circ)\):

Assume the load acts at an angle of 15° with respect to vertical \((\theta = 15^\circ)\) as shown:

\[ e_x = 16.0 \text{ in.} + 9.00 \text{ in.} \times (\tan 15^\circ) \]
\[ = 18.4 \text{ in.} \]

By interpolating AISC Manual Table 7-8, with \( \theta = 15^\circ \), \( s = 3.00 \text{ in.} \), \( e_x = 18.4 \text{ in.} \), and \( n = 6 \):

\[ C = 3.21 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_e = C \phi r_e )</td>
<td>( \frac{R_e}{\Omega} = C \frac{r_e}{\Omega} )</td>
</tr>
<tr>
<td>( = 3.21(24.3 \text{ kips}) )</td>
<td>( = 3.21(16.2 \text{ kips}) )</td>
</tr>
<tr>
<td>( = 78.0 \text{ kips} )</td>
<td>( = 52.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Thus, \( P_x \) must be less than or equal to 78.0 kips.

Thus, \( P_y \) must be less than or equal to 52.0 kips.
EXAMPLE II.A-25 ECCENTRICALLY LOADED BOLT GROUP (ELASTIC METHOD)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the bolts using the elastic method for $\theta = 0^\circ$. Compare the result with that of the previous example. Use 3/8-in.-diameter ASTM A325-N or F1852-N bolts in standard holes. Assume that bolt shear controls over bearing.

Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct shear force per bolt:</td>
<td>Direct shear force per bolt:</td>
</tr>
<tr>
<td>$r_{psu} = 0$</td>
<td>$r_{psu} = 0$</td>
</tr>
<tr>
<td>$r_{pya} = \frac{P_b}{n}$ (Manual Eq. 7-2a)</td>
<td>$r_{pya} = \frac{P_a}{n}$ (Manual Eq. 7-2b)</td>
</tr>
<tr>
<td>$= \frac{P_b}{12}$</td>
<td>$= \frac{P_a}{12}$</td>
</tr>
<tr>
<td>Additional shear force due to eccentricity:</td>
<td>Additional shear force due to eccentricity:</td>
</tr>
<tr>
<td>$I_x \approx \Sigma y^2$</td>
<td>Polar moment of inertia:</td>
</tr>
<tr>
<td>$= 4(7.50 \text{ in.})^2 + 4(4.50 \text{ in.})^2 + 4(1.50 \text{ in.})^2$</td>
<td>$I_x \approx \Sigma y^2$</td>
</tr>
<tr>
<td>$= 315 \frac{\text{in.}^4}{\text{in.}^2}$</td>
<td>$= 4(7.50 \text{ in.})^2 + 4(4.50 \text{ in.})^2 + 4(1.50 \text{ in.})^2$</td>
</tr>
<tr>
<td>$I_y \approx \Sigma x^2$</td>
<td>$= 315 \frac{\text{in.}^4}{\text{in.}^2}$</td>
</tr>
<tr>
<td>$= 12(2.75 \text{ in.})^2$</td>
<td>$I_y \approx \Sigma x^2$</td>
</tr>
<tr>
<td>$= 90.8 \frac{\text{in.}^4}{\text{in.}^2}$</td>
<td>$= 12(2.75 \text{ in.})^2$</td>
</tr>
<tr>
<td>$= 90.8 \frac{\text{in.}^4}{\text{in.}^2}$</td>
<td></td>
</tr>
</tbody>
</table>
\[
I_p \approx I_x + I_y
\]
\[
= 315 \frac{\text{in.}^4}{\text{in.}^2} + 90.8 \frac{\text{in.}^4}{\text{in.}^2}
\]
\[
= 406 \frac{\text{in.}^4}{\text{in.}^2}
\]

\[
r_{mxA} = \frac{P_a ec_y}{I_p}
\]
\[
= \frac{P_a (16.0 \text{ in.})(7.50 \text{ in.})}{406 \text{ in.}^4/\text{in.}^2}
\]
\[
= 0.296 P_a
\]

\[
r_{myA} = \frac{P_a ec_x}{I_p}
\]
\[
= \frac{P_a (16.0 \text{ in.})(2.75 \text{ in.})}{406 \text{ in.}^4/\text{in.}^2}
\]
\[
= 0.108 P_a
\]

Resultant shear force:
\[
r_u = \sqrt{(r_{pxu} + r_{mxA})^2 + (r_{pyu} + r_{myu})^2}
\]
\[
= \sqrt{(0 + 0.296 P_a)^2 + \left(\frac{P_a}{12} + 0.108 P_a\right)^2}
\]
\[
= 0.353 P_a
\]

Since \( r_u \) must be less than or equal to the available strength,
\[
P_a \leq \frac{\phi r_u}{0.353}
\]
\[
= 24.3 \text{ kips}
\]
\[
= 68.8 \text{ kips}
\]

Note: The elastic method, shown here, is more conservative than the instantaneous center of rotation method, shown in Example II.A-24.
**EXAMPLE II.A-26  ECCENTRICALLY LOADED WELD GROUP (IC METHOD)**

**Given:**

Determine the largest eccentric force, acting vertically and at a 75° angle, that can be supported by the available shear strength of the weld group, using the instantaneous center of rotation method. Use a 3/8-in. fillet weld and 70-ksi electrodes. Use AISC *Manual* Table 8-8.

**Solution A (θ = 0°):**

Assume that the load is vertical (θ = 0°) as shown:

- \( l = 10.0 \text{ in.} \)
- \( kl = 5.00 \text{ in.} \)

\[
k = \frac{kl}{l} = \frac{5.00 \text{ in.}}{10.0 \text{ in.}} = 0.500
\]

\[
xl = \frac{(kl)^2}{2(kl) + l} = \frac{(5.00 \text{ in.})^2}{2(5.00 \text{ in.}) + 10.0 \text{ in.}} = 1.25 \text{ in.}
\]

\[
xl + al = 10.0 \text{ in.}
\]

1.25 in. + \( a(10.0 \text{ in.}) = 10.0 \text{ in.} \)

\( a = 0.875 \)

By interpolating AISC *Manual* Table 8-8, with \( θ = 0° \), \( a = 0.875 \) and \( k = 0.500 \):

\( C = 1.88 \)
From AISC Manual Equation 8-13:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>0.75</td>
<td>2.00</td>
</tr>
<tr>
<td>( \phi R_e = \phi CC_i D l )</td>
<td>( = 0.75(1.88)(1.0)(6 \text{ sixteenths})(10.0 \text{ in.}) )</td>
<td>( R_a = \frac{CC_i D l}{\Omega} )</td>
</tr>
<tr>
<td></td>
<td>( = 84.6 \text{ kips} )</td>
<td>( = 1.88(1.0)(6 \text{ sixteenths})(10.0 \text{ in.}) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 2.00 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 56.4 \text{ kips} )</td>
</tr>
</tbody>
</table>

Thus, \( P_u \) must be less than or equal to 84.6 kips. Thus, \( P_a \) must be less than or equal to 56.4 kips.

Note: The eccentricity of the load significantly reduces the shear strength of this weld group as compared to the concentrically loaded case.

**Solution B (\( \theta = 75^\circ \)):**

Assume that the load acts at an angle of 75° with respect to vertical (\( \theta = 75^\circ \)) as shown:

As determined in Solution A,

\( k = 0.500 \) and \( x l = 1.25 \text{ in.} \).

\( e_x = al \)

\( = \frac{7.00 \text{ in.}}{\sin 15^\circ} \)

\( = 27.0 \text{ in.} \).

\( a = \frac{e_x}{l} \)

\( = \frac{27.0 \text{ in.}}{10.0 \text{ in.}} \)

\( = 2.70 \)
By interpolating AISC Manual Table 8-8, with $\theta = 75^o$, $a = 2.70$ and $k = 0.500$:

$$C = 1.99$$

From AISC Manual Equation 8-13:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = \phi CC_1 Dl$</td>
<td>$R_n = \frac{CC_1 Dl}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.75(1.99)(1.0)(6 \text{ sixteenths})(10.0 \text{ in.})$</td>
<td>$= 1.99(1.0)(6 \text{ sixteenths})(10.0 \text{ in.})$</td>
</tr>
<tr>
<td>$= 89.6 \text{ kips}$</td>
<td>$= 59.7 \text{ kips}$</td>
</tr>
</tbody>
</table>

Thus, $P_n$ must be less than or equal to $89.6 \text{ kips}$. Thus, $P_n$ must be less than or equal to $59.7 \text{ kips}$. 
EXAMPLE II.A-27  ECCENTRICALLY LOADED WELD GROUP (ELASTIC METHOD)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the welds in the connection, using the elastic method. Compare the result with that of the previous example. Use \(\frac{3}{8}\)-in. fillet welds and 70-ksi electrodes.

Solution:

*Direct Shear Force per Inch of Weld*

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{pax} = 0 )</td>
<td>( r_{pax} = 0 )</td>
</tr>
<tr>
<td>( r_{pay} = \frac{P_u}{I} )</td>
<td>( r_{pay} = \frac{P_u}{I} )</td>
</tr>
<tr>
<td>( = \frac{P_u}{20.0 \text{ in.}} )</td>
<td>( = \frac{P_u}{20.0 \text{ in.}} )</td>
</tr>
<tr>
<td>( = 0.0500 \frac{P_u}{\text{in.}} )</td>
<td>( = 0.0500 \frac{P_u}{\text{in.}} )</td>
</tr>
</tbody>
</table>

*Additional Shear Force due to Eccentricity*

Determine the polar moment of inertia referring to the AISC Manual Figure 8-6:

\[
I_x = \frac{I^3}{12} + 2(kl)(y^2)
\]

\[
= \frac{(10.0 \text{ in.})^3}{12} + 2(5.00 \text{ in.})(5.00 \text{ in.})^2
\]

\[
= 333 \text{ in.}^4/\text{in.}
\]

\[
I_y = \frac{2(kl)^3}{12} + 2(kl)\left(\frac{kl}{2} - x_l\right)^2 + I(x_l)^2
\]

\[
= \frac{2(5.00 \text{ in.})^3}{12} + 2(5.00 \text{ in.})(2.50 \text{ in.} - 1\frac{1}{4} \text{ in.})^2 + (10.0 \text{ in.})(1\frac{1}{4} \text{ in.})^2
\]

\[
= 52.1 \text{ in.}^4/\text{in.}
\]
\[ I_p = I_x + I_y \]
\[ = 333 \text{ in.}^4/\text{in.} + 52.1 \text{ in.}^4/\text{in.} \]
\[ = 385 \text{ in.}^4/\text{in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ r_{max} = \frac{P_a e c_y}{I_p} ] (Manual Eq. 8-9a)</td>
<td>[ r_{max} = \frac{P_a e c_x}{I_p} ] (Manual Eq. 8-9b)</td>
</tr>
<tr>
<td>[ = \frac{P_a (8.75 \text{ in.})(5.00 \text{ in.})}{385 \text{ in.}^4/\text{in.}} ]</td>
<td>[ = \frac{P_a (8.75 \text{ in.})(5.00 \text{ in.})}{385 \text{ in.}^4/\text{in.}} ]</td>
</tr>
<tr>
<td>[ = 0.114P_a ] in.</td>
<td>[ = 0.114P_a ] in.</td>
</tr>
<tr>
<td>[ r_{may} = \frac{P_a e c_y}{I_p} ] (Manual Eq. 8-10a)</td>
<td>[ r_{may} = \frac{P_a e c_x}{I_p} ] (Manual Eq. 8-10b)</td>
</tr>
<tr>
<td>[ = \frac{P_a (8.75 \text{ in.})(3.75 \text{ in.})}{385 \text{ in.}^4/\text{in.}} ]</td>
<td>[ = \frac{P_a (8.75 \text{ in.})(3.75 \text{ in.})}{385 \text{ in.}^4/\text{in.}} ]</td>
</tr>
<tr>
<td>[ = 0.0852P_a ] in.</td>
<td>[ = 0.0852P_a ] in.</td>
</tr>
</tbody>
</table>

Resultant shear force:
\[ r_u = \sqrt{(r_{pax} + r_{max})^2 + (r_{pay} + r_{may})^2} \] (Manual Eq. 8-11a)
\[ = \sqrt{\left(0 + 0.114P_a\right)^2 + \left(0.0500P_a + 0.0852P_a\right)^2} \]
\[ = \frac{0.177P_a}{\text{in.}} \]

Since \( r_u \) must be less than or equal to the available strength, from AISC Manual Equation 8-2a,
\[ r_u = 0.177P_a \leq \phi r_n \]
\[ P_a \leq \frac{\phi r_n}{0.177} \]
\[ \leq 1.392 \text{ kips/in.} \left(\frac{6 \text{ sixteens}}{0.177}\right) \]
\[ \leq 47.2 \text{ kips} \]

Resultant shear force:
\[ r_u = \sqrt{(r_{pax} + r_{max})^2 + (r_{pay} + r_{may})^2} \] (Manual Eq. 8-11b)
\[ = \sqrt{\left(0 + 0.114P_a\right)^2 + \left(0.0500P_a + 0.0852P_a\right)^2} \]
\[ = \frac{0.177P_a}{\text{in.}} \]

Since \( r_u \) must be less than or equal to the available strength, from AISC Manual Equation 8-2b,
\[ r_u = 0.177P_a \leq r_n /\Omega \]
\[ P_a \leq \frac{r_n /\Omega}{0.177} \]
\[ \leq 0.928 \text{ kip/in.} \left(\frac{6 \text{ sixteens}}{0.177}\right) \]
\[ \leq 31.5 \text{ kips} \]

Note: The strength of the weld group predicted by the elastic method, as shown here, is significantly less than that predicted by the instantaneous center of rotation method in Example II.A-26.
EXAMPLE II.A-28  ALL-BOLTED SINGLE-ANGLE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Design an all-bolted single-angle connection (Case I in Table 10-11) between an ASTM A992 W18×35 beam and an ASTM A992 W21×62 girder web, to support the following beam end reactions:

\[ R_D = 6.5 \text{ kips} \]
\[ R_L = 20 \text{ kips} \]

The top flange is cope 2 in. deep by 4 in. long, \( L_{cv} = 1\frac{1}{2} \text{ in.} \) and \( L_{eh} = 1\frac{1}{2} \text{ in.} \) (assumed to be 1¼ in. for calculation purposes to account for possible underrun in beam length). Use ¾-in.-diameter A325-N or F1852-N bolts in standard holes and an ASTM A36 angle.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
W18×35
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Girder
W21×62
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Angle
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)
From AISC *Manual* Table 1-1 and Figure 9-2, the geometric properties are as follows:

**Beam**

W18×35  
\( t_w = 0.300 \text{ in.} \)  
\( d = 17.7 \text{ in.} \)  
\( t_f = 0.425 \text{ in.} \)  
\( c = 4.00 \text{ in.} \)  
\( d_c = 2.00 \text{ in.} \)  
\( e = 5.25 \text{ in.} \)  
\( h_0 = 15.7 \text{ in.} \)

**Girder**

W21×62  
\( t_w = 0.400 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

- **LRFD ASD**
  \[ R_u = 1.2(6.5 \text{ kips}) + 1.6(20 \text{ kips}) = 39.8 \text{ kips} \]
  \[ R_d = 6.5 \text{ kips} + 20 \text{ kips} = 26.5 \text{ kips} \]

**Bolt Design**

Check eccentricity of connection.

For the 4-in. angle leg attached to the supported beam (W18x35):

\[ e = 2.75 \leq 3.00 \text{ in., therefore, eccentricity does not need to be considered for this leg.} \]

For the 3-in. angle leg attached to the supporting girder (W21x62):

\[ e = 1.75 \text{ in.} + \frac{0.300 \text{ in.}}{2} = 1.90 \text{ in.} \leq 2.50 \text{ in., therefore, AISC *Manual* Table 10-11 may conservatively be used for bolt shear.} \]

From AISC *Manual* Table 7-1, the single bolt shear strength is:

- **LRFD ASD**
  \[ \phi r_n = 17.9 \text{ kips} \]
  \[ r_n / \Omega = 11.9 \text{ kips} \]

From AISC *Manual* Table 7-5, the single bolt bearing strength on a ¾-in.-thick angle is:

- **LRFD ASD**
  \[ \phi r_n = 44.0 \text{ kips/in.}(¾ \text{ in.}) = 16.5 \text{ kips} \]
  \[ r_n / \Omega = 29.4 \text{ kips/in.}(¾ \text{ in.}) = 11.0 \text{ kips} \]

Bolt bearing is more critical than bolt shear in this example; thus, \( C_{min} = \frac{R_u}{\phi r_n} \).
\[
\begin{array}{c|c|c}
\text{LRFD} & \text{ASD} & \\
39.8 \text{ kips} & 26.5 \text{ kips} & \\
16.5 \text{ kips/bolt} & 11.0 \text{ kips/bolt} & \\
2.41 & 2.41 & \\
\end{array}
\]

Try a four-bolt connection.

From AISC Manual Table 10-11:

\[
C = 3.07 > 2.41 \quad \text{o.k.}
\]

The 3-in. leg will be shop bolted to the girder web and the 4-in. leg will be field bolted to the beam web.

**Shear Yielding of Angle**

\[
R_n = 0.60 F_v A_{gv}
\]

(Spec. Eq. J4-3) = 0.60(36 ksi)(11½ in.)(3 in.) = 93.2 kips

From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
</tbody>
</table>
| \[\phi R_n = 1.00(93.2 \text{ kips})\]
| = 93.2 kips > 39.8 kips | o.k. |

**Shear Rupture of Angle**

\[
A_{nv} = \frac{3}{8} \text{ in.}[11\frac{1}{2} \text{ in.} - 4(\frac{3}{16} \text{ in.} + \frac{3}{16} \text{ in.})]
\]

= 3.00 in.\(^2\)

\[
R_n = 0.60 F_u A_{nv}
\]

(Spec. Eq. J4-4) = 0.60(58 ksi)(3.00 in.\(^2\)) = 104 kips

From AISC Specification Section J4.2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
</tbody>
</table>
| \[\phi R_n = 0.75(104 \text{ kips})\]
| = 78.0 kips > 39.8 kips | o.k. |

**Block Shear Rupture of Angle**

\[
n = 4
\]
\[
L_{ov} = L_{oh} = 1\frac{1}{4} \text{ in.}
\]

From AISC Specification Equation J4-5:
### Tension Rupture Component

**LRFD**

\[ \phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gy}, \phi 0.60 F_u A_{ny}) \]

**ASD**

\[ R_n = \frac{U_{bs} F_u A_{nt}}{\Omega} + \min \left( \frac{0.60 F_y A_{gy}}{\Omega}, \frac{0.60 F_u A_{ny}}{\Omega} \right) \]

\( U_{bs} = 1.0 \)

- Tension rupture component from AISC Manual Table 9-3a:
  \[ \phi U_{bs} F_u A_{nt} = 1.0(35.3 \text{ kips/in.})(\% \text{ in.}) \]

- Shear yielding component from AISC Manual Table 9-3b:
  \[ \phi 0.60 F_y A_{gy} = 166 \text{ kips/in.}(\% \text{ in.}) \]

- Shear rupture component from AISC Manual Table 9-3c:
  \[ \phi 0.60 F_u A_{ny} = 188 \text{ kips/in.}(\% \text{ in.}) \]

**Result:**

\( \phi R_n = (35.3 \text{ kips/in.} + 166 \text{ kips/in.})(\% \text{ in.}) = 75.5 \text{ kips} > 39.8 \text{ kips} \)

**ASD Result:**

\( \frac{R_n}{\Omega} = (23.6 \text{ kips/in.} + 111 \text{ kips/in.})(\% \text{ in.}) = 50.5 \text{ kips} > 26.5 \text{ kips} \)

**Flexural Yielding of Support-Leg of Angle**

The required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = R_a e )</td>
<td>( M_u = R_a e )</td>
</tr>
<tr>
<td>( = 39.8 \text{ kips} \left(1\frac{1}{4} \text{ in.} + \frac{0.300 \text{ in.}}{2} \right) )</td>
<td>( = 26.5 \text{ kips} \left(1\frac{1}{4} \text{ in.} + \frac{0.300 \text{ in.}}{2} \right) )</td>
</tr>
<tr>
<td>( = 75.6 \text{ kip-in.} )</td>
<td>( = 50.4 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi M_n = \phi F_y Z_x )</td>
<td>( M_n = F_y Z_x )</td>
</tr>
<tr>
<td>( = 0.90(36 \text{ ksi}) \left[ \frac{% \text{ in.}(11\frac{1}{2} \text{ in.})^2}{4} \right] )</td>
<td>( = 36 \text{ ksi} \left[ \frac{% \text{ in.}(11\frac{1}{2} \text{ in.})^2}{4} \right] )</td>
</tr>
<tr>
<td>( = 402 \text{ kip-in.} &gt; 75.6 \text{ kip-in.} )</td>
<td>( = 267 \text{ kip-in.} &gt; 50.4 \text{ kip-in.} )</td>
</tr>
</tbody>
</table>

**Result:**

\( \phi M_n = 402 \text{ kip-in.} > 75.6 \text{ kip-in.} \)

**ASD Result:**

\( M_n = 267 \text{ kip-in.} > 50.4 \text{ kip-in.} \)
Flexural Rupture of Support-Leg of Angle

\[ Z_{net} = \frac{3}{8} \text{ in.} \left[ \frac{(11\frac{1}{2} \text{ in.})^2}{4} - 2(0.875 \text{ in.})(4.50 \text{ in.}) - 2(0.875 \text{ in.})(1.50 \text{ in.}) \right] \]

\[ = 8.46 \text{ in.}^3 \]

From AISC Manual Equation 9-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.75 )</td>
<td>( \Omega_b = 2.00 )</td>
</tr>
<tr>
<td>( \phi_b M_n = \phi_b F_u Z_{net} )</td>
<td>( M_n = \frac{F_u Z_{net}}{\Omega_b} )</td>
</tr>
<tr>
<td>( = 0.75(58 \text{ ksi})(8.46 \text{ in.}^3) )</td>
<td>( = \frac{58 \text{ ksi}(8.46 \text{ in.}^3)}{2.00} )</td>
</tr>
<tr>
<td>( = 368 \text{ kip-in.} &gt; 75.6 \text{ kip-in.} )</td>
<td>( = 245 \text{ kip-in.} &gt; 50.4 \text{ kip-in.} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Bolt Bearing and Block Shear Rupture of Beam Web

\( n = 4 \)
\( L_{eh} = L_{eh} = 1\frac{1}{2} \text{ in.} \)

\( (L_{eh} \text{ assumed to be } 1\frac{3}{4} \text{ in. for calculation purposes to provide for possible underrun in beam length.}) \)

From AISC Manual Table 10-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 257 \text{ kips/in.} (0.300 \text{ in.}) )</td>
<td>( \frac{R_n}{\Omega} )</td>
</tr>
<tr>
<td>( = 77.1 \text{ kips} &gt; 39.8 \text{ kips} )</td>
<td>( = 171 \text{ kips/in.} (0.300 \text{ in.}) )</td>
</tr>
<tr>
<td>o.k.</td>
<td>( = 51.3 \text{ kips} &gt; 26.5 \text{ kips} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note: For coped beam sections, the limit states of flexural yielding and local buckling should be checked independently per AISC Manual Part 9. The supported beam web should also be checked for shear yielding and shear rupture per AISC Specification Section J4.2. However, for the shallow cope in this example, these limit states do not govern. For an illustration of these checks, see Example II.A-4.
EXAMPLE IIA-29  BOLTED/WELDED SINGLE-ANGLE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a single-angle connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

\[ R_D = 9.0 \text{ kips} \\
R_L = 27 \text{ kips} \]

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts to connect the supported beam to an ASTM A36 single angle. Use 70-ksi electrode welds to connect the single angle to the column flange.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
W16×50
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Column
W14×90
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Angle
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)
From AISC Manual Table 1-1, the geometric properties are as follows:

**Beam**

W16×50  
\( t_w = 0.380 \text{ in.} \)  
\( d = 16.3 \text{ in.} \)  
\( t_f = 0.630 \text{ in.} \)

**Column**

W14×90  
\( t_f = 0.710 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( R_u = 1.2(9.0 \text{ kips}) + 1.6(27 \text{ kips}) \)  
\( = 54.0 \text{ kips} \) | \( R_u = 9.0 \text{ kips} + 27 \text{ kips} \)  
\( = 36.0 \text{ kips} \) |

**Single Angle, Bolts and Welds**

Check eccentricity of the connection.

For the 4-in. angle leg attached to the supported beam:

\( e = 2.75 \text{ in.} \leq 3.00 \text{ in.} \), therefore, eccentricity does not need to be considered for this leg.

For the 3-in. angle leg attached to the supporting column flange:

Since the half-web dimension of the W16×50 supported beam is less than \( \frac{1}{4} \text{ in.} \), AISC Manual Table 10-12 may conservatively be used.

Try a four-bolt single-angle (L4×3×\( \frac{3}{8} \)).

From AISC Manual Table 10-12:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| Bolt and angle available strength:  
\( \phi R_u = 71.4 \text{ kips} > 54.0 \text{ kips} \)  
\( \text{O.K.} \) | Bolt and angle available strength:  
\( \frac{R_u}{\Omega} = 47.6 \text{ kips} > 36.0 \text{ kips} \)  
\( \text{O.K.} \) |

Weld available strength:

With a \( \frac{3}{8} \)-in fillet weld size:

\( \phi R_u = 56.6 \text{ kips} > 54.0 \text{kips} \)  
\( \text{O.K.} \)  
\( \frac{R_u}{\Omega} = 37.8 \text{ kips} > 36.0 \text{kips} \)  
\( \text{O.K.} \)

**Support Thickness**

The minimum support thickness for the \( \frac{3}{8} \)-in. fillet welds is:

\[
 t_{min} = \frac{3.09 D}{F_u} \quad (\text{Manual Eq. 9-2})
\]
\[
\frac{3.09(3 \text{ sixteenths})}{65 \text{ ksi}} = 0.143 \text{ in.} < 0.710 \text{ in.} \quad \text{o.k.}
\]

Note: The minimum thickness values listed in Table 10-12 are for conditions with angles on both sides of the web.

Use a four-bolt single-angle L4×3×3\(\frac{3}{4}\). The 3-in. leg will be shop welded to the column flange and the 4-in. leg will be field bolted to the beam web.

**Supported Beam Web**

From AISC Manual Table 7-4, with \(s = 3.00 \text{ in.}, \frac{3}{4}\)-in.-diameter bolts and standard holes, the bearing strength of the beam web is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_e = \phi r_n f_w n)</td>
<td>(R_n = r_n f_w n)</td>
</tr>
<tr>
<td>(= 87.8 \text{ kips/in.}(0.380 \text{ in.})(4 \text{ bolts}))</td>
<td>(= 58.5 \text{ kips/in.}(0.380 \text{ in.})(4 \text{ bolts}))</td>
</tr>
<tr>
<td>(= 133 \text{ kips} &gt; 54.0 \text{ kips})</td>
<td>(= 88.9 \text{ kips} &gt; 36.0 \text{ kips})</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE IIA-30 ALL-BOLTED TEE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design an all-bolted tee connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

\[ R_D = 9.0 \text{ kips} \]
\[ R_L = 27 \text{ kips} \]

Use ¾-in.-diameter ASTM A325 bolts in standard holes. Try an ASTM A992 WT5×22.5 with a four-bolt connection.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam

\[ W_{16} \times 50 \]
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Column

\[ W_{14} \times 90 \]
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Tee

\[ WT_{5} \times 22.5 \]
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

From AISC Manual Tables 1-1 and 1-8, the geometric properties are as follows:
Beam
W16\times50
\quad t_w = 0.380 \text{ in.}
\quad d = 16.3 \text{ in.}
\quad t_f = 0.630 \text{ in.}

Column
W14\times90
\quad t_f = 0.710 \text{ in.}

Tee
WT5\times22.5
\quad d = 5.05 \text{ in.}
\quad b_f = 8.02 \text{ in.}
\quad t_f = 0.620 \text{ in.}
\quad t_s = 0.350 \text{ in.}
\quad k_1 = \frac{3}{16} \text{ in.} \text{ (see W10\times45 AISC Manual Table 1-1)}
\quad k_{des} = 1.12 \text{ in.}

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u )</td>
<td>1.2(9.0 \text{ kips}) + 1.6(27 \text{ kips})</td>
<td>9.0 \text{ kips} + 27 \text{ kips}</td>
</tr>
<tr>
<td></td>
<td>= 54.0 \text{ kips}</td>
<td>= 36.0 \text{ kips}</td>
</tr>
</tbody>
</table>

**Limitation on Tee Stem Thickness**


\[
t_s \max = \frac{d}{2} + \frac{\sqrt{3}}{16} \text{ in.} \quad (\text{Manual Eq. 9-38})
\]

\[
= \frac{3}{4} \text{ in.} + \frac{\sqrt{3}}{16} \text{ in.}
\]

\[
= 0.438 \text{ in.} > 0.350 \text{ in.} \quad \text{o.k.}
\]

**Limitation on Bolt Diameter for Bolts through Tee Flange**

Note: The bolts are not located symmetrically with respect to the centerline of the tee.

\[
b = \text{flexible width in connection element}
\]

\[
= 2.75 \text{ in.} - \frac{t_s}{2} - \frac{t_w}{2} - k_1
\]

\[
= 2.75 \text{ in.} - \frac{0.350 \text{ in.}}{2} - \frac{0.380 \text{ in.}}{2} - \frac{1}{16} \text{ in.}
\]

\[
= 1.57 \text{ in.}
\]

\[
d_{\min} = 0.163 t_f \sqrt{\frac{F_y}{b} \left( \frac{b^2}{L^2} + 2 \right)} \leq 0.69 \sqrt{t_s} \quad (\text{Manual Eq. 9-37})
\]

\[
= 0.163(0.620 \text{ in.}) \sqrt{\frac{50 \text{ ksi}}{1.57 \text{ in.}} \left( \frac{(1.57 \text{ in.})^2}{(11\frac{1}{2} \text{ in.})^2} + 2 \right)} \leq 0.69 \sqrt{0.350 \text{ in.}}
\]

\[
= 0.810 \text{ in.} \leq 0.408 \text{ in.}
\]
Use \( d_{\text{min}} = 0.408 \text{ in.} \)

\[ d = \frac{3}{8} \text{ in.} > d_{\text{min}} = 0.408 \text{ in.} \quad \text{o.k.} \]

Since the connection is rigid at the support, the bolts through the tee stem must be designed for shear, but do not need to be designed for an eccentric moment.

**Shear and Bearing for Bolts through Beam Web**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Since bolt shear is more critical than bolt bearing in this example, ( \phi r_n = 17.9 \text{ kips} ) from AISC Manual Table 7-1.</td>
<td>Since bolt shear is more critical than bolt bearing in this example, ( r_n/\Omega = 11.9 \text{ kips} ) from AISC Manual Table 7-1.</td>
</tr>
</tbody>
</table>
| Thus, \[ \phi R_n = n \phi r_n \]
\[ = 4 \text{ bolts}(17.9 \text{ kips}) \]
\[ = 71.6 \text{ kips} > 54.0 \text{ kips} \quad \text{o.k.} \] | Thus, \[ \frac{R_n}{\Omega} = \frac{n r_n}{\Omega} \]
\[ = 4 \text{ bolts}(11.9 \text{ kips}) \]
\[ = 47.6 \text{ kips} > 36.0 \text{ kips} \quad \text{o.k.} \]

**Flexural Yielding of Stem**

The flexural yielding strength is checked at the junction of the stem and the fillet.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( M_u = P_a e \)
\[ = (54.0 \text{ kips})(3.80 \text{ in.} - 1.12 \text{ in.}) \]
\[ = 145 \text{ kip-in.} \] | \( M_u = P_a e \)
\[ = 36.0 \text{ kips}(3.80 \text{ in.} - 1.12 \text{ in.}) \]
\[ = 96.5 \text{ kip-in.} \] |
| \( \phi = 0.90 \) | \( \Omega = 1.67 \) |
| \( \phi M_u = \phi F_x Z_x \)
\[ = 0.90(50 \text{ ksi})\left(\frac{0.350 \text{ in.}(11\frac{1}{2} \text{ in.})^2}{4}\right) \]
\[ = 521 \text{ kip-in.} > 145 \text{ kip-in.} \quad \text{o.k.} \] | \( \frac{M_u}{\Omega} = \frac{F_x Z_x}{\Omega} \)
\[ = 50 \text{ ksi}\left(\frac{0.350 \text{ in.}(11\frac{1}{2} \text{ in.})^2}{4}\right) \]
\[ = 1.67 \]
\[ = 346 \text{ kip-in.} > 96.5 \text{ kip-in.} \quad \text{o.k.} \]

**Shear Yielding of Stem**

From AISC Specification Equation J4-3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
</tbody>
</table>
| \( \phi R_n = \phi 0.60 F_y A_{gy} \)
\[ = 1.00\left[0.60(50 \text{ ksi})(11\frac{1}{2} \text{ in.})(0.350 \text{ in.})\right] \]
\[ = 121 \text{ kips} > 54.0 \text{ kips} \quad \text{o.k.} \] | \( \frac{R_n}{\Omega} = \frac{0.60 F_y A_{gy}}{\Omega} \)
\[ = 0.60(50 \text{ ksi})(11\frac{1}{2} \text{ in.})(0.350 \text{ in.}) \]
\[ = 80.5 \text{ kips} > 36.0 \text{ kips} \quad \text{o.k.} \]
Shear Rupture of Stem

\[ A_{sv} = \left[ 11\frac{1}{2} \text{ in.} - 4 \left( \frac{3}{16} \text{ in.} + \frac{3}{16} \text{ in.} \right) \right] 0.350 \text{ in.} = 2.80 \text{ in.}^2 \]

From AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>0.75</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = \phi 0.60 F_u A_{sv} )</td>
<td>( R_n = \frac{0.60 F_u A_{sv}}{\Omega} )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td></td>
<td>= 0.75(0.60)(65 ksi)(2.80 in.(^2))</td>
<td>= ( \frac{0.60(65 \text{ ksi})(2.80 \text{ in.}^2)}{2.00} )</td>
</tr>
<tr>
<td></td>
<td>= 81.9 kips &gt; 54.0 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Block Shear Rupture of Stem

\( L_{sb} = L_{sv} = 1\frac{1}{4} \text{ in.} \)

From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = \phi U_{bs} F_u A_{bst} + \min(\phi 0.60 F_y A_{gys}, \phi 0.60 F_u A_{sv}) )</td>
<td>( U_{bs} = 1.0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( U_{bs} = 1.0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tension rupture component from AISC Manual Table 9-3a:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \phi U_{bs} F_u A_{bst} = 39.6 \text{ kips/in. (0.350 in.)} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shear yielding component from AISC Manual Table 9-3b:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \phi 0.60 F_y A_{gys} = 231 \text{ kips/in. (0.350 in.)} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shear rupture component from AISC Manual Table 9-3c:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \phi 0.60 F_u A_{sv} = 210 \text{ kips/in. (0.350 in.)} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \phi R_n = (39.6 \text{ kips/in. + 210 kips/in.}) (0.350 \text{ in.}) )</td>
<td>o.k.</td>
</tr>
<tr>
<td></td>
<td>= 87.4 kips &gt; 54.0 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Shear yielding component from AISC Manual Table 9-3b:

\[ \frac{0.60 F_y A_{gys}}{\Omega} = 154 \text{ kips/in. (0.350 in.)} \]

Shear rupture component from AISC Manual Table 9-3c:

\[ \frac{0.60 F_u A_{sv}}{\Omega} = 140 \text{ kips/in. (0.350 in.)} \]

\[ R_n = \frac{(26.4 \text{ kips/in. + 140 kips/in.}) (0.350 \text{ in.})}{\Omega} \]

\[ = 58.2 \text{ kips} > 36.0 \text{ kips} \]
Since the connection is rigid at the support, the bolts attaching the tee flange to the support must be designed for the shear and the eccentric moment.

**Bolt Group at Column**

Check bolts for shear and bearing combined with tension due to eccentricity.

The following calculation follows the Case II approach in the Section “Eccentricity Normal to the Plane of the Faying Surface” in Part 7 of the AISC Manual.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tensile force per bolt, ( r_{at} ):</strong></td>
<td><strong>Tensile force per bolt, ( r_{at} ):</strong></td>
</tr>
<tr>
<td>( r_{at} = \frac{P_e}{n'd_m} ) (Manual Eq. 7-14a)</td>
<td>( r_{at} = \frac{P_e}{n'd_m} ) (Manual Eq. 7-14b)</td>
</tr>
<tr>
<td>( = \frac{54.0 \text{ kips}}{4 \text{ bolts} (6.00 \text{ in.})} )</td>
<td>( = \frac{36.0 \text{ kips}}{4 \text{ bolts} (6.00 \text{ in.})} )</td>
</tr>
<tr>
<td>( = 8.55 \text{ kips/bolt} )</td>
<td>( = 5.70 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>

**Design strength of bolts for tension-shear interaction:**

When threads are not excluded from the shear planes of ASTM A325 bolts, from AISC Manual Table 7-1, \( \phi r_n = 17.9 \text{ kips/bolt} \).

<table>
<thead>
<tr>
<th><strong>LRFD</strong></th>
<th><strong>ASD</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{av} = \frac{P_n}{n} ) (Spec. Eq. 7-13a)</td>
<td>( r_{av} = \frac{P_n}{n} ) (Spec. Eq. 7-13b)</td>
</tr>
<tr>
<td>( = \frac{54.0 \text{ kips}}{8 \text{ bolts}} )</td>
<td>( = \frac{36.0 \text{ kips}}{8 \text{ bolts}} )</td>
</tr>
<tr>
<td>( = 6.75 \text{ kips/bolt} &lt; 17.9 \text{ kips/bolt} )</td>
<td>( = 4.50 \text{ kips/bolt} &lt; 11.9 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>

\( f_{rv} = \frac{6.75 \text{ kips/bolt}}{0.442 \text{ in.}^2} \)
\( = 15.3 \text{ ksi} \)

From AISC *Specification* Table J3.2:

<table>
<thead>
<tr>
<th>( F_{nt} = 90 \text{ ksi} )</th>
<th>( F_{nv} = 54 \text{ ksi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td></td>
</tr>
<tr>
<td>( F_{nt}' = 1.3 F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt} ) (Spec. Eq. J3-3a)</td>
<td>( F_{nt}' = 1.3 F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt} ) (Spec. Eq. J3-3b)</td>
</tr>
<tr>
<td>( = 1.3(90 \text{ ksi}) - \left( \frac{90 \text{ ksi}}{0.75(54 \text{ ksi})} \right)(15.3 \text{ ksi}) )</td>
<td>( = 1.3(90 \text{ ksi}) - \left( \frac{2.00(90 \text{ ksi})}{54 \text{ ksi}} \right)(10.2 \text{ ksi}) )</td>
</tr>
<tr>
<td>( = 83.0 \text{ ksi} \leq 90 \text{ ksi} )</td>
<td>( = 83.0 \text{ ksi} &lt; 90 \text{ ksi} )</td>
</tr>
</tbody>
</table>

From AISC *Specification* Equation J3-2:

| \( \phi r_n = \phi F_{nt}' A_h \) | |

---

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IIA-112

*Design Examples V14.1*

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
With \( L_c = 1\frac{1}{4} \text{ in.} \) and \( s = 3 \text{ in.} \), the bearing strength of the tee flange exceeds the single shear strength of the bolts. Therefore, bearing strength is o.k.

**Prying Action (AISC Manual Part 9)**

By inspection, prying action in the tee will control over prying action in the column.

Note: The bolts are not located symmetrically with respect to the centerline of the tee.

\[
b = 2\frac{3}{4} \text{ in.} + \frac{0.380 \text{ in.}}{2} = 2.94 \text{ in.}
\]

\[
a = \frac{8.02 \text{ in.}}{2} - 2\frac{3}{4} \text{ in.} - \frac{0.380 \text{ in.}}{2} - \frac{0.350 \text{ in.}}{2} = 0.895 \text{ in.}
\]

\[
b' = b - \frac{d_b}{2} \quad \text{(Manual Eq. 9-21)}
\]

\[
= 2.94 \text{ in.} - \frac{3\frac{3}{4} \text{ in.}}{2} = 2.57 \text{ in.}
\]

Since \( a = 0.895 \text{ in.} \) is less than \( 1.25b = 3.68 \text{ in.} \), use \( a = 0.895 \text{ in.} \) for calculation purposes.

\[
a' = a + \frac{d_b}{2} \quad \text{(Manual Eq. 9-27)}
\]

\[
= 0.895 \text{ in.} + \frac{3\frac{3}{4} \text{ in.}}{2} = 1.27 \text{ in.}
\]

\[
\rho = \frac{b'}{a'} \quad \text{(Manual Eq. 9-26)}
\]

\[
= \frac{2.57 \text{ in.}}{1.27 \text{ in.}} = 2.02
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_u = r_{ut} = 8.55 \text{ kips/bolt} )</td>
<td>( T_u = r_{ut} = 5.70 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>( B_u = \Phi r_{n} = 27.5 \text{ kips/bolt} )</td>
<td>( B_u = r_{n} / \Omega = 18.3 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>( \beta = \frac{1}{\rho} \left( \frac{B_u}{T_u} - 1 \right) \quad \text{(Manual Eq. 9-25)} )</td>
<td>( \beta = \frac{1}{\rho} \left( \frac{B_u}{T_u} - 1 \right) \quad \text{(Manual Eq. 9-25)} )</td>
</tr>
</tbody>
</table>
Since $\beta \geq 1$, set $\alpha' = 1.0$

\[
p = 1\frac{1}{4} \text{ in.} + \frac{3.00 \text{ in.}}{2} = 2.75 \text{ in.}
\leq s = 3.00 \text{ in.}
\]

\[
\delta = 1 - \frac{d'}{p} = 1 - \frac{\frac{3}{16} \text{ in.}}{2.75 \text{ in.}} = 0.705
\]

\[
\phi = 0.90 \quad \Omega = 1.67
\]

\[
\min = \frac{4T_u b'}{\sqrt{\phi p F_{\alpha}} (1 + \delta \alpha')} \quad \min = \frac{\Omega 4T_u b'}{p F_{\alpha} (1 + \delta \alpha')}
\]

\[
= \frac{4(8.55 \text{ kips})(2.57 \text{ in.})}{\sqrt{0.90(2.75 \text{ in.})(65 \text{ ksi})[1 + (0.705)(1.0)]}} \quad = \frac{1.67(4)(5.70 \text{ kips})(2.57 \text{ in.})}{2.75 \text{ in.}(65 \text{ ksi})[1 + (0.705)(1.0)]}
\]

\[
= 0.566 \text{ in.} < 0.620 \text{ in.} \quad = 0.567 \text{ in.} < 0.620 \text{ in.}
\]

Similarly, checks of the tee flange for shear yielding, shear rupture, and block shear rupture will show that the tee flange is o.k.

**Bolt Bearing on Beam Web**

From AISC Manual Table 10-1, for four rows of ¼-in.-diameter bolts and an uncoped beam:

\[
\phi R_n = 351 \text{ kips/in.}(0.380 \text{ in.}) = 133 \text{ kips} > 54.0 \text{ kips}
\]

\[
\frac{R_n}{\Omega} = 234 \text{ kips/in.}(0.380 \text{ in.}) = 88.9 \text{ kips} > 36.0 \text{ kips}
\]

**Bolt Bearing on Column Flange**

From AISC Manual Table 10-1, for four rows of ¼-in.-diameter bolts:

\[
\phi R_n = 702 \text{ kips/in.}(0.710 \text{ in.}) = 498 \text{ kips} > 54.0 \text{ kips}
\]

\[
\frac{R_n}{\Omega} = 468 \text{ kips/in.}(0.710 \text{ in.}) = 332 \text{ kips} > 36.0 \text{ kips}
\]

Note: Although the edge distance ($a = 0.895 \text{ in.}$) for one row of bolts in the tee flange does not meet the minimum value indicated in AISC Specification Table J3.4, based on footnote [a], the edge distance provided is acceptable because the provisions of AISC Specification Section J3.10 and J4.4 have been met in this case.
EXAMPLE II.A-31  BOLTED/WELDED TEE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a tee connection bolted to an ASTM A992 W16×50 supported beam and welded to an ASTM A992 W14×90 supporting column flange, to support the following beam end reactions:

\[ R_D = 6 \text{ kips} \]
\[ R_L = 18 \text{ kips} \]

Use ¾-in.-diameter Group A bolts in standard holes and 70-ksi electrodes. Try an ASTM A992 WT5×22.5 with four bolts.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
W16×50
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Column
W14×90
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Tee
WT5×22.5
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

From AISC Manual Tables 1-1 and 1-8, the geometric properties are as follows:
Beam

W16×50

\( t_w = 0.380 \text{ in.} \)
\( d = 16.3 \text{ in.} \)
\( t_f = 0.630 \text{ in.} \)

Column

W14×90

\( t_f = 0.710 \text{ in.} \)

Tee

WT5×22.5

\( d = 5.05 \text{ in.} \)
\( b_f = 8.02 \text{ in.} \)
\( t_f = 0.620 \text{ in.} \)
\( t_s = 0.350 \text{ in.} \)
\( k_1 = \frac{1}{3/16} \text{ in.} \) (see W10×45 AISC Manual Table 1-1)
\( k_{des} = 1.12 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u )</td>
<td>1.2(6.0 kips) + 1.6(18 kips)</td>
<td>6.0 kips + 18 kips</td>
</tr>
<tr>
<td>( R_a )</td>
<td>36.0 kips</td>
<td>24.0 kips</td>
</tr>
</tbody>
</table>

**Limitation on Tee Stem Thickness**

See rotational ductility discussion at the beginning of the AISC Manual Part 9

\[ t_s \max = \frac{d}{2} + \frac{1}{16} \text{ in.} \]  
\[ = \frac{3/4 + 1/16}{2} \text{ in.} \]
\[ = 0.438 \text{ in.} > 0.350 \text{ in.} \quad \text{o.k.} \]

**Weld Design**

\[ b = \text{flexible width in connection element} \]
\[ = \frac{b_f - 2k_1}{2} \]
\[ = \frac{8.02 \text{ in.} - 2(1/16 \text{ in.})}{2} \]
\[ = 3.20 \text{ in.} \]

\[ w_{\min} = 0.0155 \left( \frac{F_y t_f}{b} \right) \left( \frac{b^2}{L^2} + 2 \right) \leq \frac{5}{8} t_s \]
\[ = 0.0155 \left[ \frac{50 \text{ ksi} (0.620 \text{ in.})^2}{3.20 \text{ in.}} \left[ \frac{(3.20 \text{ in.})^2}{(11\frac{1}{2} \text{ in.})^2} + 2 \right] \leq \frac{5}{8} (0.350 \text{ in.}) \right. \]
\[ = 0.193 \text{ in.} \leq 0.219 \text{ in.} \]

The minimum weld size is \( \frac{1}{4} \text{ in.} \) per AISC Specification Table J2.4.
Try ¼-in. fillet welds.

From AISC Manual Table 10-2, with \( n = 4, L = 11\frac{1}{2}, \) and weld \( B = ¼ \) in.:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 79.9 \text{ kips} \geq 36.0 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>( \frac{R_u}{\Omega} = 53.3 \text{ kips} \geq 24.0 \text{ kips} )</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Use ¼-in. fillet welds.

**Supporting Column Flange**

From AISC Manual Table 10-2, with \( n = 4, L = 11\frac{1}{2}, \) and weld \( B = ¼ \) in., the minimum support thickness is 0.190 in.

\[ t_f = 0.710 \text{ in.} > 0.190 \text{ in.} \quad \text{o.k.} \]

**Stem Side of Connection**

Since the connection is flexible at the support, the tee stem and bolts must be designed for eccentric shear, where the eccentricity, \( e_b \), is determined as follows:

\[
a = d - L_{eb} = 5.05 \text{ in.} - 1.25 \text{ in.} = 3.80 \text{ in.}
\]

\[ e_b = a = 3.80 \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>The tee stem and bolts must be designed for ( R_u = 36.0 \text{ kips} ) and ( R_u e_b = 36.0 \text{ kips}(3.80 \text{ in.}) = 137 \text{ kip-in.} )</td>
<td>The tee stem and bolts must be designed for ( R_u = 24.0 \text{ kips} ) and ( R_u e_b = 24.0 \text{ kips}(3.80 \text{ in.}) = 91.2 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( \phi R_u = 17.9 \text{ kips} )</td>
<td>( \frac{R_u}{\Omega} = 11.9 \text{ kips} )</td>
</tr>
<tr>
<td>Bolt Shear and Bolt Bearing on Tee Stem</td>
<td>Bolt Shear and Bolt Bearing on Tee Stem</td>
</tr>
<tr>
<td>From AISC Manual Table 7-1, the single bolt shear strength is:</td>
<td>From AISC Manual Table 7-1, the single bolt shear strength is:</td>
</tr>
<tr>
<td>( \phi R_u = 49.4 \text{ kips/in.}(0.350 \text{ in.}) = 17.3 \text{ kips} &lt; 17.9 \text{ kips} )</td>
<td>( \frac{R_u}{\Omega} = 32.9 \text{ kips/in.}(0.350 \text{ in.}) = 11.5 \text{ kips} &lt; 11.9 \text{ kips} )</td>
</tr>
<tr>
<td>Bolt bearing controls.</td>
<td>Bolt bearing controls.</td>
</tr>
<tr>
<td>Note: By inspection, bolt bearing on the beam web does not control.</td>
<td>Note: By inspection, bolt bearing on the beam web does not control.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-6 for ( \theta = 0^\circ, ) with ( s = 3 ) in., ( e_c = e_b = 3.80 \text{ in.}, ) and ( n = 4, )</td>
<td>From AISC Manual Table 7-6 for ( \theta = 0^\circ, ) with ( s = 3 ) in.,</td>
</tr>
</tbody>
</table>
Since bolt bearing is more critical than bolt shear in this example, from AISC Manual Equation 7-19,

$$\phi R_n = C r_n$$

$$= 2.45 (17.3 \text{ kips/bolt})$$

$$= 42.4 \text{ kips} > 36.0 \text{ kips}$$

o.k.

Since bolt bearing is more critical than bolt shear in this example, from AISC Manual Equation 7-19,

$$\frac{R_n}{\Omega} = C \frac{r_n}{\Omega}$$

$$= 2.45 (11.5 \text{ kips/bolt})$$

$$= 28.2 \text{ kips} > 24.0 \text{ kips}$$

o.k.

### Flexural Yielding of Tee Stem

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi M_n = \phi F_y Z_x$</td>
<td>$M_n = \frac{F_y Z_x}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.90(50 \text{ ksi}) \left[ \frac{0.350 \text{ in.}(11\frac{1}{2} \text{ in.})^2}{4} \right]$</td>
<td>$= 50 \text{ ksi} \left[ \frac{0.350 \text{ in.}(11\frac{1}{2} \text{ in.})^2}{1.67} \right]$</td>
</tr>
<tr>
<td>$= 521 \text{ kip-in.} &gt; 137 \text{ kip-in.}$</td>
<td>$= 346 \text{ kip-in.} &gt; 91.2 \text{ kip-in.}$</td>
</tr>
</tbody>
</table>

o.k.

### Flexural Rupture of Tee Stem

$$Z_{net} = 0.350 \text{ in.} \left[ \frac{(11\frac{1}{2} \text{ in.})^2}{4} - 2(\frac{3}{16} \text{ in.} + \frac{1}{16} \text{ in.})(4.50 \text{ in.}) - 2(\frac{3}{16} \text{ in.} + \frac{1}{16} \text{ in.})(1.50 \text{ in.}) \right]$$

$$= 7.90 \text{ in.}^3$$

From AISC Manual Equation 9-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi M_n = \phi F_u Z_{net}$</td>
<td>$M_n = \frac{F_u Z_{net}}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.75(65 \text{ ksi})(7.90 \text{ in.}^3)$</td>
<td>$= 65 \text{ ksi}(7.90 \text{ in.}^3)$</td>
</tr>
<tr>
<td>$= 385 \text{ kip-in.} &gt; 137 \text{ kip-in.}$</td>
<td>$= 257 \text{ kip-in.} &gt; 91.2 \text{ kip-in.}$</td>
</tr>
</tbody>
</table>

o.k.

### Shear Yielding of Tee Stem

From AISC Specification Equation J4-3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = \phi 0.60 F_y A_{gv}$</td>
<td>$R_n = \frac{0.60 F_y A_{gv}}{\Omega}$</td>
</tr>
<tr>
<td>$= 1.00(0.60)(50 \text{ ksi})(11\frac{1}{2} \text{ in.})(0.350 \text{ in.})$</td>
<td>$= 0.60(50 \text{ ksi})(11\frac{1}{2} \text{ in.})(0.350 \text{ in.})/1.50$</td>
</tr>
<tr>
<td>$= 121 \text{ kips} &gt; 36.0 \text{ kips}$</td>
<td>$= 80.5 \text{ kips} &gt; 24.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

o.k.
Shear Rupture of Tee Stem

\[ A_{sv} = \left[ 11\frac{1}{2} \text{ in.} - 4\left(1\frac{3}{4}\text{ in.} + \frac{1}{16}\text{ in.}\right) \right] (0.350 \text{ in.}) \]
\[ = 2.80 \text{ in.}^2 \]

From AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi = 0.75 ]</td>
<td>[ \Omega = 2.00 ]</td>
</tr>
<tr>
<td>[ \phi R_n = \phi 0.60 F_y A_{gy} ]</td>
<td>[ \frac{R_n}{\Omega} = \frac{0.60 F_u A_{nv}}{\Omega} ]</td>
</tr>
<tr>
<td>[ = 0.75(0.60)(65 \text{ ksi})(2.80 \text{ in.}^2) ]</td>
<td>[ = 0.60(65 \text{ ksi})(2.80 \text{ in.}^2) ]</td>
</tr>
<tr>
<td>[ = 81.9 \text{ kips} &gt; 36.0 \text{ kips} ]</td>
<td>[ = 54.6 \text{ kips} &gt; 24.0 \text{ kips} ]</td>
</tr>
</tbody>
</table>

o.k.

Block Shear Rupture of Tee Stem

\[ L_{eh} = L_{ev} = 1\frac{1}{4} \text{ in.} \]

From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi R_n = \phi U_{hs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gy}, \phi 0.60 F_u A_{nv}) ]</td>
<td>[ \frac{R_n}{\Omega} = \frac{U_{hs} F_u A_{nt} + \min\left(\frac{0.60 F_y A_{gy}}{\Omega}, \frac{0.60 F_u A_{nv}}{\Omega}\right)}{\Omega} ]</td>
</tr>
<tr>
<td>[ U_{hs} = 1.0 ]</td>
<td>[ U_{hs} = 1.0 ]</td>
</tr>
</tbody>
</table>

Tension rupture component from AISC Manual Table 9-3a:

\[ \phi U_{hs} F_u A_{nt} = 1.0(39.6 \text{ kips/in.})(0.350 \text{ in.}) \]

Shear yielding component from AISC Manual Table 9-3b:

\[ \phi 0.60 F_y A_{gy} = 231 \text{ kips/in.}(0.350 \text{ in.}) \]

Shear rupture component from AISC Manual Table 9-3c:

\[ \phi 0.60 F_u A_{nv} = 210 \text{ kips/in.}(0.350 \text{ in.}) \]

\[ \phi R_n = (39.6 \text{ kips/in.} + 210 \text{ kips/in.})(0.350\text{in.}) \]
\[ = 87.4 \text{ kips} > 36.0 \text{ kips} \]

o.k.

Tension rupture component from AISC Manual Table 9-3a:

\[ \frac{U_{hs} F_u A_{nt}}{\Omega} = 1.0(26.4 \text{ kips/in.})(0.350 \text{ in.}) \]

Shear yielding component from AISC Manual Table 9-3b:

\[ \frac{0.60 F_y A_{gy}}{\Omega} = 154 \text{ kips/in.}(0.350 \text{ in.}) \]

Shear rupture component from AISC Manual Table 9-3c:

\[ \frac{0.60 F_u A_{nv}}{\Omega} = 140 \text{ kips/in.}(0.350 \text{ in.}) \]

\[ \frac{R_n}{\Omega} = (26.4 \text{ kips/in.} + 140 \text{ kips/in.})(0.350 \text{ in.}) \]
\[ = 58.2 \text{ kips} > 24.0 \text{ kips} \]

o.k.
Chapter IIB
Fully Restrained (FR) Moment Connections

The design of fully restrained (FR) moment connections is covered in Part 12 of the AISC *Steel Construction Manual*. 
EXAMPLE II.B-1  BOLTED FLANGE-PLATED FR MOMENT CONNECTION  
(BEAM-TO-COLUMN FLANGE)

Given:

Design a bolted flange-plated FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange to transfer the following vertical shear forces and strong-axis moments:

\[ V_D = 7.0 \text{ kips} \quad M_D = 42 \text{ kip-ft} \]
\[ V_L = 21 \text{ kips} \quad M_L = 126 \text{ kip-ft} \]

Use \( \frac{3}{8} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. The flange plates are ASTM A36 material. Check the column for stiffening requirements.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]
Column  
ASTM A992  
$F_y = 50 \text{ ksi}$  
$F_u = 65 \text{ ksi}$

Plates  
ASTM A36  
$F_y = 36 \text{ ksi}$  
$F_u = 58 \text{ ksi}$

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W18×50  
$d = 18.0 \text{ in.}$  
$b_f = 7.50 \text{ in.}$  
$t_f = 0.570 \text{ in.}$  
$t_w = 0.355 \text{ in.}$  
$S_x = 88.9 \text{ in.}^3$

Column
W14×99  
$d = 14.2 \text{ in.}$  
$b_f = 14.6 \text{ in.}$  
$t_f = 0.780 \text{ in.}$  
$t_w = 0.485 \text{ in.}$  
$k_{des} = 1.38 \text{ in.}$

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(7.0 \text{ kips}) + 1.6(21 \text{ kips})$</td>
<td>$R_u = 7.0 \text{ kips} + 21 \text{ kips}$</td>
</tr>
<tr>
<td>= 42.0 kips</td>
<td>= 28.0 kips</td>
</tr>
<tr>
<td>$M_u = 1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft})$</td>
<td>$M_u = 42 \text{ kip-ft} + 126 \text{ kip-ft}$</td>
</tr>
<tr>
<td>= 252 kip-ft</td>
<td>= 168 kip-ft</td>
</tr>
</tbody>
</table>

**Flexural Strength of Beam (using AISC Specification Section F13.1)**

Use two rows of bolts in standard holes.

$$A_{fg} = b_f t_f$$

$$= 7.50 \text{ in.}(0.570 \text{ in.})$$

$$= 4.28 \text{ in.}^2$$

$$A_{fn} = A_{fg} - 2(d_b + \frac{\gamma}{10} \text{ in.}) t_f$$

$$= 4.28 \text{ in.}^2 - 2(\frac{1}{10} \text{ in.} + \frac{1}{10} \text{ in.})(0.570 \text{ in.})$$

$$= 3.14 \text{ in.}^2$$
\[ \frac{F_y}{F_u} = \frac{50 \text{ ksi}}{65 \text{ ksi}} = 0.769 \leq 0.8, \text{ therefore } Y_t = 1.0. \]

\[ F_u A_{fu} = 65 \text{ ksi} \left(3.14 \text{ in.}^2\right) = 204 \text{ kips} \]

\[ Y_t F_y A_{ft} = 1.0(50 \text{ ksi})(4.28 \text{ in.}^2) = 214 \text{ kips} > 204 \text{ kips} \]

Therefore the nominal flexural strength, \( M_n \), at the location of the holes in the tension flange is not greater than:

\[
M_n = \frac{F_u A_{fu}}{A_{ft}} S_e \quad \text{(Spec. Eq. F13-1)}
\]

\[
= 65 \text{ ksi} \left(3.14 \text{ in.}^2\right) \left(88.9 \text{ in.}^3\right) \\
= 4,240 \text{ kip-in. or 353 kip-ft}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_n = 0.90(353 \text{ kip-ft}) )</td>
<td>( \frac{M_n}{\Omega_b} = \frac{353 \text{ kip-ft}}{1.67} = 211 \text{ kip-ft} &gt; 168 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( = 318 \text{ kip-ft} &gt; 252 \text{ kip-ft} )</td>
<td>( \text{o.k.} )</td>
</tr>
</tbody>
</table>

Note: The available flexural strength of the beam may be less than that determined based on AISC Specification Equation F13-1. Other applicable provisions in AISC Specification Section F should be checked to possibly determine a lower value for the available flexural strength of the beam.

**Single-Plate Web Connection**

Try a PL\(\frac{3}{8}\times5\times0.09\)", with three \(\frac{7}{8}\)-in.-diameter ASTM A325-N bolts and \(\frac{1}{4}\)-in. fillet welds.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear strength of bolts from AISC Manual Table 7-1: ( \phi r_n = 24.3 \text{ kips/bolt} )</td>
<td>Shear strength of bolts from AISC Manual Table 7-1: ( r_n/\Omega = 16.2 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>Bearing strength of bolts:</td>
<td>Bearing strength of bolts:</td>
</tr>
<tr>
<td>Bearing on the plate controls over bearing on the beam web.</td>
<td>Bearing on the plate controls over bearing on the beam web.</td>
</tr>
<tr>
<td>Vertical edge distance = 1.50 in. ( l_e = 1.50 \text{ in.} - \frac{15/8 \text{ in.}}{2} ) = 1.03 in.</td>
<td>Vertical edge distance = 1.50 in. ( l_e = 1.50 \text{ in.} - \frac{15/8 \text{ in.}}{2} ) = 1.03 in.</td>
</tr>
</tbody>
</table>
From AISC *Specification* Equation J-36a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi r_n = \phi l_2 l t F_u \leq \phi 2dt F_u$</td>
<td>$r_n / \Omega = \frac{1.2 l_2 l t F_u}{\Omega} \leq \frac{2 Ad t F_u}{\Omega}$</td>
</tr>
<tr>
<td>0.75(1.2)(1.03 in.)(½ in.)(58 ksi)</td>
<td>1.2(1.03 in.)(½ in.)(58 ksi)</td>
</tr>
<tr>
<td>$\leq 0.75(2.4)(½ in.)(½ in.)(58 ksi)$</td>
<td>$\leq \frac{2.4(½ in.)(½ in.)(58 ksi)}{2.00}$</td>
</tr>
<tr>
<td>20.2 kips $\leq$ 34.3 kips</td>
<td>13.4 kips $\leq$ 22.8 kips</td>
</tr>
<tr>
<td>$\phi r_n = 20.2$ kips/bolt</td>
<td>$r_n / \Omega = 13.4$ kips/bolt</td>
</tr>
<tr>
<td>From AISC <em>Manual</em> Table 7-4 with $s = 3$ in.,</td>
<td>From AISC <em>Manual</em> Table 7-4 with $s = 3$ in.,</td>
</tr>
<tr>
<td>$\phi r_n = 91.4$ kips/in./bolt(½ in.)</td>
<td>$r_n / \Omega = 60.9$ kips/in./bolt(½ in.)</td>
</tr>
<tr>
<td>$= 34.3$ kips/bolt</td>
<td>$= 22.8$ kips/bolt</td>
</tr>
</tbody>
</table>

Bolt bearing strength at the top bolt controls.

Determine the coefficient for the eccentrically loaded bolt group from AISC *Manual* Table 7-6.

<table>
<thead>
<tr>
<th>$C_{min} = \frac{R_u}{\phi r_n}$</th>
<th>$C_{min} = \frac{R_u}{r_n / \Omega}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= \frac{42.0 \text{ kips}}{20.2 \text{ kips}}$</td>
<td>$= \frac{28.0 \text{ kips}}{13.4 \text{ kips}}$</td>
</tr>
<tr>
<td>$= 2.08$</td>
<td>$= 2.09$</td>
</tr>
</tbody>
</table>

Using $e = 3.00$ in./2 = 1.50 in. and $s = 3.00$ in.,

| $C = 2.23 > 2.08$ o.k. | $C = 2.23 > 2.09$ o.k. |

Plate shear yielding, from AISC *Specification* Equation J4-3:

<table>
<thead>
<tr>
<th>$\phi = 1.00$</th>
<th>$\Omega = 1.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = \phi (0.60 F_s A_{sv})$</td>
<td>$R_u = 0.60 F_s A_{sv}$</td>
</tr>
<tr>
<td>$= 1.00(0.60)(36 \text{ ksi})(9.00 \text{ in.})(½ \text{ in.})$</td>
<td>$= \frac{0.60(36 \text{ ksi})(9.00 \text{ in.})(½ \text{ in.})}{1.50}$</td>
</tr>
<tr>
<td>$= 72.9$ kips $&gt; 42.0$ kips</td>
<td>$= 48.6$ kips $&gt; 28.0$ kips o.k.</td>
</tr>
</tbody>
</table>

Plate shear rupture, from AISC *Specification* Equation J4-4:

Total length of bolt holes:
### Block Shear Rupture Strength of the Web Plate (using AISC Specification Equation J4-5)

$L_{eb} = 2$ in.; $L_{ev} = 1\frac{1}{2}$ in.; $U_{bs} = 1.0$; $n = 3$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = \phi U_{bs} F_u A_{ut} + \min \left( \phi_0.60 F_y A_{gv}, \phi_0.60 F_u A_{nv} \right)$</td>
<td>$R_n = \frac{U_{bs} F_u A_{ut}}{\Omega} + \min \left( \frac{0.60 F_y A_{gv}}{\Omega}, \frac{0.60 F_u A_{nv}}{\Omega} \right)$</td>
</tr>
<tr>
<td>$U_{bs} = 1.0$</td>
<td>$U_{bs} = 1.0$</td>
</tr>
<tr>
<td><strong>Tension rupture component from AISC Manual Table 9-3a:</strong></td>
<td><strong>Tension rupture component from AISC Manual Table 9-3a:</strong></td>
</tr>
<tr>
<td>$\phi U_{bs} F_u A_{ut} = 1.0(65.3 \text{ kips/in.})(% \text{ in.})$</td>
<td>$U_{bs} F_u A_{ut}/\Omega = 1.0(43.5 \text{ kips/in.})(% \text{ in.})$</td>
</tr>
<tr>
<td><strong>Shear yielding component from AISC Manual Table 9-3b:</strong></td>
<td><strong>Shear yielding component from AISC Manual Table 9-3b:</strong></td>
</tr>
<tr>
<td>$\phi_0.60 F_y A_{gv} = 121 \text{ kips/in.}(% \text{ in.})$</td>
<td>$0.60 F_y A_{gv}/\Omega = 81.0 \text{ kips/in.}(% \text{ in.})$</td>
</tr>
<tr>
<td><strong>Shear rupture component from AISC Manual Table 9-3c:</strong></td>
<td><strong>Shear rupture component from AISC Manual Table 9-3c:</strong></td>
</tr>
<tr>
<td>$\phi_0.60 F_u A_{nv} = 131 \text{ kips/in.}(% \text{ in.})$</td>
<td>$0.60 F_u A_{nv}/\Omega = 87.0 \text{ kips/in.}(% \text{ in.})$</td>
</tr>
<tr>
<td>$\phi R_n = (65.3 \text{ kips/in.} + 121 \text{ kips/in.})(% \text{ in.})$</td>
<td>$R_n = (43.5 \text{ kips/in.} + 81.0 \text{ kips/in.})(% \text{ in.})$</td>
</tr>
<tr>
<td>$69.9 \text{ kips} &gt; 42.0 \text{ kips}$</td>
<td>$46.7 \text{ kips} &gt; 28.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

### Web Plate to Column Flange Weld Shear Strength (using AISC Manual Part 8)

From AISC Manual Equation 8-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 1.392 D(2)$</td>
<td>$R_n = 0.928 D(2)$</td>
</tr>
<tr>
<td>$= 1.392(4 \text{ sixteenths})(9.00 \text{ in.})(2)$</td>
<td>$= 0.928(4 \text{ sixteenths})(9.00 \text{ in.})(2)$</td>
</tr>
<tr>
<td>$= 100 \text{ kips} &gt; 42.0 \text{ kips}$</td>
<td>$= 66.8 \text{ kips} &gt; 28.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

**o.k.**
Note: By inspection, the available shear yielding, shear rupture and block shear rupture strengths of the beam web are o.k.

**Web Plate Rupture Strength at Welds (using AISC Manual Part 9)**

\[ t_{min} = \frac{0.6F_{EXX} \left( \frac{\sqrt{5}}{2} \right) \left( \frac{D}{16} \right)}{0.6F_u} \]

\[ = \frac{3.09D}{F_u} \quad \text{for} \quad F_{EXX} = 70.0 \text{ ksi} \]

\[ = \frac{(3.09)(4 \text{ sixteenths})}{65 \text{ ksi}} = 0.190 \text{ in.} < 0.780 \text{ in. column flange} \quad \text{o.k.} \]

**Tension Flange Plate and Connection**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{df} = \frac{M_u}{d} = \frac{252 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.}} )</td>
<td>( P_{df} = \frac{M_u}{d} = \frac{168 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.}} )</td>
</tr>
<tr>
<td>= 168 kips</td>
<td>= 112 kips</td>
</tr>
</tbody>
</table>

Try a PL \( \frac{3}{4} \times 7 \).

Determine critical bolt strength.

Bolt shear using AISC Manual Table 7-1,

\( \phi_{r_a} = 24.3 \text{ kips/bolt} \)

Bearing on flange using AISC Manual Table 7-5,

Edge distance = 1½ in. (Use 1¼ in. to account for possible underrun in beam length.)

\( \phi_{r_a} = 45.7 \text{ kips/bolt/in.}(t_f) \)

\[ = 45.7 \text{ kips/bolt/in.}(0.570 \text{ in.}) \]

\[ = 26.0 \text{ kips/bolt} \]

Bearing on plate using AISC Manual Table 7-5,

Edge distance = 1½ in. (Conservatively, use 1¼ in. value from table.)

\( \phi_{r_a} = 40.8 \text{ kips/bolt/in.}(t_p) \)

\[ = 40.8 \text{ kips/bolt/in.}(\frac{3}{4} \text{ in.}) \]

\[ = 30.6 \text{ kips/bolt} \]

Bolt shear controls, therefore the number of bolts required is as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_n / \Omega = 16.2 \text{ kips/bolt} )</td>
<td>( r_n / \Omega = 30.5 \text{ kips/bolt/in.}(t_f) )</td>
</tr>
<tr>
<td>[ = 30.5 \text{ kips/bolt/in.}(0.570 \text{ in.}) ]</td>
<td>[ = 17.4 \text{ kips/bolt} ]</td>
</tr>
</tbody>
</table>

Bearing on plate using AISC Manual Table 7-5,

Edge distance = 1½ in. (Conservatively, use 1¼ in. value from table.)

\( r_n / \Omega = 27.2 \text{ kips/bolt/in.}(t_p) \)

\[ = 27.2 \text{ kips/bolt/in.}(\frac{3}{4} \text{ in.}) \]

\[ = 20.4 \text{ kips/bolt} \]

Bolt shear controls, therefore the number of bolts required is as follows:
Flange Plate Tensile Yielding

\[ R_n = F_n A_e \]
\( = 36 \text{ ksi}(7.00 \text{ in.})(\frac{3}{8} \text{ in.}) \)
\( = 189 \text{ kips} \)

\[ n_{\text{min}} = \frac{P_{af}}{\phi r_n} = \frac{168 \text{ kips}}{24.3 \text{ kips/bolt}} = 6.91 \text{ bolts} \]
Use 8 bolts.

\[ n_{\text{min}} = \frac{P_{af}}{r_n / \Omega} = \frac{112 \text{ kips}}{16.2 \text{ kips/bolt}} = 6.91 \text{ bolts} \]
Use 8 bolts.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ P_{af} = \frac{M_u}{d + t_p} = \frac{252 \text{ kip-ft}(12 \text{ in./ft})}{(18.0 \text{ in.} + \frac{3}{4} \text{ in.})} = 161 \text{ kips} \] | \[ \Omega = 1.67 \]
| \( \phi = 0.90 \) | \( R_n = 189 \text{ kips} \)
| \( \phi R_n = 0.90(189 \text{ kips}) \) | \( \Omega = 1.67 \)
| = 170 kips > 161 kips | = 113 kips > 108 kips o.k. |

Flange Plate Tensile Rupture

\[ A_e = A_e \text{ from AISC Specification Section J4.1(b)} \]

\[ A_e = \left[ B - 2(d_h + \frac{1}{16} \text{ in.}) \right] t_p \]
\( = [7.00 \text{ in.} - 2(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{3}{8} \text{ in.}) \)
\( = 3.75 \text{ in.}^2 \)

\[ A_e = 3.75 \text{ in.}^2 \]

\[ R_n = F_n A_e \]
(\( \text{Spec. Eq. J4-2} \))
\( = 58 \text{ ksi}(3.75 \text{ in.}^2) \)
\( = 218 \text{ kips} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( \phi = 0.75 \) | \( \Omega = 2.00 \)
| \( \phi R_n = 0.75(218 \text{ kips}) \) | \( R_n = 218 \text{ kips} \)
| = 164 kips > 161 kips o.k. | \( \Omega = 2.00 \)
| \( = 109 \text{ kips} > 108 \text{ kips} \) o.k. |

Flange Plate Block Shear Rupture

There are three cases for which block shear rupture must be checked (see Figure IIB-1). The first case involves the tearout of the two blocks outside the two rows of bolt holes in the flange plate; for this case \( L_{eb} = 1\frac{1}{2} \text{ in.} \) and \( L_{ev} = 1\frac{1}{2} \text{ in.} \). The second case involves the tearout of the block between the two rows of the holes in the flange plate. AISC Manual Tables 9-3a, 9-3b, and 9-3c may be adapted for this calculation by considering the 4 in. width to be comprised of two, 2 in. wide blocks where \( L_{eb} = 2 \text{ in.} \) and \( L_{ev} = 1\frac{1}{2} \text{ in.} \). The first case is more critical than the second case because \( L_{eb} \) is smaller. The third case involves a shear failure through one row of bolts and a tensile failure through the two bolts closest to the column.
Fig. IIB-1. Three cases for block shear rupture.

<table>
<thead>
<tr>
<th>Case 1:</th>
<th>Case 1:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LRFD</strong></td>
<td><strong>ASD</strong></td>
</tr>
<tr>
<td>From AISC Specification Equation J4-5:</td>
<td>From AISC Specification Equation J4-5:</td>
</tr>
<tr>
<td>$R_n = \phi U_{bs} F_u A_{at} + \min \left( \phi 0.60 F_y A_{gv}, \phi 0.60 F_u A_{nv} \right)$</td>
<td>$R_n = \frac{U_{bs} F_u A_{at}}{\Omega} + \min \left( \frac{0.60 F_y A_{gv}}{\Omega}, \frac{0.60 F_u A_{nv}}{\Omega} \right)$</td>
</tr>
<tr>
<td>$U_{bs} = 1.0, L_{cv} = 1\frac{1}{2} \text{ in.}$</td>
<td>$U_{bs} = 1.0, L_{cv} = 1\frac{1}{2} \text{ in.}$</td>
</tr>
<tr>
<td>Tension component from AISC Manual Table 9-3a:</td>
<td>Tension component from AISC Manual Table 9-3a:</td>
</tr>
<tr>
<td>$\phi U_{bs} F_u A_{at} = 1.0(43.5 \text{ kips/in.})(\frac{3}{4} \text{ in.})(2)$</td>
<td>$U_{bs} F_u A_{at} / \Omega = 1.0(29.0 \text{ kips/in.})(\frac{3}{4} \text{ in.})(2)$</td>
</tr>
<tr>
<td>Shear yielding component from AISC Manual Table 9-3b:</td>
<td>Shear yielding component from AISC Manual Table 9-3b:</td>
</tr>
<tr>
<td>$\phi 0.60 F_y A_{gv} = 170 \text{ kips/in.}(\frac{3}{4} \text{ in.})(2)$</td>
<td>$0.60 F_y A_{gv} / \Omega = 113 \text{ kips/in.}(\frac{3}{4} \text{ in.})(2)$</td>
</tr>
<tr>
<td>Shear rupture component from AISC Manual Table 9-3c:</td>
<td>Shear rupture component from AISC Manual Table 9-3c:</td>
</tr>
<tr>
<td>$\phi 0.60 F_u A_{nv} = 183 \text{ kips/in.}(\frac{3}{4} \text{ in.})(2)$</td>
<td>$0.60 F_u A_{nv} / \Omega = 122 \text{ kips/in.}(\frac{3}{4} \text{ in.})(2)$</td>
</tr>
<tr>
<td>Shear yielding controls, thus,</td>
<td>Shear yielding controls, thus,</td>
</tr>
<tr>
<td>$R_n = \left( \frac{43.5 \text{ kips}}{\text{in.}} + \frac{170 \text{ kips}}{\text{in.}} \right)(\frac{3}{4} \text{ in.})(2)$</td>
<td>$R_n = \left( \frac{29.0 \text{ kips}}{\text{in.}} + \frac{113 \text{ kips}}{\text{in.}} \right)(\frac{3}{4} \text{ in.})(2)$</td>
</tr>
<tr>
<td>$= 320 \text{ kips} &gt; 161 \text{ kips}$</td>
<td>$= 213 \text{ kips} &gt; 108 \text{ kips}$</td>
</tr>
</tbody>
</table>

**o.k.**
Case 3:
From AISC Specification Equation J4-5:
\[ \phi R_n = \phi U_{bs} F_u A_{nt} + \min \left( \phi 0.60 F_y A_{gy}, \phi 0.60 F_u A_{nv} \right) \]
\[ U_{bs} = 1.0 \]

Tension component:
\[ A_{nt} = [5.50 \text{ in.} - 1.5(\frac{5}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{3}{4} \text{ in}) = 3.00 \text{ in}^2 \]
\[ \phi U_{bs} F_u A_{nt} = 0.75 (1.0)(58 \text{ ksi})(3.00 \text{ in}^2) = 131 \text{ kips} \]

Shear yielding component from AISC Manual Table 9-3b with \( L_{cv} = 1\frac{1}{2} \text{ in.} \):
\[ \phi 0.60 F_y A_{gy} = 170 \text{ kips/in.}(\frac{3}{4} \text{ in}) \]

Shear rupture component from AISC Manual Table 9-3c with \( L_{cv} = 1\frac{1}{2} \text{ in.} \):
\[ \phi 0.60 F_u A_{nv} = 183 \text{ kips/in.}(\frac{3}{4} \text{ in}) \]

Shear yielding controls, thus,
\[ \phi R_n = 131 \text{ kips} + 170 \text{ kips/in.}(\frac{3}{4} \text{ in.}) = 259 \text{ kips} > 161 \text{ kips} \text{ o.k.} \]

\[ R_n = 87.0 \text{ kips} + 113 \text{ kips/in.}(\frac{3}{4} \text{ in.}) \]
\[ = 172 \text{ kips} > 108 \text{ kips} \text{ o.k.} \]
Shear yielding component from AISC Manual Table 9-3b:
\[ \phi 0.6F_y A_{pv} = (231 \text{ kips/in.})(0.570 \text{ in.})(2) \]

Shear rupture component from AISC Manual Table 9-3c:
\[ \phi 0.6F_u A_{nv} = (197 \text{ kips/in.})(0.570 \text{ in.})(2) \]

Shear rupture controls, thus,
\[ \phi R_e = \left( \frac{60.9 \text{ kips}}{\text{in.}} + \frac{197 \text{ kips}}{\text{in.}} \right)(0.570 \text{ in.})(2) \]
\[ = 294 \text{ kips} \times 168 \text{ kips} \]

Shear yielding component from AISC Manual Table 9-3b:
\[ 0.6F_y A_{pv} / \Omega = (154 \text{ kips/in.})(0.570 \text{ in.})(2) \]

Shear rupture component from AISC Manual Table 9-3c:
\[ 0.6F_u A_{nv} / \Omega = (132 \text{ kips/in.})(0.570 \text{ in.})(2) \]

Shear rupture controls, thus,
\[ R_n / \Omega = \left( \frac{40.6 \text{ kips}}{\text{in.}} + \frac{132 \text{ kips}}{\text{in.}} \right)(0.570 \text{ in.})(2) \]
\[ = 197 \text{ kips} \times 112 \text{ kips} \]

Fillet Weld to Supporting Column Flange

The applied load is perpendicular to the weld length; therefore \( \theta = 90^\circ \) and \( 1.0 + 0.50 \sin^{1.5} \theta = 1.5 \).

From AISC Manual Equation 8-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ D_{min} = \frac{P_{af}}{2(1.5)(1.392)l} ] 161 kips</td>
<td>[ D_{min} = \frac{P_{af}}{2(1.5)(0.928)l} ] 108 kips</td>
</tr>
<tr>
<td>[ = \frac{2(1.5)(1.392)(7.00 \text{ in.})}{161 \text{ kips}} ] 5.51 sixteenths</td>
<td>[ = \frac{2(1.5)(0.928)(7.00 \text{ in.})}{108 \text{ kips}} ] 5.54 sixteenths</td>
</tr>
<tr>
<td>Use ( % )-in. fillet welds, ( 6 &gt; 5.51 ) ( \text{o.k.} )</td>
<td>Use ( % )-in. fillet welds, ( 6 &gt; 5.54 ) ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

Connecting Elements Rupture Strength at Welds (using AISC Manual Part 9)

\[ t_{min} = \frac{3.09D}{F_u} \text{ for } F_{EXX} = 70 \text{ ksi} \]

\[ = \frac{3.09(5.54 \text{ sixteenths})}{65 \text{ ksi}} \]

\[ = 0.263 \text{ in.} < 0.780 \text{ in. column flange} \text{ o.k.} \]

Compression Flange Plate and Connection

Try PL \( \frac{3}{4} \times 7 \).

\[ K = 0.65 \text{ from AISC Specification Commentary Table C-A-7.1} \]
\[ L = 2.00 \text{ in. (1} \frac{1}{2} \text{ in. edge distance and } \frac{1}{2} \text{ in. setback)} \]
\[
\frac{KL}{r} = \frac{0.65(2.00 \text{ in.})}{\frac{\text{1/4 in.}}{\sqrt{12}}} = 6.00 \leq 25
\]

Therefore, \(F_{cr} = F_y\) from AISC Specification Section J4.4.

\[A_k = 7.00 \text{ in.}(\text{1/4 in.}) = 5.25 \text{ in.}^2\]

From AISC Specification Equation J4-6:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.90)</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(\phi P_n = \phi F_y A_g = 0.90(36 \text{ ksi})(5.25 \text{ in.}^2))</td>
<td>(P_n = \frac{F_y A_g}{\Omega})</td>
</tr>
<tr>
<td>= 0.90 (36 ksi)(5.25 in.(^2))</td>
<td>= \frac{36 \text{ ksi}(5.25 \text{ in.}^2)}{1.67}</td>
</tr>
<tr>
<td>= 170 kips &gt; 161 kips</td>
<td>= 113 kips &gt; 108 kips</td>
</tr>
</tbody>
</table>

The compression flange plate will be identical to the tension flange plate; a \(1/4\)-in. x 7-in. plate with eight bolts in two rows of four bolts on a 4 in. gage and \(5/8\)-in. fillet welds to the supporting column flange.

Note: The bolt bearing and shear checks are the same as for the tension flange plate and are o.k. by inspection. Tension due to load reversal must also be considered in the design of the fillet weld to the supporting column flange.

Flange Local Bending of Column (AISC Specification Section J10.1)

Assume the concentrated force to be resisted is applied at a distance from the member end that is greater than \(10t_f\).

\[10t_f = 10(0.780 \text{ in.}) = 7.80 \text{ in.}\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_a = 6.25 F_{yf} t_f^2) (Spec. Eq. J10-1)</td>
<td>(R_a = 6.25 F_{yf} t_f^2) (Spec. Eq. J10-1)</td>
</tr>
<tr>
<td>= 6.25(50 ksi)(0.780 in.)(^2)</td>
<td>= 6.25(50 ksi)(0.780 in.)(^2)</td>
</tr>
<tr>
<td>= 190 kips</td>
<td>= 190 kips</td>
</tr>
<tr>
<td>(\phi = 0.90)</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(\phi R_a = 0.90(190 \text{ kips}))</td>
<td>(R_a = \frac{190 \text{ kips}}{1.67})</td>
</tr>
<tr>
<td>= 171 kips &gt; 161 kips</td>
<td>= 114 kips &gt; 108 kips</td>
</tr>
</tbody>
</table>

Web Local Yielding of Column (AISC Specification Section J10.2)

Assume the concentrated force to be resisted is applied at a distance from the member that is greater than the depth of the member, \(d\).

From AISC Manual Table 9-4:
Web Crippling (AISC Specification Section J10.3)

Assume the concentrated force to be resisted is applied at a distance from the member end that is greater than or equal to $d/2$.

From AISC Manual Table 9-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 2(\phi R_1) + l_b(\phi R_2)$</td>
<td>$R_n = 2\left(\frac{R_1}{\Omega}\right) + l_b\left(\frac{R_2}{\Omega}\right)$</td>
</tr>
<tr>
<td>$(Manual\ Eq.\ 9-46a)$</td>
<td>$(Manual\ Eq.\ 9-46b)$</td>
</tr>
<tr>
<td>$= 2(83.7\ \text{kips}) + 0.750\ \text{in.}(24.3\ \text{kips/in.})$</td>
<td>$= 2(55.8\ \text{kips}) + 0.750\ \text{in.}(16.2\ \text{kips/in.})$</td>
</tr>
<tr>
<td>$= 186\ \text{kips} &gt; 161\ \text{kips}$</td>
<td>$= 124\ \text{kips} &gt; 108\ \text{kips}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note: Web compression buckling (AISC Specification Section J10.5) must be checked if another beam is framed into the opposite side of the column at this location.

Web panel zone shear (AISC Specification Section J10.6) should also be checked for this column.

For further information, see AISC Design Guide 13 Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications (Carter, 1999).
EXAMPLE II.B-2  WELDED FLANGE-PLATED FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a welded flange-plated FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange to transfer the following vertical shear forces and strong-axis moments:

\[ \begin{align*}
V_D &= 7.0 \text{ kips} \\
M_D &= 42 \text{ kip-ft} \\
V_L &= 21 \text{ kips} \\
M_L &= 126 \text{ kip-ft}
\end{align*} \]

Use \%'-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. The flange plates are A36 material. Check the column for stiffening requirements.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)
Column
W14×99
ASTM A992
$F_y = 50 \text{ ksi}$
$F_u = 65 \text{ ksi}$

Plates
ASTM A36
$F_y = 36 \text{ ksi}$
$F_u = 58 \text{ ksi}$

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
\[
\begin{align*}
  d &= 18.0 \text{ in.} \\
  b_f &= 7.50 \text{ in.} \\
  t_f &= 0.570 \text{ in.} \\
  t_w &= 0.355 \text{ in.} \\
  Z_x &= 101 \text{ in.}^3
\end{align*}
\]

Column
W14×99
\[
\begin{align*}
  d &= 14.2 \text{ in.} \\
  b_f &= 14.6 \text{ in.} \\
  t_f &= 0.780 \text{ in.} \\
  t_w &= 0.485 \text{ in.} \\
  k_{des} &= 1.38 \text{ in.}
\end{align*}
\]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$</td>
<td>$1.2(7.0 \text{ kips}) + 1.6(21 \text{ kips}) = 42.0 \text{ kips}$</td>
<td>$R_u = 7.0 \text{ kips} + 21 \text{ kips} = 28.0 \text{ kips}$</td>
</tr>
<tr>
<td>$M_u$</td>
<td>$1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft}) = 252 \text{ kip-ft}$</td>
<td>$M_u = 42 \text{ kip-ft} + 126 \text{ kip-ft} = 168 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

The single-plate web connection is verified in Example II-B-1.

Note: By inspection, the available shear yielding, shear rupture and block shear rupture strengths of the beam web are o.k.

*Tension Flange Plate and Connection*

Determine the flange force using AISC *Manual* Part 12.

The top flange width, $b_f = 7.50$ in. Assume a shelf dimension of $\frac{3}{4}$ in. on both sides of the plate. The plate width, then, is $7.50$ in. $- 2(\frac{3}{4} \text{ in.}) = 6.25$ in. Try a 1 in. $\times 6 \frac{3}{4}$ in. flange plate. Assume a $\frac{3}{4}$-in. bottom flange plate.

From AISC *Manual* Equation 12-1:
$P_{af} = \frac{M_u}{d + t_p}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{af} = \frac{M_u}{d + t_p}$</td>
<td>$P_{af} = \frac{M_u}{d + t_p}$</td>
</tr>
<tr>
<td>= 252 kip-ft (12 in./ft)</td>
<td>= 168 kip-ft (12 in./ft)</td>
</tr>
<tr>
<td>18.0 in. + 0.875 in.</td>
<td>18.0 in. + 0.875 in.</td>
</tr>
<tr>
<td>= 160 kips</td>
<td>= 107 kips</td>
</tr>
</tbody>
</table>

Flange Plate Tensile Yielding

$R_n = F_y A_t$

= 36 ksi (6.25 in.)(1.00 in.)

= 225 kips

$\phi = 0.90$

$\phi R_n = 0.90 (225 \text{ kips})$

= 203 kips

$\Omega = 1.67$

$R_n = 225 \text{ kips}$

$\Omega = 1.67$

= 135 kips

203 kips > 160 kips \textbf{O.K.}

135 kips > 107 kips \textbf{O.K.}

Determine the force in the welds.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{af} = \frac{M_u}{d}$</td>
<td>$P_{af} = \frac{M_u}{d}$</td>
</tr>
<tr>
<td>= 252 kip-ft (12 in./ft)</td>
<td>= 168 kip-ft (12 in./ft)</td>
</tr>
<tr>
<td>18.0 in.</td>
<td>18.0 in.</td>
</tr>
<tr>
<td>= 168 kips</td>
<td>= 112 kips</td>
</tr>
</tbody>
</table>

Required Weld Size and Length for Fillet Welds to Beam Flange (using AISC Manual Part 8)

Try a 7/8-in. fillet weld. The minimum length of weld, $l_{min}$, is determined as follows.

For weld compatibility, disregard the increased capacity due to perpendicular loading of the end weld (see Part 8 discussion under “Effect of Load Angle”).

From AISC Manual Equation 8-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{min} = \frac{P_{af}}{1.392D}$</td>
<td>$l_{min} = \frac{P_{af}}{0.928D}$</td>
</tr>
<tr>
<td>= 168 kips</td>
<td>= 112 kips</td>
</tr>
<tr>
<td>1.392(5)</td>
<td>0.928(5)</td>
</tr>
<tr>
<td>= 24.1 in.</td>
<td>= 24.1 in.</td>
</tr>
</tbody>
</table>

Use 9 in. of weld along each side and 6 ¼ in. of weld along the end of the flange plate.

Use 9 in. of weld along each side and 6¼ in. of weld along the end of the flange plate.
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

\[
\begin{align*}
I &= 2(9.00 \text{ in.}) + 6.25 \text{ in.} \\
&= 24.3 \text{ in.} > 24.1 \text{ in.} & o.k.
\end{align*}
\]

Connecting Elements Rupture Strength at Welds (Top Flange)

\[
t_{min} = \frac{3.09 D}{F_u} \quad \text{for } F_{EXX} = 70 \text{ ksi}
\]

\[
t_{min} = \frac{3.09(5 \text{ sixteenths})}{65 \text{ ksi}} = 0.238 \text{ in.} < 0.570 \text{ in. beam flange} \quad o.k.
\]

\[
t_{min} = \frac{3.09 D}{F_u} \quad \text{(Manual Eq. 9-2)}
\]

\[
t_{min} = \frac{3.09(5 \text{ sixteenths})}{58 \text{ ksi}} = 0.266 \text{ in.} < 1.00 \text{ in. top flange plate} \quad o.k.
\]

Required Fillet Weld Size at Top Flange Plate to Column Flange (using AISC Manual Part 8)

The applied tensile load is perpendicular to the weld, therefore,

\[\theta = 90^\circ \text{ and } 1.0 + 0.50\sin^{1.5} \theta = 1.5.\]

From AISC Manual Equation 8-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[D_{min} = \frac{P_{uf}}{2(1.5)(1.392)l} = \frac{160 \text{ kips}}{2(1.5)(1.392)(6.25 \text{ in.})} = 6.13 \text{ sixteenths}}\right| \text{Use } \frac{\gamma_{lo}}{6}-\text{in. fillet welds, } 7 > 6.13 \quad o.k.
| \[D_{min} = \frac{P_{uf}}{2(1.5)(0.928)l} = \frac{107 \text{ kips}}{2(1.5)(0.928)(6.25 \text{ in.})} = 6.15 \text{ sixteenths}}\right| \text{Use } \frac{\gamma_{lo}}{6}-\text{in. fillet welds, } 7 > 6.15 \quad o.k.

Connecting Elements Rupture Strength at Welds

\[
t_{min} = \frac{3.09 D}{F_u} \quad \text{for } F_{EXX} = 70 \text{ ksi}
\]

\[
t_{min} = \frac{3.09(6.15 \text{ sixteenths})}{65 \text{ ksi}} = 0.292 \text{ in.} < 0.780 \text{ in. column flange} \quad o.k.
\]

Compression Flange Plate and Connection

Assume a shelf dimension of \(\frac{3}{8}\) in. The plate width, then, is 7.50 in. + 2(\(\frac{3}{8}\) in.) = 8.75 in.

Try a \(\frac{3}{8}\) in. \(\times\) 8\(\frac{3}{4}\) in. compression flange plate.
Assume $K = 0.65$ from AISC Specification Commentary Table C-A-7.1, and $L = 1.00$ in (1-in. setback).

\[
\frac{KL}{r} = \frac{0.65(1.00 \text{ in.})}{\frac{\frac{3}{4} \text{ in.}}{\sqrt{12}}} = 3.00 < 25
\]

Therefore, $F_{cr} = F_y$ from AISC Specification Section J4.4.

\[
A_g = 8\frac{3}{4} \text{ in.}(\frac{3}{4} \text{ in.})
= 6.56 \text{ in.}^2
\]

From AISC Specification Equation J4-6:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.90</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_n = \phi F_y A_g$</td>
<td>$R_n = \frac{F_y A_g}{\Omega}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 0.90(36 \text{ ksi})(6.56 \text{ in.}^2)$</td>
<td>$= \frac{36 \text{ ksi}(6.56 \text{ in.}^2)}{1.67}$</td>
</tr>
<tr>
<td></td>
<td>$= 213 \text{ kips} &gt; 160 \text{ kips}$</td>
<td>$= 141 \text{ kips} &gt; 107 \text{ kips}$</td>
</tr>
</tbody>
</table>

Required Weld Size and Length for Fillet Welds to Beam Flange (using AISC Manual Part 8)

Based upon the weld length required for the tension flange plate, use 5/6 in. fillet weld and 12 1/2 in. of weld along each side of the beam flange.

Connecting Elements Rupture Strength at Welds (Bottom Flange)

\[
t_{min} = \frac{3.09 D}{F_u} \text{ for } F_{exx} = 70 \text{ ksi} \quad \text{(Manual Eq. 9-2)}
\]

\[
= \frac{3.09(5 \text{ sixteenths})}{65 \text{ ksi}}
= 0.238 \text{ in.} < 0.570 \text{ in. beam flange} \quad \text{o.k.}
\]

\[
t_{min} = \frac{3.09 D}{F_u} \text{ for } F_{exx} = 58 \text{ ksi} \quad \text{(Manual Eq. 9-2)}
\]

\[
= \frac{3.09(5 \text{ sixteenths})}{58 \text{ ksi}}
= 0.266 \text{ in.} < \frac{3}{8} \text{ in. bottom flange plate} \quad \text{o.k.}
\]

Note: Tension due to load reversal must also be considered in the design of the fillet weld to the supporting column flange.

Required Fillet Weld Size at Bottom Flange Plate to Column Flange (using AISC Manual Part 8)

The applied tensile load is perpendicular to the weld; therefore
0 = 90° and 1.0 + 0.50\sin^{1.5} \theta = 1.5.

From AISC *Manual* Equation 8-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{\text{min}} = \frac{P_{uf}}{2(1.5)(1.392)l} )</td>
<td>( D_{\text{min}} = \frac{P_{uf}}{2(1.5)(0.928)l} )</td>
</tr>
<tr>
<td>160 kips</td>
<td>107 kips</td>
</tr>
<tr>
<td>( = \frac{2(1.5)(1.392)(8\frac{3}{4} \text{ in.})}{2(1.5)(0.928)(8\frac{3}{4} \text{ in.})} )</td>
<td>( = \frac{4.38 \text{ sixteenths}}{4.39 \text{ sixteenths}} )</td>
</tr>
</tbody>
</table>

Use \( \frac{3}{16} \)-in. fillet welds, 5 > 4.38 **O.K.**. Use \( \frac{3}{16} \)-in. fillet welds, 5 > 4.39 **O.K.**.

**Connecting Elements Rupture Strength at Welds**

\[
I_{\text{max}} = \frac{3.09D}{F_u} \quad \text{for } F_{EXX} = 70 \text{ ksi} \\
= \frac{3.09(4.39 \text{ sixteenths})}{65 \text{ ksi}} \\
= 0.209 \text{ in.} < 0.780 \text{ in. column flange} \quad \text{**O.K.**}
\]

See Example II.B-1 for checks of the column under concentrated forces. For further information, see AISC Design Guide 13 *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications*. (Carter, 1999).
EXAMPLE II.B-3  DIRECTLY WELDED FLANGE FR MOMENT CONNECTION
(BEAM-TO-COLUMN FLANGE)

Given:

Design a directly welded flange FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange to transfer the following vertical shear forces and strong-axis moments:

\[ V_D = 7.0 \text{ kips} \quad M_D = 42 \text{ kip-ft} \]
\[ V_L = 21 \text{ kips} \quad M_L = 126 \text{ kip-ft} \]

Use \( \frac{3}{8} \)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. Check the column for stiffening requirements.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Column
W14×99
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)
From AISC Manual Table 1-1, the geometric properties are as follows:

**Beam**

W18×50

\[ d = 18.0 \text{ in.} \]
\[ b_f = 7.50 \text{ in.} \]
\[ t_f = 0.570 \text{ in.} \]
\[ t_w = 0.355 \text{ in.} \]
\[ Z_x = 101 \text{ in.}^3 \]

**Column**

W14×99

\[ d = 14.2 \text{ in.} \]
\[ b_f = 14.6 \text{ in.} \]
\[ t_f = 0.780 \text{ in.} \]
\[ t_w = 0.485 \text{ in.} \]
\[ k_{des} = 1.38 \text{ in.} \]

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_u = 1.2(7.0 \text{ kips}) + 1.6(21 \text{ kips}) )</td>
<td>( R_u = 7.0 \text{ kips} + 21 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>= 42.0 kips</td>
<td>= 28.0 kips</td>
</tr>
<tr>
<td></td>
<td>( M_u = 1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft}) )</td>
<td>( M_u = 42 \text{ kip-ft} + 126 \text{ kip-ft} )</td>
</tr>
<tr>
<td></td>
<td>= 252 kip-ft</td>
<td>= 168 kip-ft</td>
</tr>
</tbody>
</table>

The single-plate web connection is verified in Example II.B-1.

Note: By inspection, the available shear yielding, shear rupture, and block shear rupture strengths of the beam web are o.k.

**Weld of Beam Flange to Column**

A complete-joint-penetration groove weld will transfer the entire flange force in tension and compression. It is assumed that the beam is adequate for the applied moment and will carry the tension and compression forces through the flanges.

See Example II.B-1 for checks of the column under concentrated forces. For further information, see AISC Design Guide 13 *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications.* (Carter, 1999).
EXAMPLE II.B-4 FOUR-BOLT UNSTIFFENED EXTENDED END-PLATE FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a four-bolt unstiffened extended end-plate FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange to transfer the following vertical shear forces and strong-axis moments:

\[
\begin{align*}
V_D &= 7 \text{ kips} \\
V_L &= 21 \text{ kips} \\
M_D &= 42 \text{ kip-ft} \\
M_L &= 126 \text{ kip-ft}
\end{align*}
\]

Use ASTM A325-N snug-tight bolts in standard holes and 70-ksi electrodes. The plate is ASTM A36 material.

a. Use the design procedure from AISC Steel Design Guide 4 Extended End-Plate Moment Connections Seismic and Wind Applications (Murray and Sumner, 2003).

b. Use design procedure 2 (thin end-plate and larger diameter bolts) from AISC Design Guide 16, Flush and Extended Multiple-Row Moment End-Plate Connections (Murray and Shoemaker, 2002).

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
\[F_y = 50 \text{ ksi}\]
\[F_u = 65 \text{ ksi}\]
Column
W14×99
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W18×50
\( d = 18.0 \text{ in.} \)
\( b_f = 7.50 \text{ in.} \)
\( t_f = 0.570 \text{ in.} \)
\( t_w = 0.355 \text{ in.} \)
\( S_x = 88.9 \text{ in.}^3 \)

Column
W14×99
\( d = 14.2 \text{ in.} \)
\( b_f = 14.6 \text{ in.} \)
\( t_f = 0.780 \text{ in.} \)
\( t_w = 0.485 \text{ in.} \)
\( k_{des} = 1.38 \text{ in.} \)

Solution a:

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2(7.0 \text{ kips}) + 1.6(21 \text{ kips}) ) = 42.0 kips</td>
<td>( R_u = 7.0 \text{ kips} + 21 \text{ kips} ) = 28.0 kips</td>
</tr>
<tr>
<td>( M_u = 1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft}) ) = 252 kip-ft</td>
<td>( M_u = 42 \text{ kip-ft} + 126 \text{ kip-ft} ) = 168 kip-ft</td>
</tr>
</tbody>
</table>

Extended end-plate geometric properties are as follows:

\( b_p = 7\frac{1}{2} \text{ in.} \)
\( g = 4 \text{ in.} \)
\( p_{fo} = 1\frac{1}{2} \text{ in.} \)
\( p_{fo} = 1\frac{1}{2} \text{ in.} \)

Additional dimensions are as follows:

\[ h_0 = d + p_{fo} - \frac{t_f}{2} \]
\[ = 18.0 \text{ in.} + 1\frac{1}{2} \text{ in.} - \frac{0.570 \text{ in.}}{2} \]
\[ = 19.2 \text{ in.} \]
\[ h_i = d - p_{bij} - t_f - \frac{t_f}{2} \]
\[ = 18.0 \text{ in.} - 1\frac{1}{2} \text{ in.} - 0.570 \text{ in.} - \frac{0.570 \text{ in.}}{2} \]
\[ = 15.6 \text{ in.} \]

**Required Bolt Diameter Assuming No Prying Action**

From AISC Specification Table J3.2, \( F_{nt} = 90 \text{ ksi} \), for ASTM A325-N bolts.

From Design Guide 4 Equation 3.5, determine the required bolt diameter:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( d_{b,\text{Reqd}} = \sqrt{\frac{2M_u}{\pi \phi F_{nt} (h_o + h_i)}} )</td>
<td>( d_{b,\text{Reqd}} = \sqrt{\frac{2M_u \Omega}{\pi F_{nt} (h_o + h_i)}} )</td>
</tr>
<tr>
<td>[ = \sqrt{\frac{2(252 \text{ kip-ft})(12 \text{ in./ft})}{\pi (0.75)(90 \text{ ksi})(19.2 \text{ in.} + 15.6 \text{ in.})}} ]</td>
<td>[ = \sqrt{\frac{2(168 \text{ kip-ft})(12 \text{ in./ft})(2.00)}{\pi (90 \text{ ksi})(19.2 \text{ in.} + 15.6 \text{ in.})}} ]</td>
</tr>
<tr>
<td>= 0.905 \text{ in.}</td>
<td>= 0.905 \text{ in.}</td>
</tr>
</tbody>
</table>

Use 1-in.-diameter ASTM A325-N snug-tightened bolts.

**Required End-Plate Thickness**

The end-plate yield line mechanism parameter is:

\[ s = \frac{\sqrt{b_pg}}{2} \]
\[ = \frac{\sqrt{7\frac{1}{2} \text{ in.}(4.00 \text{ in.})}}{2} \]
\[ = 2.74 \text{ in.} \]

\( p_{bij} = 1.50 \text{ in.} \leq s = 2.74 \text{ in.} \), therefore, use \( p_{bij} = 1.50 \text{ in.} \).

From Design Guide 4 Table 3.1:

\[ Y_p = \frac{b_o}{2} \left[ h_i \left( \frac{1}{p_{bij}} + \frac{1}{s} \right) + h_0 \left( \frac{1}{p_{f,0}} - \frac{1}{2} \right) + \frac{2}{g} \left( h_i (p_{bij} + s) \right) \right] \]
\[ = \frac{7\frac{1}{2} \text{ in.}}{2} \left[ 15.6 \text{ in.} \left( \frac{1}{1\frac{1}{2} \text{ in.}} + \frac{1}{2.74 \text{ in.}} \right) + 19.2 \text{ in.} \left( \frac{1}{1\frac{1}{2} \text{ in.}} - \frac{1}{2} \right) + \frac{2}{4.00 \text{ in.}} \left( 15.6 \text{ in.}(1\frac{1}{2} \text{ in.} + 2.74 \text{ in.}) \right) \right] \]
\[ = 140 \text{ in.} \]

\[ P_t = F_{nt} \left( \frac{\pi d_b^2}{4} \right) \] (Design Guide 4 Eq. 3.9)
\[ = 90 \text{ ksi} \left( \frac{\pi (1.00 \text{ in.})^2}{4} \right) \]

\[ = 70.7 \text{ kips} \]

\[ M_{np} = 2P \left( h_0 + h_1 \right) \]

\[ = 2(70.7 \text{ kips})(19.2 \text{ in.} + 15.6 \text{ in.}) \]

\[ = 4,920 \text{ kip-in.} \]

The no prying bolt available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi M_{np} = 0.75(4,920 \text{ kip-in.}) )</td>
<td>( M_{np} = 4,920 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( = 3,690 \text{ kip-in.} )</td>
<td>( \Omega = \frac{2.00}{2.00} )</td>
</tr>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \frac{t_p \text{ Req'd}}{\phi_b F_{yp} Y_p} = \sqrt{\frac{1.11 \phi M_{np}}{\phi_b F_{yp} Y_p}} ) (Design Guide 4 Eq. 3.10)</td>
<td>( t_p \text{ Req'd} = \sqrt{\frac{1.11}{\Omega_b}} \left( \frac{M_{np}}{\Omega} \right) ) (from Design Guide 4 Eq. 3.10)</td>
</tr>
<tr>
<td>( = \sqrt{\frac{0.90(36 \text{ ksi})(140 \text{ in.})}{0.90(36 \text{ ksi})(140 \text{ in.})}} )</td>
<td>( = \sqrt{\frac{0.90(36 \text{ ksi})(140 \text{ in.})}{0.90(36 \text{ ksi})(140 \text{ in.})}} )</td>
</tr>
<tr>
<td>( = 0.950 \text{ in.} )</td>
<td>( = 0.951 \text{ in.} )</td>
</tr>
</tbody>
</table>

Use a 1-in.-thick end-plate.

With a 1-in.-thick end-plate, the design strength is:

\[ \phi_b = 0.90 \]

\[ \phi_b M_{pl} = \frac{\phi_b F_{yp} t_p^2 Y_p}{1.11} \]

\[ = \frac{0.90(36 \text{ ksi})(1.00 \text{ in.})^2(140 \text{ in.})}{1.11} \]

\[ = 4,090 \text{ kip-in.} \]

Use a 1-in.-thick end-plate.

With a 1-in.-thick end-plate, the allowable strength is:

\[ \Omega_b = 1.67 \]

\[ M_{pl} = \frac{F_{yp} t_p^2 Y_p}{\Omega_b} \]

\[ \Omega_b = \frac{36 \text{ ksi}(1.00 \text{ in.})^2(140 \text{ in.})}{1.11(1.67)} \]

\[ = 2,720 \text{ kip-in.} \]
**Beam Flange Force**

The required force applied to the end plate through the beam flange is:

\[
F_{fa} = \frac{M_a}{d - t_f}
\]

\[
= \frac{252 \text{ kip-ft}}{18.0 \text{ in.} - 0.570 \text{ in.}}
= 173 \text{ kips}
\]

**Shear Yielding of the Extended End-Plate**

The available strength due to shear yielding on the extended portion of the end-plate is determined as follows.

\[
\phi = 0.90
\]

\[
\phi R_a = \phi 0.6 F_{yp} b_p t_p > \frac{F_{fa}}{2} \quad \text{(Design Guide 4 Eq. 3.12)}
\]

\[
= 0.90(0.6)(36 \text{ ksi})(7\frac{1}{2} \text{ in.})(1.00 \text{ in.}) > \frac{173 \text{ kips}}{2}
= 146 \text{ kips} > 86.5 \text{ kips}
\]

\[
= \text{o.k.}
\]

\[
\Omega = 1.67
\]

\[
\frac{R_a}{\Omega} = 0.6 F_{yp} b_p t_p > \frac{F_{fa}}{2} \quad \text{(from Design Guide 4 Eq. 3.12)}
\]

\[
\Omega = \frac{0.6(36 \text{ ksi})(7\frac{1}{2} \text{ in.})(1.00 \text{ in.})}{1.67} > \frac{116 \text{ kips}}{2}
= 97.0 \text{ kips} > 58.0 \text{ kips}
\]

\[
= \text{o.k.}
\]

**Shear Rupture of the Extended End-Plate**

The available strength due to shear rupture on the extended portion of the end-plate is determined as follows.

\[
A_p = [7\frac{1}{2} \text{ in.} - 2(1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})](1.00 \text{ in.})
= 5.25 \text{ in.}^2
\]

\[
\phi = 0.75
\]

\[
\phi R_a = \phi 0.6 F_{up} A_n > \frac{F_{fa}}{2} \quad \text{(from Design Guide 4 Eq. 3.13)}
\]

\[
= 0.75(0.6)(58 \text{ ksi})(5.25 \text{ in.}^2) > \frac{173 \text{ kips}}{2}
= 137 \text{ kips} > 86.5 \text{ kips}
\]

\[
= \text{o.k.}
\]

\[
\Omega = 2.00
\]

\[
\frac{R_a}{\Omega} = 0.6 F_{up} A_n > \frac{F_{fa}}{2} \quad \text{(from Design Guide 4 Eq. 3.13)}
\]

\[
= \frac{0.6(58 \text{ ksi})(5.25 \text{ in.}^2)}{2.00} > \frac{116 \text{ kips}}{2}
= 91.4 \text{ kips} > 58.0 \text{ kips}
\]

\[
= \text{o.k.}
\]

Note: For the vertical shear forces, the shear yielding strength, shear rupture strength, and flexural yielding strength of the end-plate are all adequate by inspection.

**Bolt Shear and Bearing**

Try the minimum of four bolts at the tension flange and two bolts at the compression flange.

Note: Based on common practice, the compression bolts are assumed to resist all of the shear force.
Determine the required size of the beam web to end-plate fillet weld in the tension-bolt region to develop the yield strength of the beam web. The minimum weld size required to match the shear rupture strength of the weld to the tension yield strength of the beam web, per unit length, is:

\[
D_{\text{min}} = \frac{\phi \frac{F_y}{t_w} (1 \text{ in.})}{2(1.5)(1.392)(1 \text{ in.})} = \frac{0.90(50 \text{ ksi})(0.355 \text{ in.})(1 \text{ in.})}{2(1.5)(1.392)(1 \text{ in.})} = 3.83
\]

Use \( \frac{1}{4} \)-in. fillet welds on both sides

Use \( \frac{1}{4} \)-in. fillet welds on both sides of the beam web from the inside face of the beam tension flange to the centerline of the inside bolt holes plus two bolt diameters. Note that the 1.5 factor is from AISC Specification Section J2.4 and accounts for the increased strength of a transversely loaded fillet weld.

**Weld Size Required for the End Reaction**

The end reaction, \( R_u \) or \( R_a \), is resisted by the lesser of the beam web-to-end-plate weld 1) between the mid-depth of the beam and the inside face of the compression flange, or 2) between the inner row of tension bolts plus two bolt diameters and the inside face of the beam compression flange. By inspection, the former governs for this example.

\[
l = \frac{d}{2} - t_f
\]

\[
= \frac{18.0 \text{ in.}}{2} - 0.570 \text{ in.}
\]

\[
= 8.43 \text{ in.}
\]

From AISC *Manual* Equations 8-2:
The minimum fillet weld size from AISC Specification Table J2.4 is \( \frac{1}{6} \) in. Use a \( \frac{1}{6} \)-in. fillet weld on both sides of the beam web below the tension-bolt region.

**Connecting Elements Rupture Strength at Welds**

\[
\begin{align*}
t_{\min} &= \frac{6.19D}{F_u} \\
&= \frac{6.19(1.79 \text{ sixteenths})}{65 \text{ ksi}} \\
&= 0.170 \text{ in.} < 0.355 \text{ in. beam web o.k.}
\end{align*}
\]

\[
\begin{align*}
t_{\min} &= \frac{3.09D}{F_u} \\
&= \frac{3.09(1.79 \text{ sixteenths})}{58 \text{ ksi}} \\
&= 0.0954 \text{ in.} < 1.00 \text{ in. end-plate o.k.}
\end{align*}
\]

**Required Fillet Weld Size for the Beam Flange to End-Plate Connection**

\[
l = 2(b_f) - t_w
\]

\[
= 2(7.50 \text{ in.}) - 0.355 \text{ in.}
\]

\[
= 14.6 \text{ in.}
\]

From AISC Manual Part 8:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{fu} = 173 \text{ kips} )</td>
<td>( F_{fu} = 116 \text{ kips} )</td>
</tr>
<tr>
<td>( D_{min} = \frac{F_{fu}}{1.5(1.392)l} )</td>
<td>( D_{min} = \frac{F_{fu}}{1.5(0.928)l} )</td>
</tr>
<tr>
<td>( = \frac{173 \text{ kips}}{1.5(1.392)(14.6 \text{ in.})} )</td>
<td>( = \frac{116 \text{ kips}}{1.5(0.928)(14.6 \text{ in.})} )</td>
</tr>
<tr>
<td>( = 5.67 \rightarrow 6 \text{ sixteenths} )</td>
<td>( = 5.71 \rightarrow 6 \text{ sixteenths} )</td>
</tr>
</tbody>
</table>

Note that the 1.5 factor is from AISC Specification J2.4 and accounts for the increased strength of a transversely loaded fillet weld.

Use \( \frac{1}{6} \)-in. fillet welds at the beam tension flange. Welds at the compression flange may be \( \frac{1}{4} \)-in. fillet welds (minimum size per AISC Specification Table J2.4).
Connecting Elements Rupture Strength at Welds

Shear rupture strength of base metal

\[ t_{\text{min}} = \frac{3.09D}{F_u} \]

\[ = 3.09 \times (5.71 \text{ sixteenths}) \]

\[ = \frac{58 \text{ ksi}}{3.09(5.71 \text{ sixteenths})} \]

\[ = 0.304 \text{ in.} < 1.00 \text{ in. end-plate} \quad \text{o.k.} \]

Solution b:

Only those portions of the design that vary from the Solution “a” calculations are presented here.

**Required End-Plate Thickness**

\[ \phi_b = 0.90 \]

\[ t_{p,\text{req}} = \frac{\gamma_f M_u}{\phi_b F_y Y} \]

\[ = \frac{1.0(252 \text{ kip-ft})(12 \text{ in./ft})}{0.90(36 \text{ ksi})(140 \text{ in.})} \]

\[ = 0.816 \text{ in.} \quad \text{Use } t_p = \frac{3}{16} \text{ in.} \]

\[ \Omega_b = 1.67 \]

\[ t_{p,\text{req}} = \frac{\gamma_f M_u \Omega_b}{F_y Y} \]

\[ = \frac{1.0(168 \text{ kip-ft})(12 \text{ in./ft})(1.67)}{(36 \text{ ksi})(140 \text{ in.})} \]

\[ = 0.817 \text{ in.} \quad \text{Use } t_p = \frac{3}{16} \text{ in.} \]
**Trial Bolt Diameter and Maximum Prying Forces**

Try 1-in.-diameter bolts.

\[ w' = \frac{b_p}{2} - \left( d_h + \frac{\gamma}{2} \right) \text{ in.} \]
\[ = \frac{7.50}{2} \text{ in.} - (1.00 \text{ in.} + \frac{\gamma}{2}) \text{ in.} \]
\[ = 2.69 \text{ in.} \]

\[ a_i = 3.62 \left( \frac{d_h}{d_p} \right)^3 - 0.085 \]
\[ = 3.62 \left( \frac{\gamma}{2} \text{ in.}}{1.00 \text{ in.}} \right)^3 - 0.085 \]
\[ = 2.34 \text{ in.} \]

\[ F_i' = \frac{t_p^2 F_{py} \left[ 0.85 \left( \frac{b_p}{2} \right) + 0.80 w' \right] + \pi d_h^2 F_{nt}}{4 p_{f,i}} \]
\[ = \frac{3.62 \left( \frac{7.50}{2} \text{ in.} \right) + 0.80 (2.69 \text{ in.}) + \pi (1.00 \text{ in.})^3 (90 \text{ ksi})}{4 (1\frac{1}{2} \text{ in.})} \]
\[ = 30.4 \text{ kips} \]

\[ Q_{\text{max,i}} = \frac{w' t_p^2}{4 a_i} \sqrt{F_{py} - 3 \left( \frac{F_i'}{w' t_p} \right)^2} \]
\[ = \frac{2.69 \text{ in.} \left( \frac{\gamma}{2} \text{ in.} \right)^2}{4 (2.38 \text{ in.})} \sqrt{(36 \text{ ksi})^2 - 3 \left( \frac{30.4 \text{ kips}}{2.69 \text{ in.} \left( \frac{\gamma}{2} \text{ in.} \right)} \right)^2} \]
\[ = 6.10 \text{ kips} \]

\[ a_o = \min \left[ a_i, p_{\text{ext}} - p_{f,o} \right] \]
\[ = \min [2.34 \text{ in.}, (3.00 \text{ in.} - 1\frac{1}{2} \text{ in.})] \]
\[ = 1.50 \text{ in.} \]

From AISC Design Guide 16 Equation 2-17:

\[ F_{o,i}' = F_i' \left( \frac{p_{f,i}}{p_{f,o}} \right) \]
\[ = 30.4 \text{ kips} \left( \frac{1\frac{1}{2} \text{ in.}}{1\frac{1}{2} \text{ in.}} \right) \]
\[ = 30.4 \text{ kips} \]
\[
Q_{\text{max},o} = \frac{w' t_p^2}{4 a_o} \sqrt{F_p' - 3 \left( \frac{F_o' w't_p}{w't_p} \right)^2}
\]

(Design Guide 16 Eq. 2-15)

\[
= \frac{2.69 \text{ in.} (76 \text{ in.})^2}{4(1.50 \text{ in.})} \sqrt{36 \text{ ksi}^2 - 3 \left( \frac{30.4 \text{ kips}}{2.69 \text{ in.} (76 \text{ in.})} \right)^2}
\]

= 9.68 kips

**Bolt Rupture with Prying Action**

\[
P_t = \frac{\pi d_b^2 F_{ut}}{4}
\]

\[
= \frac{\pi (1.00 \text{ in.})^2 (90 \text{ ksi})}{4}
\]

= 70.7 kips

From AISC Specification Table J3.1, the unmodified bolt pretension, \( T_{0o} \), = 51 kips.

Modify bolt pretension for the snug-tight condition.

\[
T_b = 0.25(T_{0o}) \text{ from AISC Design Guide 16 Table 4-1.}
\]

= 0.25(51 kips)

= 12.8 kips

From AISC Design Guide Equation 2-19:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi ) = 0.75</td>
<td>( \Omega = 2.00 )</td>
</tr>
</tbody>
</table>
| \( \phi M_q = \max \left\{ \begin{array}{c}
\phi \left[ 2(P_t - Q_{\text{max},o})d_0 + 2(P_t - Q_{\text{max},i})d_1 \right] \\
\phi \left[ 2(P_t - Q_{\text{max},o})d_0 + 2(T_b)d_1 \right] \\
\phi \left[ 2(P_t - Q_{\text{max},i})d_1 + 2(T_b)d_0 \right] \\
\phi \left[ 2(T_b)(d_0 + d_1) \right]
\end{array} \right\} \) |
| \( \frac{M_q}{\Omega} = \max \left\{ \begin{array}{c}
\frac{1}{\Omega} \left[ 2(P_t - Q_{\text{max},o})d_0 + 2(P_t - Q_{\text{max},i})d_1 \right] \\
\frac{1}{\Omega} \left[ 2(P_t - Q_{\text{max},o})d_0 + 2(T_b)d_1 \right] \\
\frac{1}{\Omega} \left[ 2(P_t - Q_{\text{max},i})d_1 + 2(T_b)d_0 \right] \\
\frac{1}{\Omega} \left[ 2(T_b)(d_0 + d_1) \right]
\end{array} \right\} \) |

\[
0.75 \left[ 2(70.7 \text{ kips} - 9.68 \text{ kips})(19.2 \text{ in.}) \right] \\
+2(70.7 \text{ kips} - 6.10 \text{ kips})(15.6 \text{ in.})
\]

\[
0.75 \left[ 2(70.7 \text{ kips} - 9.68 \text{ kips})(19.2 \text{ in.}) \right] \\
+2(12.8 \text{ kips})(15.6 \text{ in.})
\]

\[
0.75 \left[ 2(70.7 \text{ kips} - 6.10 \text{ kips})(15.6 \text{ in.}) \right] \\
+2(12.8 \text{ kips})(19.2 \text{ in.})
\]

\[
0.75 \left[ 2(12.8 \text{ kips})(19.2 \text{ in.} + 15.6 \text{ in.}) \right]
\]

\[
0.75 \left[ 2(70.7 \text{ kips} - 9.68 \text{ kips})(19.2 \text{ in.}) \right] \\
+2(70.7 \text{ kips} - 6.10 \text{ kips})(15.6 \text{ in.})
\]

\[
0.75 \left[ 2(12.8 \text{ kips})(19.2 \text{ in.} + 15.6 \text{ in.}) \right]
\]

\[
0.75 \left[ 2(70.7 \text{ kips} - 6.10 \text{ kips})(15.6 \text{ in.}) \right] \\
+2(12.8 \text{ kips})(19.2 \text{ in.})
\]

\[
0.75 \left[ 2(12.8 \text{ kips})(19.2 \text{ in.} + 15.6 \text{ in.}) \right]
\]
For Example II.B-4, the design procedure from Design Guide 4 produced a design with a 1-in.-thick end-plate and 1-in. diameter bolts. Design procedure 2 from Design Guide 16 produced a design with a ³⁄₈-in.-thick end-plate and 1-in.-diameter bolts. Either design is acceptable. The first design procedure did not produce a smaller bolt diameter for this example, although in general it should result in a thicker plate and smaller diameter bolt than the second design procedure. Note that the bolt stress is lower in the first design procedure than in the second design procedure.
CHAPTER IIB DESIGN EXAMPLE REFERENCES


Chapter IIC
Bracing and Truss Connections

The design of bracing and truss connections is covered in Part 13 of the AISC Steel Construction Manual.
EXAMPLE II.C-1  TRUSS SUPPORT CONNECTION

Given:

Based on the configuration shown in Figure II.C-1-1, determine:

a. The connection requirements between the gusset and the column
b. The required gusset size and the weld requirements connecting the diagonal to the gusset

The reactions on the truss end connection are:

\[ R_D = 16.6 \text{ kips} \]
\[ R_L = 53.8 \text{ kips} \]

Use ½-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. The top chord and column are ASTM A992 material. The diagonal member, gusset plate and clip angles are ASTM A36 material.

Fig. II.C-1-1. Truss support connection.
Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

- **Top Chord**
  - WT8×38.5
  - ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

- **Column**
  - W12×50
  - ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

- **Diagonal**
  - 2L4×3½×¾
  - ASTM A36
  - $F_y = 36$ ksi
  - $F_u = 58$ ksi

- **Gusset Plate**
  - ASTM A36
  - $F_y = 36$ ksi
  - $F_u = 58$ ksi

- **Clip Angles**
  - 2L4×4×¾
  - ASTM A36
  - $F_y = 36$ ksi
  - $F_u = 58$ ksi

From AISC Manual Tables 1-1, 1-8 and 1-15, the geometric properties are as follows:

- **Top Chord**
  - WT8×38.5
  - $d = 8.26$ in.
  - $t_w = 0.455$ in.

- **Column**
  - W12×50
  - $d = 12.2$ in.
  - $t_f = 0.640$ in.
  - $b_f = 8.08$ in.
  - $t_w = 0.370$ in.

- **Diagonal**
  - 2L4×3½×¾
  - $t = 0.375$ in.
  - $A = 5.36$ in.$^2$
  - $\bar{x} = 0.947$ in. for single angle

From Chapter 2 of ASCE/SEI 7, the required strength is:
Brace axial load:

\[ R_u = 168 \text{ kips} \]

Truss end reaction:

\[ R_u = 1.2(16.6 \text{ kips}) + 1.6(53.8 \text{ kips}) = 106 \text{ kips} \]

Top chord axial load:

\[ R_u = 131 \text{ kips} \]

As ASD:

Brace axial load:

\[ R_u = 112 \text{ kips} \]

Truss end reaction:

\[ R_u = 16.6 \text{ kips} + 53.8 \text{ kips} = 70.4 \text{ kips} \]

Top chord axial load:

\[ R_u = 87.2 \text{ kips} \]

**Weld Connecting the Diagonal to the Gusset Plate**

Note: AISC Specification Section J1.7 requiring that the center of gravity of the weld group coincide with the center of gravity of the member does not apply to end connections of statically loaded single angle, double angle and similar members.

For \(\frac{3}{8}\)-in. angles, \(D_{\text{min}} = 3\) from AISC Specification Table J2.4.

Try \(\frac{3}{4}\)-in. fillet welds, \(D = 4\). From AISC Manual Equation 8-2, the required length is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_{\text{req}} = \frac{R_u}{4D(1.392)})</td>
<td>(L_{\text{req}} = \frac{R_u}{4D(0.928)})</td>
</tr>
<tr>
<td>[= \frac{168 \text{ kips}}{4(4)(1.392)}] = 7.54 in.</td>
<td>[= \frac{112 \text{ kips}}{4(4)(0.928)}] = 7.54 in.</td>
</tr>
</tbody>
</table>

Use 8 in. at the heel and 8 in. at the toe of each angle.

**Tensile Yielding of Diagonal**

\[ R_u = F_y A_g \quad (\text{Spec. Eq. J4-1}) \]

\[ = 36 \text{ ksi} \left(5.36 \text{ in.}^2\right) \]

\[ = 193 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.90)</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(\phi R_u = 0.90(193 \text{ kips}) = 174 \text{ kips} &gt; 168 \text{ kips})</td>
<td>(R_u = 193 \text{ kips})</td>
</tr>
</tbody>
</table>

| \(\Omega = 1.67\) |
| \(\Omega = 116 \text{ kips} > 112 \text{ kips}\) |

\[A_g = A_g = 5.36 \text{ in.}^2\]
\[ U = 1 - \frac{X}{I} \] from AISC Specification Table D3.1 Case 2
\[ = 1 - \frac{0.947 \text{ in.}}{8.00 \text{ in.}} \]
\[ = 0.882 \]

\[ A_e = A_n U \]
\[ = 5.36 \text{ in.}^2 \times 0.882 \]
\[ = 4.73 \text{ in.}^2 \]

\[ R_n = F_u A_e \]
\[ = 58 \text{ ksi}(4.73 \text{ in.}^2) \]
\[ = 274 \text{ kips} \]

\( \phi = 0.75 \)

\[ \phi R_n = 0.75(274 \text{ kips}) \]
\[ = 206 \text{ kips} > 168 \text{ kips} \]

\[ \Omega = 2.00 \]

\[ \frac{R_n}{\Omega} = \frac{274 \text{ kips}}{2.00} \]
\[ = 137 \text{ kips} > 112 \text{ kips} \]

\[ \Omega = 2.00 \]

\[ \frac{R_n}{\Omega} = \frac{70.4 \text{ kips}}{16.2 \text{ kips/bolt}} \]
\[ = 4.35 \text{ bolts} \]

Use a \( \frac{3}{8} \)-in. gusset plate. With the diagonal to gusset welds determined, a gusset plate layout as shown in Figure II.C-1-1(a) can be made.

**Bolts Connecting Clip Angles to Column (Shear and Tension)**

From AISC Manual Table 7-1, the number of \( \frac{3}{8} \)-in.-diameter ASTM A325-N bolts required for shear only is:

\[ n_{\text{min}} = \frac{R_u}{\phi_f r_n} \]
\[ = \frac{106 \text{ kips}}{24.3 \text{ kips/bolt}} \]
\[ = 4.36 \text{ bolts} \]

Try a clip angle thickness of \( s \) in. For a trial calculation, the number of bolts was increased to 10 in pairs at 3-in. spacing. This is done to “square off” the gusset plate.

With 10 bolts:

\[ f_{rv} = \frac{R_u}{n A_b} \]
\[ = \frac{106 \text{ kips}}{10 \text{ bolts}(0.601 \text{ in.}^2)} \]
\[ = 17.6 \text{ ksi} \]

\[ f_{rv} = \frac{R_u}{n A_b} \]
\[ = \frac{70.4 \text{ kips}}{10 \text{ bolts}(0.601 \text{ in.}^2)} \]
\[ = 11.7 \text{ ksi} \]

The eccentric moment about the workpoint (WP) in Figure II.C-1-1 at the faying surface (face of column flange) is determined as follows. The eccentricity, \( e \), is half of the column depth, \( d \), equal to 12.1 in.
For the bolt group, the Case II approach in AISC Manual Part 7 can be used. Thus, the maximum tensile force per bolt, $T$, is given by the following:

\[ n' = \text{number of bolts on tension side of the neutral axis (the bottom in this case)} = 4 \text{ bolts} \]

\[ d_m = \text{moment arm between resultant tensile force and resultant compressive force} = 9.00 \text{ in.} \]

From AISC Manual Equations 7-14a and 7-14b:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{u} = R_a e )</td>
<td>( M_{u} = R_a e )</td>
</tr>
<tr>
<td>( = 106 \text{ kips (6.10 in.)} )</td>
<td>( = 70.4 \text{ kips (6.10 in.)} )</td>
</tr>
<tr>
<td>( = 647 \text{ kip-in.} )</td>
<td>( = 429 \text{ kip-in.} )</td>
</tr>
</tbody>
</table>

\[ T_u = \frac{M_u}{n'd_m} \]

\[ = \frac{647 \text{ kip-in.}}{4 \text{ bolts (9.00 in.)}} \]

\[ = 18.0 \text{ kips/bolt} \]

\[ T_a = \frac{M_a}{n'd_m} \]

\[ = \frac{429 \text{ kip-in.}}{4 \text{ bolts (9.00 in.)}} \]

\[ = 11.9 \text{ kips/bolt} \]

Tensile strength of bolts:

From AISC Specification Table J3.2:

\[ F_{nt} = 90 \text{ ksi} \]

\[ F_{nv} = 54 \text{ ksi} \]

\[ \phi = 0.75 \]

\[ F'_{nt} = 1.3F_{nt} - \phi F'_{nv} f_{rv} \leq F_{nt} \quad \text{(Spec. Eq. J3-3a)} \]

\[ = 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{0.75(54 \text{ ksi})} (17.6 \text{ ksi}) \]

\[ = 77.9 \text{ ksi} < 90 \text{ ksi} \]

\( \text{o.k.} \)

\[ b = \frac{\phi F'_{nt} A_b}{\Omega} \]

\[ = 0.75(77.9 \text{ ksi})(0.601 \text{ in.}^2) \]

\[ = 35.1 \text{ kips} > 18.0 \text{ kips} \]

\( \text{o.k.} \)

\[ \Omega = 2.00 \]

\[ F'_{nt} = 1.3F_{nt} - \frac{\Omega F'_{nt}}{F_{nv}} f_{rv} \leq F_{nt} \quad \text{(Spec. Eq. J3-3b)} \]

\[ = 1.3(90 \text{ ksi}) - \frac{2.00(90 \text{ ksi})}{54 \text{ ksi}} (11.7 \text{ ksi}) \]

\[ = 78.0 \text{ ksi} < 90 \text{ ksi} \]

\( \text{o.k.} \)

\[ B = \frac{F'_{nt}}{\Omega} A_b \]

\[ = 78.0 \text{ ksi} \frac{0.601 \text{ in.}^2}{2.00} \]

\[ = 23.4 \text{ kips} > 11.9 \text{ kips} \]

\( \text{o.k.} \)

Prying Action on Clip Angles (AISC Manual Part 9)

\[ p = 3.00 \text{ in.} \]

\[ b = 2.00 \text{ in.} - \frac{5k}{2} \]

\[ = 1.69 \text{ in.} \]
Note: 1¾ in. entering and tightening clearance from AISC Manual Table 7-15 is accommodated and the column fillet toe is cleared.

\[
    a = \frac{8.08 \text{ in.} - 4\frac{1}{2} \text{ in.}}{2} = 1.79 \text{ in.}
\]

Note: \(a\) was calculated based on the column flange width in this case because it is less than the double angle width.

\[
b' = b - \frac{d_b}{2}
\]

\[
= 1.69 \text{ in.} - \frac{7\frac{1}{2} \text{ in.}}{2}
\]

\[
= 1.25 \text{ in.}
\]

\[
a' = a + \frac{d_b}{2} \leq 1.25b + \frac{d_b}{2}
\]

\[
= 1.79 \text{ in.} + \frac{7\frac{1}{2} \text{ in.}}{2} \leq 1.25(1.69 \text{ in.}) + \frac{7\frac{1}{2} \text{ in.}}{2}
\]

\[
= 2.23 \text{ in.} \leq 2.55 \text{ in.}
\]

\[
\rho = \frac{b'}{a'}
\]

\[
= \frac{1.25 \text{ in.}}{2.23 \text{ in.}} = 0.561
\]

\[
\delta = 1 - \frac{d'}{p}
\]

\[
= 1 - \frac{3\frac{1}{2} \text{ in.}}{3.00 \text{ in.}} = 0.688
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.90)</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(t_c = \frac{4Bb'}{\phi pF_u}) ((\text{Manual Eq. 9-30a}))</td>
<td>(t_c = \frac{\Omega 4Bb'}{pF_u}) ((\text{Manual Eq. 9-30b}))</td>
</tr>
<tr>
<td>(= \frac{4(35.1 \text{kips})(1.25 \text{ in.})}{0.90(3.00 \text{ in.})(58 \text{ ksi})})</td>
<td>(= \frac{1.67(4)(23.4 \text{kips})(1.25 \text{ in.})}{3.00 \text{ in.}(58 \text{ ksi})})</td>
</tr>
<tr>
<td>(= 1.06 \text{ in.})</td>
<td>(= 1.06 \text{ in.})</td>
</tr>
</tbody>
</table>

\[
\alpha' = \frac{1}{\delta(1+\rho)} \left[ \left( \frac{t_c}{t} \right)^2 - 1 \right]
\]

\[
= \frac{1}{0.688(1+0.561)} \left[ \left( \frac{1.06 \text{ in.}}{\frac{3}{8} \text{ in.}} \right)^2 - 1 \right]
\]
Because $\alpha' > 1$,

$$Q = \left( \frac{t}{t_c} \right)^2 (1 + \delta)$$  

$(Manual\ Eq.\ 9-34)$

$$= \left( \frac{\frac{3}{8} \text{ in.}}{1.06 \text{ in.}} \right)^2 (1 + 0.688)$$

$$= 0.587$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{avail}} = BQ$ $(Manual\ Eq.\ 9-31)$</td>
<td>$T_{\text{avail}} = BQ$ $(Manual\ Eq.\ 9-31)$</td>
</tr>
<tr>
<td>$= 35.1\ \text{kips}(0.587)$</td>
<td>$= 23.4\ \text{kips}(0.587)$</td>
</tr>
<tr>
<td>$= 20.6\ \text{kips} &gt; 18.0\ \text{kips}$ o.k.</td>
<td>$= 13.7\ \text{kips} &gt; 11.9\ \text{kips}$ o.k.</td>
</tr>
</tbody>
</table>

**Shear Yielding of Clip Angles**

From AISC Specification Equation J4-3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = \phi 0.60 F_y A_{gy}$</td>
<td>$R_n = \frac{0.60 F_y A_{gy}}{\Omega}$</td>
</tr>
<tr>
<td>$= 1.00 (0.60) (36\ \text{ksi}) \left[ 2 (15.0\ \text{in.}) \left( \frac{3}{8} \text{ in.} \right) \right]$</td>
<td>$= \frac{0.60 (36\ \text{ksi}) \left[ 2 (15.0\ \text{in.}) \left( \frac{3}{8} \text{ in.} \right) \right]}{1.50}$</td>
</tr>
<tr>
<td>$= 405\ \text{kips} &gt; 106\ \text{kips}$ o.k.</td>
<td>$= 270\ \text{kips} &gt; 70.4\ \text{kips}$ o.k.</td>
</tr>
</tbody>
</table>

**Shear Rupture of Clip Angles**

$$A_{nv} = 2 \left[ 15.0\ \text{in.} - 5 \left( \frac{3}{8} \text{ in.} + \frac{1}{16} \text{ in.} \right) \right] \left( \frac{3}{8} \text{ in.} \right)$$

$$= 12.5\ \text{in.}^2$$

From AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = \phi 0.60 F_y A_{nv}$</td>
<td>$R_n = \frac{0.60 F_y A_{nv}}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.75 (0.60) (58\ \text{ksi}) \left( 12.5\ \text{in.}^2 \right)$</td>
<td>$= \frac{0.60 (58\ \text{ksi}) \left( 12.5\ \text{in.}^2 \right)}{2.00}$</td>
</tr>
<tr>
<td>$= 326\ \text{kips} &gt; 106\ \text{kips}$ o.k.</td>
<td>$= 218\ \text{kips} &gt; 70.4\ \text{kips}$ o.k.</td>
</tr>
</tbody>
</table>

**Block Shear Rupture of Clip Angles**

Assume uniform tension stress, so use $U_{bs} = 1.0$. 

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AMERICAN INSTITUTE OF STEEL CONSTRUCTION
\[ A_{gv} = 2(15.0 \text{ in.} - 1\frac{1}{2} \text{ in.}) \left( \frac{\%}{\text{in.}} \right) \]
\[ = 16.9 \text{ in.}^2 \]
\[ A_{av} = 16.9 \text{ in.}^2 - 2 \left[ 4.5 \left( \frac{15}{64} \text{ in.} + \frac{1}{64} \text{ in.} \right) \right] \left( \frac{\%}{\text{in.}} \right) \]
\[ = 11.3 \text{ in.}^2 \]
\[ A_{nt} = 2 \left[ (2.00 \text{ in.}) \left( \frac{\%}{\text{in.}} \right) - 0.5 \left( \frac{15}{64} \text{ in.} + \frac{1}{64} \text{ in.} \right) \right] \left( \frac{\%}{\text{in.}} \right) \]
\[ = 1.88 \text{ in.}^2 \]

From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi = 0.75 ]</td>
<td>[ \Omega = 2.00 ]</td>
</tr>
<tr>
<td>[ \phi R_u = \phi \left[ U_{ls} F_u A_{nt} + \min \left{ 0.60 F_y A_{gv}, 0.60 F_u A_{av} \right} \right] ]</td>
<td>[ R_u = \frac{U_{ls} F_u A_{nt} + \min \left{ 0.60 F_y A_{gv}, 0.60 F_u A_{av} \right}}{\Omega} ]</td>
</tr>
<tr>
<td>[ = 0.75 \left[ 0.4 (58 \text{ ksi}) (1.88 \text{ in.}^2) \right] + \min \left{ 0.60 \left( 36 \text{ ksi} \right) (16.9 \text{ in.}^2) \right} ]</td>
<td>[ \Omega = \left[ 0.75 \left( 1.03 \text{ in.} \right) \right] \left( \frac{1.03 \text{ in.}}{%} \right) \left( 58 \text{ ksi} \right) ]</td>
</tr>
<tr>
<td>[ = 356 \text{ kips} &gt; 106 \text{ kips} ] o.k.</td>
<td>[ = ] [ 2.00 \left[ 0.4 (58 \text{ ksi}) (1.88 \text{ in.}^2) \right] + \min \left{ 0.60 \left( 36 \text{ ksi} \right) (16.9 \text{ in.}^2) \right} ]</td>
</tr>
<tr>
<td>[ = 337 \text{ kips} &gt; 70.4 \text{ kips} ] o.k.</td>
<td>[ = ] [ 22.4 \text{ kips} &lt; 16.2 \text{ kips} ]</td>
</tr>
</tbody>
</table>

**Bearing and Tearout on Clip Angles**

The clear edge distance, \( l_c \), for the top bolts is \( l_c = L_e - d/2 \), where \( L_e \) is the distance to the center of the hole.

\[ l_c = \frac{1}{2} \text{ in.} - \frac{15}{64} \text{ in.}}{2} \]
\[ = 1.03 \text{ in.} \]

The available strength due to bearing/tearout of the top bolt is as follows:

From AISC Specification Equation J3-6a:

\[ \phi = 0.75 \]
\[ \phi R_u = \phi \left[ 1.2 l \left( \frac{F_y}{2} \right) - \phi 2.4 dt F_u \right] \]
\[ = 0.75 (1.2) (1.03 \text{ in.}) \left( \frac{\%}{\text{in.}} \right) \left( 58 \text{ ksi} \right) \]
\[ \leq 0.75 (2.4) \left( \frac{\%}{\text{in.}} \right) \left( \frac{\%}{\text{in.}} \right) \left( 58 \text{ ksi} \right) \]
\[ = 33.6 \text{ kips} \leq 57.1 \text{ kips} \]

33.6 kips/bolt > 24.3 kips/bolt

Bolt shear controls.
By inspection, the bearing capacity of the remaining bolts does not control. 

Use 2L4x4x⅜ clip angles.

_Prying Action on Column Flange (AISC Manual Part 9)_

Using the same procedure as that for the clip angles, the tensile strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{avail}$</td>
<td>18.7 kips &gt; 18.0 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>$T_{avail}$</td>
<td>12.4 kips &gt; 11.9 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

_Bearing and Tearout on Column Flange_

By inspection, these limit states will not control.

_Clip Angle-to-Gusset Plate Connection_

From AISC _Specification_ Table J2.4, the minimum weld size is $\frac{3}{16}$ in. with the top chord slope being $\frac{1}{2}$ on 12, the horizontal welds are as shown in Figure II.C-1-1(c) due to the square cut end. Use the average length.

$l = 15.0$ in.

$$kl = \frac{3\frac{3}{8} \text{ in.} + 2\frac{3}{4} \text{ in.}}{2} = 3.06 \text{ in.}$$

$$k = \frac{kl}{l} = \frac{3.06 \text{ in.}}{15.0 \text{ in.}} = 0.204$$

$$x_l = \frac{(kl)^2}{(l + 2kl)} = \frac{(3.06 \text{ in.})^2}{15.0 \text{ in.} + 2(3.06 \text{ in.})} = 0.443 \text{ in.}$$

$$a_l + x_l = 6.10 \text{ in.} + 4.00 \text{ in.} = 10.1 \text{ in.}$$

$$a_l = 10.1 \text{ in.} - x_l = 10.1 \text{ in.} - 0.443 \text{ in.} = 9.66 \text{ in.}$$

$$a = \frac{a_l}{l} = \frac{9.66 \text{ in.}}{15.0 \text{ in.}} = 0.644$$
By interpolating AISC Manual Table 8-8 with $\theta = 0^\circ$:

$$C = 1.50$$

From AISC Manual Table 8-8:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td></td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$D_{req} = \frac{R_g}{2(\phi CGl)}$</td>
<td>$\frac{106 \text{ kips}}{2(0.75)(1.50)(1.0)(15.0 \text{ in.})} = 3.14 \rightarrow 4 \text{ sixteenths}$</td>
<td>$D_{req} = \frac{\Omega R_g}{2(CGl)} = \frac{2.00(70.4 \text{ kips})}{2(1.50)(1.0)(15.0 \text{ in.})} = 3.13 \rightarrow 4 \text{ sixteenths}$</td>
</tr>
</tbody>
</table>

Use $\frac{1}{4}$-in. fillet welds.

Note: Using the average of the horizontal weld lengths provides a reasonable solution when the horizontal welds are close in length. A conservative solution can be determined by using the smaller of the horizontal weld lengths as effective for both horizontal welds. For this example, using $kl = 2.75 \text{ in.}$, $C = 1.43$ and $D_{req} = 3.29 \text{ sixteenths}$.

Tensile Yielding of Gusset Plate on the Whitmore Section (AISC Manual Part 9)

The gusset plate thickness should match or slightly exceed that of the tee stem. This requirement is satisfied by the $\frac{1}{2}$-in. plate previously selected.

The width of the Whitmore section is:

$$l_w = 4.00 \text{ in.} + 2(8.00 \text{ in.}) \tan 30^\circ$$
$$= 13.2 \text{ in.}$$

From AISC Specification Equation J4-1, the available strength due to tensile yielding is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td></td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_n = \phi F_y A_g$</td>
<td>$R_n = \frac{F_y A_g}{\Omega} = \frac{36 \text{ ksi}(13.2 \text{ in.})}{1.67} = 142 \text{ kips} &gt; 112 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
**Gusset Plate-to-Tee Stem Weld**

The interface forces are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizontal shear between gusset and WT:</strong></td>
<td><strong>Horizontal shear between gusset and WT:</strong></td>
</tr>
<tr>
<td>( H_{ab} = 131 \text{ kips} - 4 \text{ bolts}(18.0 \text{ kips/bolt}) )</td>
<td>( H_{ab} = 87.2 \text{ kips} - 4 \text{ bolts}(11.9 \text{ kips/bolt}) )</td>
</tr>
<tr>
<td>= 59.0 kips</td>
<td>= 39.6 kips</td>
</tr>
<tr>
<td><strong>Vertical tension between gusset and WT:</strong></td>
<td><strong>Vertical tension between gusset and WT:</strong></td>
</tr>
<tr>
<td>( V_{ab} = 106 \text{ kips}(4 \text{ bolts/10 bolts}) )</td>
<td>( V_{ab} = 70.4 \text{ kips}(4 \text{ bolts/10 bolts}) )</td>
</tr>
<tr>
<td>= 42.4 kips</td>
<td>= 28.2 kips</td>
</tr>
<tr>
<td><strong>Compression between WT and column:</strong></td>
<td><strong>Compression between WT and column:</strong></td>
</tr>
<tr>
<td>( C_{ab} = 4 \text{ bolts}(18.0 \text{ kips/bolt}) )</td>
<td>( C_{ab} = 4 \text{ bolts}(11.9 \text{ kips/bolt}) )</td>
</tr>
<tr>
<td>= 72.0 kips</td>
<td>= 47.6 kips</td>
</tr>
</tbody>
</table>

Summing moments about the face of the column at the workline of the top chord:

\[
M_{ub} = C_{ab}(2\frac{1}{2} \text{ in.} + 1.50 \text{ in.}) + H_{ab}(d - \bar{y})
- V_{ab} (\text{gusset plate width/2 + setback})
= 72.0 \text{ kips}(4.00 \text{ in.}) + 59.0 \text{ kips}(8.26 \text{ in.} - 1\frac{3}{16} \text{ in.})
- 42.4 \text{ kips}(15.0 \text{ in./2 + 1/2 in.})
= 340 \text{ kip-in.}
\]

A CJP weld should be used along the interface between the gusset plate and the tee stem. The weld should be ground smooth under the clip angles.

The gusset plate width depends upon the diagonal connection. From a scaled layout, the gusset plate must be 1 ft 3 in. wide.

The gusset plate depth depends upon the connection angles. From a scaled layout, the gusset plate must extend 12 in. below the tee stem.

Use a PL\(\frac{3}{4}\times12\text{ in.}\times1\text{ ft 3 in.}
EXAMPLE II.C-2 BRACING CONNECTION

Given:

Design the diagonal bracing connection between the ASTM A992 W12×87 brace and the ASTM A992 W18×106 beam and the ASTM A992 W14×605 column.

Brace Axial Load  \( T_u = 675 \text{ kips} \)  \( T_d = 450 \text{ kips} \)
Beam End Reaction  \( R_u = 15 \text{ kips} \)  \( R_d = 10 \text{ kips} \)
Column Axial Load  \( P_u = 421 \text{ kips} \)  \( P_d = 280 \text{ kips} \)
Beam Axial Load  \( P_u = 528 \text{ kips} \)  \( P_d = 352 \text{ kips} \)

Use \( \frac{3}{8}\)-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. The gusset plate and angles are ASTM A36 material.

Fig. II.C-2-1. Diagonal bracing connection.
Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

**Brace**
W12×87
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

**Beam**
W18×106
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

**Column**
W14×605
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

**Gusset Plate**
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

**Angles**
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

**Brace**
W12×87

\[ A = 25.6 \text{ in.}^2 \]
\[ d = 12.5 \text{ in.} \]
\[ t_w = 0.515 \text{ in.} \]
\[ b_f = 12.1 \text{ in.} \]
\[ t_f = 0.810 \text{ in.} \]

**Beam**
W18×106

\[ d = 18.7 \text{ in.} \]
\[ t_w = 0.590 \text{ in.} \]
\[ b_f = 11.2 \text{ in.} \]
\[ t_f = 0.940 \text{ in.} \]
\[ k_{des} = 1.34 \]

**Column**
W14×605

\[ d = 20.9 \text{ in.} \]
\[ t_w = 2.60 \text{ in.} \]
\[ b_f = 17.4 \text{ in.} \]
$t_f = 4.16$ in.

**Brace-to-Gusset Connection**

Distribute brace force in proportion to web and flange areas.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Force in one flange:</strong></td>
<td><strong>Force in one flange:</strong></td>
</tr>
<tr>
<td>$P_{af} = \frac{P_u b_f t_f}{A}$</td>
<td>$P_{af} = \frac{P_u b_f t_f}{A}$</td>
</tr>
<tr>
<td>$= \frac{675 \text{ kips}(12.1 \text{ in.})(0.810 \text{ in.})}{25.6 \text{ in.}^2}$</td>
<td>$= \frac{450 \text{ kips}(12.1 \text{ in.})(0.810 \text{ in.})}{25.6 \text{ in.}^2}$</td>
</tr>
<tr>
<td>$= 258 \text{ kips}$</td>
<td>$= 172 \text{ kips}$</td>
</tr>
<tr>
<td><strong>Force in web:</strong></td>
<td><strong>Force in web:</strong></td>
</tr>
<tr>
<td>$P_{aw} = P_u - 2P_{af}$</td>
<td>$P_{aw} = P_u - 2P_{af}$</td>
</tr>
<tr>
<td>$= 675 \text{ kips} - 2(258 \text{ kips})$</td>
<td>$= 450 \text{ kips} - 2(172 \text{ kips})$</td>
</tr>
<tr>
<td>$= 159 \text{ kips}$</td>
<td>$= 106 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Brace-Flange-to-Gusset Connection**


Determine number of $\frac{3}{8}$-in.-diameter ASTM A325-N bolts required on the brace side of the brace-flange-to-gusset connection for single shear.

From AISC Manual Table 7-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{min} = \frac{P_{af}}{\phi r_n}$</td>
<td>$n_{min} = \frac{P_{af}}{r_n/\Omega}$</td>
</tr>
<tr>
<td>$= \frac{258 \text{ kips}}{24.3 \text{ kips/bolt}}$</td>
<td>$= \frac{172 \text{ kips}}{16.2 \text{ kips/bolt}}$</td>
</tr>
<tr>
<td>$= 10.6 \text{ bolts} \rightarrow \text{ use 12 bolts}$</td>
<td>$= 10.6 \text{ bolts} \rightarrow \text{ use 12 bolts}$</td>
</tr>
</tbody>
</table>

On the gusset side, since the bolts are in double shear, half as many bolts will be required. Try six rows of two bolts through each flange, six bolts per flange through the gusset, and 2L4x4x1/4 angles per flange.

From AISC Manual Tables 1-7 and 1-15, the geometric properties are as follows:

$A = 10.9 \text{ in.}^2$

$\bar{x} = 1.27 \text{ in.}$
Tensile Yielding of Angles

From AISC Specification Equation J4-1:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\phi R_y = \phi F_y A_g$</td>
<td>$R_n = \frac{F_y A_g}{\Omega}$</td>
</tr>
<tr>
<td></td>
<td>$= 0.90(36 \text{ ksi})(10.9 \text{ in.}^2)$</td>
<td>$= \frac{36 \text{ ksi}(10.9 \text{ in.}^2)}{1.67}$</td>
</tr>
<tr>
<td></td>
<td>$= 353 \text{ kips} &gt; 258 \text{ kips}$</td>
<td>$= 235 \text{ kips} &gt; 172 \text{ kips}$</td>
</tr>
</tbody>
</table>

Tensile Rupture of Angles

$U = 1 - \frac{x}{l}$ from AISC Specification Table D3.1 Case 2

$= 1 - \frac{1.27 \text{ in.}}{15.0 \text{ in.}}$

$= 0.915$

$A_e = A_u U$

$= [10.9 \text{ in.}^2 - 2\left(\frac{3}{4} \text{ in.}\right)(\frac{1}{8} \text{ in.} + \frac{1}{8} \text{ in.})](0.915)$

$= 8.60 \text{ in.}^2$

From AISC Specification Equation J4-2:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\phi R_u = \phi F_u A_e$</td>
<td>$R_n / \Omega = \frac{F_u A_e}{\Omega}$</td>
</tr>
<tr>
<td></td>
<td>$= 0.75(58 \text{ ksi})(8.60 \text{ in.}^2)$</td>
<td>$= \frac{(58 \text{ ksi})(8.60 \text{ in.}^2)}{2.00}$</td>
</tr>
<tr>
<td></td>
<td>$= 374 \text{ kips} &gt; 258 \text{ kips}$</td>
<td>$= 249 \text{ kips} &gt; 172 \text{ kips}$</td>
</tr>
</tbody>
</table>

Block Shear Rupture of Angles

Use $n = 6$, $L_e = 1\frac{1}{2} \text{ in.}$, $L_{eh} = 1\frac{1}{2} \text{ in.}$ and $U_{hs} = 1.0$.

From AISC Specification Equation J4-5:
The flange thickness is greater than the angle thickness, the yield and tensile strengths of the flange are greater than that of the angles, \( L_{\text{ew}} = 1\frac{3}{4} \text{ in.} \) for the brace is greater than 1\( \frac{1}{2} \) in. for the angles and \( L_{\text{eh}} = 3\frac{3}{8} \text{ in.} \) for the brace is greater than 1\( \frac{1}{2} \) in. for the angles.

Therefore, by inspection, the block shear rupture strength of the brace flange is o.k.

**Brace-Web-to-Gusset Connection**

Determine number of \( \frac{3}{8} \)-in.-diameter ASTM A325-N bolts required on the brace side (double shear) for shear.

From AISC Manual Table 7-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{\text{min}} = \frac{P_{\text{aw}}}{\phi r_n} )</td>
<td>( n_{\text{min}} = \frac{P_{\text{aw}}}{r_n/\Omega} )</td>
</tr>
<tr>
<td>( = \frac{159 \text{ kips}}{48.7 \text{ kips/bolt}} )</td>
<td>( = \frac{106 \text{ kips}}{32.5 \text{ kips/bolt}} )</td>
</tr>
<tr>
<td>( = 3.26 \text{ bolts} \rightarrow 4 \text{ bolts} )</td>
<td>( = 3.26 \text{ bolts} \rightarrow 4 \text{ bolts} )</td>
</tr>
</tbody>
</table>

On the gusset side, the same number of bolts are required. Try two rows of two bolts and two PL\( \frac{3}{8} \times 9 \).
Tensile Yielding of Plates

From AISC Specification Equation J4-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = \phi F_{u} A_e )</td>
<td>( R_n = F_{u} A_e )</td>
</tr>
<tr>
<td>( = 0.90 \left( 36 \text{ ksi} \right) \left( 2 \frac{\text{in.}}{\text{in.}} \right) \left( 9.00 \text{ in.} \right) )</td>
<td>( \frac{36 \text{ ksi} \left( 2 \frac{\text{in.}}{\text{in.}} \right) \left( 9.00 \text{ in.} \right)}{1.67} )</td>
</tr>
<tr>
<td>( = 219 \text{ kips} &gt; 159 \text{ kips} ) \text{ o.k.}</td>
<td>( = 146 \text{ kips} &gt; 106 \text{ kips} ) \text{ o.k.}</td>
</tr>
</tbody>
</table>

Tensile Rupture of Plates

From AISC Specification Section J4.1, take \( A_e \) as the lesser of \( A_n \) and \( 0.85 A_g \).

\[
A_e = \min \left( A_n, 0.85 A_g \right)
\]

\[
= \min \left( \left( \frac{3}{8} \text{ in.} \right) \left( 2 \text{ in.} \right) - 4 \left( 1.00 \text{ in.} \right), 0.85 \left( \frac{3}{8} \text{ in.} \right) \left( 9.00 \text{ in.} \right) \right)
\]

\( = 5.25 \text{ in.}^2 \)

From AISC Specification Equation J4-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = \phi F_{u} A_e )</td>
<td>( R_n = F_{u} A_e )</td>
</tr>
<tr>
<td>( = 0.75 \left( 58 \text{ ksi} \right) \left( 5.25 \text{ in.}^2 \right) )</td>
<td>( \frac{58 \text{ ksi} \left( 5.25 \text{ in.}^2 \right)}{2.00} )</td>
</tr>
<tr>
<td>( = 228 \text{ kips} &gt; 159 \text{ kips} ) \text{ o.k.}</td>
<td>( = 152 \text{ kips} &gt; 106 \text{ kips} ) \text{ o.k.}</td>
</tr>
</tbody>
</table>

Block Shear Rupture of Plates (Outer Blocks)

Use \( n = 2, L_{cy} = 1\frac{1}{2} \text{ in.}, L_{cy} = 1\frac{1}{2} \text{ in.} \) and \( U_{sy} = 1.0 \).

From AISC Specification Equation J4-5:
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AMERICAN INSTITUTE OF STEEL CONSTRUCTION

Similarly, by inspection, because the tension area is larger for the interior blocks, the block shear rupture strength of the interior blocks of the brace-web plates is o.k.

Block Shear Rupture of Brace Web

Use \( n = 2 \), \( L_{ew} = 1\frac{1}{4} \) in., but use \( 1\frac{1}{2} \) in. for calculations to account for possible underrun in brace length, and \( L_{eh} = 3 \) in.

From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = \phi U_{bs} F_u A_{mt} + \min \left( \phi \frac{0.60 F_y A_{gy}}{\Omega} , \phi \frac{0.60 F_u A_{nv}}{\Omega} \right) )</td>
<td>( R_u = \frac{U_{bs} F_u A_{mt}}{\Omega} + \min \left( \frac{0.60 F_y A_{gy}}{\Omega} , \frac{0.60 F_u A_{nv}}{\Omega} \right) )</td>
</tr>
<tr>
<td>Tension rupture component from AISC Manual Table 9-3a:</td>
<td>Tension rupture component from AISC Manual Table 9-3a:</td>
</tr>
<tr>
<td>( \phi U_{bs} F_u A_{mt} = 1.0 \left( 43.5 \text{ kips/in.} \right) \left( \frac{3}{8} \text{ in.} \right) \left( 4 \right) )</td>
<td>( \frac{U_{bs} F_u A_{mt}}{\Omega} = 1.0 \left( 29.0 \text{ kips/in.} \right) \left( \frac{3}{8} \text{ in.} \right) \left( 4 \right) )</td>
</tr>
<tr>
<td>Shear yielding component from AISC Manual Table 9-3b:</td>
<td>Shear yielding component from AISC Manual Table 9-3b:</td>
</tr>
<tr>
<td>( \phi 0.60 F_y A_{gy} = 72.9 \text{ kips/in.} \left( \frac{3}{8} \text{ in.} \right) \left( 4 \right) )</td>
<td>( \phi \frac{0.60 F_y A_{gy}}{\Omega} = 48.6 \text{ kips/in.} \left( \frac{3}{8} \text{ in.} \right) \left( 4 \right) )</td>
</tr>
<tr>
<td>Shear rupture component from AISC Manual Table 9-3c:</td>
<td>Shear rupture component from AISC Manual Table 9-3c:</td>
</tr>
<tr>
<td>( \phi 0.60 F_u A_{nv} = 78.3 \text{ kips/in.} \left( \frac{3}{8} \text{ in.} \right) \left( 4 \right) )</td>
<td>( \phi \frac{0.60 F_u A_{nv}}{\Omega} = 52.2 \text{ kips/in.} \left( \frac{3}{8} \text{ in.} \right) \left( 4 \right) )</td>
</tr>
<tr>
<td></td>
<td>( R_u = \frac{(29.0 \text{ kips} + 48.6 \text{ kips})}{\left( \frac{3}{8} \text{ in.} \right) \left( 4 \right)} = 116 \text{ kips} &gt; 106 \text{ kips} \quad \text{o.k.} )</td>
</tr>
<tr>
<td>( \phi 0.60 F_u A_{nv} = (43.5 \text{ kips} + 72.9 \text{ kips}) \left( \frac{3}{8} \text{ in.} \right) \left( 4 \right) )</td>
<td>( \phi \frac{0.60 F_u A_{nv}}{\Omega} = (72.9 \text{ kips} + 78.3 \text{ kips}) \left( \frac{3}{8} \text{ in.} \right) \left( 4 \right) )</td>
</tr>
<tr>
<td>( = 175 \text{ kips} &gt; 159 \text{ kips} \quad \text{o.k.} )</td>
<td>( = 159 \text{ kips} &gt; 159 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>
Shear rupture component from AISC Manual Table 9-3c:

\[ 0.60 F_y A_{gy} = 101 \text{kips/in.}(0.515 \text{in.})^2 \]

\[ \frac{0.60 F_y A_{gy}}{\Omega} = 67.5 \text{kips/in.}(0.515 \text{in.})^2 \]

Shear rupture component from AISC Manual Table 9-3c:

\[ 0.60 F_u A_{nu} = 87.8 \text{kips/in.}(0.515 \text{in.})^2 \]

\[ \frac{0.60 F_u A_{nu}}{\Omega} = 58.5 \text{kips/in.}(0.515 \text{in.})^2 \]

\[ \phi R_e = (122 \text{kips} + 87.8 \text{kips})(0.515 \text{in.})^2 = 216 \text{kips} > 159 \text{kips} \text{ o.k.} \]

\[ \frac{R_e}{\Omega} = (81.3 \text{kips} + 58.5 \text{kips})(0.515 \text{in.})^2 = 144 \text{kips} > 106 \text{kips} \text{ o.k.} \]

Tensile Yielding of Brace

From AISC Specification Equation J4-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_e = \phi F_y A_g )</td>
<td>( \frac{R_e}{\Omega} = \frac{F_y A_g}{\Omega} )</td>
</tr>
<tr>
<td>( = 0.90(50 \text{ksi})(25.6 \text{in.}^2) )</td>
<td>( = \frac{50 \text{ksi}(25.6 \text{in.}^2)}{1.67} )</td>
</tr>
<tr>
<td>( = 1150 \text{kips} &gt; 675 \text{kips} \text{ o.k.} )</td>
<td>( = 766 \text{kips} &gt; 450 \text{kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

Tensile Rupture of Brace

Because the load is transmitted to all of the cross-sectional elements, \( U = 1.0 \) and \( A_e = A_n \).

\[ A_e = A_n = 25.6 \text{in.}^2 - [4(0.810 \text{in.}) + 2(0.515 \text{in.})](\frac{5}{32} \text{in.} + \frac{3}{16} \text{in.}) \]

\[ = 21.3 \text{in.}^2 \]

From AISC Specification Equation J4-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_e = \phi F_u A_e )</td>
<td>( \frac{R_e}{\Omega} = \frac{F_u A_e}{\Omega} )</td>
</tr>
<tr>
<td>( = 0.75(65 \text{ksi})(21.3 \text{in.}^2) )</td>
<td>( = \frac{65 \text{ksi}(21.3 \text{in.}^2)}{2.00} )</td>
</tr>
<tr>
<td>( = 1040 \text{kips} &gt; 675 \text{kips} \text{ o.k.} )</td>
<td>( = 692 \text{kips} &gt; 450 \text{kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

Gusset Plate

From edge distance, spacing and thickness requirements of the angles and web plates, try PL \( \frac{3}{4} \).

For the bolt layout in this example, because the gusset plate thickness is equal to the sum of the web plate thicknesses, block shear rupture of the gusset plate for the web force is o.k., by inspection.
Block Shear Rupture of Gusset Plate for Total Brace Force

$U_{bs} = 1.0$

From gusset plate geometry:

$A_{gv} = 25.1 \text{ in.}^2$

$A_{nv} = 16.9 \text{ in.}^2$

$A_{nt} = 12.4 \text{ in.}^2$

*Fig. II.C-2-2. Block shear rupture area for gusset.*
From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ϕ</strong> = 0.75</td>
<td></td>
<td><strong>Ω</strong> = 2.00</td>
</tr>
<tr>
<td><strong>ϕRa</strong> = <strong>ϕUbaAvt</strong> + min ( (0.60F_{uy}), ( \phi U_{ba}A_{vt} = 0.75(1.0)(58 \text{ ksi})(12.4 \text{ in.}^2) ) = 539 kips</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Tension rupture component:</strong></td>
<td></td>
<td><strong>Tension rupture component:</strong></td>
</tr>
<tr>
<td><strong>ϕU_{ba}F_{u}A_{vt} = 0.75(1.0)(58 \text{ ksi})(12.4 \text{ in.}^2) = 539 kips</strong></td>
<td></td>
<td><strong>ϕU_{ba}F_{u}A_{vt} = 1.0(58 \text{ ksi})(12.4 \text{ in.}^2) = 360 kips</strong></td>
</tr>
<tr>
<td><strong>Shear yielding component:</strong></td>
<td></td>
<td><strong>Shear yields component:</strong></td>
</tr>
<tr>
<td><strong>ϕ0.60F_{uy}A_{vt} = 0.75(0.60)(36 \text{ ksi})(25.1 \text{ in.}^2) = 407 kips</strong></td>
<td></td>
<td><strong>ϕ0.60F_{uy}A_{vt} = 0.60(36 \text{ ksi})(25.1 \text{ in.}^2) = 271 kips</strong></td>
</tr>
<tr>
<td><strong>Shear rupture component:</strong></td>
<td></td>
<td><strong>Shear rupture component:</strong></td>
</tr>
<tr>
<td><strong>ϕ0.60F_{u}A_{vt} = 0.75(0.60)(58 \text{ ksi})(16.9 \text{ in.}^2) = 441 kips</strong></td>
<td></td>
<td><strong>ϕ0.60F_{u}A_{vt} = 0.60(58 \text{ ksi})(16.9 \text{ in.}^2) = 294 kips</strong></td>
</tr>
<tr>
<td><strong>ϕRa</strong> = 539 kips + min (407 kips, 441 kips) ( = 946 kips &gt; 675 kips**</td>
<td></td>
<td><strong>ϕRa</strong> = 360 kips + min (271 kips, 294 kips) ( = 631 kips &gt; 450 kips**</td>
</tr>
</tbody>
</table>

**Tensile Yielding on Whitmore Section of Gusset Plate**

The Whitmore section, as illustrated with dashed lines in Figure II.C-2-1(b), is 34.8 in. long; 30.9 in. occurs in the gusset and 3.90 in. occurs in the beam web.

From AISC Specification Equation J4-1:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ϕ</strong> = 0.90</td>
<td></td>
<td><strong>Ω</strong> = 1.67</td>
</tr>
<tr>
<td><strong>ϕRa</strong> = <strong>ϕF_{uy}A_{g}</strong> ( = 0.90(36 \text{ ksi})(30.9 \text{ in.})(0.750 \text{ in.}) + (50 \text{ ksi})(3.90 \text{ in.})(0.590 \text{ in.}) ) = 854 kips &gt; 675 kips</td>
<td></td>
<td><strong>ϕRa</strong> = <strong>ϕF_{uy}A_{g}</strong> ( = (36 \text{ ksi})(30.9 \text{ in.})(0.750 \text{ in.}) + (50 \text{ ksi})(3.90 \text{ in.})(0.590 \text{ in.}) ) = 1.67 ( = 568 kips &gt; 450 kips**</td>
</tr>
</tbody>
</table>

Note: The beam web thickness is used, conservatively ignoring the larger thickness in the beam flange and the flange-to-web fillet area.
Bolt Bearing Strength of Angles, Brace Flange and Gusset Plate

By inspection, bolt bearing on the gusset plate controls.

For an edge bolt,

\[ l_e = 1\frac{1}{4} \text{ in.} - (0.5)(\frac{1}{16} \text{ in.}) = 1.28 \text{ in.} \]

From AISC Specification Equation J3-6a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_n = \phi 1.2l_e t_{F_u} \leq \phi 2.4dt_{F_u} )</td>
<td>( r_n \leq \frac{1.2l_e t_{F_u}}{\Omega} \leq \frac{2.4dt_{F_u}}{\Omega} )</td>
</tr>
<tr>
<td>= 0.75(1.2)(1.28 \text{ in.})(\frac{3}{16} \text{ in.})(58 \text{ ksi})</td>
<td>= \frac{1.2(1.28 \text{ in.})(\frac{3}{16} \text{ in.})(58 \text{ ksi})}{2.00}</td>
</tr>
<tr>
<td>\leq (0.75)2.4(\frac{3}{16} \text{ in.})(\frac{3}{16} \text{ in.})(58 \text{ ksi})</td>
<td>\leq \frac{2.4(\frac{3}{16} \text{ in.})(\frac{3}{16} \text{ in.})(58 \text{ ksi})}{2.00}</td>
</tr>
<tr>
<td>= 50.1 \text{ kips} \leq 68.5 \text{ kips}</td>
<td>= 33.4 \text{ kips} \leq 45.7 \text{ kips}</td>
</tr>
<tr>
<td>= 50.1 \text{ kips/bolt}</td>
<td>= 33.4 \text{ kips/bolt}</td>
</tr>
</tbody>
</table>

For an interior bolt,

\[ l_e = 3.00 \text{ in.} - (1)(\frac{1}{16} \text{ in.}) = 2.06 \text{ in.} \]

From AISC Specification Equation J3-6a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_n = \phi 1.2l_e t_{F_u} \leq \phi 2.4dt_{F_u} )</td>
<td>( r_n \leq \frac{1.2l_e t_{F_u}}{\Omega} \leq \frac{2.4dt_{F_u}}{\Omega} )</td>
</tr>
<tr>
<td>= 0.75(1.2)(2.06 \text{ in.})(\frac{3}{16} \text{ in.})(58 \text{ ksi})</td>
<td>= \frac{1.2(2.06 \text{ in.})(\frac{3}{16} \text{ in.})(58 \text{ ksi})}{2.00}</td>
</tr>
<tr>
<td>\leq 0.75(2.4)(\frac{3}{16} \text{ in.})(\frac{3}{16} \text{ in.})(58 \text{ ksi})</td>
<td>\leq \frac{2.4(\frac{3}{16} \text{ in.})(\frac{3}{16} \text{ in.})(58 \text{ ksi})}{2.00}</td>
</tr>
<tr>
<td>= 80.6 \text{ kips} \leq 68.5 \text{ kips}</td>
<td>= 53.8 \text{ kips} \leq 45.7 \text{ kips}</td>
</tr>
<tr>
<td>= 68.5 \text{ kips/bolt}</td>
<td>= 45.7 \text{ kips/bolt}</td>
</tr>
</tbody>
</table>

The total bolt bearing strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 1 \text{ bolt } (50.1 \text{ kips/bolt}) + 5 \text{ bolts } (68.5 \text{ kips/bolt}) )</td>
<td>( \frac{R_n}{\Omega} = 1 \text{ bolt } (33.4 \text{ kips/bolt}) + 5 \text{ bolts } (45.7 \text{ kips/bolt}) )</td>
</tr>
<tr>
<td>= 393 \text{ kips} &gt; 258 \text{ kips}</td>
<td>= 262 \text{ kips} &gt; 172 \text{ kips}</td>
</tr>
</tbody>
</table>
Note: If any of these bearing strengths were less than the bolt double shear strength; the bolt shear strength would need to be rechecked.

Bolt Bearing Strength of Brace Web, Web Plates and Gusset Plate

The total web plate thickness is the same as the gusset plate thickness, but since the web plates and brace web have a smaller edge distance due to possible underrun in brace length they control over the gusset plate for bolt bearing strength.

Accounting for a possible \( \frac{1}{4} \) in. underrun in brace length, the brace web and web plates have the same edge distance. Therefore, the controlling element can be determined by finding the minimum \( t F_u \). For the brace web, \( 0.515 \text{ in.}(65 \text{ ksi}) = 33.5 \text{ kip/in} \). For the web plates, \( \frac{3}{8} \text{ in.}(58 \text{ ksi}) = 43.5 \text{ kip/in} \). The brace web controls for bolt bearing strength.

For an edge bolt,

\[
l_c = 1\frac{3}{4} \text{ in.} - \frac{1}{4} \text{ in.} - 0.5(\% \text{ in.}) = 1.03 \text{ in.}
\]

From AISC Specification Equation J3-6a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi_{tc} = 0.75(1.2)(1.03 \text{ in.})(0.515 \text{ in.})(65 \text{ ksi}) \leq 0.75(2.4)(% \text{ in.})(0.515 \text{ in.})(65 \text{ ksi}) )</td>
<td>( \frac{1.2l_c t F_u}{\Omega} \leq \frac{2.4 t F_u}{\Omega} )</td>
</tr>
<tr>
<td>( = 31.0 \text{ kips} \leq 52.7 \text{ kips} )</td>
<td>( = 20.7 \text{ kips} \leq 35.1 \text{ kips} )</td>
</tr>
<tr>
<td>( = 31.0 \text{ kips} )</td>
<td>( = 35.1 \text{ kips} )</td>
</tr>
</tbody>
</table>

For an interior bolt,

\[
l_c = 3.00 \text{ in.} - 1.0(\% \text{ in.}) = 2.06 \text{ in.}
\]

From AISC Specification Equation J3-6a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi_{tc} = 0.75(1.2)(2.06 \text{ in.})(0.515 \text{ in.})(65 \text{ ksi}) \leq 0.75(2.4)(% \text{ in.})(0.515 \text{ in.})(65 \text{ ksi}) )</td>
<td>( \frac{1.2l_c t F_u}{\Omega} \leq \frac{2.4 t F_u}{\Omega} )</td>
</tr>
<tr>
<td>( = 62.1 \text{ kips} \leq 52.7 \text{ kips} )</td>
<td>( = 41.4 \text{ kips} \leq 35.1 \text{ kips} )</td>
</tr>
<tr>
<td>( = 52.7 \text{ kips} )</td>
<td>( = 35.1 \text{ kips} )</td>
</tr>
</tbody>
</table>
The total bolt bearing strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_e = 2 \text{ bolts (31.0 kips/bolt)} + 2 \text{ bolts (52.7 kips/bolt)} )</td>
<td>( R_e = 2 \text{ bolts (20.7 kips/bolt)} + 2 \text{ bolts (35.1 kips/bolt)} )</td>
</tr>
<tr>
<td>( = 167 \text{ kips} &gt; 159 \text{ kips} )</td>
<td>( = 112 \text{ kips} &gt; 106 \text{ kips} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note: The bearing strength for the edge bolts is less than the double shear strength of the bolts; therefore, the bolt group strength must be rechecked. Using the minimum of the bearing strength and the bolt shear strength for each bolt the revised bolt group strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_e = 2 \text{ bolts (31.0 kips/bolt)} + 2 \text{ bolts (48.7 kips/bolt)} )</td>
<td>( R_e = 2 \text{ bolts (20.7 kips/bolt)} + 2 \text{ bolts (32.5 kips/bolt)} )</td>
</tr>
<tr>
<td>( = 159 \text{ kips} \geq 159 \text{ kips} )</td>
<td>( = 106 \text{ kips} \geq 106 \text{ kips} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note: When a brace force is compressive, gusset plate buckling would have to be checked. Refer to the comments at the end of this example.

**Distribution of Brace Force to Beam and Column (AISC Manual Part 13)**

From the member geometry:

\[
\begin{align*}
\varepsilon_b &= \frac{d_{\text{beam}}}{2} \\
&= \frac{18.7 \text{ in.}}{2} \\
&= 9.35 \text{ in.} \\
\varepsilon_c &= \frac{d_{\text{column}}}{2} \\
&= \frac{20.9 \text{ in.}}{2} \\
&= 10.5 \text{ in.} \\
\tan \theta &= \frac{12}{9\%} \\
&= 1.25 \\
\varepsilon_b \tan \theta - \varepsilon_c &= 9.35 \text{ in.}(1.25) - 10.5 \text{ in.} \\
&= 1.19 \text{ in.}
\end{align*}
\]

Try gusset PL\(\frac{3}{4}\) \(\times\) 42 in. horizontally \(\times\) 33 in. vertically (several intermediate gusset dimensions were inadequate). Place connection centroids at the midpoint of the gusset plate edges.

\[
\bar{\alpha} = \frac{42.0 \text{ in.}}{2} + 0.500 \text{ in.} \\
= 21.5 \text{ in.}
\]

\(\frac{1}{2}\) in. is allowed for the setback between the gusset plate and the column.
\[ \bar{\beta} = \frac{33.0 \text{ in.}}{2} = 16.5 \text{ in.} \]

Choosing \( \beta = \bar{\beta} \), the \( \alpha \) required for the uniform forces from AISC Manual Equation 13-1 is:

\[ \alpha = e_b \tan \theta - e_c + \beta \tan \theta = 1.19 \text{ in.} + 16.5 \text{ in.} (1.25) = 21.8 \text{ in.} \]

The resulting eccentricity is \( \alpha - \bar{\alpha} \).

\[ \alpha - \bar{\alpha} = 21.8 \text{ in.} - 21.5 \text{ in.} = 0.300 \text{ in.} \]

Since this slight eccentricity is negligible, use \( \alpha = 21.8 \text{ in.} \) and \( \beta = 16.5 \text{ in.} \).

**Gusset Plate Interface Forces**

\[ r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \]

\[ = \sqrt{(21.8 \text{ in.} + 10.5 \text{ in.})^2 + (16.5 \text{ in.} + 9.35 \text{ in.})^2} \]

\[ = 41.4 \text{ in.} \]

On the gusset-to-column connection,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{ac} = \frac{e_c P_u}{r} ) \hspace{1cm} (from Manual Eq. 13-3)</td>
<td>( H_{ac} = \frac{e_c P_u}{r} ) \hspace{1cm} (from Manual Eq. 13-3)</td>
</tr>
<tr>
<td>( = \frac{10.5 \text{ in.} (675 \text{ kips})}{41.4 \text{ in.}} ) \hspace{1cm} = 171 \text{ kips}</td>
<td>( = \frac{10.5 \text{ in.} (450 \text{ kips})}{41.4 \text{ in.}} ) \hspace{1cm} = 114 \text{ kips}</td>
</tr>
<tr>
<td>( V_{ac} = \frac{\beta P_u}{r} ) \hspace{1cm} (from Manual Eq. 13-2)</td>
<td>( V_{ac} = \frac{\beta P_u}{r} ) \hspace{1cm} (from Manual Eq. 13-2)</td>
</tr>
<tr>
<td>( = \frac{16.5 \text{ in.} (675 \text{ kips})}{41.4 \text{ in.}} ) \hspace{1cm} = 269 \text{ kips}</td>
<td>( = \frac{16.5 \text{ in.} (450 \text{ kips})}{41.4 \text{ in.}} ) \hspace{1cm} = 179 \text{ kips}</td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{ab} = \frac{\alpha P_u}{r} ) \hspace{1cm} (from Manual Eq. 13-5)</td>
<td>( H_{ab} = \frac{\alpha P_u}{r} ) \hspace{1cm} (from Manual Eq. 13-5)</td>
</tr>
<tr>
<td>( = \frac{21.8 \text{ in.} (675 \text{ kips})}{41.4 \text{ in.}} ) \hspace{1cm} = 355 \text{ kips}</td>
<td>( = \frac{21.8 \text{ in.} (450 \text{ kips})}{41.4 \text{ in.}} ) \hspace{1cm} = 237 \text{ kips}</td>
</tr>
</tbody>
</table>
**Gusset Plate-to-Column Connection**

The forces involved are:

\[ V_{uc} = 269 \text{ kips} \text{ and } V_{ac} = 179 \text{ kips shear} \]
\[ H_{uc} = 171 \text{ kips} \text{ and } H_{ac} = 114 \text{ kips tension} \]

Try 2L5×3\(\frac{1}{2}\)×\(\frac{3}{4}\)×2 ft 6 in. welded to the gusset plate and bolted with 10 rows of \(\frac{3}{8}\)-in.-diameter A325-N bolts in standard holes to the column flange.

The required tensile strength per bolt is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ T_u = \frac{H_{ac}}{n} \]
\[ = \frac{171 \text{ kips}}{20 \text{ bolts}} \]
\[ = 8.55 \text{ kips/bolt} \]
| \[ T_a = \frac{H_{ac}}{n} \]
\[ = \frac{114 \text{ kips}}{20 \text{ bolts}} \]
\[ = 5.70 \text{ kips/bolt} \]

Design strength of bolts for tension-shear interaction is determined from AISC Specification Section J3.7 as follows:

\[ r_{av} = \frac{V_{uc}}{n} \]
\[ = \frac{269 \text{ kips}}{20 \text{ bolts}} \]
\[ = 13.5 \text{ kips/bolt} \]

From AISC Manual Table 7-1, the bolt available shear strength is:

24.3 kips/bolt > 13.5 kips/bolt \textbf{O.K.}

\[ f_{av} = \frac{r_{av}}{A_b} \]
\[ = \frac{13.5 \text{ kips}}{0.601 \text{ in.}^2} \]
\[ = 22.5 \text{ ksi} \]

\[ \phi = 0.75 \]

From AISC Specification Table J3.2:

\[ \Omega = 2.00 \]

Allowable strength of bolts for tension-shear interaction is determined from AISC Specification Section J3.7 as follows:

\[ r_{av} = \frac{V_{ac}}{n} \]
\[ = \frac{179 \text{ kips}}{20 \text{ bolts}} \]
\[ = 8.95 \text{ kips/bolt} \]

From AISC Manual Table 7-1, the bolt available shear strength is:

16.2 kips/bolt > 8.95 kips/bolt \textbf{O.K.}

\[ f_{av} = \frac{r_{av}}{A_b} \]
\[ = \frac{8.95 \text{ kips}}{0.601 \text{ in.}^2} \]
\[ = 14.9 \text{ ksi} \]
Bolt Bearing Strength on Double Angles at Column Flange

From AISC Specification Equation J3-6a using \( L_c = 1\frac{1}{2} \) in. – 0.5(\% in.) = 1.03 in. for an edge bolt.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{nv} = 54 \text{ ksi and } F_{nt} = 90 \text{ ksi} )</td>
<td>( F_{nv} = 54 \text{ ksi and } F_{nt} = 90 \text{ ksi} )</td>
</tr>
<tr>
<td>( F'<em>{nt} = 1.3 F</em>{nt} - \frac{F_{nt}}{\phi}\frac{F_{nv}}{F_{nv}} \leq F_{nt} ) (Spec. Eq. J3-3a)</td>
<td>( F'<em>{nt} = 1.3 F</em>{nt} - \frac{F_{nt}}{\phi}\frac{F_{nv}}{F_{nv}} \leq F_{nt} ) (Spec. Eq. J3-3b)</td>
</tr>
<tr>
<td>( = 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{0.75(54 \text{ ksi})} (22.5 \text{ ksi}) )</td>
<td>( = 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi} (2.00)}{54 \text{ ksi}} (14.9 \text{ ksi}) )</td>
</tr>
<tr>
<td>( = 67.0 \text{ ksi } \leq 90 \text{ ksi} )</td>
<td>( = 67.3 \text{ ksi } \leq 90 \text{ ksi} )</td>
</tr>
<tr>
<td>( B_u = \phi F'_{nt} A_b ) (Spec. Eq. J3-2)</td>
<td>( B_u = \frac{F'_{nt} A_b}{\Omega} ) (Spec. Eq. J3-2)</td>
</tr>
<tr>
<td>( = 0.75(67.0 \text{ ksi})\left(0.601 \text{ in.}^2\right) )</td>
<td>( = 67.3 \text{ ksi}\left(0.601 \text{ in.}^2\right) )</td>
</tr>
<tr>
<td>( = 30.2 \text{ kips/bolt } &gt; 8.55 \text{ kips/bolt} ) o.k.</td>
<td>( \frac{2.00}{2.00} )</td>
</tr>
<tr>
<td>( = 20.2 \text{ kips/bolt } &gt; 5.70 \text{ kips/bolt} ) o.k.</td>
<td>( = 22.4 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>

Since this edge bolt value exceeds the single bolt shear strength of 24.3 kips, and the actual shear per bolt of 13.5 kips, bolt shear and bolt bearing strengths are o.k.

The bearing strength of the interior bolts on the double angle will not control.

Prying Action on Double Angles (AISC Manual Part 9)

\[ b = g - \frac{t}{2} \]
\[ = 3.00 \text{ in. } - \frac{\% \text{ in.}}{2} \]
\[ = 2.69 \text{ in.} \]

\[ a = 5.00 \text{ in. } - g \]
\[ = 5.00 \text{ in. } - 3.00 \text{ in.} \]
\[ = 2.00 \text{ in.} \]
\[ b' = b - \frac{d_h}{2} \]  
\[ = 2.69 \text{ in.} - \frac{\frac{\frac{3}{4} \text{ in.}}{2}}{2} \]
\[ = 2.25 \text{ in.} \]  

\[ a' = a + \frac{d_h}{2} \leq \left( 1.25b + \frac{d_h}{2} \right) \]  
\[ = 2.00 \text{ in.} + \frac{\frac{\frac{3}{4} \text{ in.}}{2}}{2} \leq 1.25(2.69 \text{ in.}) + \frac{\frac{3}{4} \text{ in.}}{2} \]
\[ = 2.44 \text{ in.} \leq 3.80 \text{ in.} \]

\[ \rho = \frac{b'}{a'} \]  
\[ = \frac{2.25 \text{ in.}}{2.44 \text{ in.}} \]
\[ = 0.922 \]  

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ \beta = \frac{1}{\rho} \left( \frac{B_u}{T_u} - 1 \right) \]  
\[ = \frac{1}{0.922} \left( \frac{30.2 \text{ kips/bolt}}{8.55 \text{ kips/bolt}} - 1 \right) \]
\[ = 2.75 \]  
| \[ \beta = \frac{1}{\rho} \left( \frac{B_u}{T_u} - 1 \right) \]  
\[ = \frac{1}{0.922} \left( \frac{20.2 \text{ kips/bolt}}{5.70 \text{ kips/bolt}} - 1 \right) \]
\[ = 2.76 \]  

Because \( \beta > 1 \), set \( \alpha' = 1.0 \).

\[ \delta = 1 - \frac{d'}{\rho} \]  
\[ = 1 - \frac{\frac{\frac{15}{64} \text{ in.}}{3.00 \text{ in.}}}{0.688} \]
\[ = 0.688 \]  

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ \phi = 0.90 \]  
\[ t_{req} = \frac{4T_u b'}{\phi p F_{tu} (1 + \delta \alpha')} \]  
\[ = \frac{4(8.55 \text{ kips/bolt})(2.25 \text{ in.})}{0.90(3.00 \text{ in.})(58 \text{ ksi})(1 + 0.688(1.0))} \]
\[ = 0.540 \text{ in.} < 0.625 \text{ in.} \]  
| \[ \Omega = 1.67 \]  
\[ t_{req} = \frac{\Omega T_u b'}{p F_{tu} (1 + \delta \alpha')} \]  
\[ = \frac{1.67(4)(5.70 \text{ kips/bolt})(2.25 \text{ in.})}{3.00 \text{ in.}(58 \text{ ksi})(1 + 0.688(1.0))} \]
\[ = 0.540 \text{ in.} < 0.625 \text{ in.} \]  

Use the 2L5x3\(\frac{3}{4}\)x\% for the gusset plate-to-column connection.

Weld Design

Try fillet welds around the perimeter (three sides) of both angles.
The resultant required force on the welds is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{uc} = \sqrt{H_{uc}^2 + V_{uc}^2}$</td>
<td>$P_{ac} = \sqrt{H_{ac}^2 + V_{ac}^2}$</td>
</tr>
<tr>
<td>$= \sqrt{(171 \text{ kips})^2 + (269 \text{ kips})^2}$</td>
<td>$= \sqrt{(114 \text{ kips})^2 + (179 \text{ kips})^2}$</td>
</tr>
<tr>
<td>$= 319 \text{ kips}$</td>
<td>$= 212 \text{ kips}$</td>
</tr>
</tbody>
</table>

$\theta = \tan^{-1}\left(\frac{H_{uc}}{V_{uc}}\right)$

$= \tan^{-1}\left(\frac{171 \text{ kips}}{269 \text{ kips}}\right)$

$= 32.4^\circ$  

$= \tan^{-1}\left(\frac{114 \text{ kips}}{179 \text{ kips}}\right)$

$= 32.5^\circ$

$l = 30.0 \text{ in.}$

$kl = 3.00 \text{ in.}, \text{ therefore, } k = 0.100$

$xl = \frac{(kl)^2}{(l + 2kl)}$

$= \frac{(3.00 \text{ in.})^2}{30.0 \text{ in.} + 2(3.00 \text{ in.})}$

$= 0.250 \text{ in.}$

$al = 3.50 \text{ in.} - xl$

$= 3.50 \text{ in.} - 0.250 \text{ in.}$

$= 3.25 \text{ in.}$

$a = 0.108$

By interpolating AISC Manual Table 8-8 with $\theta = 30^\circ$, $C = 2.55$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
</tbody>
</table>

$D_{req} = \frac{P_{ac}}{\phi CC \bar{l}}$

$= \frac{319 \text{ kips}}{0.75(2.55)(1.0)(2 \text{ welds})(30.0 \text{ in.})}$

$= 2.78 \rightarrow 3 \text{ sixteenths}$

$D_{req} = \frac{\Omega P_{ac}}{CC \bar{l}}$

$= \frac{(2.00)(212 \text{ kips})}{2.55(1.0)(2 \text{ welds})(30.0 \text{ in.})}$

$= 2.77 \rightarrow 3 \text{ sixteenths}$

From AISC Specification Table J2.4, minimum fillet weld size is $\frac{1}{4}$ in. Use $\frac{1}{4}$-in. fillet welds.
Gusset Plate Thickness

\[ t_{\text{min}} = \frac{6.19 D}{F_u} \]
\[ = \frac{6.19 (2.78 \text{ sixteenths})}{58 \text{ ksi}} \]
\[ = 0.297 \text{ in.} < \frac{3}{8} \text{ in.} \quad \text{o.k.} \]

Shear Yielding of Angles (due to \( V_{uc} \) or \( V_{ac} \))

\[ A_{gv} = 2 \left( 30.0 \text{ in.} \right) \left( \frac{3}{8} \text{ in.} \right) \]
\[ = 37.5 \text{ in}.^2 \]

From AISC Specification Equation J4-3:

\[
\begin{array}{c|c|c}
\text{LRFD} & \text{ASD} \\
\hline
\phi = 1.00 & \Omega = 1.50 \\
\phi R_u = \phi 0.60 F_y A_{gv} & \frac{R_u}{\Omega} = 0.60 F_y A_{gv} \\
\quad = 1.00 \left( 0.60 \right) \left( 36 \text{ ksi} \right) \left( 37.5 \text{ in}.^2 \right) & \quad = \frac{0.60 \left( 36 \text{ ksi} \right) \left( 37.5 \text{ in}.^2 \right)}{1.50} \\
\quad = 810 \text{ kips} > 269 \text{ kips} & \quad = 540 \text{ kips} > 179 \text{ kips} \\
\text{o.k.} & \text{o.k.} \\
\end{array}
\]

Similarly, shear yielding of the angles due to \( H_{uc} \) and \( H_{ac} \) is not critical.

Shear Rupture of Angles

\[ A_{nv} = \frac{3}{8} \text{ in.} \left[ 2 \left( 30.0 \text{ in.} \right) - 2 \left( 1\frac{5}{8} \text{ in.} + \frac{1}{16} \text{ in.} \right) \right] \]
\[ = 25.0 \text{ in}.^2 \]

From AISC Specification Equation J4-4:

\[
\begin{array}{c|c|c}
\text{LRFD} & \text{ASD} \\
\hline
\phi = 0.75 & \Omega = 2.00 \\
\phi R_u = \phi 0.60 F_y A_{nv} & \frac{R_u}{\Omega} = 0.60 F_y A_{nv} \\
\quad = 0.75 \left( 0.60 \right) \left( 58 \text{ ksi} \right) \left( 25.0 \text{ in}.^2 \right) & \quad = \frac{0.60 \left( 58 \text{ ksi} \right) \left( 25.0 \text{ in}.^2 \right)}{2.00} \\
\quad = 653 \text{ kips} > 269 \text{ kips} & \quad = 435 \text{ kips} > 179 \text{ kips} \\
\text{o.k.} & \text{o.k.} \\
\end{array}
\]

Block Shear Rupture of Angles

Use \( n = 10, L_{cv} = 1\frac{1}{2} \text{ in.} \) and \( L_{ch} = 2 \text{ in.} \)
From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th>Component</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension rupture component from AISC Manual Table 9-3a:</td>
<td>$\phi R_a = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv}, \phi 0.60 F_u A_{mv})$</td>
<td>$R_a = \frac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\frac{0.60 F_y A_{gv}}{\Omega}, \frac{0.60 F_u A_{mv}}{\Omega}\right)$</td>
</tr>
<tr>
<td>$U_{bs} = 1.0$</td>
<td>$U_{bs} = 1.0$</td>
<td>$U_{bs} = 1.0$</td>
</tr>
<tr>
<td>Shear yielding component from AISC Manual Table 9-3b:</td>
<td>$\phi U_{bs} F_u A_{nt} = 1.0(65.3 \text{ kips/in.})(% \text{ in.})(2)$</td>
<td>$\frac{U_{bs} F_u A_{nt}}{\Omega} = 1.0(43.5 \text{ kips/in.})(% \text{ in.})(2)$</td>
</tr>
<tr>
<td>$0.60 F_y A_{gv} = 462 \text{ kips/in.}(% \text{ in.})(2)$</td>
<td>$U_{bs} F_u A_{nt} = 1.0(308 \text{ kips/in.})(% \text{ in.})(2)$</td>
<td>$0.60 F_y A_{gv} = 308 \text{ kips/in.}(% \text{ in.})(2)$</td>
</tr>
<tr>
<td>Shear rupture component from AISC Manual Table 9-3c:</td>
<td>$\phi 0.60 F_u A_{mv} = 496 \text{ kips/in.}(% \text{ in.})(2)$</td>
<td>$0.60 F_u A_{mv} = 331 \text{ kips/in.}(% \text{ in.})(2)$</td>
</tr>
<tr>
<td>$\phi R_a = (65.3 \text{ kips} + 462 \text{ kips})(% \text{ in.})(2)$</td>
<td>$R_a = (43.5 \text{ kips} + 308 \text{ kips})(% \text{ in.})(2)$</td>
<td>$R_a = 439 \text{ kips} &gt; 179 \text{ kips}$</td>
</tr>
<tr>
<td>$= 659 \text{ kips} &gt; 269 \text{ kips}$</td>
<td>$= 439 \text{ kips} &gt; 179 \text{ kips}$</td>
<td></td>
</tr>
</tbody>
</table>

Column Flange

By inspection, the 4.16-in.-thick column flange has adequate flexural strength, stiffness and bearing strength.

Gusset Plate-to-Beam Connection

The forces involved are:

- $V_{ub} = 152 \text{ kips}$ and $V_{ab} = 102 \text{ kips}$
- $H_{ub} = 355 \text{ kips}$ and $H_{ab} = 237 \text{ kips}$

This edge of the gusset plate is welded to the beam. The distribution of force on the welded edge is known to be nonuniform. The Uniform Force Method, as shown in AISC Manual Part 13, used here assumes a uniform distribution of force on this edge. Fillet welds are known to have limited ductility, especially if transversely loaded. To account for this, the required strength of the gusset edge weld is amplified by a factor of 1.25 to allow for the redistribution of forces on the weld as discussed in Part 13 of the AISC Manual.
The stresses on the gusset plate at the welded edge are as follows:

From AISC Specification Sections J4.1(a) and J4.2:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ua} = \frac{V_{ab}}{t} \leq \phi F_y$</td>
<td></td>
<td>$f_{ua} = \frac{V_{ab}}{t} \leq F_y$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{152 \text{ kips}}{\frac{3}{4} \text{ in.}(42.0 \text{ in.})} \leq 0.90(36 \text{ ksi})$</td>
<td>$= \frac{102 \text{ kips}}{\frac{3}{4} \text{ in.}(42.0 \text{ in.})} \leq 36 \text{ ksi}$</td>
</tr>
<tr>
<td></td>
<td>$= 4.83 \text{ ksi} &lt; 32.4 \text{ ksi}$</td>
<td>$= 3.24 \text{ ksi} &lt; 21.6 \text{ ksi}$</td>
</tr>
<tr>
<td></td>
<td>$f_{uv} = \frac{H_{ab}}{t} \leq \phi 0.60 F_y$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{355 \text{ kips}}{\frac{3}{4} \text{ in.}(42.0 \text{ in.})} \leq 1.00(0.60)(36 \text{ ksi})$</td>
<td>$= \frac{237 \text{ kips}}{\frac{3}{4} \text{ in.}(42.0 \text{ in.})} \leq 0.60(36 \text{ ksi})$</td>
</tr>
<tr>
<td></td>
<td>$= 11.3 \text{ ksi} &lt; 21.6 \text{ ksi}$</td>
<td>$= 7.52 \text{ ksi} &lt; 14.4 \text{ ksi}$</td>
</tr>
</tbody>
</table>

The weld strength per $\frac{1}{16}$ in. is as follows from AISC Manual Equation 8-2a:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = \tan^{-1}\left(\frac{V_{ab}}{H_{ab}}\right)$</td>
<td>$\theta = \tan^{-1}\left(\frac{V_{ab}}{H_{ab}}\right)$</td>
<td>$= 23.2^\circ$</td>
</tr>
<tr>
<td></td>
<td>$= \tan^{-1}\left(\frac{152 \text{ kips}}{355 \text{ kips}}\right)$</td>
<td>$= \tan^{-1}\left(\frac{102 \text{ kips}}{237 \text{ kips}}\right)$</td>
</tr>
<tr>
<td></td>
<td>$= 23.2^\circ$</td>
<td>$= 23.3^\circ$</td>
</tr>
</tbody>
</table>

From AISC Specification Equation J2-5 and AISC Manual Part 8:

$\mu = 1.0 + 0.50 \sin^{1.5} \theta$

$= 1.0 + 0.50 \sin^{1.5}(23.2^\circ)$

$= 1.12$

The peak weld stress is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{u \ peak} = \left(\frac{t}{2}\right)\sqrt{\left(f_{ua} + f_{ub}\right)^2 + f_{uv}^2}$</td>
<td>$f_{a \ peak} = \left(\frac{t}{2}\right)\sqrt{\left(f_{ua} + f_{ab}\right)^2 + f_{av}^2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \left(\frac{\frac{3}{4} \text{ in.}}{2}\right)\sqrt{(4.83 \text{ ksi} + 0 \text{ ksi})^2 + (11.3 \text{ ksi})^2}$</td>
<td>$= \left(\frac{\frac{3}{4} \text{ in.}}{2}\right)\sqrt{(3.24 \text{ ksi} + 0 \text{ ksi})^2 + (7.52 \text{ ksi})^2}$</td>
</tr>
<tr>
<td></td>
<td>$= 4.61 \text{ kips/in.}$</td>
<td>$= 3.07 \text{ kips/in.}$</td>
</tr>
</tbody>
</table>
The average stress is:

\[
\frac{1}{2} \left[ \sqrt{\left( f_{ua} - f_{ub} \right)^2 + f_{av}^2} \right] + \frac{1}{2} \left[ \sqrt{\left( f_{ua} + f_{ub} \right)^2 + f_{av}^2} \right]
\]

Because \( f_{ub} = 0 \text{ ksi} \) (there is no moment on the edge),

\( f_{u\ ave} = f_{u\ peak} = 4.61 \text{ kips/in.} \)

The design weld stress is:

\[ f_{u\ weld} = \max \left\{ f_{u\ peak} \cdot 1.25 \cdot f_{u\ ave} \right\} \]

\[ = \max \left\{ 4.61 \text{ kips/in.,} \cdot 1.25 \left( 4.61 \text{ kips/in.} \right) \right\} \]

\[ = 5.76 \text{ kips/in.} \]

The required weld size is:

\[ D_{req} = \frac{f_{u\ weld}}{\phi_{w}} \]

\[ = \frac{5.76 \text{ kips/in.}}{1.56 \text{ kips/in.}} \]

\[ = 3.69 \rightarrow 4 \text{ sixteenths} \]

From AISC Specification Table J2.4, the minimum fillet weld size is \( \frac{1}{4} \text{ in.} \). Use a \( \frac{1}{4} \)-in. fillet weld.

Web Local Yielding of Beam

From AISC Manual Table 9-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi R_{e} = \phi R_{t} + l_{b} (\phi R_{c}) ]</td>
<td>[ R_{e} = \frac{R_{t}}{\Omega} + l_{b} \left( \frac{R_{c}}{\Omega} \right) ]</td>
</tr>
<tr>
<td>= 98.8 kips + 42.0 in.(29.5 kips/in.)</td>
<td>= 65.9 kips + 42.0 in.(19.7 kips/in.)</td>
</tr>
<tr>
<td>= 1,340 kips &gt; 152 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>o.k.</td>
<td>= 893 kips &gt; 102 kips</td>
</tr>
</tbody>
</table>

Web Crippling of Beam

\[ l_{b} = 42.0 \text{ in.} \]

\[ \frac{l_{b}}{d} = 18.7 \text{ in.} \]

\[ = 2.25 > 0.2 \]
From AISC Manual Table 9-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_e = \phi R_s + l_0 (\phi R_s)$</td>
<td>$R_e = R_s + l_0 \left( \frac{R_s}{\Omega} \right)$</td>
</tr>
<tr>
<td>$=-143 \text{kips} + 42.0 \text{ in.}(16.9 \text{kips/in.})$</td>
<td>$= 95.3 \text{kips} + 42.0 \text{ in.}(11.3 \text{kips/in.})$</td>
</tr>
<tr>
<td>$= 853 \text{kips} &gt; 152 \text{kips}$</td>
<td>$= 570 \text{kips} &gt; 102 \text{kips}$</td>
</tr>
</tbody>
</table>

**Beam-to-Column Connection**

Since the brace is in tension, the required strength of the beam-to-column connection is as follows.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>The required shear strength is:</td>
<td>The required shear strength is:</td>
</tr>
<tr>
<td>$R_{ab} + V_{ab} = 15 \text{kips} + 152 \text{kips}$</td>
<td>$R_{ab} + V_{ab} = 10 \text{kips} + 102 \text{kips}$</td>
</tr>
<tr>
<td>$= 167 \text{kips}$</td>
<td>$= 112 \text{kips}$</td>
</tr>
<tr>
<td>$H_a - H_{ab} = H_{ac}$</td>
<td>$H_a - H_{ab} = H_{ac}$</td>
</tr>
<tr>
<td>$= 171 \text{kips}$</td>
<td>$= 114 \text{kips}$</td>
</tr>
</tbody>
</table>

The required axial strength is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ab} + (H_a - H_{ab}) = 0 \text{kips} + 171 \text{kips}$</td>
<td>$A_{ab} + (H_a - H_{ab}) = 0 \text{kips} + 114 \text{kips}$</td>
</tr>
<tr>
<td>$= 171 \text{kips compression}$</td>
<td>$= 114 \text{kips compression}$</td>
</tr>
</tbody>
</table>

Try 2L8×6×7/8×1 ft 2½ in. LLBB (leg gage = 3/16 in.) welded to the beam web, bolted with five rows of 7/8-in.-diameter A325-N bolts in standard holes to the column flange.

Since the connection is in compression in this example, the bolts resist shear only, no tension. If the bolts were in tension, the angles would also have to be checked for prying action.

**Bolt Shear**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_a = \frac{167 \text{kips}}{10 \text{ bolts}}$</td>
<td>$r_a = \frac{112 \text{kips}}{10 \text{ bolts}}$</td>
</tr>
<tr>
<td>$= 16.7 \text{kips/bolt}$</td>
<td>$= 11.2 \text{kips/bolt}$</td>
</tr>
</tbody>
</table>

From AISC Manual Table 7-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi r_a = 24.3 \text{kips/bolt} &gt; 16.7 \text{kips/bolt}$</td>
<td>$r_a = 16.2 \text{kips/bolt} &gt; 11.2 \text{kips/bolt}$</td>
</tr>
</tbody>
</table>

**Bolt Bearing**

Bearing on the angles controls over bearing on the column flange.

\[ l_c = 1\frac{1}{4} \text{ in.} - \frac{15/16 \text{ in.}}{2} \]
= 0.781 in.

From AISC Specification Equation J3-6a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi_{n} = \phi(1.2) t_{Fw} &lt; \phi 2.4 t_{Fu}$</td>
<td>$\frac{r_{n}}{\Omega} \leq \frac{2.4 t_{Fw}}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.75(1.2)(0.781 \text{ in.})(\frac{58 \text{ ksi}}{% \text{ in.}})$</td>
<td>$= \frac{1.2(0.781 \text{ in.})(\frac{58 \text{ ksi}}{% \text{ in.}})}{2.00}$</td>
</tr>
<tr>
<td>$\leq 0.75(2.4)(\frac{58 \text{ ksi}}{% \text{ in.}})$</td>
<td>$\leq \frac{2.4(% \text{ in.})(\frac{58 \text{ ksi}}{% \text{ in.}})}{2.00}$</td>
</tr>
<tr>
<td>$= 35.7 \text{ kips} &lt; 79.9 \text{ kips}$</td>
<td>$= 23.8 \text{ kips} &lt; 53.3 \text{ kips}$</td>
</tr>
<tr>
<td>$= 35.7 \text{ kips/bolt}$</td>
<td>$= 23.8 \text{ kips/bolt}$</td>
</tr>
</tbody>
</table>

Since this edge bolt value exceeds the single bolt shear strength of 24.3 kips, bearing does not control.

Since this edge bolt value exceeds the single bolt shear strength of 16.2 kips, bearing does not control.

Weld Design

Try fillet welds around perimeter (three sides) of both angles.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{ac}} = \sqrt{(171 \text{ kips})^2 + (167 \text{ kips})^2}$</td>
<td>$P_{\text{ac}} = \sqrt{(114 \text{ kips})^2 + (112 \text{ kips})^2}$</td>
</tr>
<tr>
<td>= 239 kips</td>
<td>= 160 kips</td>
</tr>
<tr>
<td>$\theta = \tan^{-1} \left( \frac{171 \text{ kips}}{167 \text{ kips}} \right)$</td>
<td>$\theta = \tan^{-1} \left( \frac{114 \text{ kips}}{112 \text{ kips}} \right)$</td>
</tr>
<tr>
<td>= 45.7°</td>
<td>= 45.5°</td>
</tr>
</tbody>
</table>

$l = 14.5 \text{ in.}, kl = 7.50 \text{ in.} \text{ and } k = 0.517$

$xl = \frac{(kl)^2}{l + 2kl}$

$= \frac{(7.50 \text{ in.})^2}{14.5 \text{ in.} + 2(7.50 \text{ in.})}$

$= 1.91 \text{ in.}$

$al = 8.00 \text{ in.} - xl$

$= 8.00 \text{ in.} - 1.91 \text{ in.}$

$= 6.09 \text{ in.}$

$a = 0.420$

By interpolating AISC Manual Table 8-8 for $\theta = 45^\circ$:

$C = 3.55$
Design Examples V14.1
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<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$D_{req} = \frac{P_{ac}}{\phi CC_l}$</td>
<td>$D_{req} = \frac{\Omega P_{ac}}{CC_I}$</td>
</tr>
<tr>
<td>= 239 kips</td>
<td>= 160 kips</td>
</tr>
<tr>
<td>$0.75(3.55)(1.0)(2 \text{ welds})(14.5 \text{ in.})$</td>
<td>$3.55(1.0)(2 \text{ welds})(14.5 \text{ in.})$</td>
</tr>
<tr>
<td>= 3.10 $\rightarrow$ 4 sixteenths</td>
<td>= 3.11 $\rightarrow$ 4 sixteenths</td>
</tr>
</tbody>
</table>

From AISC Specification Table J2.4, the minimum fillet weld size is $\frac{1}{4}$ in. Use $\frac{1}{4}$-in. fillet welds.

Beam Web Thickness

$$t_{min} = \frac{6.19D}{F_y}$$

$$= 6.19(3.11 \text{ sixteenths})$$

$$= \frac{65 \text{ ksi}}{6.19(3.11 \text{ sixteenths})}$$

$$= 0.296 \text{ in.} < 0.590 \text{ in.} \quad \text{o.k.}$$

Shear Yielding of Angles

$$A_{gv} = 2(14.5 \text{ in.})(\frac{7}{8} \text{ in.})$$

$$= 25.4 \text{ in.}^2$$

From AISC Specification Equation J4-3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_a = \phi 0.60 F_y A_{gv}$</td>
<td>$R_a = \frac{0.60 F_y A_{gv}}{\Omega}$</td>
</tr>
<tr>
<td>= 1.00(0.60)(36 ksi)(25.4 in.$^2$)</td>
<td>= 0.60(36 ksi)(25.4 in.$^2$)</td>
</tr>
<tr>
<td>= 549 kips &gt; 167 kips</td>
<td>= 1.50</td>
</tr>
<tr>
<td></td>
<td>= 366 kips &gt; 112 kips</td>
</tr>
</tbody>
</table>

Similarly, shear yielding of the angles due to $H_{uc}$ and $H_{ac}$ is not critical.

Shear Rupture of Angles

$$A_{sy} = \frac{7}{8} \text{ in.} \left[ 2(14.5 \text{ in.}) - 10(\frac{1}{16} \text{ in.} + \frac{1}{4} \text{ in.}) \right]$$

$$= 16.6 \text{ in.}^2$$
From AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
<td>( \frac{R_a}{\Omega} = \frac{0.60F_u A_{nv}}{\Omega} )</td>
</tr>
<tr>
<td>( \phi R_a = \phi 0.60F_u A_{nv} )</td>
<td>( 0.75(0.60)(58 \text{ ksi})(16.6 \text{ in.}^2) )</td>
<td>( 433 \text{ kips} &gt; 167 \text{ kips} )</td>
</tr>
</tbody>
</table>

Block Shear Rupture of Angles

Use \( n = 5 \), \( L_{ev} = 1 \frac{1}{8} \text{ in.} \) and \( L_{eh} = 2.94 \text{ in.} \)

\[
A_{nt} = \left[ (2.94 \text{ in.}) - (0.5)(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}) \right] (\frac{3}{8} \text{ in.})(2) = 4.27 \text{ in.}^2
\]

\[
A_{gv} = 13.3 \text{ in.} \left( \frac{1}{8} \text{ in.} \right)(2) = 23.3 \text{ in.}^2
\]

\[
A_{nv} = \left[ (13.3 \text{ in.}) - (4.50)(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}) \right] (\frac{3}{8} \text{ in.})(2) = 15.4 \text{ in.}^2
\]

\[
U_{bs} F_u A_{nt} = 1.0(58 \text{ ksi})(4.27 \text{ in.}^2) = 248 \text{ kips}
\]

\[
0.60F_y A_{gv} = 0.60(36 \text{ ksi})(23.3 \text{ in.}^2) = 503 \text{ kips} \quad \text{controls}
\]

\[
0.60F_u A_{nv} = 0.60(58 \text{ ksi})(15.4 \text{ in.}^2) = 536 \text{ kips}
\]

From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
<td>( \frac{R_a}{\Omega} = \frac{U_{bs} F_u A_{nt}}{\Omega} + \min \left( \frac{0.60F_y A_{gv}}{\Omega}, \frac{0.60F_u A_{nv}}{\Omega} \right) )</td>
</tr>
<tr>
<td>( \phi R_a = \phi U_{bs} F_u A_{nt} + \min \left( \phi 0.60F_y A_{gv}, \phi 0.60F_u A_{nv} \right) )</td>
<td>( 0.75(248 \text{ kips} + 503 \text{ kips}) )</td>
<td>( 563 \text{ kips} &gt; 167 \text{ kips} )</td>
</tr>
</tbody>
</table>

Column Flange

By inspection, the 4.16-in.-thick column flange has adequate flexural strength, stiffness and bearing strength.
Note: When the brace is in compression, the buckling strength of the gusset would have to be checked as follows:

\[
\phi R_n = \phi_c F_{cr} A_w
\]

In the preceding equation, \(\phi_c F_{cr}\) or \(F_{cr}/\Omega_c\) may be determined with \(KL_1/r\) from AISC Specification Section J4.4, where \(l_1\) is the perpendicular distance from the Whitmore section to the interior edge of the gusset plate. Alternatively, the average value of \(l = (l_1 + l_2 + l_3)/3\) may be substituted, where these quantities are illustrated in the figure. Note that for this example, \(l_2\) is negative since part of the Whitmore section is in the beam web.

The effective length factor \(K\) has been established as 0.5 by full scale tests on bracing connections (Gross, 1990). It assumes that the gusset plate is supported on both edges. In cases where the gusset plate is supported on one edge only, such as illustrated in Example II.C-3, Figure (d), the brace can more readily move out-of-plane and a sidesway mode of buckling can occur in the gusset. For that case, \(K\) should be taken as 1.2.

### Gusset Plate Buckling

The area of the Whitmore section is:

\[
A_w = 30.9 \text{ in.}^2 + 3.90 \text{ in.}(0.590 \text{ in.})\left(\frac{50}{36} \text{ ksi}\right)
\]

\[
= 26.4 \text{ in.}^2
\]

In the preceding equation, the area in the beam web is multiplied by the ratio 50/36 to convert the area to an equivalent area of ASTM A36 plate. Assume \(l_1 = 17.0 \text{ in.}\).

\[
\frac{KL_1}{r} = \frac{0.5(17.0 \text{ in.})(\sqrt{12})}{\frac{3}{4} \text{ in.}}
\]

\[
= 39.3
\]

Because \(KL_1/r > 25\), use AISC Specification Section E3:

\[
F_{cr} = \frac{\pi^2 E}{\left(\frac{KL_1}{r}\right)^2}
\]

\[
= \frac{\pi^2 (29,000 \text{ ksi})}{39.3^2}
\]

\[
= 185 \text{ ksi}
\]

\[
4.71 \sqrt{\frac{E}{F_{cr}}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}}
\]

\[
= 134
\]
\[ F_{cr} = \left[ \frac{F_y}{F_{cr}} \right] F_y \]  
\[
= \left[ \frac{36 \text{ ksi}}{185 \text{ ksi}} \right] (36 \text{ ksi}) 
= 33.2 \text{ ksi} 
\]

\[ R_n = F_{cr} A_g \]  
\[
= 33.2 \text{ ksi} \left( 26.4 \text{ in.}^2 \right) 
= 876 \text{ kips} 
\]

From AISC *Specification* Section E1:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi )</td>
<td>0.90</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>( \psi \cdot R_n )</td>
<td>0.90(876 kips)</td>
<td>o.k.</td>
</tr>
<tr>
<td>( = 788 \text{ kips} &gt; 675 \text{ kips} )</td>
<td>( R_n = 876 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{R_n}{\Omega_c} = \frac{876 \text{ kips}}{1.67} )</td>
<td>( = 525 \text{ kips} &gt; 450 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
<td></td>
</tr>
</tbody>
</table>

Reference:

EXAMPLE II.C-3 BRACING CONNECTION

Given:

Each of the four designs shown for the diagonal bracing connection between the W14×68 brace, W24×55 beam and W14×211 column web have been developed using the Uniform Force Method (the General Case and Special Cases 1, 2, and 3).

For the given values of \( \alpha \) and \( \beta \), determine the interface forces on the gusset-to-column and gusset-to-beam connections for the following:

a. General Case of Figure (a)
b. Special Case 1 of Figure (b)
c. Special Case 2 of Figure (c)
d. Special Case 3 of Figure (d)

Brace Axial Load \( P_a = 195 \text{ kips} \) \( P_a = 130 \text{ kips} \)
Beam End Reaction \( R_a = 44 \text{ kips} \) \( R_a = 29 \text{ kips} \)
Beam Axial Load \( A_a = 26 \text{ kips} \) \( A_a = 17 \text{ kips} \)
Fig. II.C-3-1. Bracing connection configurations for Example II.C-3.
From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

**Brace**
W14×68
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

**Beam**
W24×55
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

**Column**
W14×211
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

**Gusset Plate**
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC *Manual* Table 1-1, the geometric properties are as follows:

**Brace**
W14×68
\( A = 20.0 \text{ in.}^2 \)
\( d = 14.0 \text{ in.} \)
\( t_w = 0.415 \text{ in.} \)
\( b_f = 10.0 \text{ in.} \)
\( t_f = 0.720 \text{ in.} \)

**Beam**
W24×55
\( d = 23.6 \text{ in.} \)
\( t_w = 0.395 \text{ in.} \)
\( b_f = 7.01 \text{ in.} \)
\( t_f = 0.505 \text{ in.} \)
\( k_{des} = 1.01 \text{ in.} \)

**Column**
W14×211
\( d = 15.7 \text{ in.} \)
\( t_w = 0.980 \text{ in.} \)
\( b_f = 15.8 \text{ in.} \)
\( t_f = 1.56 \text{ in.} \)

**Solution A (General Case):**

Assume \( \beta = \bar{\beta} = 3.00 \text{ in.} \).
From AISC Manual Equation 13-1:

\[
\alpha = e_b \tan \theta - e_c + \beta \tan \theta
\]

\[
= 11.8 \text{ in.} \left( \frac{12}{11\frac{1}{16}} \right) - 0 + 3.00 \text{ in.} \left( \frac{12}{11\frac{1}{16}} \right)
\]

\[
= 16.1 \text{ in.}
\]

Since \( \alpha \neq \alpha \), an eccentricity exists on the gusset-to-beam connection.

**Interface Forces**

\[
r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2}
\]

\[
= \sqrt{(16.1 \text{ in.} + 0 \text{ in.})^2 + (3.00 \text{ in.} + 11.8 \text{ in.})^2}
\]

\[
= 21.9 \text{ in.}
\]

On the gusset-to-column connection:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{ac} = \frac{\beta}{r} P_a )</td>
<td>(Manual Eq. 13-2)</td>
<td>( V_{ac} = \frac{\beta}{r} P_a )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{3.00 \text{ in.}}{21.9 \text{ in.}} (195 \text{ kips}) )</td>
<td>( = \frac{3.00 \text{ in.}}{21.9 \text{ in.}} (130 \text{ kips}) )</td>
</tr>
<tr>
<td>( H_{ac} = \frac{e_c}{r} P_a )</td>
<td>(Manual Eq. 13-3)</td>
<td>( H_{ac} = \frac{e_c}{r} P_a )</td>
</tr>
<tr>
<td></td>
<td>( = 0 \text{ kips} )</td>
<td>( = 0 \text{ kips} )</td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{ab} = \frac{e_b}{r} P_a )</td>
<td>(Manual Eq. 13-4)</td>
<td>( V_{ab} = \frac{e_b}{r} P_a )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{11.8 \text{ in.}}{21.9 \text{ in.}} (195 \text{ kips}) )</td>
<td>( = \frac{11.8 \text{ in.}}{21.9 \text{ in.}} (130 \text{ kips}) )</td>
</tr>
<tr>
<td>( H_{ab} = \frac{\alpha}{r} P_a )</td>
<td>(Manual Eq. 13-5)</td>
<td>( H_{ab} = \frac{\alpha}{r} P_a )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{16.1 \text{ in.}}{21.9 \text{ in.}} (195 \text{ kips}) )</td>
<td>( = \frac{16.1 \text{ in.}}{21.9 \text{ in.}} (130 \text{ kips}) )</td>
</tr>
<tr>
<td>( M_{ab} = V_{ab} (\alpha - \overline{\alpha}) )</td>
<td></td>
<td>( M_{ab} = V_{ab} (\alpha - \overline{\alpha}) )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{105 \text{ kips} \left(16.1 \text{ in.} - 15\frac{3}{4} \text{ in.}\right)}{12 \text{ in./ft}} )</td>
<td>( = 70.0 \text{ kips} \left(16.1 \text{ in.} - 15\frac{3}{4} \text{ in.}\right) )</td>
</tr>
</tbody>
</table>
In this case, this small moment is negligible.

On the beam-to-column connection, the required shear strength is:

\[
R_{ub} + V_{ub} = 44.0 \text{ kips} + 105 \text{ kips} = 149 \text{ kips}
\]

\[
R_{ab} + V_{ab} = 29.0 \text{ kips} + 70.0 \text{ kips} = 99.0 \text{ kips}
\]

The required axial strength is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{ub} + H_{uc} ) = 26.0 \text{ kips} + 0 \text{ kips} = 26.0 \text{ kips}</td>
<td>( A_{ab} + H_{ac} ) = 17.0 \text{ kips} + 0 \text{ kips} = 17.0 \text{ kips}</td>
</tr>
</tbody>
</table>

For a discussion of the sign use between \( A_{ub} \) and \( H_{uc} \) (\( A_{ab} \) and \( H_{ac} \) for ASD), refer to Part 13 of the AISC Manual.

**Solution B (Special Case 1):**

In this case, the centroidal positions of the gusset edge connections are irrelevant; \( \overline{\alpha} \) and \( \overline{\beta} \) are given to define the geometry of the connection, but are not needed to determine the gusset edge forces.

The angle of the brace from the vertical is

\[
\theta = \tan^{-1}\left( \frac{12}{10} \right) = 49.8^\circ
\]

The horizontal and vertical components of the brace force are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_u = P_u \sin \theta ) = (195 \text{ kips}) \sin 49.8^\circ = 149 \text{ kips}</td>
<td>( H_a = P_a \sin \theta ) = (130 \text{ kips}) \sin 49.8^\circ = 99.3 \text{ kips}</td>
</tr>
<tr>
<td>( V_u = P_u \cos \theta ) = (195 \text{ kips}) \cos 49.8^\circ = 126 \text{ kips}</td>
<td>( V_a = P_a \cos \theta ) = (130 \text{ kips}) \cos 49.8^\circ = 83.9 \text{ kips}</td>
</tr>
</tbody>
</table>

On the gusset-to-column connection:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{uc} = V_u = 126 \text{ kips} )</td>
<td>( V_{ac} = V_a = 83.9 \text{ kips} )</td>
</tr>
<tr>
<td>( H_{uc} = 0 \text{ kips} ) (Manual Eq. 13-10)</td>
<td>( H_{ac} = 0 \text{ kips} ) (Manual Eq. 13-10)</td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{ub} = 0 \text{ kips} ) (Manual Eq. 13-8)</td>
<td>( V_{ab} = 0 \text{ kips} ) (Manual Eq. 13-8)</td>
</tr>
<tr>
<td>( H_{ub} = H_u = 149 \text{ kips} )</td>
<td>( H_{ab} = H_a = 99.3 \text{ kips} )</td>
</tr>
</tbody>
</table>
On the beam-to-column connection:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ab}$</td>
<td>44.0 kips (shear)</td>
<td>29.0 kips (shear)</td>
</tr>
<tr>
<td>$A_{ab}$</td>
<td>26.0 kips (axial transfer force)</td>
<td>17.0 kips (axial transfer force)</td>
</tr>
</tbody>
</table>

In addition to the forces on the connection interfaces, the beam is subjected to a moment $M_{ub}$ or $M_{ab}$.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{ab} = H_{ab} \beta$</td>
<td>$149 \text{ kips (11.8 in.)}$</td>
<td>$99.3 \text{ kips (11.8 in.)}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{149 \text{ kips (11.8 in.)}}{12 \text{ in./ft}}$</td>
<td>$= \frac{99.3 \text{ kips (11.8 in.)}}{12 \text{ in./ft}}$</td>
</tr>
<tr>
<td>$= 147 \text{ kip-ft}$</td>
<td>$= 97.6 \text{ kip-ft}$</td>
<td></td>
</tr>
</tbody>
</table>

This moment, as well as the beam axial load $H_{ab} = 149$ kips or $H_{ab} = 99.3$ kips and the moment and shear in the beam associated with the end reaction $R_{ab}$ or $R_{ab}$, must be considered in the design of the beam.

**Solution C (Special Case 2):**

Assume $\beta = \bar{\beta} = 10\frac{1}{2}$ in.

From AISC *Manual* Equation 13-1:

\[
\alpha = \varepsilon_b \tan \theta - \varepsilon_c + \beta \tan \theta \\
= 11.8 \text{ in.} \left( \frac{12}{11\frac{1}{2}} \right) - 0 + 10\frac{1}{2} \text{ in.} \left( \frac{12}{11\frac{1}{2}} \right) \\
= 24.2 \text{ in.}
\]

Calculate the interface forces for the general case before applying Special Case 2.

\[
r = \sqrt{\left(\alpha + \varepsilon_c\right)^2 + \left(\beta + \varepsilon_b\right)^2} \\
= \sqrt{(24.2 \text{ in.} + 0 \text{ in.})^2 + (10\frac{1}{2} \text{ in.} + 11.8 \text{ in.})^2} \\
= 32.9 \text{ in.}
\]

On the gusset-to-column connection:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{ac} = \frac{\beta}{r} P_a$</td>
<td>$10\frac{1}{2} \text{ in.} (195 \text{ kips})$</td>
<td>$10\frac{1}{2} \text{ in.} (130 \text{ kips})$</td>
</tr>
<tr>
<td>$= \frac{10\frac{1}{2} \text{ in.} (195 \text{ kips})}{32.9 \text{ in.}}$</td>
<td>$= \frac{10\frac{1}{2} \text{ in.} (130 \text{ kips})}{32.9 \text{ in.}}$</td>
<td></td>
</tr>
<tr>
<td>$= 62.2 \text{ kips}$</td>
<td>$= 41.5 \text{ kips}$</td>
<td></td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection:
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{ub} = \frac{e_h}{r} P_t$  ((\text{Manual Eq. 13-4}))</td>
<td>$V_{ab} = \frac{e_h}{r} P_t$  ((\text{Manual Eq. 13-4}))</td>
</tr>
<tr>
<td>$= 11.8 \text{ in.} \left(195 \text{ kips}\right)$</td>
<td>$= 11.8 \text{ in.} \left(130 \text{ kips}\right)$</td>
</tr>
<tr>
<td>$= 69.9 \text{ kips}$</td>
<td>$= 46.6 \text{ kips}$</td>
</tr>
<tr>
<td>$H_{ub} = \frac{\alpha}{r} P_t$  ((\text{Manual Eq. 13-5}))</td>
<td>$H_{ab} = \frac{\alpha}{r} P_t$  ((\text{Manual Eq. 13-5}))</td>
</tr>
<tr>
<td>$= 24.2 \text{ in.} \left(195 \text{ kips}\right)$</td>
<td>$= 24.2 \text{ in.} \left(130 \text{ kips}\right)$</td>
</tr>
<tr>
<td>$= 143 \text{ kips}$</td>
<td>$= 95.6 \text{ kips}$</td>
</tr>
</tbody>
</table>

On the beam-to-column connection, the shear is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ub} + V_{ub} = 44.0 \text{ kips} + 69.9 \text{ kips}$</td>
<td>$R_{ub} + V_{ub} = 29.0 \text{ kips} + 46.6 \text{ kips}$</td>
</tr>
<tr>
<td>$= 114 \text{ kips}$</td>
<td>$= 75.6 \text{ kips}$</td>
</tr>
</tbody>
</table>

The axial force is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ub} + H_{uc} = 26.0 \text{ kips} + 0 \text{ kips}$</td>
<td>$A_{ub} + H_{uc} = 17.0 \text{ kips} + 0 \text{ kips}$</td>
</tr>
<tr>
<td>$= 26.0 \text{ kips}$</td>
<td>$= 17.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

Next, applying Special Case 2 with $\Delta V_{ub} = V_{ub} = 69.9 \text{ kips}$ \((\Delta V_{ab} = V_{ab} = 46.6 \text{ kips} \text{ for ASD})\), calculate the interface forces.

On the gusset-to-column connection (where $V_{uc}$ is replaced by $V_{uc} + \Delta V_{ub}$) or (where $V_{ac}$ is replaced by $V_{ac} + \Delta V_{ab}$ for ASD):

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{uc} = 62.2 \text{ kips} + 69.9 \text{ kips}$</td>
<td>$V_{ac} = 41.5 \text{ kips} + 46.6 \text{ kips}$</td>
</tr>
<tr>
<td>$= 132 \text{ kips}$</td>
<td>$= 88.1 \text{ kips}$</td>
</tr>
<tr>
<td>$H_{uc} = 0 \text{ kips} \text{ (unchanged)}$</td>
<td>$H_{ac} = 0 \text{ kips} \text{ (unchanged)}$</td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection (where $V_{ub}$ is replaced by $V_{ub} - \Delta V_{ub}$) or (where $V_{ab}$ is replaced by $V_{ab} - \Delta V_{ab}$):

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{ub} = 143 \text{ kips} \text{ (unchanged)}$</td>
<td>$H_{ab} = 95.6 \text{ kips} \text{ (unchanged)}$</td>
</tr>
<tr>
<td>$V_{ub} = 69.9 \text{ kips} - 69.9 \text{ kips}$</td>
<td>$V_{ab} = 46.6 \text{ kips} - 46.6 \text{ kips}$</td>
</tr>
<tr>
<td>$= 0 \text{ kips}$</td>
<td>$= 0 \text{ kips}$</td>
</tr>
<tr>
<td>$M_{ub} = \left(\Delta V_{ub}\right) \alpha$  ((\text{Manual Eq. 13-13}))</td>
<td>$M_{ab} = \left(\Delta V_{ab}\right) \alpha$  ((\text{Manual Eq. 13-13}))</td>
</tr>
<tr>
<td>$= 69.9 \text{ kips} \left(24.2 \text{ in.}\right)$</td>
<td>$= 46.6 \text{ kips} \left(24.2 \text{ in.}\right)$</td>
</tr>
<tr>
<td>$= 12 \text{ in./ft}$</td>
<td>$= 12 \text{ in./ft}$</td>
</tr>
<tr>
<td>$= 141 \text{ kip-ft}$</td>
<td>$= 94.0 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>
On the beam-to-column connection, the shear is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ab} + V_{ab} - \Delta V_{ab}$</td>
<td>$R_{ab} + V_{ab} - \Delta V_{ab}$</td>
</tr>
<tr>
<td>$= 44.0 \text{ kips} + 69.9 \text{ kips} - 69.9 \text{ kips}$</td>
<td>$= 29.0 \text{ kips} + 46.6 \text{ kips} - 46.6 \text{ kips}$</td>
</tr>
<tr>
<td>$= 44.0 \text{ kips}$</td>
<td>$= 29.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

The axial force is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ab} + H_{ac} = 26.0 \text{ kips} \pm 0 \text{ kips}$</td>
<td>$A_{ab} + H_{ac} = 17.0 \text{ kips} \pm 0 \text{ kips}$</td>
</tr>
<tr>
<td>$= 26.0 \text{ kips}$</td>
<td>$= 17.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Solution D (Special Case 3):**

Set $\beta = \bar{\beta} = 0 \text{ in.}$

$\alpha = e_b \tan \theta$

$= 11.8 \text{ in.} \left(\frac{12}{11/16}\right)$

$= 12.8 \text{ in.}$

Since, $\alpha \neq \bar{\alpha}$, an eccentricity exists on the gusset-to-beam connection.

**Interface Forces**

From AISC Manual Equation 13-6:

$r = \sqrt{\alpha^2 + e_b^2}$

$= \sqrt{(12.8 \text{ in.})^2 + (11.8 \text{ in.})^2}$

$= 17.4 \text{ in.}$

On the gusset-to-beam connection:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{ab} = \frac{\alpha}{r} P_a$ (Manual Eq. 13-5)</td>
<td>$H_{ab} = \frac{\alpha}{r} P_a$ (Manual Eq. 13-5)</td>
</tr>
<tr>
<td>$= \frac{12.8 \text{ in.}}{17.4 \text{ in.}} (195 \text{ kips})$</td>
<td>$= \frac{12.8 \text{ in.}}{17.4 \text{ in.}} (130 \text{ kips})$</td>
</tr>
<tr>
<td>$= 143 \text{ kips}$</td>
<td>$= 95.6 \text{ kips}$</td>
</tr>
<tr>
<td>$V_{ab} = \frac{e_b}{r} P_a$ (Manual Eq. 13-4)</td>
<td>$V_{ab} = \frac{e_b}{r} P_a$ (Manual Eq. 13-4)</td>
</tr>
<tr>
<td>$= \frac{11.8 \text{ in.}}{17.4 \text{ in.}} (195 \text{ kips})$</td>
<td>$= \frac{11.8 \text{ in.}}{17.4 \text{ in.}} (130 \text{ kips})$</td>
</tr>
<tr>
<td>$= 132 \text{ kips}$</td>
<td>$= 88.2 \text{ kips}$</td>
</tr>
<tr>
<td>$M_{ab} = V_{ab} (\alpha - \bar{\alpha})$ (Manual Eq. 13-14)</td>
<td>$M_{ab} = V_{ab} (\alpha - \bar{\alpha})$ (Manual Eq. 13-14)</td>
</tr>
<tr>
<td>$= 132 \text{ kips}(12.8 \text{ in.} - 13\frac{1}{2} \text{ in.})$</td>
<td>$= 88.2 \text{ kips}(12.8 \text{ in.} - 13\frac{1}{2} \text{ in.})$</td>
</tr>
<tr>
<td>$= \frac{12 \text{ in./ft}}{12 \text{ in./ft}}$</td>
<td>$= \frac{12 \text{ in./ft}}{12 \text{ in./ft}}$</td>
</tr>
</tbody>
</table>
In this case, this small moment is negligible.

On the beam-to-column connection, the shear is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ub} + V_{ub} = 44.0 \text{ kips} + 132 \text{ kips}$</td>
<td>$R_{ub} + V_{ub} = 29.0 \text{ kips} + 88.2 \text{ kips}$</td>
</tr>
<tr>
<td>= 176 kips</td>
<td>= 117 kips</td>
</tr>
</tbody>
</table>

The axial force is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ub} + H_{uc} = 26 \text{ kips} + 0 \text{ kips}$</td>
<td>$A_{ub} + H_{uc} = 17 \text{ kips} + 0 \text{ kips}$</td>
</tr>
<tr>
<td>= 26.0 kips</td>
<td>= 17.0 kips</td>
</tr>
</tbody>
</table>

Note: Designs by Special Case 1 result in moments on the beam and/or column that must be considered.
EXAMPLE II.C-4  TRUSS SUPPORT CONNECTION

Given:

Design the truss support connections at the following joints:

a. Joint $L_1$

b. Joint $U_1$

Use 70-ksi electrodes, ASTM A36 plate, ASTM A992 bottom and top chords, and ASTM A36 double angles.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

- Top Chord
  - WT8×38.5
  - ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

---

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
Bottom Chord
WT8×28.5
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Diagonal UdL_1
2L4×3\frac{1}{2}×\frac{3}{8}
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Web UdL_1
2L3\frac{1}{2}×3\times\frac{3}{16}
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Diagonal UdL_2
2L3\frac{1}{2}×2\frac{1}{2}×\frac{3}{16}
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Plate
PL\frac{3}{8}×4×1'−10''
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC Manual Tables 1-7, 1-8 and 1-15, the geometric properties are as follows:

Top Chord
WT8×38.5
t_w = 0.455 in.
d = 8.26 in.

Bottom Chord
WT8×28.5
t_w = 0.430 in.
d = 8.22 in.

Diagonal UdL_1
2L4×3\frac{1}{2}×\frac{3}{8}
A = 5.36 in.²
\bar{x} = 0.947 in.

Web UdL_1
2L3\frac{1}{2}×3\times\frac{3}{16}
A = 3.90 in.²
Diagonal $U_1L_2$
$2L3\frac{1}{2}\times2\frac{1}{2}\times\frac{3}{16}$
$A = 3.58 \text{ in.}^2$
$\bar{x} = 0.632 \text{ in.}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web $U_1L_1$ load:</td>
<td>Web $U_1L_1$ load:</td>
</tr>
<tr>
<td>$R_u = -104 \text{ kips}$</td>
<td>$R_u = -69.2 \text{ kips}$</td>
</tr>
<tr>
<td>Diagonal $U_0L_1$ load:</td>
<td>Diagonal $U_0L_1$ load:</td>
</tr>
<tr>
<td>$R_u = +165 \text{ kips}$</td>
<td>$R_u = +110 \text{ kips}$</td>
</tr>
<tr>
<td>Diagonal $U_1L_2$ load:</td>
<td>Diagonal $U_1L_2$ load:</td>
</tr>
<tr>
<td>$R_u = +114 \text{ kips}$</td>
<td>$R_u = +76.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Solution a:**

*Shear Yielding of Bottom Chord Tee Stem (on Section A-A)*

$$R_u = 0.60F_y A_{gy}$$

$$= 0.60(50 \text{ ksi})(8.22 \text{ in.})(0.430 \text{ in.})$$

$$= 106 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_u = 1.00(106 \text{ kips})$</td>
<td>$\frac{R_u}{\Omega} = \frac{106 \text{ kips}}{1.50}$</td>
</tr>
<tr>
<td>$= 106 \text{ kips} &gt; 104 \text{ kips}$</td>
<td>$= 70.7 \text{ kips} &gt; 69.2 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Welds for Member $U_1L_1$**

Note: AISC *Specification* Section J1.7 requiring that the center of gravity of the weld group coincide with the center of gravity of the member does not apply to end connections of statically loaded single angle, double angle and similar members.

From AISC *Specification* Table J2.4, the minimum weld size is $w_{\min} = \frac{3}{16}$ in. The maximum weld size is $w_{\max} = \text{thickness} - \frac{3}{16}$ in. = $\frac{3}{4}$ in.

Using AISC *Manual* Part 8, Equations 8-2, the minimum length of $\frac{3}{16}$-in. fillet weld is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{\min} = \frac{R_u}{1.392D}$</td>
<td>$L_{\min} = \frac{R_u}{0.928D}$</td>
</tr>
<tr>
<td>$= \frac{104 \text{ kips}}{1.392(3 \text{ sixteenths})}$</td>
<td>$= \frac{69.2 \text{ kips}}{0.928(3 \text{ sixteenths})}$</td>
</tr>
<tr>
<td>$= 24.9 \text{ in.}$</td>
<td>$= 24.9 \text{ in.}$</td>
</tr>
</tbody>
</table>
Use 6\(\frac{1}{2}\) in. of \(\frac{3}{16}\)-in. weld at the heel and toe of both angles for a total of 26 in.

**Minimum Angle Thickness to Match the Required Shear Rupture Strength of Welds**

\[
t_{\text{min}} = \frac{3.09D}{F_v} = \frac{3.09(3 \text{ sixteenths})}{58 \text{ ksi}} = 0.160 \text{ in.} < \frac{3}{16} \text{ in.} \quad \text{o.k.}
\]

**Minimum Stem Thickness to Match the Required Shear Rupture Strength of Welds**

\[
t_{\text{min}} = \frac{6.19D}{F_v} = \frac{6.19(3 \text{ sixteenths})}{65 \text{ ksi}} = 0.286 \text{ in.} < 0.430 \text{ in.} \quad \text{o.k.}
\]

Top and bottom chords are o.k.

**Welds for Member U_0L_1**

From AISC Specification Table J2.4, the minimum weld size is \(w_{\text{min}} = \frac{3}{16} \text{ in.}\). The maximum weld size is \(w_{\text{max}} = \text{thickness} - \frac{3}{16} \text{ in.} = \frac{3}{16} \text{ in.}\).

Using AISC Manual Part 8, Equations 8-2, the minimum length of \(\frac{3}{16}\)-in. fillet weld is:

<table>
<thead>
<tr>
<th>(L_{\text{min}})</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{R_v}{1.392D}) = (165 \text{ kips} / 1.392(3 \text{ sixteenths})) = (39.5 \text{ in.})</td>
<td>(\frac{R_v}{0.928D}) = (110 \text{ kips} / 0.928(3 \text{ sixteenths})) = (39.5 \text{ in.})</td>
<td></td>
</tr>
</tbody>
</table>

Use 10 in. of \(\frac{3}{16}\)-in. weld at the heel and toe of both angles for a total of 40 in.

Note: A plate will be welded to the stem of the WT to provide room for the connection. Based on the preceding calculations for the minimum angle and stem thicknesses, by inspection the angles, stems, and stem plate extension have adequate strength.

**Tensile Yielding of Diagonal U_0L_1**

\[R_e = F_v A_e = 36 \text{ ksi} \cdot (5.36 \text{ in.}^2) = 193 \text{ kips}\]
### Tensile Rupture of Diagonal $U_0L_1$

$U = 1 - \frac{x}{l}$ from AISC Specification Table D3.1 Case 2

\[
= 1 - \frac{0.947 \text{ in.}}{10.0 \text{ in.}} = 0.905
\]

\[R_u = F_u A_u \quad (\text{Spec. Eq. J4-2})\]

\[= 58 \text{ ksi} (0.905)(5.36 \text{ in.}^2) = 281 \text{ kips}\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_u = 0.90(193 \text{ kips}) = 174 \text{ kips} &gt; 165 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$R_u = 193 \text{ kips}$</td>
<td>$\frac{R_u}{\Omega} = \frac{193 \text{ kips}}{1.67} = 116 \text{ kips} &gt; 110 \text{ kips}$</td>
</tr>
</tbody>
</table>

### Block Shear Rupture of Bottom Chord

\[A_w = 4.0 \text{ in.}(0.430 \text{ in.}) = 1.72 \text{ in.}^2\]

\[A_{gw} = 10.0 \text{ in.}(0.430 \text{ in.})(2) = 8.60 \text{ in.}^2\]

From AISC Specification Equation J4-5:

\[R_u = U_h F_y A_w + 0.60 F_y A_{gw} = 1.0(65 \text{ ksi})(1.72 \text{ in.}^2) + 0.60(36 \text{ ksi})(8.60 \text{ in.}^2) = 298 \text{ kips}\]

Because an ASTM A36 plate is used for the stem extension plate, use $F_y = 36 \text{ ksi}$.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_u = 0.75(281 \text{ kips}) = 211 \text{ kips} &gt; 165 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$R_u = 281 \text{ kips}$</td>
<td>$\frac{R_u}{\Omega} = \frac{281 \text{ kips}}{2.00} = 141 \text{ kips} &gt; 110 \text{ kips}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_u = 0.75(298 \text{ kips}) = 224 \text{ kips} &gt; 165 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$R_u = 298 \text{ kips}$</td>
<td>$\frac{R_u}{\Omega} = \frac{298 \text{ kips}}{2.00} = 149 \text{ kips} &gt; 110 \text{ kips}$</td>
</tr>
</tbody>
</table>
Solution b:

Shear Yielding of Top Chord Tee Stem (on Section B-B)

\[ R_y = 0.60F_yA_{gy} \]
\[ = 0.60 \times (50 \text{ ksi}) \times (8.26 \text{ in.}) \times (0.455 \text{ in.}) \]
\[ = 113 \text{ kips} \]

\[
\begin{array}{|c|c|}
\hline
LRFD & ASD \\
\hline
\phi = 1.00 & \Omega = 1.50 \\
\phi R_y = 1.00(113 \text{ kips}) & \frac{R_y}{\Omega} = \frac{113 \text{ kips}}{1.50} \\
= 113 \text{ kips} > 74.0 \text{ kips} & = 75.3 \text{ kips} > 49.2 \text{ kips} \\
\text{o.k.} & \text{o.k.} \\
\hline
\end{array}
\]

Welds for Member \( U_1 L_1 \)

As calculated previously in Solution a, use 6\( \frac{1}{2} \) in. of 7/16-in. weld at the heel and toe of both angles for a total of 26 in.

Welds for Member \( U_1 L_2 \)

From AISC Specification Table J2.4, the minimum weld size is \( w_{\text{min}} = \frac{3}{16} \) in. The maximum weld size is \( w_{\text{max}} = \frac{3}{4} \) in.

Using AISC Manual Part 8, Equations 8-2, the minimum length of \( \frac{3}{4} \)-in. fillet weld is:

\[
\begin{array}{|c|c|}
\hline
LRFD & ASD \\
\hline
L_{\text{min}} = \frac{R_y}{1.392D} & L_{\text{min}} = \frac{R_y}{0.928D} \\
= \frac{114 \text{ kips}}{1.392(4 \text{ sixteenths})} & = \frac{76.0 \text{ kips}}{0.928(4 \text{ sixteenths})} \\
= 20.5 \text{ in.} & = 20.5 \text{ in.} \\
\hline
\end{array}
\]

Use 7\( \frac{1}{2} \) in. of 7/16-in. fillet weld at the heel and 4 in. of \( \frac{1}{4} \)-in. fillet weld at the toe of each angle for a total of 23 in.

Minimum Angle Thickness to Match the Required Shear Rupture Strength of Welds

\[
t_{\text{min}} = \frac{3.09D}{F_y} \]
\[ = \frac{3.09(4 \text{ sixteenths})}{58 \text{ ksi}} \]
\[ = 0.213 \text{ in.} < \frac{3}{16} \text{ in.} \text{ o.k.} \]

Minimum Stem Thickness to Match the Required Shear Rupture Strength of Welds

\[
t_{\text{min}} = \frac{6.19D}{F_y} \]
\[ = \frac{6.19(4 \text{ sixteenths})}{65 \text{ ksi}} \]
= 0.381 in. < 0.455 in.  o.k.

_Tensile Yielding of Diagonal $U_1L_2$

\[
R_y = F_y A_y \\
= 36 \text{ ksi}(3.58 \text{ in.}^2) \\
= 129 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_y = 0.90(129 \text{ kips})$</td>
<td>$R_y = \frac{129 \text{ kips}}{1.67}$</td>
</tr>
<tr>
<td>$= 116 \text{ kips} &gt; 114 \text{ kips}$</td>
<td>$= 77.2 \text{ kips} &gt; 76.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

_Tensile Rupture of Diagonal $U_1L_2$

\[
U = 1 - \frac{X}{l} \text{ from AISC Specification Table D3.1 Case 2} \\
= 1 - \frac{0.632 \text{ in.}}{\left(\frac{7.50 \text{ in.} + 4.00 \text{ in.}}{2}\right)} \\
= 0.890
\]

\[
R_e = F_n A_e \\
= 58 \text{ ksi}(0.890)(3.58 \text{ in.}^2) \\
= 185 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_e = 0.75(185 \text{ kips})$</td>
<td>$R_e = \frac{185 \text{ kips}}{2.00}$</td>
</tr>
<tr>
<td>$= 139 \text{ kips} &gt; 114 \text{ kips}$</td>
<td>$= 92.5 \text{ kips} &gt; 76.0 \text{ kips}$</td>
</tr>
</tbody>
</table>
Example II.C-5  HSS Chevron Brace Connection

Given:

Verify that the chevron brace connection shown in Figure II.C-5-1 is adequate for the loading shown. The ASTM A36, 3/8-in.-thick gusset plate is welded with 70-ksi electrode welds to an ASTM A992 W18×35 beam. The braces are ASTM A500 Grade B HSS6×6×½.

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×35
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Brace
HSS 6×6×½
ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

Gusset Plate
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi
\( t_p = \frac{3}{4} \text{ in.} \)

From the AISC Manual Tables 1-1 and 1-12, the geometric properties are as follows:

**Beam**

W18×35

- \( d = 17.7 \text{ in.} \)
- \( t_w = 0.300 \text{ in.} \)
- \( t_f = 0.425 \text{ in.} \)
- \( k_{des} = 0.827 \text{ in.} \)
- \( b_f = 6.00 \text{ in.} \)

**Brace**

HSS 6×6×\( \frac{3}{16} \)

- \( H = B = 6.00 \text{ in.} \)
- \( A = 9.74 \text{ in.}^2 \)
- \( t = 0.465 \text{ in.} \)

**Solution:**

Calculate the interface forces (at the beam-gusset plate interface).

\[ \Delta = \frac{1}{2}(L_2 - L_1) = 0 \quad \text{(Note: } \Delta \text{ is negative if } L_2 < L_1; \text{ see Figure II.C-5-2.)} \]

As shown in Figure II.C-5-1, the work point is at the concentric location at the beam gravity axis, \( e_b = 8.85 \text{ in.} \)

The brace bevels and loads are equal, thus the gusset will be symmetrical and \( \Delta = 0. \)

Brace forces may both act in tension or compression, but the most common case is for one to be in tension and the other to be in compression, as shown for this example in Figure II.C-5-1.

From Figure II.C-5-1:

\[ e_b = \frac{d}{2} = \frac{17.7}{2} = 8.85 \text{ in.} \]

\[ \theta = \tan^{-1} \left( \frac{12}{10 \cdot \frac{3}{16}} \right) = 48.0' \]

\( L = 44.0 \text{ in.} \)
\( L_1 = L_2 = 22.0 \text{ in.} \)
\( h = 11.0 \text{ in.} \)

Determine the brace component forces and moments as indicated in the general case in Figure II.C-5-2.
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_{a1} = 158 \text{kips}</td>
<td>P_{a1} = 105 \text{kips}</td>
</tr>
<tr>
<td>H_{a1} = 158 \sin 48^\circ \text{kips}</td>
<td>H_{a1} = 105 \sin 48^\circ \text{kips}</td>
</tr>
<tr>
<td>= 117 \text{kips}</td>
<td>= 78.0 \text{kips}</td>
</tr>
<tr>
<td>V_{a1} = 158 \cos 48^\circ \text{kips}</td>
<td>V_{a1} = 105 \cos 48^\circ \text{kips}</td>
</tr>
<tr>
<td>= 106 \text{kips}</td>
<td>= 70.3 \text{kips}</td>
</tr>
<tr>
<td>P_{a2} = -158 \text{kips}</td>
<td>P_{a2} = -105 \text{kips}</td>
</tr>
<tr>
<td>H_{a2} = -117 \text{kips}</td>
<td>H_{a2} = -78.0 \text{kips}</td>
</tr>
<tr>
<td>V_{a2} = -106 \text{kips}</td>
<td>V_{a2} = -70.3 \text{kips}</td>
</tr>
<tr>
<td>M_{a1} = H_{a1}e_b + V_{a1}\Delta</td>
<td>M_{a1} = H_{a1}e_b + V_{a1}\Delta</td>
</tr>
<tr>
<td>= 117 \text{kips}(8.85 \text{ in.}) + 106 \text{kips}(0 \text{ in.})</td>
<td>= 78.0 \text{kips}(8.85 \text{ in.}) + 70.3 \text{kips}(0 \text{ in.})</td>
</tr>
<tr>
<td>= 1,040 \text{ kip-in.}</td>
<td>= 690 \text{ kip-in.}</td>
</tr>
<tr>
<td>M_{a2} = H_{a2}e_b - V_{a2}\Delta</td>
<td>M_{a2} = H_{a2}e_b - V_{a2}\Delta</td>
</tr>
<tr>
<td>= -1,040 \text{ kip-in.}</td>
<td>= -690 \text{ kip-in.}</td>
</tr>
<tr>
<td>M_{a1}' = \frac{1}{8}V_{a1}L - \frac{1}{4}H_{a1}h - \frac{1}{2}M_{a1}</td>
<td>M_{a1}' = \frac{1}{8}V_{a1}L - \frac{1}{4}H_{a1}h - \frac{1}{2}M_{a1}</td>
</tr>
<tr>
<td>= \frac{1}{8}(106 \text{kips})(44.0 \text{ in.})</td>
<td>= \frac{1}{8}(70.3 \text{kips})(44.0 \text{ in.})</td>
</tr>
<tr>
<td>- \frac{1}{4}(117 \text{kips})(11.0 \text{ in.})</td>
<td>- \frac{1}{4}(78.0 \text{kips})(11.0 \text{ in.})</td>
</tr>
<tr>
<td>- \frac{1}{2}(1,040 \text{ kip-in.})</td>
<td>- \frac{1}{2}(690 \text{ kip-in.})</td>
</tr>
<tr>
<td>= -259 \text{ kip-in.}</td>
<td>= -173 \text{ kip-in.}</td>
</tr>
<tr>
<td>M_{a2}' = \frac{1}{8}V_{a2}L - \frac{1}{4}H_{a2}h - \frac{1}{2}M_{a2}</td>
<td>M_{a2}' = \frac{1}{8}V_{a2}L - \frac{1}{4}H_{a2}h - \frac{1}{2}M_{a2}</td>
</tr>
<tr>
<td>= \frac{1}{8}(-106 \text{kips})(44.0 \text{ in.})</td>
<td>= \frac{1}{8}(-70.3 \text{kips})(44.0 \text{ in.})</td>
</tr>
<tr>
<td>- \frac{1}{4}(-117 \text{kips})(11.0 \text{ in.})</td>
<td>- \frac{1}{4}(-78.0 \text{kips})(11.0 \text{ in.})</td>
</tr>
<tr>
<td>- \frac{1}{2}(-1,040 \text{ kip-in.})</td>
<td>- \frac{1}{2}(-690 \text{ kip-in.})</td>
</tr>
<tr>
<td>= 259 \text{ kip-in.}</td>
<td>= 173 \text{ kip-in.}</td>
</tr>
</tbody>
</table>

Note: The signs on the variables represent the directions of the forces shown on the Figures IIC-5-2,3 and 4. Forces are positive if in the directions shown in these figures, the forces are negative otherwise, and the signs must be used consistently in the formulas.
Forces for Section a-a (Gusset Edge Forces),

Determine the forces and moments at Section a-a in Figure II.C-5-1, as indicated in the general case in Figure II.C-5-3.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial</td>
<td>Axial</td>
</tr>
<tr>
<td>$N_a = V_{u1} + V_{u2}$</td>
<td>$N_a = V_{a1} + V_{a2}$</td>
</tr>
<tr>
<td>= 106 kips + (−106 kips)</td>
<td>= 70.3 kips + (−70.3 kips)</td>
</tr>
<tr>
<td>= 0 kips</td>
<td>= 0 kips</td>
</tr>
<tr>
<td>Shear</td>
<td>Shear</td>
</tr>
<tr>
<td>$V_a = H_{a1} - H_{a2}$</td>
<td>$V_a = H_{a1} - H_{a2}$</td>
</tr>
<tr>
<td>= 117 kips − (−117 kips)</td>
<td>= 78.0 kips − (−78.0 kips)</td>
</tr>
<tr>
<td>= 234 kips</td>
<td>= 156 kips</td>
</tr>
</tbody>
</table>

Moment

Fig. II.C-5-2. Chevron brace gusset forces—general case.
(Also see Figures II.C-5-3 and II.C-5-4 for forces and moments at Sections a-a and b-b.)
\[
M_u = M_{a1} - M_{a2} \\
= 1,040 \text{kip-in.} - (-1,040 \text{kip-in.}) \\
= 2,080 \text{kip-in.}
\]

\[
M_d = M_{a1} - M_{a2} \\
= 690 \text{kip-in.} - (-690 \text{kip-in.}) \\
= 1,380 \text{kip-in.}
\]

Forces for Section b-b (Gusset Internal Forces)

Determine the forces and moments at Section b-b in Figure II.C-5-1, as indicated in the general case in Figure II.C-5-4.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Axial</strong></td>
<td><strong>Axial</strong></td>
</tr>
<tr>
<td>( N'<em>u = \frac{1}{2} (H</em>{a1} + H_{a2}) )</td>
<td>( N'<em>d = \frac{1}{2} (H</em>{a1} + H_{a2}) )</td>
</tr>
<tr>
<td>( = \frac{1}{2} [117 \text{kips} + (-117 \text{kips})] )</td>
<td>( = \frac{1}{2} [78.0 \text{kips} + (-78.0 \text{kips})] )</td>
</tr>
</tbody>
</table>
Fig. II.C-5-3. Forces on Section a-a (positive directions shown)—general case.

Forces on Section a-a
Axial: \( N = V_1 + V_2 \)
Shear: \( V = H_1 - H_2 \)
Moment: \( M = M_1 - M_2 \)

Fig. II.C-5-4. Forces on Section b-b (positive directions shown)—general case.

Forces on Section b-b:
Axial: \( N' = \frac{1}{2}(H_1 + H_2) \)
Shear: \( V' = \frac{1}{2}(V_1 - V_2) - \frac{1}{2}(M) \)
Moment: \( M' = M'_1 + M'_2 \)
Design Brace-to-Gusset Connection

This part of the connection should be designed first because it will give a minimum required size of the gusset plate.

Brace Gross Tension Yielding

From AISC Specification Equation J4-1, determine the available strength due to tensile yielding in the gross section as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = \phi F_y A_t$</td>
<td>$R_u = F_y A_t / \Omega$</td>
</tr>
<tr>
<td>$= 0.90(46 \text{ ksi})(9.74 \text{ in.}^2)$</td>
<td>$= (46 \text{ ksi})(9.74 \text{ in.}^2)$</td>
</tr>
<tr>
<td>$= 403 \text{ kips} &gt; 158 \text{ kips}$ <strong>O.K.</strong></td>
<td>$= 1.67$</td>
</tr>
</tbody>
</table>

Therefore, $R_u = 268 \text{ kips} > 105 \text{ kips}$ **O.K.**

Brace Shear Rupture

Because net tension rupture involves shear lag, first determine the weld length, $l$, required for shear rupture of the brace.

From AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = \phi 0.6 F_y A_w$</td>
<td>$R_u = 0.6 F_y A_w / \Omega$</td>
</tr>
<tr>
<td>$158 \text{ kips} = 0.75(0.60) (58 \text{ ksi})(0.465 \text{ in.})(l)(4)$</td>
<td>$105 \text{ kips} = 0.60(58 \text{ ksi})(0.465 \text{ in.})(l)(4)$</td>
</tr>
<tr>
<td>Therefore, $l = 3.25 \text{ in.}$</td>
<td>Therefore, $l = 3.24 \text{ in.}$</td>
</tr>
</tbody>
</table>

Assume a $5/16$-in. fillet weld. The weld length, $l_w$, required is determined from AISC Manual Equation 8-2:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_w = \frac{\phi R_u}{1.392(D)(4)}$</td>
<td>$l_w = \frac{R_u / \Omega}{0.928(D)(4)}$</td>
</tr>
<tr>
<td>$= \frac{158 \text{ kips}}{1.392(5)(4)}$</td>
<td>$= \frac{105 \text{ kips}}{0.928(5)(4)}$</td>
</tr>
<tr>
<td>$= 5.68 \text{ in.}$</td>
<td>$= 5.66 \text{ in.}$</td>
</tr>
</tbody>
</table>

Use 6-in.-long $5/16$-in. fillet welds.

Brace Tension Rupture (Assume $3/16$-in.-thick gusset plate)

Determine the shear lag factor, $U$, from AISC Specification Table D3.1, Case 6. For a single concentric gusset plate:
\[ \bar{x} = \frac{B^2 + 2BH}{4(B + H)} \]
\[ = \frac{(6.00 \text{ in.})^2 + 2(6.00 \text{ in.})(6.00 \text{ in.})}{4(6.00 \text{ in.} + 6.00 \text{ in.})} \]
\[ = 2.25 \text{ in.} \]

\[ U = 1 - \frac{\bar{x}}{t} \]
\[ = 1 - \frac{2.25 \text{ in.}}{6.00 \text{ in.}} \]
\[ = 0.625 \]

\[ A_s = A_k - 2t_{slot} \quad d_{slot} = \text{slot width} \]

\[ A_k = 9.74 \text{ in.}^2 - 2(0.465 \text{ in.})(1/4 \text{ in.} + 1/6 \text{ in.}) \]
\[ = 8.93 \text{ in.}^2 \]

\[ A_v = A_k U \]
\[ = (8.93 \text{ in.}^2)(0.625) \]
\[ = 5.58 \text{ in.}^2 \]

The nominal tensile rupture of the brace is:

\[ R_n = F_u A_v \]
\[ = 58 \text{ ksi}(5.58 \text{ in.}^2) \]
\[ = 324 \text{ kips} \]

The available tensile rupture strength of the brace is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( \phi R_n = 0.75(324 \text{ kips}) \)
| = 243 \text{ kips} > 158 \text{ kips}  \textbf{o.k.} |
| \( R_n = 324 \text{ kips} \)
| \( \Omega = 2.00 \)
| = 162 \text{ kips} > 105 \text{ kips}  \textbf{o.k.} |

Check Block Shear on Gusset

\[ R_u = 0.60 F_y A_{uv} + U_{bs} F_y A_{ut} \leq 0.60 F_y A_{gg} + U_{bs} F_y A_{gt} \]
\( U_{bs} = 1.0 \)

\[ A_{gg} = A_{ut} \]
\[ = 2t_u l_w \]
\[ = 2(1/4 \text{ in.})(6.00 \text{ in.}) \]
\[ = 9.00 \text{ in.}^2 \]

\[ A_{gg} = A_{bt} \]
\[ = s u = 4.50 \text{ in.}^2 \]

\[ 4.50 \text{ in.}^2 \]

Design Examples V14.1
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\[ F_u A_u = 58 \text{ ksi}(4.50 \text{ in.}^2) \]
\[ = 261 \text{ kips} \]
\[ F_y A_{gy} = 36 \text{ ksi}(9.00 \text{ in.}^2) \]
\[ = 324 \text{ kips} \]
\[ F_u A_{uw} = 58 \text{ ksi}(9.00 \text{ in.}^2) \]
\[ = 522 \text{ kips} \]

Shear Yielding:
\[ R_y = 0.60 F_y A_{gy} \]
\[ = 0.60(324 \text{ kips}) \]
\[ = 194 \text{ kips} \quad \text{goes} \]

Shear Rupture:
\[ R_u = 0.60 F_u A_{uw} \]
\[ = 0.60(522 \text{ kips}) \]
\[ = 313 \text{ kips} \]

The available block shear rupture strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi R_u = 0.75 \left( 0.60 F_y A_{gy} + U_{ku} F_u A_{uw} \right) ]</td>
<td></td>
</tr>
<tr>
<td>[ R_u \frac{1}{\Omega} = \frac{1}{2.00} \left( 0.60 F_y A_{gy} + U_{ku} F_u A_{uw} \right) ]</td>
<td></td>
</tr>
<tr>
<td>[ = 0.75 \left[ 194 \text{ kips} + 1.0(261 \text{ kips}) \right] ]</td>
<td></td>
</tr>
<tr>
<td>[ = \frac{1}{2.00} \left[ 194 \text{ kips} + 1.0(261 \text{ kips}) \right] ]</td>
<td></td>
</tr>
<tr>
<td>[ = 341 \text{ kips} &gt; 158 \text{ kips} \quad \text{o.k.} ]</td>
<td></td>
</tr>
<tr>
<td>[ = 228 \text{ kips} &gt; 105 \text{ kips} \quad \text{o.k.} ]</td>
<td></td>
</tr>
</tbody>
</table>

*Whitmore Tension Yield and Compression Buckling of Gusset Plate (AISC Manual Part 9)*

Determine whether AISC *Specification* Equation J4-6 is applicable \((KL/r \leq 25)\).

\[ r = \frac{t_g}{\sqrt{12}} \]
\[ = \frac{3}{4} \text{ in.} \]
\[ = \frac{0.217 \text{ in.}}{\sqrt{12}} \]

Assume \(K = 0.65\), from Dowswell (2012).

From geometry, the unbraced gusset plate length is \(L = 6.50 \text{ in.}\).

\[ \frac{KL}{r} = \frac{0.65(6.50 \text{ in.})}{0.217 \text{ in.}} \]
\[ = 19.5 \]
Based on AISC Specification Section J4.4, Equation J4-6 is applicable because $KL/r \leq 25$.

Determine the length of the Whitmore Section

$$l_w = B + 2(\text{connection length}) \tan 30^\circ$$
$$= 6.00 \text{ in.} + 2(6.00 \text{ in.}) \tan 30^\circ$$
$$= 12.9 \text{ in.}$$

$$A_w = l_w t_p$$
$$= 12.9 \text{ in. (w in.)}$$
$$= 9.68 \text{ in.}^2$$  
(Whitmore section is assumed to be entirely in gusset.)

$$P_n = F/A_w$$  
(from Spec. Eq. J4-6)
$$= 36 \text{ ksi}(9.68 \text{ in.}^2)$$
$$= 348 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_n = 0.90(348 \text{ kips})$</td>
<td>$P_n = \frac{348 \text{ kips}}{\Omega}$</td>
</tr>
<tr>
<td>$= 313 \text{ kips} &gt; 158 \text{ kips} \text{ o.k.}$</td>
<td>$= 208 \text{ kips} &gt; 105 \text{ kips} \text{ o.k.}$</td>
</tr>
</tbody>
</table>

**Gusset-to-Beam Connection**

As determined previously, the forces and moment at Section a-a of Figure II.C-5-1:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear : $V_a = 234 \text{ kips}$</td>
<td>Shear : $V_a = 156 \text{ kips}$</td>
</tr>
<tr>
<td>Axial : $N_a = 0 \text{ kips}$</td>
<td>Axial : $N_a = 0 \text{ kips}$</td>
</tr>
<tr>
<td>Moment : $M_a = 2,080 \text{ kip-in.}$</td>
<td>Moment : $M_a = 1,380 \text{ kip-in.}$</td>
</tr>
</tbody>
</table>

The gusset stresses are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear : $f_v = \frac{V_a}{t_p L}$</td>
<td>Shear : $f_v = \frac{V_a}{t_p L}$</td>
</tr>
<tr>
<td>$= \frac{234 \text{ kips}}{\frac{3}{4} \text{ in.}(44.0 \text{ in.})}$</td>
<td>$= \frac{156 \text{ kips}}{\frac{3}{4} \text{ in.}(44.0 \text{ in.})}$</td>
</tr>
<tr>
<td>$= 7.09 \text{ ksi}$</td>
<td>$= 4.73 \text{ ksi}$</td>
</tr>
</tbody>
</table>

From AISC Specification Equation J4-3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 1.00(0.60)(36 \text{ ksi})$</td>
<td>$R_n = \frac{0.60(36 \text{ ksi})}{\Omega}$</td>
</tr>
<tr>
<td>$= 21.6 \text{ ksi} &gt; 7.09 \text{ ksi} \text{ o.k.}$</td>
<td>$= 14.4 \text{ ksi} &gt; 4.73 \text{ ksi} \text{ o.k.}$</td>
</tr>
</tbody>
</table>
Axial: 
\[ f_a = \frac{N_u}{t_p L} \]
\[ = 0 \text{ kips} \]
\[ = 0 \text{ kips} \]

Moment: 
\[ f_b = \frac{M_u}{Z_e} \]
\[ = \frac{M_u}{t_p L^2 / 4} \]
\[ = \frac{4(2,080 \text{ kip-in.})}{\frac{3}{4} \text{ in.}(44.0 \text{ in.})^2} \]
\[ = 5.73 \text{ ksi} \]

Total Axial Stress: 
\[ f_a = f_a + f_b \]
\[ = 0 \text{ ksi} + 5.73 \text{ ksi} \]
\[ = 5.73 \text{ ksi} \]

Axial: 
\[ f_a = \frac{N_u}{t_p L} \]
\[ = 0 \text{ kips} \]
\[ = 0 \text{ kips} \]

Moment: 
\[ f_b = \frac{M_u}{Z_e} \]
\[ = \frac{M_u}{t_p L^2 / 4} \]
\[ = \frac{4(1,380 \text{ kip-in.})}{\frac{3}{4} \text{ in.}(44.0 \text{ in.})^2} \]
\[ = 3.80 \text{ ksi} \]

Total Axial Stress: 
\[ f_a = f_a + f_b \]
\[ = 0 \text{ ksi} + 3.80 \text{ ksi} \]
\[ = 3.80 \text{ ksi} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare the total axial stress to the available stress from AISC Specification Section J4.1(a) for the limit state of tensile yielding.</td>
<td>Compare the total axial stress to the available stress from AISC Specification Section J4.1(a) for the limit state of tensile yielding.</td>
</tr>
<tr>
<td>5.73 ksi ( \leq 0.90 ) (36 ksi) = 32.4 ksi \textbf{O.K.}</td>
<td>3.80 ksi ( \leq \frac{36 \text{ ksi}}{1.67} ) = 21.6 ksi \textbf{O.K.}</td>
</tr>
</tbody>
</table>

\textbf{Weld of Gusset to Beam}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ R = \sqrt{V_a^2 + N_a^2} ] [ = \sqrt{(234 \text{ kips})^2 + (0 \text{ kips})^2} ] [ = 234 \text{ kips} ]</td>
<td>[ R = \sqrt{V_a^2 + N_a^2} ] [ = \sqrt{(156 \text{ kips})^2 + (0 \text{ kips})^2} ] [ = 156 \text{ kips} ]</td>
</tr>
</tbody>
</table>

The 234 kips (LRFD) and 156 kips (ASD) shear force is actually at the center of the beam; this is a function of the moment (2,080 kip-in. (LRFD) and 1,380 kips (ASD)). The effective eccentricity of \( V \) is:
Note that \( e \) is \( e_b \). The LRFD and ASD values for \( e \) differ due to rounding. Continue the problem with \( e = 8.89 \) in.

\[
e = 8.89 \text{ in.}
\]

\[
\theta = 0^\circ
\]

\[
k = 0
\]

\[
a = \frac{e}{L} = \frac{8.89}{44.0} \text{ in.} = 0.202
\]

From AISC Manual Table 8-4: \( C = 3.50, \ C_1 = 1.00 \)

Applying a ductility factor of 1.25 as discussed in Part 13 of the AISC Manual to AISC Manual Equation 8-13, the weld required is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{\text{req}}' = \frac{V_a (1.25)}{\phi CC_1L} )</td>
<td>( D_{\text{req}}' = \frac{V_a (1.25)}{CC_1L} )</td>
</tr>
<tr>
<td>( = \frac{234 \text{kips}(1.25)}{0.75(3.50)(1.00)(44 \text{ in.})} )</td>
<td>( = \frac{2.00(156 \text{kips})(1.25)}{3.50(1.00)(44 \text{ in.})} )</td>
</tr>
<tr>
<td>( = 2.53 )</td>
<td>( = 2.53 )</td>
</tr>
</tbody>
</table>

A \( \frac{1}{8}\)-in. fillet weld is required.

Verifying this with AISC Specification Table J2.4, the material thickness of the thinner part joined is \( t_f = 0.425 \) in. and the minimum size fillet weld is \( \frac{3}{8}\)-in. in.

Therefore, use a \( \frac{1}{8}\)-in. fillet weld.

**Gusset Internal Stresses**

The gusset internal forces and moment at Section b-b of Figure II.C-5-1, as determined previously, are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear : ( V_a' = 11.5 ) kips</td>
<td>Shear : ( V_a' = 7.57 ) kips</td>
</tr>
<tr>
<td>Axial : ( N_a' = 0 ) kips</td>
<td>Axial : ( N_a' = 0 ) kips</td>
</tr>
<tr>
<td>Moment : ( M_a' = 0 ) kip-in.</td>
<td>Moment : ( M_a' = 0 ) kip-in.</td>
</tr>
</tbody>
</table>
Gusset Stresses

The limit of shear yielding of the gusset is checked using AISC Specification Equation J4-3 as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_v = \frac{V'_{w}}{t_ph}$</td>
<td>$f_v = \frac{V'_{a}}{t_ph}$</td>
</tr>
<tr>
<td>$\frac{11.5 \text{ kips}}{\text{in.} \times (11.0 \text{ in.})} = 1.39 \text{ ksi}$</td>
<td>$\frac{7.57 \text{ kips}}{\text{in.} \times (11.0 \text{ in.})} = 0.918 \text{ ksi}$</td>
</tr>
<tr>
<td>$\leq 1.00 \times (0.60)(36 \text{ ksi}) = 21.6 \text{ ksi} \quad \text{O.K.}$</td>
<td>$\leq \frac{0.60 \times (36 \text{ ksi})}{1.50} = 14.4 \text{ ksi} \quad \text{O.K.}$</td>
</tr>
<tr>
<td>$N'<em>{w} = M'</em>{w} = 0$</td>
<td>$N'<em>{a} = M'</em>{a} = 0$</td>
</tr>
<tr>
<td>No check is required</td>
<td>No check is required</td>
</tr>
</tbody>
</table>

If $N'$ and $M'$ are greater than zero, it is possible for a compressive stress to exist on the gusset free edge at Section b-b. In this case, the gusset should be checked for buckling under this stress. The procedure in AISC Manual Part 9 for buckling of a coped beam can be used. If gusset plate buckling controls, an edge stiffener could be added or a thicker plate used.

Check Web Local Yielding of Beam Under Normal Force

The limit state of web local yielding is checked using AISC Specification Equation J10-2, with $l_b = L = 44\text{ in.}$ and $k = k_{des}$, as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{max}} =</td>
<td>N_{u}</td>
</tr>
<tr>
<td>$= 0 \text{ kips} + \frac{4(2,080 \text{ kip-in.})}{44.0 \text{ in.}}$</td>
<td>$= 0 \text{ kips} + \frac{4(1,380 \text{ kip-in.})}{44.0 \text{ in.}}$</td>
</tr>
<tr>
<td>$= 189 \text{ kips}$</td>
<td>$= 125 \text{ kips}$</td>
</tr>
<tr>
<td>$\phi R_{w} = \phi F_{yw,t_{w}}(5k + l_b)$</td>
<td>$R_{w} = F_{yw,t_{w}}(5k + l_b)$</td>
</tr>
<tr>
<td>$= 1.00(50 \text{ ksi})(0.300 \text{ in.})[5(0.827 \text{ in.}) + 44.0 \text{ in.}]$</td>
<td>$= \frac{(50 \text{ ksi})(0.300 \text{ in.})[5(0.827 \text{ in.}) + 44.0 \text{ in.}]}{1.50}$</td>
</tr>
<tr>
<td>$= 722 \text{ kips} \geq 189 \text{ kips} \quad \text{O.K.}$</td>
<td>$= 481 \text{ kips} \geq 125 \text{ kips} \quad \text{O.K.}$</td>
</tr>
</tbody>
</table>

Web Crippling Under Normal Load

From AISC Specification Equation J10-4:
Check of Beam for Horizontal Shear

The length of beam web effective in carrying the shear on Section a-a can be determined based on the assumption that the critical section in the web is $L + 5k$ plus some web area length over which the force in the beam flange is transferred into the web area, that is determined as follows. If the flange area of the beam is $A_f = b_ft_f$, the nominal tensile yielding strength of the beam flange is $2F_y A_f$ from AISC Specification Equation J4-1, where the factor of 2 comes from the two effective flange areas, one on each side of the chevron gusset coinciding with the two braces. This force can be taken out of the flange and into the web shear area, where the length of the web, $L_w$, is determined by setting the tensile yielding strength of the flange equal to the shear yielding strength of the web from AISC Specification Equation J4-3 and solving for $L_w$, as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\phi_y F_y b_f t_f = \phi_y 0.60 F_y t_y L_w$</td>
<td>$2F_y b_f t_f \Omega_y = 0.60 F_y t_y L_w$</td>
</tr>
</tbody>
</table>

and therefore:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_w = \frac{2\phi_y b_f t_f}{\phi_y 0.60 t_y}$</td>
<td>$L_w = \frac{2\Omega_y b_f t_f}{\Omega_y 0.60 t_y}$</td>
</tr>
</tbody>
</table>

The total web length effective in transferring the shear into the beam is:

$$L_{eff} = L + 5k + L_w$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{eff} = L + 5k + \frac{2\phi_y b_f t_f}{\phi_y 0.60 t_y}$</td>
<td>$L_{eff} = L + 5k + \frac{2\Omega_y b_f t_f}{\Omega_y 0.60 t_y}$</td>
</tr>
</tbody>
</table>

$$= 44.0 \text{ in.} + 5(0.827 \text{ in.}) + 2(0.90)(6.00 \text{ in})(0.425 \text{ in.})$$

$$+ \frac{2(0.90)(6.00 \text{ in})(0.425 \text{ in.})}{1.00(0.60)(0.300 \text{ in.})}$$

$$= 73.6 \text{ in.}$$

From AISC Specification Equation J4-3, the available web shear strength is:


\[
\phi R_a = \phi_0 0.60 F_y t_w L_{\text{eff}} = 1.00(0.60)(50 \text{ ksi})(0.300 \text{ in.})(73.6 \text{ in.}) = 662 \text{ kips} > V_u = 234 \text{ kips} \quad \text{o.k.}
\]

\[
\frac{R_a}{\Omega} = \frac{0.60 F_y t_w L_{\text{eff}}}{\Omega} = \frac{(0.60)(50 \text{ ksi})(0.300 \text{ in.})(73.6 \text{ in.})}{1.50} = 442 \text{ kips} > V_a = 156 \text{ kips} \quad \text{o.k.}
\]

**Transverse Section Web Yielding**

The transverse shear induced in the beam at the centerline of the gusset (Section b-b) is calculated and compared to the available shear yielding limit state determined from AISC *Specification* Equation G2-1, with \(C_v = 1.0\).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_a = 106 \text{ kips} – 11.5 \text{ kips})</td>
<td>(V_a = 70.3 \text{ kips} – 7.57 \text{ kips})</td>
</tr>
<tr>
<td>(= 94.5 \text{ kips})</td>
<td>(= 62.7 \text{ kips})</td>
</tr>
<tr>
<td>(\phi R_a = \phi_0 0.60 F_y d t_w C_v)</td>
<td>(R_a = \frac{0.60 F_y d t_w C_v}{\Omega_v})</td>
</tr>
<tr>
<td>(= 1.00(0.6)(50 \text{ ksi})(17.7 \text{ in.})(0.300 \text{ in.})(1.0))</td>
<td>(= \frac{0.60(50 \text{ ksi})(17.7 \text{ in.})(0.300 \text{ in.})(1.0)}{1.50})</td>
</tr>
<tr>
<td>(= 159 \text{ kips} \geq 94.5 \text{ kips} \quad \text{o.k.})</td>
<td>(= 106 \text{ kips} \geq 62.7 \text{ kips} \quad \text{o.k.})</td>
</tr>
</tbody>
</table>

**Reference**

EXAMPLE II.C-6  HEAVY WIDE FLANGE COMPRESSION CONNECTION (FLANGES ON THE OUTSIDE)

Given:

This truss has been designed with nominal 14-in. ASTM A992 W-shapes, with the flanges to the outside of the truss. Beams framing into the top chord and lateral bracing are not shown but can be assumed to be adequate.

Based on multiple load cases, the critical dead and live load forces for this connection were determined. A typical top chord connection and the dead load and live load forces are shown as follows in detail A. Design this typical connection using 1-in.-diameter ASTM A325 slip-critical bolts in standard holes with a Class A faying surface and ASTM A36 gusset plates.
Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

- **W-shapes**
  - ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

- **Gusset Plates**
  - ASTM A36
  - \( F_y = 36 \text{ ksi} \)
  - \( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

- **W14x109**
  - \( d = 14.3 \text{ in.} \)
  - \( b_f = 14.6 \text{ in.} \)
  - \( t_f = 0.860 \text{ in.} \)

- **W14x61**
  - \( d = 13.9 \text{ in.} \)
  - \( b_f = 10.0 \text{ in.} \)
  - \( t_f = 0.645 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Left top chord:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_u = 1.2(262 \text{ kips}) + 1.6(262 \text{ kips}) )</td>
<td>( P_u = 734 \text{ kips} )</td>
<td>( P_u = 262 \text{ kips} + 262 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Right top chord:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_u = 1.2(345 \text{ kips}) + 1.6(345 \text{ kips}) )</td>
<td>( P_u = 966 \text{ kips} )</td>
<td>( P_u = 345 \text{ kips} + 345 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vertical Web:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_u = 1.2(102 \text{ kips}) + 1.6(102 \text{ kips}) )</td>
<td>( P_u = 286 \text{ kips} )</td>
<td>( P_u = 102 \text{ kips} + 102 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Diagonal Web:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_u = 1.2(113 \text{ kips}) + 1.6(113 \text{ kips}) )</td>
<td>( T_u = 316 \text{ kips} )</td>
<td>( T_u = 113 \text{ kips} + 113 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Single Bolt Shear Strength (AISC Specification Section J3.8)*

- \( d = 1.00 \text{ in.} \)
- ASTM A325-SC bolts
- Class A faying surface
\[ \mu = 0.30 \]
\[ d_h = 1\frac{3}{8} \text{ in. (diameter of holes at gusset plates)} \]
\[ h_f = 1.0 \text{ (factor for fillers)} \]
\[ T_b = 51 \text{ kips from AISC Specification Table J3.1} \]
\[ D_u = 1.13 \]

\[ R_u = \mu D_u h_f T_b n_s \]
\[ = 0.30(1.13)(1.0)(51 \text{ kips})(1) \]
\[ = 17.3 \text{ kips} \]

For standard holes, determine the available slip resistance and available bolt shear rupture strength:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi = 1.00 & \Omega = 1.50 \\
\phi R_u = 1.00(17.3 \text{ kips}) & \frac{R_u}{\Omega} = 17.3 \text{ kips} \\
= 17.3 \text{ kips/bolt} & = 11.5 \text{ kips/bolt} \\
\hline
\end{array}
\]

From AISC Manual Table 7-1, the shear strength of an ASTM A325-N bolt is:
\[ \phi r_u = 31.8 \text{ kips} > 17.3 \text{ kips} \quad \text{o.k.} \]

From AISC Manual Table 7-1, the shear strength of an ASTM A325-N bolt is:
\[ \frac{r_u}{\Omega} = 21.2 \text{ kips} > 11.5 \text{ kips} \quad \text{o.k.} \]

Note: Standard holes are used in both plies for this example. Other hole sizes may be used and should be considered based on the preferences of the fabricator or erector on a case-by-case basis.

### Diagonal Connection

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 316 \text{ kips} )</td>
<td>( P_u = 226 \text{ kips} )</td>
</tr>
<tr>
<td>( 316 \text{ kips} / 17.3 \text{ kips/bolt} = 18.3 \text{ bolts} )</td>
<td>( 226 \text{ kips} / 11.5 \text{ kips/bolt} = 19.7 \text{ bolts} )</td>
</tr>
<tr>
<td>( 2 \text{ lines both sides} = 18.3 \text{ bolts} / 4 = 4.58 )</td>
<td>( 2 \text{ lines both sides} = 19.7 \text{ bolts} / 4 = 4.93 )</td>
</tr>
<tr>
<td>Therefore, use 5 rows at min. 3-in. spacing.</td>
<td>Therefore, use 5 rows at min. 3-in. spacing.</td>
</tr>
</tbody>
</table>

Whitmore Section in Gusset Plate (AISC Manual Part 9)

\[
\text{Whitmore section} = \text{gage of the bolts} + \tan 30^\circ(\text{length of the bolt group}) (2) \\
= \frac{5}{2} \text{ in.} + \tan 30^\circ[(4 \text{ spaces})(3.00 \text{ in.})] (2) \\
= 19.4 \text{ in.} \\
\]

Try 3/8-in.-thick plate

\[ A_g = \frac{3}{8} \text{ in.}(19.4 \text{ in.}) \\
= 7.28 \text{ in.}^2 \]

Tensile Yielding of Gusset Plate
From AISC Specification Equation J4-1:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.90</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_t = \phi F_y A_t$</td>
<td>$= 0.90(36 \text{ ksi})(7.28 \text{ in.}^2)(2)$</td>
<td>$= \frac{36 \text{ ksi}(7.28 \text{ in.}^2)(2)}{1.67}$</td>
</tr>
<tr>
<td></td>
<td>$= 472 \text{ kips} &gt; 316 \text{ kips}$</td>
<td>$= 314 \text{ kips} &gt; 226 \text{ kips}$</td>
</tr>
</tbody>
</table>

Block Shear Rupture of Gusset Plate

Tension stress is uniform, therefore, $U_{ls} = 1.0$. Assume a 2-in. edge distance on the diagonal gusset plate connection.

$t_p = \frac{3}{8} \text{ in.}$
$A_{gv} = \frac{3}{8} \text{ in.} \cdot \{2 \text{ lines}(4 \text{ spaces})(3 \text{ in.}) + 2 \text{ in.}\}$
$= 10.5 \text{ in.}^2$

$A_{nt} = 10.5 \text{ in.}^2 - (\frac{3}{8} \text{ in.})(2 \text{ lines})(4.5 \text{ bolts})(1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})$
$= 6.70 \text{ in.}^2$

$A_{nt} = \frac{3}{8} \text{ in.}[5.50 \text{ in.} - (1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})]$
$= 1.64 \text{ in.}^2$

From AISC Specification Equation J4-5:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.75</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_t = \phi U_{ls} F_y A_{nt} + \min(\phi 0.60 F_y A_{gv}, \phi 0.60 F_y A_{nt})$</td>
<td>$\frac{R_n}{\Omega} = \frac{U_{ls} F_y A_{nt}}{\Omega} + \min\left(\frac{0.60 F_y A_{gv}}{\Omega}, \frac{0.60 F_y A_{nt}}{\Omega}\right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{\phi U_{ls} F_y A_{nt}}{\Omega} = 0.75(1.0)(58 \text{ ksi})(1.64 \text{ in.}^2)$</td>
<td>Tension rupture component: $\frac{U_{ls} F_y A_{nt}}{\Omega} = \frac{1.0(58 \text{ ksi})(1.64 \text{ in.}^2)}{2.00}$ $= 47.6 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 71.3 \text{ kips}$</td>
<td>Shear yielding component: $\frac{0.6 F_y A_{gv}}{\Omega} = \frac{0.6(36 \text{ ksi})(10.5 \text{ in.}^2)}{2.00}$ $= 113 \text{ kips}$</td>
</tr>
<tr>
<td>Shear yielding component:</td>
<td>$\frac{0.6 F_y A_{gv}}{\Omega} = 0.75(0.6)(36 \text{ ksi})(10.5 \text{ in.}^2)$</td>
<td>$= 170 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 170 \text{ kips}$</td>
<td>Shear rupture component: $\frac{0.6 F_y A_{nt}}{\Omega} = \frac{0.6(58 \text{ ksi})(6.70 \text{ in.}^2)}{2.00}$ $= 117 \text{ kips}$</td>
</tr>
<tr>
<td>LRFD</td>
<td>ASD</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-----</td>
<td></td>
</tr>
<tr>
<td>( \phi R_n = 71.3 \text{ kips} + 170 \text{ kips/in.} = 241 \text{ kips} &gt; 316 \text{ kips/2} = 158 \text{ kips} )</td>
<td>o.k.</td>
<td></td>
</tr>
<tr>
<td>( \frac{R_n}{\Omega} = 47.6 \text{ kips} + 113 \text{ kips} = 161 \text{ kips} &gt; 226 \text{ kips/2} = 113 \text{ kips} )</td>
<td>o.k.</td>
<td></td>
</tr>
</tbody>
</table>

**Block Shear Rupture of Diagonal Flange**

By inspection, block shear rupture on the diagonal flange will not control.

**Bolt Bearing on Gusset Plate**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From AISC Manual Table 7-4 with ( s = 3 \text{ in.} ) and standard holes, ( \phi r_n = 101 \text{ kips/in.}(% \text{ in.}) = 37.9 \text{ kips} &gt; 17.3 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-5 with ( L_e = 2 \text{ in.} ) and standard holes, ( \phi r_n = 76.7 \text{ kips/in.}(% \text{ in.}) = 28.8 \text{ kips} &gt; 17.3 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-4 with ( s = 3 \text{ in.} ) and standard holes, ( \frac{r_n}{\Omega} = 67.4 \text{ kips/in.}(% \text{ in.}) = 25.3 \text{ kips} &gt; 11.5 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-5 with ( L_e = 2 \text{ in.} ) and standard holes, ( \frac{r_n}{\Omega} = 51.1 \text{ kips/in.}(% \text{ in.}) = 19.2 \text{ kips} &gt; 11.5 \text{ kips} )</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Bolt Bearing on Diagonal Flange**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From AISC Manual Table 7-4 with ( s = 3 \text{ in.} ) and standard holes, ( \phi r_n = 113 \text{ kips/in.}(0.645 \text{ in.}) = 72.9 \text{ kips} &gt; 17.3 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-5 with ( L_e = 2 \text{ in.} ) and standard holes, ( \phi r_n = 85.9 \text{ kips/in.}(0.645 \text{ in.}) = 55.4 \text{ kips} &gt; 17.3 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-4 with ( s = 3 \text{ in.} ) and standard holes, ( \frac{r_n}{\Omega} = 75.6 \text{ kips/in.}(0.645 \text{ in.}) = 48.8 \text{ kips} &gt; 11.5 \text{ kips} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>From AISC Manual Table 7-5 with ( L_e = 2 \text{ in.} ) and standard holes, ( \frac{r_n}{\Omega} = 57.3 \text{ kips/in.}(0.645 \text{ in.}) = 37.0 \text{ kips} &gt; 11.5 \text{ kips} )</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Horizontal Connection**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required strength: ( P_u = 966 \text{ kips} - 734 \text{ kips} = 232 \text{ kips} )</td>
<td>Required strength: ( P_a = 690 \text{ kips} - 524 \text{ kips} = 166 \text{ kips} )</td>
</tr>
<tr>
<td>As determined previously, the design bolt shear strength is 17.3 kips/bolt.</td>
<td>As determined previously, the allowable bolt shear strength is 11.5 kips/bolt.</td>
</tr>
<tr>
<td>232 kips / 17.3 kips/bolt = 13.4 bolts</td>
<td>166 kips / 11.5 kips/bolt = 14.4 bolts</td>
</tr>
<tr>
<td>2 lines both sides = 13.4 bolts / 4 = 3.35</td>
<td>2 lines both sides = 14.4 bolts / 4 = 3.60</td>
</tr>
<tr>
<td>Use 4 rows on each side.</td>
<td>Use 4 rows on each side.</td>
</tr>
</tbody>
</table>

For members not subject to corrosion the maximum bolt spacing is calculated using AISC Specification Section J3.5(a):

Maximum bolt spacing  = 24(⅛ in.)  
= 9.00 in.

Due to the geometry of the gusset plate, the use of 4 rows of bolts in the horizontal connection will exceed the maximum bolt spacing; instead use 5 rows of bolts in two lines.

Shear Yielding of Plate

Try plate with, \( t_p = \frac{3}{8} \) in.

\[ A_{gy} = \frac{3}{8} \text{ in.}(32 \text{ in.}) = 12.0 \text{ in.}^2 \]

From AISC Specification Equation J4-3:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_u = \phi 0.60 F_y A_{gy} )</td>
<td>( R_u = 0.60 F_y A_{gy} )</td>
</tr>
<tr>
<td>( = 1.00 \times (0.60)(36 \text{ ksi})(12.0 \text{ in.}^2) )</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>( = 259 \text{ kips} &gt; 232 \text{ kips}/2 = 116 \text{ kips} )</td>
<td>( = \frac{0.60(36 \text{ ksi})(12.0 \text{ in.}^2)}{1.50} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>( = 173 \text{ kips} &gt; 166 \text{ kips}/2 = 83.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Shear Rupture of Plate

\[ A_{nv} = 12.0 \text{ in.}^2 - \frac{3}{8} \text{ in.}(5 \text{ bolts})(1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}) = 9.89 \text{ in.}^2 \]

From AISC Specification Equation J4-4:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_u = \phi 0.60 F_y A_{nv} )</td>
<td>( R_u = 0.60 F_y A_{nv} )</td>
</tr>
<tr>
<td>( = 0.75(0.60)(58 \text{ ksi})(9.89 \text{ in.}^2) )</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>( = 258 \text{ kips} &gt; 232 \text{ kips}/2 = 116 \text{ kips} )</td>
<td>( = \frac{0.60(58 \text{ ksi})(9.89 \text{ in.}^2)}{2.00} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>( = 172 \text{ kips} &gt; 166 \text{ kips}/2 = 83.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Bolt Bearing on Gusset Plate and Horizontal Flange
By comparison to the preceding calculations for the diagonal connection, bolt bearing does not control.

**Vertical Connection**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Required axial strength:</strong></td>
<td><strong>Required axial strength:</strong></td>
</tr>
<tr>
<td>( P_u = 286 ) kips</td>
<td>( P_a = 204 ) kips</td>
</tr>
<tr>
<td>As determined previously, the design bolt shear strength is 17.3 kips/bolt.</td>
<td>As determined previously, the allowable bolt shear strength is 11.5 kips/bolt.</td>
</tr>
<tr>
<td>286 kips / 17.3 kips/bolt = 16.5 bolts</td>
<td>204 kips / 11.5 kips/bolt = 17.7 bolts</td>
</tr>
<tr>
<td>2 lines both sides = 16.5 bolts / 4 = 4.13</td>
<td>2 lines both sides = 17.7 bolts / 4 = 4.43</td>
</tr>
<tr>
<td>Use 5 bolts per line.</td>
<td>Use 5 bolts per line.</td>
</tr>
</tbody>
</table>

**Shear Yielding of Plate**

Try plate with, \( t_p = \frac{3}{8} \) in.

\[ A_{sv} = \frac{3}{8} \text{ in.}(31.75 \text{ in.}) = 11.9 \text{ in.}^2 \]

From AISC Specification Equation J4-3:

\[ \phi = 1.00 \]

\[ \phi R_u = \phi R_{sv} A_{sv} = 1.00(0.60)(36 \text{ ksi})(11.9 \text{ in.}^2) = 257 \text{ kips} > 286 \text{ kips}/2 = 143 \text{ kips} \]

\[ \Omega = 1.50 \]

\[ R_u \frac{\Omega}{\Omega} = \frac{0.60 F_y A_{sv}}{1.50} = 171 \text{ kips} > 204 \text{ kips}/2 = 102 \text{ kips} \]

\[ \Phi = 0.60(36 \text{ ksi})(11.9 \text{ in.}^2) \]

**Shear Rupture of Plate**

\[ A_{sv} = 11.9 \text{ in.}^2 = \frac{3}{8} \text{ in.}(7 \text{ bolts})(1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}) = 8.95 \text{ in.}^2 \]

From AISC Specification Equation J4-4:

\[ \Phi = 0.75 \]

\[ \phi R_u = \phi R_{sv} A_{sv} = 0.75(0.60)(58 \text{ ksi})(8.95 \text{ in.}^2) = 234 \text{ kips} > 286 \text{ kips}/2 = 143 \text{ kips} \]

\[ \Omega = 2.00 \]

\[ R_u \frac{\Omega}{\Omega} = \frac{0.60 F_y A_{sv}}{2.00} = 156 \text{ kips} > 204 \text{ kips}/2 = 102 \text{ kips} \]
Bolt Bearing on Gusset Plate and Vertical Flange

By comparison to the preceding calculations for the diagonal connection, bolt bearing does not control.

The final layout for the connection is as follows:

Note that because of the difference in depths between the top chord and the vertical and diagonal members, \(\frac{\gamma}{16}\)-in. loose shims are required on each side of the shallower members.
Chapter IID
Miscellaneous Connections

This section contains design examples on connections in the AISC Steel Construction Manual that are not covered in other sections of the AISC Design Examples.
EXAMPLE IID-1 PRYING ACTION IN TEES AND IN SINGLE ANGLES

Given:

Design an ASTM A992 WT hanger connection between an ASTM A36 2L3×3×⅜ tension member and an ASTM A992 W24×94 beam to support the following loads:

\[ P_D = 13.5 \text{ kips} \]
\[ P_L = 40 \text{ kips} \]

Use ⅛-in.-diameter ASTM A325-N or F1852-N bolts and 70-ksi electrodes.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Hanger
WT
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Beam
W24×94
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Angles
2L3×3×⅜
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]
From AISC Manual Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W24×94
\(d = 24.3 \text{ in.}\)
\(t_w = 0.515 \text{ in.}\)
\(b_y = 9.07 \text{ in.}\)
\(t_f = 0.875 \text{ in.}\)

Angles
2L3×3×\(\frac{3}{16}\)
\(A = 3.56 \text{ in.}^2\)
\(\bar{x} = 0.860 \text{ in.} \text{ for single angle}\)

From Chapter 2 of ASCE/SEI 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_d)</td>
<td>(1.2(13.5 \text{ kips}) + 1.6(40 \text{ kips}))</td>
<td>(P_d = 13.5 \text{ kips} + 40 \text{ kips})</td>
</tr>
<tr>
<td></td>
<td>(= 80.2 \text{ kips})</td>
<td>(= 53.5 \text{ kips})</td>
</tr>
</tbody>
</table>

Tensile Yielding of Angles

\(P_d = F_y A_g\)  \(\text{(Spec. Eq. D2-1)}\)
\(= 36 \text{ ksi}(3.56 \text{ in.}^2)\)
\(= 128 \text{ kips}\)

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi)</td>
<td>0.90</td>
<td>o.k.</td>
</tr>
<tr>
<td>(\phi P_s)</td>
<td>0.90(128 \text{ kips})</td>
<td>(\frac{P_d}{\Omega} = 128 \text{ kips})</td>
</tr>
<tr>
<td></td>
<td>(= 115 \text{ kips} &gt; 80.2 \text{ kips})</td>
<td>(= 76.6 \text{ kips} &gt; 53.5 \text{ kips})</td>
</tr>
</tbody>
</table>

From AISC Specification Table J2.4, the minimum size of fillet weld based on a material thickness of \(\frac{3}{16}\) in. is \(\frac{3}{16}\) in.

From AISC Specification Section J2.2b, the maximum size of fillet weld is:

\(w_{\text{max}} = \text{thickness} - \frac{3}{16} \text{ in.}\)
\(= \frac{3}{16} \text{ in.} - \frac{3}{16} \text{ in.}\)
\(= \frac{3}{16} \text{ in.}\)
\(= \frac{3}{16} \text{ in.}\)

Try \(\frac{3}{16}\)-in. fillet welds.
From AISC Manual Part 8, Equations 8-2:

\[ l_{\min} = \frac{P_a}{1.392D} \]

\[ = \frac{80.2 \text{ kips}}{1.392(4 \text{ sixteenths})} \]

\[ = 14.4 \text{ in.} \]

\[ l_{\min} = \frac{P_a}{0.928D} \]

\[ = \frac{53.5 \text{ kips}}{0.928(4 \text{ sixteenths})} \]

\[ = 14.4 \text{ in.} \]

Use four 4-in. welds (16 in. total), one at the toe and heel of each angle.

**Tensile Rupture Strength of Angles**

\[ U = 1 - \frac{x}{L} \] from AISC Specification Table D3.1 case 2

\[ = 1 - \frac{0.860 \text{ in.}}{4.00 \text{ in.}} \]

\[ = 0.785 \]

\[ A_c = A_u U \] (Spec. Eq. D3-1)

\[ = 3.56 \text{ in.}^2 (0.785) \]

\[ = 2.79 \text{ in.}^2 \]

\[ P_n = F_u A_c \] (Spec. Eq. D2-2)

\[ = 58 \text{ ksi} (2.79 \text{ in.}^2) \]

\[ = 162 \text{ kips} \]

Use four 4-in. welds (16 in. total), one at the toe and heel of each angle.

**Preliminary WT Selection Using Beam Gage**

\[ g = 4 \text{ in.} \]

Try four $\frac{3}{4}$-in.-diameter ASTM A325-N bolts.

From AISC Manual Table 7-2:

\[ T = r_{ul} = \frac{P_n}{n} \]

\[ = \frac{80.2 \text{ kips}}{4} = 20.1 \text{ kips/bolt} \]

\[ B = \phi r_n = 29.8 \text{ kips} > 20.1 \text{ kips} \] o.k.

\[ T = r_{ul} = \frac{P_n}{n} \]

\[ = \frac{53.5 \text{ kips}}{4} = 13.4 \text{ kips/bolt} \]

\[ B = \phi r_n = 19.9 \text{ kips} > 13.4 \text{ kips} \] o.k.
Determine tributary length per pair of bolts, $p$, using AISC Manual Figure 9-4 and assuming a ½-in. web thickness.

\[
p = \frac{4.00 \text{ in.} - \frac{1}{2} \text{ in.}}{2} + \frac{8.00 \text{ in.} - 4\frac{1}{2} \text{ in.}}{2} = 3.50 \text{ in.} \leq 4\frac{1}{2} \text{ in.}
\]

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 bolts (20.1 kips/bolt)</td>
<td>$\frac{11.5 \text{kips/ft}}{3.50 \text{ in.}}$</td>
<td>$\frac{7.66 \text{kips/ft}}{3.50 \text{ in.}}$</td>
</tr>
</tbody>
</table>

From AISC Manual Table 15-2b, with an assumed $b = (4.00 \text{ in.} - \frac{1}{2} \text{ in.})/2 = 1.75 \text{ in.}$, the flange thickness, $t = t_f$, of the WT hanger should be approximately ½ in.

The minimum depth WT that can be used is equal to the sum of the weld length plus the weld size plus the $k$-dimension for the selected section. From AISC Manual Table 1-8 with an assumed $b = 1.75 \text{ in.}$, $t_f \approx \frac{1}{2} \text{ in.}$, and $d_{min} = 4 \text{ in.} + \frac{3}{4} \text{ in.} + k \approx 6 \text{ in.}$, appropriate selections include:

- WT6×25
- WT7×26.5
- WT8×25
- WT9×27.5

Try a WT6×25.

From AISC Manual Table 1-8, the geometric properties are as follows:

\[
\begin{align*}
  b_f &= 8.08 \text{ in.} \\
  t_f &= 0.640 \text{ in.} \\
  t_w &= 0.370 \text{ in.}
\end{align*}
\]

**Prying Action Using AISC Manual Part 9**

The beam flange is thicker than the WT flange; therefore, prying in the tee flange will control over prying in the beam flange.

\[
\begin{align*}
  b &= \frac{g - t_w}{2} \\
  &= \frac{4.00 \text{ in.} - 0.370 \text{ in.}}{2} \\
  &= 1.82 \text{ in.} > 1\frac{3}{4} \text{-in. entering and tightening clearance, and the fillet toe is cleared}
\end{align*}
\]

\[
\begin{align*}
  a &= \frac{b_f - g}{2} \\
  &= \frac{8.08 \text{ in.} - 4.00 \text{ in.}}{2} \\
  &= 2.04 \text{ in.}
\end{align*}
\]

\[
b' = b - \frac{d_b}{2}
\]

(Manual Eq. 9-21)
$$= 1.82 \text{ in.} - \left(\frac{\frac{3}{4} \text{ in.}}{2}\right)$$
$$= 1.45 \text{ in.}$$

$$a' = \left(\frac{a + \frac{d}{2}}{2}\right) \leq \left(\frac{1.25b + \frac{d}{2}}{2}\right) \quad (\text{Manual Eq. 9-27})$$
$$= 2.04 \text{ in.} + \left(\frac{\frac{3}{4} \text{ in.}}{2}\right) \leq 1.25(1.82 \text{ in.}) + \frac{\frac{3}{4} \text{ in.}}{2}$$
$$= 2.42 \text{ in.} \leq 2.65 \text{ in.}$$

$$\rho = \frac{b'}{a'} \quad (\text{Manual Eq. 9-26})$$
$$= \frac{1.45 \text{ in.}}{2.42 \text{ in.}} = 0.599$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = \frac{1}{\rho} \left(\frac{B}{T} - 1\right) \quad (\text{Manual Eq. 9-25})$</td>
<td>$\beta = \frac{1}{\rho} \left(\frac{B}{T} - 1\right) \quad (\text{Manual Eq. 9-25})$</td>
</tr>
<tr>
<td>$= \frac{1}{0.599} \left(\frac{29.8 \text{ kips/bolt}}{20.1 \text{ kips/bolt}} - 1\right)$</td>
<td>$= \frac{1}{0.599} \left(\frac{19.9 \text{ kips/bolt}}{13.4 \text{ kips/bolt}} - 1\right)$</td>
</tr>
<tr>
<td>$= 0.806$</td>
<td>$= 0.810$</td>
</tr>
</tbody>
</table>

$$\delta = 1 - \frac{d'}{\rho} \quad (\text{Manual Eq. 9-24})$$
$$= 1 - \frac{\frac{3}{4} \text{ in.} + \frac{\delta}{16} \text{ in.}}{3.50 \text{ in.}}$$
$$= 0.768$$

Since $\beta < 1.0$,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha' = \frac{1}{\delta} \left(\frac{\beta}{1 - \beta}\right) \leq 1.0$</td>
<td>$\alpha' = \frac{1}{\delta} \left(\frac{\beta}{1 - \beta}\right) \leq 1.0$</td>
</tr>
<tr>
<td>$= \frac{1}{0.768} \left(\frac{0.806}{1 - 0.806}\right)$</td>
<td>$= \frac{1}{0.768} \left(\frac{0.810}{1 - 0.810}\right)$</td>
</tr>
<tr>
<td>$= 5.41$, therefore, $\alpha' = 1.0$</td>
<td>$= 5.55$, therefore, $\alpha' = 1.0$</td>
</tr>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$t_{\min} = \frac{4Tb'}{\sqrt{\phi pF_u(1 + \delta \alpha')}} \quad (\text{Manual Eq. 9-23a})$</td>
<td>$t_{\min} = \frac{\Omega 4Tb'}{\sqrt{pF_u(1 + \delta \alpha')}} \quad (\text{Manual Eq. 9-23b})$</td>
</tr>
<tr>
<td>$= \frac{4(20.1 \text{ kips/bolt})(1.45 \text{ in.})}{\sqrt{0.90(3.50 \text{ in.})(65 \text{ ksi})[1 + (0.768)(1.0)]}}$</td>
<td>$= \frac{1.67(4)(13.4 \text{ kips/bolt})(1.45 \text{ in.})}{\sqrt{3.50 \text{ in.}(65 \text{ ksi})[1 + (0.768)(1.0)]}}$</td>
</tr>
<tr>
<td>$= 0.567 \text{ in.} &lt; t_f = 0.640 \text{ in.}$</td>
<td>$= 0.568 \text{ in.} &lt; t_f = 0.640 \text{ in.}$</td>
</tr>
</tbody>
</table>
Tensile Yielding of the WT Stem on the Whitmore Section Using AISC Manual Part 9

The effective width of the WT stem (which cannot exceed the actual width of 8 in.) is:

\[ l_w = 3.00 \text{ in.} + 2(4.00 \text{ in.})(\tan 30°) \leq 8.00 \text{ in.} \]
\[ = 7.62 \text{ in.} \]

The nominal strength is determined as:

\[ R_n = F_y A_g \]
\[ = 50 \text{ ksi}(7.62 \text{ in.})(0.370 \text{ in.}) \]
\[ = 141 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(141 \text{ kips}) )</td>
<td>( R_n = 141 \text{ kips} )</td>
</tr>
<tr>
<td>( = 127 \text{ kips} &gt; 80.2 \text{ kips} )</td>
<td>( = 84.4 \text{ kips} &gt; 53.5 \text{ kips} )</td>
</tr>
</tbody>
</table>

Shear Rupture of the WT Stem Base Metal

\[ t_{min} = \frac{6.19 D}{F_u} \]
\[ = 6.19 \left( \frac{4 \text{ sixteenths}}{65 \text{ ksi}} \right) \]
\[ = 0.381 \text{ in.} > 0.370 \text{ in.} \]

**shear rupture strength of WT stem controls over weld rupture strength**

Block Shear Rupture of the WT Stem

\[ A_{gv} = (2 \text{ shear planes})(4.00 \text{ in.})(0.370 \text{ in.}) \]
\[ = 2.96 \text{ in.}^2 \]

Tension stress is uniform, therefore \( U_{hs} = 1.0 \).

\[ A_{nt} = A_{gt} = 3.00 \text{ in.}(0.370 \text{ in.}) \]
\[ = 1.11 \text{ in.}^2 \]

\[ R_n = 0.60 F_y A_{nv} + U_{hs} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{hs} F_u A_{nt} \]
\[ \text{(Spec. Eq. J4-5)} \]

Because the angles are welded to the WT-hanger, shear yielding on the gross area will control (that is, the portion of the block shear rupture equation that addresses shear rupture on the net area does not control).

\[ R_n = 0.60 F_y A_{gv} + U_{hs} F_u A_{nt} \]
\[ = 0.60(50 \text{ ksi})(2.96 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.11 \text{ in.}^2) \]
\[ = 161 \text{ kips} \]
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75(161 \text{ kips})$</td>
<td>$R_n = 161 \text{ kips}$</td>
</tr>
<tr>
<td>$= 121 \text{ kips} &gt; 80.2 \text{ kips}$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>o.k.</td>
<td>$= 80.5 \text{ kips} &gt; 53.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

Note: As an alternative to the preceding calculations, the designer can use a simplified procedure to select a WT hanger with a flange thick enough to reduce the effect of prying action to an insignificant amount, i.e., $q \approx 0$. Assuming $b' = 1.45$ in.

From AISC *Manual* Part 9:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$t_{\text{min}} = \frac{4Tb'}{\phi pF_u}$ (Manual Eq. 9-20a)</td>
<td>$t_{\text{min}} = \frac{\Omega 4Tb'}{pF_u}$ (Manual Eq. 9-20b)</td>
</tr>
<tr>
<td>$= \frac{4(20.1 \text{ kips/bolt})(1.45 \text{ in.})}{0.90(3.50 \text{ in./bolt})(65 \text{ ksi})}$</td>
<td>$= \frac{1.67(4)(13.4 \text{ kips/bolt})(1.45 \text{ in.})}{(3.50 \text{ in./bolt})(65 \text{ ksi})}$</td>
</tr>
<tr>
<td>$= 0.755 \text{ in.}$</td>
<td>$= 0.755 \text{ in.}$</td>
</tr>
</tbody>
</table>

A WT6x25, with $t_f = 0.640$ in. < 0.755 in., does not have a sufficient flange thickness to reduce the effect of prying action to an insignificant amount. In this case, the simplified approach requires a WT section with a thicker flange.
EXAMPLE II.D-2 BEAM BEARING PLATE

Given:

An ASTM A992 W18×50 beam with a dead load end reaction of 15 kips and a live load end reaction of 45 kips is supported by a 10-in.-thick concrete wall. Assuming the concrete has $f'_{c}=3$ ksi, and the bearing plate is ASTM A36 material determine the following:

a. If a bearing plate is required if the beam is supported by the full wall thickness
b. The bearing plate required if $l_b=10$ in. (the full wall thickness)
c. The bearing plate required if $l_b=6\frac{1}{2}$ in. and the bearing plate is centered on the thickness of the wall

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
$F_y=50$ ksi
$F_u=65$ ksi

Bearing Plate (if required)
ASTM A36
$F_y=36$ ksi
$F_u=58$ ksi

Concrete Wall
$f'_c=3$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W18×50

$d=18.0$ in.
$tw=0.355$ in.
$b_f=7.50$ in.
$tr=0.570$ in.
$k_{des}=0.972$ in.
$k_1=\frac{1}{16}$ in.
Concrete Wall  
$h = 10.0$ in.

Solution a:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate required strength.</td>
<td>Calculate required strength.</td>
</tr>
</tbody>
</table>
| $R_u = 1.2(15 \text{ kips}) + 1.6(45 \text{ kips})$ 
$= 90.0 \text{ kips}$ | $R_u = 15 \text{ kips} + 45 \text{ kips}$ 
$= 60.0 \text{ kips}$ |
| $l_{b \text{req}} = \frac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ 
$= \frac{90.0 \text{ kips} - 43.1 \text{ kips}}{17.8 \text{ kips/in.}} \geq 0.972 \text{ in.}$ 
$= 2.63 \text{ in.} < 10.0 \text{ in.}$ | $l_{b \text{req}} = \frac{R_u - R_1 / \Omega}{R_2 / \Omega} \geq k_{des}$ 
$= \frac{60.0 \text{ kips} - 28.8 \text{ kips}}{11.8 \text{ kips/in.}} \geq 0.972 \text{ in.}$ 
$= 2.64 \text{ in.} < 10.0 \text{ in.}$ |
| $l_b = \frac{10.0 \text{ in.}}{18.0 \text{ in.}}$ 
$= 0.556$ | $l_b = \frac{10.0 \text{ in.}}{18.0 \text{ in.}}$ 
$= 0.556$ |
| Since $\frac{l_b}{d} > 0.2$, use Manual Equation 9-48a. | Since $\frac{l_b}{d} > 0.2$, use Manual Equation 9-48b. |
| $l_{b \text{req}} = \frac{R_u - \phi R_5}{\phi R_6}$ 
$= \frac{90.0 \text{ kips} - 52.0 \text{ kips}}{6.30 \text{ kips/in.}}$ 
$= 6.03 \text{ in.} < 10.0 \text{ in.}$ | $l_{b \text{req}} = \frac{R_u - R_3 / \Omega}{R_6 / \Omega}$ 
$= \frac{60.0 \text{ kips} - 34.7 \text{ kips}}{4.20 \text{ kips/in.}}$ 
$= 6.02 \text{ in.} < 10.0 \text{ in.}$ |
| Verify $\frac{l_b}{d} > 0.2,$ | Verify $\frac{l_b}{d} > 0.2,$ |
| $l_b = 6.03 \text{ in.}$ 
$d = 18.0 \text{ in.}$ 
$= 0.335 > 0.2$ | $l_b = 6.02 \text{ in.}$ 
$d = 18.0 \text{ in.}$ 
$= 0.334 > 0.2$ |
<p>| Check the bearing strength of concrete. | Check the bearing strength of concrete. |
| Note that AISC Specification Equation J8-1 is used because $A_2$ is not larger than $A_1$ in this case. | Note that AISC Specification Equation J8-1 is used because $A_2$ is not larger than $A_1$ in this case. |
| $P_p = 0.85f_c , 'A_1$ | $P_p = 0.85f_c , 'A_1$ |</p>
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.65 )</td>
<td>( \Omega_c = 2.31 )</td>
</tr>
<tr>
<td>( \phi P_p = \phi (0.85f'_c A_1) )</td>
<td>( P_p = 0.85f'_c A_1 )</td>
</tr>
<tr>
<td>( = 0.65(0.85)(3 \text{ ksi})(7.50 \text{ in.})(10.0 \text{ in.}) )</td>
<td>( \Omega_c = \frac{\Omega_c}{\Omega_c} )</td>
</tr>
<tr>
<td>( = 124 \text{ kips} &gt; 90.0 \text{ kips} )</td>
<td>( = 0.85(3 \text{ ksi})(7.50 \text{ in.})(10.0 \text{ in.}) )</td>
</tr>
<tr>
<td>( = 124 \text{ kips} &gt; 90.0 \text{ kips} )</td>
<td>( = 82.8 \text{ kips} &gt; 60.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Beam Flange Thickness Check Using AISC Manual Part 14**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the cantilever length from Manual Equation 14-1.</td>
<td>Determine the cantilever length from Manual Equation 14-1.</td>
</tr>
<tr>
<td>( n = \frac{b_f}{2} - k_{des} )</td>
<td>( n = \frac{b_f}{2} - k_{des} )</td>
</tr>
<tr>
<td>( = \frac{7.50 \text{ in.}}{2} - 0.972 \text{ in.} )</td>
<td>( = \frac{7.50 \text{ in.}}{2} - 0.972 \text{ in.} )</td>
</tr>
<tr>
<td>( = 2.78 \text{ in.} )</td>
<td>( = 2.78 \text{ in.} )</td>
</tr>
</tbody>
</table>

Determine bearing pressure.

\[ f_p = \frac{R_u}{A_1} \]

Determine the minimum beam flange thickness required if no bearing plate is provided. The beam flanges along the length, \( n \), are assumed to be fixed end cantilevers with a minimum thickness determined using the limit state of flexural yielding.

\[ M_a = \frac{f_p n^2}{2} = \frac{R_u n^2}{2A_1} \]

\[ Z = \frac{1}{4} t^2 \]

\[ M_a \leq \phi F_y Z = \phi F_y \left( \frac{t^2}{4} \right) \]

\[ t_{min} = \sqrt{\frac{4M_a}{\phi F_y}} = \sqrt{\frac{2R_u n^2}{\phi A_1 F_y}} \]

\( \phi = 0.90 \)

\[ t_{min} = \sqrt{\frac{2R_u n^2}{\phi A_1 F_y}} \]

\[ \Omega = 1.67 \]

\[ t_{min} = \sqrt{\frac{\Omega 2 R_u n^2}{A_1 F_y}} \]
A bearing plate is required. See note following.

Note: The designer may assume a bearing width narrower than the beam flange in order to justify a thinner flange. In this case, if 5.44 in. ≤ bearing width ≤ 6.56 in., a 0.570 in. flange thickness is ok and the concrete has adequate bearing strength.

Solution b:

\( I_b = 10 \text{ in.} \)

From Solution a, web local yielding and web local crippling are o.k.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho = 0.65 )</td>
<td>( \Omega = 2.31 )</td>
</tr>
<tr>
<td>( A_{1,req} = \frac{R_u}{\rho f'c} = \frac{90.0 \text{ kips}}{0.65(0.85)(3 \text{ ksi})} = 54.3 \text{ in.}^2 )</td>
<td>( A_{1,req} = \frac{R_u \Omega_c}{0.85 f'c} = \frac{60.0 \text{ kips}(2.31)}{(0.85)(3 \text{ ksi})} = 54.4 \text{ in.}^2 )</td>
</tr>
<tr>
<td>( B_{req} = \frac{A_{1,req}}{N} = \frac{54.3 \text{ in.}^2}{10.0 \text{ in.}} = 5.43 \text{ in.} )</td>
<td>( B_{req} = \frac{A_{1,req}}{N} = \frac{54.4 \text{ in.}^2}{10.0 \text{ in.}} = 5.44 \text{ in.} )</td>
</tr>
<tr>
<td>Use ( B = 8 \text{ in.} ) (selected as the least whole-inch dimension that exceeds ( b_f )).</td>
<td>Use ( B = 8 \text{ in.} ) (selected as the least whole-inch dimension that exceeds ( b_f )).</td>
</tr>
</tbody>
</table>


\( n = \frac{B}{2} - k_{des} \) (Manual Eq. 14-1)

\( = \frac{8.00 \text{ in.}}{2} - 0.972 \text{ in.} \)

\( = 3.03 \text{ in.} \)

\( t_{min} = \frac{2R_u n^2}{\rho A_f F_y} \)

\( = \frac{\Omega 2R_u n^2}{A_f F_y} \)
\[
\frac{2(90.0 \text{ kips})(3.03 \text{ in.})^2}{\sqrt{0.90(10.0 \text{ in.})(8.00 \text{ in.})(36 \text{ ksi})}} = 0.798 \text{ in.}
\]

Use PL \( \frac{9}{8} \) in. \( \times \) 10 in. \( \times 0 \) ft 8 in.

\[
\frac{1.67(2)(60.0 \text{ kips})(3.03 \text{ in.})^2}{\sqrt{(10.0 \text{ in.})(8.00 \text{ in.})(36 \text{ ksi})}} = 0.799 \text{ in.}
\]

Use PL \( \frac{9}{8} \) in. \( \times \) 10 in. \( \times 0 \) ft 8 in.

Note: The calculations for \( t_{\text{min}} \) are conservative. Taking the strength of the beam flange into consideration results in a thinner required bearing plate or no bearing plate at all.

**Solution c:**

\( I_b = N = 6.50 \text{ in.} \)

From Solution a, web local yielding and web local crippling are o.k.

Try \( B = 8 \) in.

\[
A_1 = BN
= 8.00 \text{ in.}(6.50 \text{ in.})
= 52.0 \text{ in.}^2
\]

To determine the dimensions of the area \( A_2 \), the load is spread into the concrete until an edge or the maximum condition \( \sqrt{\frac{A_2}{A_1}} = 2 \) is met. There is also a requirement that the area, \( A_2 \), be geometrically similar to \( A_1 \) or, in other words, have the same aspect ratio as \( A_1 \).

\[
N_1 = 6.50 \text{ in.} + 2(1.75 \text{ in.})
= 10.0 \text{ in.}
\]

\[
\frac{B}{N} = \frac{8.00 \text{ in.}}{6.50 \text{ in.}}
= 1.23
\]

\[
B_1 = 1.23(10.0 \text{ in.})
= 12.3 \text{ in.}
\]

\[
A_2 = B_1N_1
= 12.3 \text{ in.}(10.0 \text{ in.})
= 123 \text{ in.}^2
\]

Check \( \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{123 \text{ in.}^2}{52.0 \text{ in.}^2}} \)

\[
= 1.54 \leq 2 \quad \text{o.k.}
\]

\[
P_b = 0.85f'c'A_1\sqrt{\frac{A_2}{A_1}} \leq 1.7f'c'A_1 \quad \text{(Spec. Eq. J8-2)}
\]

\[
= 0.85(3 \text{ ksi})(52.0 \text{ in.}^2)(1.54) \leq 1.7(3 \text{ ksi})(52.0 \text{ in.}^2)
\]

\[
= 204 \text{ kips} \leq 265 \text{ kips}
\]
\[ \phi_P = 0.65(204 \text{ kips}) = 133 \text{ kips} \]

133 kips > 90.0 kips \text{ o.k.}


\[ n = \frac{B - k}{2} \quad (\text{Manual Eq. 14-1}) \]

= \frac{8.00 \text{ in.}}{2} - 0.972 \text{ in.}

= 3.03 \text{ in.}

\[ t_{\text{min}} = \sqrt{\frac{2R_n n^2}{\phi_4F_y}} \]

= \sqrt{\frac{2(90.0 \text{ kips})(3.03 \text{ in.})^2}{0.90(6.50 \text{ in.})(8.00 \text{ in.})(36 \text{ ksi})}}

= 0.990 \text{ in.}

Use PL 1 in.\times 6\frac{1}{2} \text{ in.} \times 0 \text{ ft 8 in.}

Note: The calculations for \( t_{\text{min}} \) are conservative. Taking the strength of the beam flange into consideration results in a thinner required bearing plate or no bearing plate at all.
EXAMPLE II.D-3 SLIP-CRITICAL CONNECTION WITH OVERSIZED HOLES

Given:

Design the connection of an ASTM A36 2L3×3×½¢ tension member to an ASTM A36 plate welded to an ASTM A992 beam as shown in Figure II.D-3-1 for a dead load of 15 kips and a live load of 45 kips. The angles have standard holes and the plate has oversized holes per AISC Specification Table J3.3. Use ⅝-in.-diameter ASTM A325-SC bolts with Class A surfaces.

\[ P_D = 15 \text{ kips} \]
\[ P_L = 45 \text{ kips} \]

Solution:

From AISC Manual Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×26
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

Hanger
2L3×3×½¢
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)
Plate
ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W16×26
\( t_f = 0.345 \text{ in.} \)
\( t_w = 0.250 \text{ in.} \)
\( k_{des} = 0.747 \text{ in.} \)

Hanger
2L3×3×3⁄8
\( A = 3.56 \text{ in.}^2 \)
\( \bar{x} = 0.860 \text{ in.} \) for single angle

Plate
\( t_p = 0.500 \text{ in.} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate required strength.</td>
<td>Calculate required strength.</td>
</tr>
<tr>
<td>( R_u = (1.2)(15 \text{ kips}) + (1.6)(45 \text{ kips}) )</td>
<td>( R_u = 15 \text{ kips} + 45 \text{ kips} )</td>
</tr>
<tr>
<td>= 90.0 kips</td>
<td>= 60.0 kips</td>
</tr>
<tr>
<td>Check the available slip resistance of the bolts using AISC Manual Table 7-3.</td>
<td>Check the available slip resistance of the bolts using AISC Manual Table 7-3.</td>
</tr>
<tr>
<td>For ( \frac{3}{8} )-in.-diameter ASTM A325-SC bolts with Class A faying surfaces in oversized holes and double shear:</td>
<td>For ( \frac{3}{8} )-in.-diameter ASTM A325-SC bolts with Class A faying surfaces in oversized holes and double shear:</td>
</tr>
<tr>
<td>( \phi r_n = 16.1 \text{ kips/bolt} )</td>
<td>( r_n = 10.8 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>( n = \frac{R_u}{\phi r_n} = \frac{90.0 \text{ kips}}{16.1 \text{ kips/bolt}} )</td>
<td>( n = \frac{R_u}{(r_n / \bar{x})} = \frac{60.0 \text{ kips}}{10.8 \text{ kips/bolt}} )</td>
</tr>
<tr>
<td>= 5.59 ( \rightarrow ) 6 bolts</td>
<td>= 5.56 ( \rightarrow ) 6 bolts</td>
</tr>
<tr>
<td>Slip-critical connections must also be designed for the limit states of bearing-type connections. Check bolt shear strength using AISC Manual Table 7-1.</td>
<td>Slip-critical connections must also be designed for the limit states of bearing-type connections. Check bolt shear strength using AISC Manual Table 7-1.</td>
</tr>
<tr>
<td>( \phi R_n = \phi F_c A_b = 35.8 \text{ kips/bolt} )</td>
<td>( r_n = \frac{F_c A_b}{\bar{x}} = 23.9 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>( \frac{\phi R_n}{\Omega} = \frac{\phi r_n n}{\Omega} )</td>
<td>( \frac{r_n}{\Omega} = \frac{r_n n}{\Omega} )</td>
</tr>
<tr>
<td>= (35.8 kips/bolt)(6 bolts)</td>
<td>= (23.9 kips/bolt)(6 bolts)</td>
</tr>
<tr>
<td>= 215 kips &gt; 90.0 kips</td>
<td>= 143 kips &gt; 60.0 kips</td>
</tr>
</tbody>
</table>
**Tensile Yielding Strength of the Angles**

\[ P_t = F_y A_t \]
\[ = 36 \text{ ksi} \left( 3.56 \text{ in.}^2 \right) \]
\[ = 128 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega_e = 1.67 )</td>
</tr>
<tr>
<td>( \phi P_t = 0.90(128 \text{ kips}) )</td>
<td>( \frac{P_t}{\Omega} = \frac{128 \text{ kips}}{1.67} )</td>
</tr>
<tr>
<td>= 115 kips &gt; 90.0 kips</td>
<td>= 76.6 kips &gt; 60.0 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

**Tensile Rupture Strength of the Angles**

\[ U = 1 - \frac{3}{l} \text{ from AISC Specification Table D3.1 Case 2} \]
\[ = 1 - \frac{0.860 \text{ in.}}{15.0 \text{ in.}} \]
\[ = 0.943 \]

\[ A_e = A_u U \]
\[ = \left[ 3.56 \text{ in.}^2 - 2 \left( \frac{3}{16} \text{ in.} \right) \left( \frac{3}{16} \text{ in.} + \frac{3}{32} \text{ in.} \right) \right] (0.943) \]
\[ = 2.84 \text{ in.}^2 \]

\[ P_n = F_u A_e \]
\[ = 58 \text{ ksi}(2.84 \text{ in.}^2) \]
\[ = 165 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega_e = 2.00 )</td>
</tr>
<tr>
<td>( \phi P_n = 0.75(165 \text{ kips}) )</td>
<td>( \frac{P_n}{\Omega} = \frac{165 \text{ kips}}{2.00} )</td>
</tr>
<tr>
<td>= 124 kips &gt; 90.0 kips</td>
<td>= 82.5 kips &gt; 60.0 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

**Block Shear Rupture Strength of the Angles**

Use a single vertical row of bolts.

\[ U_{hs} = 1, \quad n = 6, \quad L_{cv} = 1\frac{1}{2} \text{ in.}, \quad \text{and} \quad L_{cb} = 1\frac{3}{4} \text{ in.} \]

\[ R_n = 0.60 F_u A_{nv} + U_{ha} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{ha} F_u A_{nt} \] (Spec. Eq. J4-5)

**Shear Yielding Component**

\[ A_{gv} = \left[ 5 \left( \frac{3.00 \text{ in.}}{16} \right) + 1.50 \text{ in.} \right] \left( \frac{3}{16} \text{ in.} \right) \]
\[ = 5.16 \text{ in.}^2 \text{ per angle} \]
0.60F_yA_{grv} = 0.60(36 \text{ ksi})(5.16 \text{ in.}^2)
= 111 \text{ kips per angle}

Shear Rupture Component

A_{sv} = 5.16 \text{ in.}^2 - 5.5\left(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}\right)\left(\frac{1}{16} \text{ in.}\right)
= 3.66 \text{ in.}^2 \text{ per angle}

0.60F_uA_{sv} = 0.60(58 \text{ ksi})(3.66 \text{ in.}^2)
= 127 \text{ kips per angle}

Shear yielding controls over shear rupture.

Tension Rupture Component

A_{nt} = \left[1.25 \text{ in.} - 0.5\left(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}\right)\right]\left(\frac{1}{16} \text{ in.}\right)
= 0.254 \text{ in.}^2 \text{ per angle}

U_{bt}F_uA_{nt} = 1.0(58 \text{ ksi})(0.254 \text{ in.}^2)
= 14.7 \text{ kips per angle}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.75)</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>(\phi R_n = 0.75\left(2\right)(111 \text{ kips} + 14.7 \text{ kips}))</td>
<td>(R_n = \frac{2(111 \text{ kips} + 14.7 \text{ kips})}{2.00})</td>
</tr>
<tr>
<td>= 189 \text{ kips} &gt; 90.0 \text{ kips}</td>
<td>= 126 \text{ kips} &gt; 60.0 \text{ kips}</td>
</tr>
</tbody>
</table>

Bearing / Tear Out Strength of the Angles

Holes are standard \(\frac{3}{16}\)-in. diameter.

Check strength for edge bolt.

\(l_c = 1.50 \text{ in.} - \frac{\frac{3}{8} \text{ in.} + \frac{1}{16} \text{ in.}}{2}\)
= 1.09 in.

\(r_n = 1.2l_c tF_u \leq 2.4dtF_u\)  
= 1.2(1.09 in.)(\frac{1}{16} \text{ in.})(2)(58 \text{ ksi}) \leq 2.4(\frac{3}{8} \text{ in.})(\frac{1}{16} \text{ in.})(2)(58 \text{ ksi})
= 47.4 \text{ kips} \leq 65.3 \text{ kips}

Check strength for interior bolts.

\(l_c = 3.00 \text{ in.} - (\frac{3}{4} \text{ in.} + \frac{1}{16} \text{ in.})\)
= 2.19 in.

\(r_n = 1.2l_c tF_u \leq 2.4dtF_u\)  
= 1.2(2.19 in.)(\frac{1}{16} \text{ in.})(2)(58 \text{ ksi}) \leq 2.4(\frac{3}{8} \text{ in.})(\frac{1}{16} \text{ in.})(2)(58 \text{ ksi})
= 71.1 \text{ kips} \leq 65.3 \text{ kips}
Total strength for all bolts.

\( r_n = 1(47.4 \text{ kips}) + 5(65.3 \text{ kips}) \)
\( = 374 \text{ kips} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \Phi r_n = 0.75(374 \text{ kips}) )</td>
<td>( r_n = 374 \text{ kips} )</td>
</tr>
<tr>
<td>( = 281 \text{ kips} &gt; 90.0 \text{ kips} )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>o.k.</td>
<td>( = 187 \text{ kips} &gt; 60.0 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Tensile Yielding Strength of the \( \frac{1}{2} \)-in. Plate**

By inspection, the Whitmore section includes the entire width of the \( \frac{1}{2} \)-in. plate.

\[ R_n = F_y A_t \]
\[ = 36 \text{ ksi}(\frac{1}{2} \text{ in.})(6.00 \text{ in.}) \]
\[ = 108 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \Phi R_n = 0.90(108 \text{ kips}) )</td>
<td>( R_n = 108 \text{ kips} )</td>
</tr>
<tr>
<td>( = 97.2 \text{ kips} &gt; 90.0 \text{ kips} )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>o.k.</td>
<td>( = 64.7 \text{ kips} &gt; 60.0 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Tensile Rupture Strength of the \( \frac{1}{2} \)-in. Plate**

Holes are oversized \( \frac{1}{16} \)-in. diameter.

Calculate the effective net area.

\[ A_e = A_n \leq 0.85 A_g \text{ from AISC Specification Section J4.1} \]
\[ \leq 0.85(3.00 \text{ in.}^2) \]
\[ \leq 2.55 \text{ in.}^2 \]

\[ A_n = 3.00 \text{ in.}^2 - (\frac{1}{2} \text{ in.})(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.}) \]
\[ = 2.50 \text{ in.}^2 \leq 2.55 \text{ in.}^2 \]

\[ A_e = A_n U \]
\[ = 2.50 \text{ in.}^2 (1.0) \]
\[ = 2.50 \text{ in.}^2 \]

\[ R_n = F_y A_e \]
\[ = 58 \text{ ksi}(2.50 \text{ in.}^2) \]
\[ = 145 \text{ kips} \]
### Block Shear Rupture Strength of the ½-in. Plate

Use a single vertical row of bolts.

\[ U_{bs} = 1.0, \quad n = 6, \quad L_{ev} = 1\frac{1}{2} \text{ in.}, \text{ and } L_{eb} = 3 \text{ in.} \]

\[ R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad (\text{Spec. Eq. J4-5}) \]

**Shear Yielding Component**

\[ A_{gv} = \left[ 5 \left( 3.00 \text{ in.} \right) + 1.50 \text{ in.} \right] \left( \frac{1}{2} \text{ in.} \right) = 8.25 \text{ in.}^2 \]

\[ 0.60F_y A_{gv} = 0.60 \left( 36 \text{ ksi} \right) \left( 8.25 \text{ in.}^2 \right) = 178 \text{ kips} \]

**Shear Rupture Component**

\[ A_{nv} = 8.25 \text{ in.}^2 - 5.5 \left( \frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.} \right) \left( \frac{1}{2} \text{ in.} \right) = 5.50 \text{ in.}^2 \]

\[ 0.60F_u A_{nv} = 0.60 \left( 58 \text{ ksi} \right) \left( 5.50 \text{ in.}^2 \right) = 191 \text{ kips} \]

Shear yielding controls over shear rupture.

**Tension Rupture Component**

\[ A_{nt} = \left[ 3.00 \text{ in.} - 0.5 \left( \frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.} \right) \right] \left( \frac{1}{2} \text{ in.} \right) = 1.25 \text{ in.}^2 \]

\[ U_{bs} F_u A_{nt} = 1.0 \left( 58 \text{ ksi} \right) \left( 1.25 \text{ in.}^2 \right) = 72.5 \text{ kips} \]
Bearing/Tear Out Strength of the ½-in. Plate

Holes are oversized ⅛-in. in diameter.

Check strength for edge bolt.

\[ l_c = 1.50 \text{ in.} - \frac{\frac{1}{8} \text{ in.}}{2} = 1.03 \text{ in.} \]

\[ r_n = 1.2l_c t F_u \leq 2.4dt F_u \]
\[ = 1.2(1.03 \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) \leq 2.4(\frac{3}{4} \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) \]
\[ = 35.8 \text{ kips} \leq 52.2 \text{ kips} \]

Check strength for interior bolts.

\[ l_c = 3.00 \text{ in.} - \frac{\frac{1}{8} \text{ in.}}{2} = 2.06 \text{ in.} \]

\[ r_n = 1.2l_c t F_u \leq 2.4dt F_u \]
\[ = 1.2(2.06 \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) \leq 2.4(\frac{3}{4} \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) \]
\[ = 71.7 \text{ kips} \leq 52.2 \text{ kips} \]

Total strength for all bolts.

\[ r_n = 1(35.8 \text{ kips}) + 5(52.2 \text{ kips}) = 297 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
</tbody>
</table>
| \( \phi r_n = 0.75(297 \text{ kips}) = 223 \text{ kips} > 90.0 \text{ kips} \) | o.k. | \( r_n = 297 \text{ kips} \) | \( \Omega = 2.00 \)
| | = 149 kips $>$ 60.0 kips | o.k. |

Fillet Weld Required for the ½-in. Plate to the W-Shape Beam

Because the angle of the force relative to the axis of the weld is 90°, the strength of the weld can be increased by the following factor from AISC Specification Section J2.4.

\[(1.0 + 0.50 \sin^{1.5} \theta) = (1.0 + 0.50 \sin^{1.5} 90°) = 1.50\]

From AISC Manual Equations 8-2,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{req} = \frac{R_a}{1.50(1.392/l)} )</td>
<td>( D_{req} = \frac{P_a}{1.50(0.928/l)} )</td>
</tr>
<tr>
<td>= 90.0 kips</td>
<td>= 60.0 kips</td>
</tr>
<tr>
<td>= 1.50(1.392)(2)(6.00 in.)</td>
<td>= 1.50(0.928)(2)(6.00 in.)</td>
</tr>
<tr>
<td>= 3.59 sixteenths</td>
<td>= 3.59 sixteenths</td>
</tr>
</tbody>
</table>
From AISC *Manual* Table J2.4, the minimum fillet weld size is $\frac{3}{8}$ in.

Use a $\frac{3}{8}$-in. fillet weld on both sides of the plate.

**Beam Flange Base Metal Check**

$$t_{\text{min}} = \frac{3.09D}{F_u}$$

$$= \frac{3.09(3.59 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.171 \text{ in.} < 0.345 \text{ in.} \quad \text{o.k.}$$

**Concentrated Forces Check for W16x26 Beam**

Check web local yielding. (Assume the connection is at a distance from the member end greater than the depth of the member, $d$.)

$$R_n = F_{yw}t_w(5k_{dex} + l_b)$$

$$= 50 \text{ ksi}(\frac{3}{8} \text{ in.})[5(0.747 \text{ in.}) + 6.00 \text{ in.}]$$

$$= 122 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = 1.00(122 \text{ kips})$</td>
<td>$R_n = \frac{122 \text{ kips}}{1.50}$</td>
</tr>
<tr>
<td>$= 122 \text{ kips} &gt; 90.0 \text{ kips}$</td>
<td>$= 81.3 \text{ kips} &gt; 60.0 \text{ kips}$</td>
</tr>
</tbody>
</table>
Part III
System Design Examples
EXAMPLE III-1  Design of Selected Members and Lateral Analysis of a Four-Story Building

INTRODUCTION

This section illustrates the load determination and selection of representative members that are part of the gravity and lateral frame of a typical four-story building. The design is completed in accordance with the 2010 AISC Specification for Structural Steel Buildings and the 14th Edition AISC Steel Construction Manual. Loading criteria are based on ASCE/SEI 7-10 (ASCE, 2010).

This section includes:
- Analysis and design of a typical steel frame for gravity loads
- Analysis and design of a typical steel frame for lateral loads
- Examples illustrating three methods for satisfying the stability provisions of AISC Specification Chapter C

The building being analyzed in this design example is located in a Midwestern city with moderate wind and seismic loads. The loads are given in the description of the design example. All members are ASTM A992 steel.

CONVENTIONS

The following conventions are used throughout this example:

1. Beams or columns that have similar, but not necessarily identical, loads are grouped together. This is done because such grouping is generally a more economical practice for design, fabrication and erection.

2. Certain calculations, such as design loads for snow drift, which might typically be determined using a spreadsheet or structural analysis program, are summarized and then incorporated into the analysis. This simplifying feature allows the design example to illustrate concepts relevant to the member selection process.

3. Two commonly used deflection calculations, for uniform loads, have been rearranged so that the conventional units in the problem can be directly inserted into the equation for steel design. They are as follows:

Simple Beam:  \[ \Delta = \frac{5}{384} \frac{w \text{kip/in.}(L \text{ in.})^4}{(29,000 \text{ ksi})(I \text{ in.}^4)} = \frac{w \text{kip/ft}(L \text{ ft})^4}{1,290(I \text{ in.}^4)} \]

Beam Fixed at both Ends:  \[ \Delta = \frac{w \text{kip/in.}(L \text{ in.})^4}{384(29,000 \text{ ksi})(I \text{ in.}^4)} = \frac{w \text{kip/ft}(L \text{ ft})^4}{6,440(I \text{ in.}^4)} \]
DESIGN SEQUENCE

The design sequence is presented as follows:

1. General description of the building including geometry, gravity loads and lateral loads
2. Roof member design and selection
3. Floor member design and selection
4. Column design and selection for gravity loads
5. Wind load determination
6. Seismic load determination
7. Horizontal force distribution to the lateral frames
8. Preliminary column selection for the moment frames and braced frames
9. Seismic load application to lateral systems
10. Stability (P-Δ) analysis
GENERAL DESCRIPTION OF THE BUILDING

Geometry

The design example is a four-story building, comprised of seven bays at 30 ft in the East-West (numbered grids) direction and bays of 45 ft, 30 ft and 45 ft in the North-South (lettered grids) direction. The floor-to-floor height for the four floors is 13 ft 6 in. and the height from the fourth floor to the roof (at the edge of the building) is 14 ft 6 in. Based on discussions with fabricators, the same column size will be used for the whole height of the building.

Basic Building Layout

The plans of these floors and the roof are shown on Sheets S2.1 thru S2.3, found at the end of this Chapter. The exterior of the building is a ribbon window system with brick spandrels supported and back-braced with steel and infilled with metal studs. The spandrel wall extends 2 ft above the elevation of the edge of the roof. The window and spandrel system is shown on design drawing Sheet S4.1.

The roof system is 1½-in. metal deck on bar joists. These bar joists are supported on steel beams as shown on Sheet S2.3. The roof slopes to interior drains. The middle 3 bays have a 6 ft tall screen wall around them and house the mechanical equipment and the elevator over run. This area has steel beams, in place of steel bar joists, to support the mechanical equipment.

The three elevated floors have 3 in. of normal weight concrete over 3-in. composite deck for a total slab thickness of 6 in. The supporting beams are spaced at 10 ft on center. These beams are carried by composite girders in the East-West direction to the columns. There is a 30 ft by 29 ft opening in the second floor, to create a two-story atrium at the entrance. These floor layouts are shown on Sheets S2.1 and S2.2. The first floor is a slab on grade and the foundation consists of conventional spread footings.

The building includes both moment frames and braced frames for lateral resistance. The lateral system in the North-South direction consists of chevron braces at the end of the building, located adjacent to the stairways. In
the East-West direction there are no locations in which chevron braces can be concealed; consequently, the lateral system in the East-West direction is composed of moment frames at the North and South faces of the building.

This building is sprinklered and has large open spaces around it, and consequently does not require fireproofing for the floors.

**Wind Forces**

The Basic Wind Speed is 90 miles per hour (3 second gust). Because it is sited in an open, rural area, it will be analyzed as Wind Exposure Category C. Because it is an ordinary (Risk Category II) office occupancy, the wind importance factor is 1.0.

**Seismic Forces**

The sub-soil has been evaluated and the site class has been determined to be Category D. The area has a short period $S_s = 0.121g$ and a one-second period $S_1 = 0.060g$. The seismic importance factor is 1.0, that of an ordinary office occupancy (Risk Category II).

**Roof and Floor Loads**

*Roof loads:*

The ground snow load ($p_g$) is 20 psf. The slope of the roof is $1/4$ in./ft or more at all locations, but not exceeding $1/2$ in./ft; consequently, 5 psf rain-on-snow surcharge is to be considered, but ponding instability design calculations are not required. This roof can be designed as a fully exposed roof, but, per ASCE/SEI 7 Section 7.3, cannot be designed for less than $p_f = (I)p_g = 20$ psf uniform snow load. Snow drift will be applied at the edges of the roof and at the screen wall around the mechanical area. The roof live load for this building is 20 psf, but may be reduced per ASCE/SEI 7 Section 4.8 where applicable.

*Floor Loads:*

The basic live load for the floor is 50 psf. An additional partition live load of 20 psf is specified. Because the locations of partitions and, consequently, corridors are not known, and will be subject to change, the entire floor will be designed for a live load of 80 psf. This live load will be reduced, based on type of member and area per the ASCE provisions for live-load reduction.

*Wall Loads:*

A wall load of 55 psf will be used for the brick spandrels, supporting steel, and metal stud back-up. A wall load of 15 psf will be used for the ribbon window glazing system.

**ROOF MEMBER DESIGN AND SELECTION**

Calculate dead load and snow load.

**Dead Load**

<table>
<thead>
<tr>
<th>Item</th>
<th>Load (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roofing</td>
<td>5</td>
</tr>
<tr>
<td>Insulation</td>
<td>2</td>
</tr>
<tr>
<td>Deck</td>
<td>2</td>
</tr>
<tr>
<td>Beams</td>
<td>3</td>
</tr>
<tr>
<td>Joists</td>
<td>3</td>
</tr>
<tr>
<td>Misc.</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>20</strong></td>
</tr>
</tbody>
</table>

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Snow Load from ASCE/SEI 7 Section 7.3 and 7.10

Snow = 20 psf
Rain on Snow = 5 psf
Total = 25 psf

Note: In this design, the rain and snow load is greater than the roof live load.

The deck is 1½ in., wide rib, 22 gage, painted roof deck, placed in a pattern of three continuous spans minimum. The typical joist spacing is 6 ft on center. At 6 ft on center, this deck has an allowable total load capacity of 89 psf. The roof diaphragm and roof loads extend 6 in. past the centerline of grid as shown on Sheet S4.1.

From Section 7.7 of ASCE/SEI 7, the following drift loads are calculated:

Flat roof snow load = 20 psf, Density $\gamma = 16.6 \text{ lbs/ft}^3$, $h_b = 1.20 \text{ ft}$

### Summary of Drifts

<table>
<thead>
<tr>
<th>Upwind Roof Length ($L_u$)</th>
<th>Proj. Height</th>
<th>Max. Drift Load</th>
<th>Max Drift Width ($W$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side Parapet 121 ft</td>
<td>2 ft</td>
<td>13.2 psf</td>
<td>6.36 ft</td>
</tr>
<tr>
<td>End Parapet 211 ft</td>
<td>2 ft</td>
<td>13.2 psf</td>
<td>6.36 ft</td>
</tr>
<tr>
<td>Screen Wall 60.5 ft</td>
<td>6 ft</td>
<td>30.5 psf</td>
<td>7.35 ft</td>
</tr>
</tbody>
</table>

The snow drift at the penthouse was calculated for the maximum effect, using the East-West wind and an upwind fetch from the parapet to the centerline of the columns at the penthouse. This same drift is conservatively used for wind in the North-South direction. The precise location of the drift will depend upon the details of the penthouse construction, but will not affect the final design in this case.

### SELECT ROOF JOISTS

Layout loads and size joists.

User Note: Joists may be specified using ASD or LRFD but are most commonly specified by ASD as shown here.

The 45-ft side joist with the heaviest loads is shown below along with its end reactions and maximum moment:

Because the load is not uniform, select a 24KCS4 joist from the Steel Joist Institute load tables (SJI, 2005). This joist has an allowable moment of 92.3 kip-ft, an allowable shear of 8.40 kips, a gross moment of inertia of 453 in.$^4$ and weighs 16.6 plf.

The first joist away from the end of the building is loaded with snow drift along the length of the member. Based on analysis, a 24KCS4 joist is also acceptable for this uniform load case.
As an alternative to directly specifying the joist sizes on the design document, as done in this example, loading diagrams can be included on the design documents to allow the joist manufacturer to economically design the joists.

The typical 30-ft joist in the middle bay will have a uniform load of

\[ w = (20 \text{ psf} + 25 \text{ psf})(6 \text{ ft}) = 270 \text{ plf} \]

\[ w_{SL} = (25 \text{ psf})(6 \text{ ft}) = 150 \text{ plf} \]

From the Steel Joist Institute load tables, select an 18K5 joist which weighs approximately 7.7 plf and satisfies both strength and deflection requirements.

Note: the first joist away from the screen wall and the first joist away from the end of the building carry snow drift. Based on analysis, an 18K7 joist will be used in these locations.
SELECT ROOF BEAMS

Calculate loads and select beams in the mechanical area.

For the beams in the mechanical area, the mechanical units could weigh as much as 60 psf. Use 40 psf additional dead load, which will account for the mechanical units and the screen wall around the mechanical area. Use 15 psf additional snow load, which will account for any snow drift which could occur in the mechanical area. The beams in the mechanical area are spaced at 6 ft on center.

Per AISC Design Guide 3 (West et al., 2003), calculate the minimum $I_x$ to limit deflection to $l/360 = 1.00$ in. because a plaster ceiling will be used in the lobby area. Use 40 psf as an estimate of the snow load, including some drifting that could occur in this area, for deflection calculations.

Note: The beams and supporting girders in this area should be rechecked when the final weights and locations for the mechanical units have been determined.

$$I_{req} = \frac{0.240 \, \text{kip/ft} \cdot (30.0 \, \text{ft})^4}{1,290 (1.00 \, \text{in.})} = 151 \, \text{in.}^4$$

Calculate the required strengths from Chapter 2 of ASCE/SEI 7 and select the beams in the mechanical area.

**LRFD**

- $w_a = 6.00 \, \text{ft} \left[ 1.2 \left( 0.020 \, \text{kip/ft}^2 + 0.040 \, \text{kip/ft}^2 \right) + 1.6 \left( 0.025 \, \text{kip/ft}^2 + 0.015 \, \text{kip/ft}^2 \right) \right] = 0.816 \, \text{kip/ft}$
- $R_a = \frac{30.0 \, \text{ft}}{2} \cdot (0.816 \, \text{kip/ft}) = 12.2 \, \text{kips}$
- $M_a = \frac{0.816 \, \text{kip/ft} \cdot (30.0 \, \text{ft})^2}{8} = 91.8 \, \text{kip-ft}$

Assuming the beam has full lateral support, use Manual Table 3-2, select an ASTM A992 W14×22, which has a design flexural strength of 125 kip-ft, a design shear strength of 94.5 kips, and an $I_x$ of 199 in.$^4$

**ASD**

- $w_a = 6.00 \, \text{ft} \left[ 0.020 \, \text{kip/ft}^2 + 0.040 \, \text{kip/ft}^2 + 0.025 \, \text{kip/ft}^2 + 0.015 \, \text{kip/ft}^2 \right] = 0.600 \, \text{kip/ft}$
- $R_a = \frac{30.0 \, \text{ft}}{2} \cdot (0.600 \, \text{kip/ft}) = 9.00 \, \text{kips}$
- $M_a = \frac{0.600 \, \text{kip/ft} \cdot (30.0 \, \text{ft})^2}{8} = 67.5 \, \text{kip-ft}$

Assuming the beam has full lateral support, use Manual Table 3-2, select an ASTM A992 W14×22, which has an allowable flexural strength of 82.8 kip-ft, an allowable shear strength of 63.0 kips and an $I_x$ of 199 in.$^4$
SELECT ROOF BEAMS AT THE END (EAST & WEST) OF THE BUILDING

The beams at the ends of the building carry the brick spandrel panel and a small portion of roof load. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to \( L/600 \) or \( \frac{a}{4} \) in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to \( L/600 \) or 0.3 in. max to accommodate the brick and \( L/360 \) or \( \frac{a}{4} \) in. max to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to \( L/600 \) or \( \frac{a}{4} \) in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. In calculating the wall loads, the spandrel panel weight is taken as 55 psf. The spandrel panel weight is approximately:

\[
\begin{align*}
wp & = 7.50 \text{ ft}(0.055 \text{ kip/ft}^2) \\
& = 0.413 \text{ kip/ft}
\end{align*}
\]

The dead load from the roof is equal to:

\[
\begin{align*}
w_D & = 3.50 \text{ ft}(0.020 \text{ kip/ft}^2) \\
& = 0.070 \text{ kip/ft}
\end{align*}
\]

Use 8 psf for the initial dead load.

\[
\begin{align*}
w_D^{\text{(initial)}} & = 3.50 \text{ ft}(0.008 \text{ kip/ft}^2) \\
& = 0.0280 \text{ kip/ft}
\end{align*}
\]

Use 12 psf for the superimposed dead load.

\[
\begin{align*}
w_D^{\text{(super)}} & = 3.50 \text{ ft}(0.012 \text{ kip/ft}^2) \\
& = 0.0420 \text{ kip/ft}
\end{align*}
\]

The snow load from the roof can be conservatively taken as:

\[
\begin{align*}
w_S & = 3.50 \text{ ft}(0.025 \text{ kip/ft}^2 + 0.0132 \text{ kip/ft}^2) \\
& = 0.134 \text{ kip/ft}
\end{align*}
\]

to account for the maximum snow drift as a uniform load.

Assume the beams are simple spans of 22.5 ft.

Calculate minimum \( I_x \) to limit the superimposed dead and live load deflection to \( \frac{a}{4} \) in.

\[
I_{req} = \frac{0.176 \text{ kip/ft}(22.5 \text{ ft})^4}{1,290(\frac{a}{4} \text{ in.})} = 140 \text{ in.}^4
\]

Calculate minimum \( I_x \) to limit the cladding and initial dead load deflection to \( \frac{a}{3} \) in.

\[
I_{req} = \frac{0.441 \text{ kip/ft}(22.5 \text{ ft})^4}{1,290(\frac{a}{3} \text{ in.})} = 234 \text{ in.}^4
\]

The beams are full supported by the deck as shown in Detail 4 on Sheet S4.1. The loading diagram is as follows:
Calculate the required strengths from Chapter 2 of ASCE/SEI 7 and select the beams for the roof ends.

\[ w_u = 0.413 + 0.070 = 0.483 \text{ kip/ft} \]
\[ w_a = 0.134 \text{ kip/ft} \]

![Beam Loading & Bracing Diagram](Diagram)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.2(0.070 \text{ kip/ft} + 0.413 \text{ kip/ft}) + 1.6(0.134 \text{ kip/ft}) )</td>
<td>( w_a = (0.070 \text{ kip/ft} + 0.413 \text{ kip/ft}) + 0.134 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( = 0.794 \text{ kip/ft} )</td>
<td>( = 0.617 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( R_u = \frac{22.5 \text{ ft}}{2}(0.794 \text{ kip/ft}) )</td>
<td>( R_a = \frac{22.5 \text{ ft}}{2}(0.617 \text{ kip/ft}) )</td>
</tr>
<tr>
<td>( = 8.93 \text{ kips} )</td>
<td>( = 6.94 \text{ kips} )</td>
</tr>
<tr>
<td>( M_u = \frac{0.794 \text{ kip/ft}(22.5 \text{ ft})^2}{8} )</td>
<td>( M_a = \frac{0.617 \text{ kip/ft}(22.5 \text{ ft})^2}{8} )</td>
</tr>
<tr>
<td>( = 50.2 \text{ kip-ft} )</td>
<td>( = 39.0 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Assuming the beam has full lateral support, use Manual Table 3-2, select an ASTM A992 W16x26, which has a design flexural strength of 166 kip-ft, a design shear strength of 106 kips, and an \( I_x \) of 301 in.\(^4\)
SELECT ROOF BEAMS ALONG THE SIDE (NORTH & SOUTH) OF THE BUILDING

The beams along the side of the building carry the spandrel panel and a substantial roof dead load and live load. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to \( L/600 \) or \( \frac{a}{6} \) in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to \( L/600 \) or 0.3 in. max to accommodate the brick and \( L/360 \) or \( \frac{4}{5} \) in. max to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to \( L/600 \) or \( \frac{4}{5} \) in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. These beams will be part of the moment frames on the side of the building and therefore will be designed as fixed at both ends. The roof dead load and snow load on this edge beam is equal to the joist end dead load and snow load reaction. Treating this as a uniform load, divide this by the joist spacing.

\[
\begin{align*}
w_D &= 2.76 \text{ kips/6.00 ft} \\
&= 0.460 \text{ kip/ft} \\
w_S &= 3.73 \text{ kips/6.00 ft} \\
&= 0.622 \text{ kip/ft} \\
w_{D(\text{initial})} &= 23.0 \text{ ft} (0.008 \text{ kip/ft}^2) \\
&= 0.184 \text{ kip/ft} \\
w_{D(\text{super})} &= 23.0 \text{ ft} (0.012 \text{ kip/ft}^2) \\
&= 0.276 \text{ kip/ft}
\end{align*}
\]

Calculate the minimum \( I_x \) to limit the superimposed dead and live load deflection to \( \frac{4}{5} \) in.

\[
I_{req} = \frac{(0.898 \text{ kip/ft})(30.0 \text{ ft})^4}{6,440(\frac{4}{5} \text{ in.})} = 452 \text{ in.}^4
\]

Calculate the minimum \( I_x \) to limit the cladding and initial dead load deflection to \( \frac{3}{5} \) in.

\[
I_{req} = \frac{(0.597 \text{ kip/ft})(30.0 \text{ ft})^4}{6,440(\frac{3}{5} \text{ in.})} = 200 \text{ in.}^4
\]
Calculate the required strengths from Chapter 2 of ASCE/SEI 7 and select the beams for the roof sides.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.2(0.460 \text{ kip/ft} + 0.413 \text{ kip/ft}) + 1.6(0.622 \text{ kip/ft}) )</td>
<td>( w_u = (0.460 \text{ kip/ft} + 0.413 \text{ kip/ft}) + 0.622 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( = 2.04 \text{ kip/ft} )</td>
<td>( = 1.50 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( R_u = \frac{30.0 \text{ ft}}{2}(2.04 \text{ kip/ft}) )</td>
<td>( R_u = \frac{30.0 \text{ ft}}{2}(1.50 \text{ kip/ft}) )</td>
</tr>
<tr>
<td>( = 30.6 \text{ kips} )</td>
<td>( = 22.5 \text{ kips} )</td>
</tr>
</tbody>
</table>

Calculate \( C_b \) for compression in the bottom flange braced at the midpoint and supports using AISC Specification Equation F1-1.

From AISC Manual Table 3-23,

\[
M_{uAM} = \frac{2.04 \text{ kip/ft}(30.0 \text{ ft})^2}{12} = 153 \text{ kip-ft at supports}
\]

\[
M_u = \frac{2.04 \text{ kip/ft}(30.0 \text{ ft})^2}{24} = 76.5 \text{ kip-ft at midpoint}
\]

\[
M_{uBM} = \frac{2.04 \text{ kip/ft}(7.50 \text{ ft})}{12} = 19.1 \text{ kip-ft at midpoint of unbraced segment}
\]

\[
M_{uCM} = \frac{2.04 \text{ kip/ft}(11.3 \text{ ft})}{12} = 62.5 \text{ kip-ft at three quarter point of unbraced segment}
\]

\[
M_{uAM} = \frac{1.50 \text{ kip/ft}(30.0 \text{ ft})^2}{12} = 113 \text{ kip-ft at supports}
\]

\[
M_u = \frac{1.50 \text{ kip/ft}(30.0 \text{ ft})^2}{24} = 56.3 \text{ kip-ft at midpoint}
\]

\[
M_{uBM} = \frac{1.50 \text{ kip/ft}(7.50 \text{ ft})}{12} = 14.1 \text{ kip-ft at midpoint of unbraced segment}
\]

\[
M_{uCM} = \frac{1.50 \text{ kip/ft}(11.3 \text{ ft})}{12} = 46.0 \text{ kip-ft at three quarter point of unbraced segment}
\]
Using AISC *Specification* Equation F1-1,  
\[
C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_d + 4M_b + 3M_C}
\]
\[
= \frac{12.5(153 \text{ kip-ft})}{2.5(153 \text{ kip-ft}) + 3(52.6 \text{ kip-ft}) + 4(19.1 \text{ kip-ft}) + 3(62.5 \text{ kip-ft})}
\]
\[
= 2.38
\]

From AISC *Manual* Table 3-10, select W18×35.

For \(L_b = 6 \text{ ft and } C_b = 1.0\)
\[
\phi_b M_n = 229 \text{ kip-ft} > 76.5 \text{ kip-ft} \quad \text{ o.k.}
\]

For \(L_b = 15 \text{ ft and } C_b = 2.38\),
\[
\phi_b M_n = 109 \text{ kip-ft}\times 2.38 = 259 \text{ kip-ft} \quad \phi_b M_p = 249 \text{ kip-ft} > 153 \text{ kip-ft} \quad \text{ o.k.}
\]

From AISC *Manual* Table 3-2, a W18×35 has a design shear strength of 159 kips and an \(I_x\) of 510 in.⁴ \quad \text{ o.k.}

\[\text{Note: This roof beam may need to be upsized during the lateral load analysis to increase the stiffness and strength of the member and improve lateral frame drift performance.}\]
SELECT THE ROOF BEAMS ALONG THE INTERIOR LINES OF THE BUILDING

There are three individual beam loadings that occur along grids C and D. The beams from 1 to 2 and 7 to 8 have a uniform snow load except for the snow drift at the end at the parapet. The snow drift from the far ends of the 45-ft joists is negligible. The beams from 2 to 3 and 6 to 7 are the same as the first group, except they have snow drift at the screen wall. The loading diagrams are shown below. A summary of the moments, left and right reactions, and required $I$, to keep the live load deflection to equal or less than the span divided by 240 (or 1.50 in.) is given below.

Calculate required strengths from Chapter 2 of ASCE/SEI 7 and required moment of inertia.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grids 1 to 2 and 7 to 8 (opposite hand)</td>
<td>$R_u$ (left) = 1.2(11.6 kips) + 1.6(16.0 kips) = 39.5 kips</td>
<td>$R_u$ (left) = 11.6 kips + 16.0 kips = 27.6 kips</td>
</tr>
<tr>
<td></td>
<td>$R_u$ (right) = 1.2(11.2 kips) + 1.6(14.2 kips) = 36.2 kips</td>
<td>$R_u$ (right) = 11.2 kips + 14.2 kips = 25.4 kips</td>
</tr>
<tr>
<td></td>
<td>$M_u$ = 1.2(84.3 kip-ft) + 1.6(107 kip-ft) = 272 kip-ft</td>
<td>$M_u$ = 84.3 kip-ft + 107 kip-ft = 191 kip-ft</td>
</tr>
</tbody>
</table>
III-15

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

\[
I_{x \text{ req'd}} = \frac{(0.938 \text{ kip/ft})(30.0 \text{ ft})^4}{1,290(1.50 \text{ in.})} = 393 \text{ in.}^4
\]

From AISC Manual Table 3-10, for \(L_b = 6 \text{ ft}\) and \(C_b = 1.0\), select \(W21 \times 44\) which has a design flexural strength of 332 kip-ft, a design shear strength of 217 kips, and \(I_x = 843 \text{ in.}^4\).

Grids 2 to 3 and 6 to 7(opposite hand)

\[
R_a \text{ (left)} = 1.2(11.3 \text{ kips}) + 1.6(14.4 \text{ kips}) = 36.6 \text{ kips}
\]

\[
R_a \text{ (right)} = 1.2(11.3 \text{ kips}) + 1.6(17.9 \text{ kips}) = 42.2 \text{ kips}
\]

\[
M_a = 1.2(84.4 \text{ kip-ft}) + 1.6(111 \text{ kip-ft}) = 279 \text{ kip-ft}
\]

\[
I_{x \text{ req'd}} = \frac{(0.938 \text{ kip/ft})(30.0 \text{ ft})^4}{1,290(1.50 \text{ in.})} = 393 \text{ in.}^4
\]

From AISC Manual Table 3-10, for \(L_b = 6 \text{ ft}\) and \(C_b = 1.0\), select \(W21 \times 44\) which has an allowable flexural strength of 221 kip-ft, an allowable shear strength of 145 kips, and \(I_x = 843 \text{ in.}^4\).

The third individual beam loading occurs at the beams from 3 to 4, 4 to 5, and 5 to 6. This is the heaviest load.
SELECT THE ROOF BEAMS ALONG THE SIDES OF THE MECHANICAL AREA

The beams from 3 to 4, 4 to 5, and 5 to 6 have a uniform snow load outside the screen walled area, except for the snow drift at the parapet ends and the screen wall ends of the 45-ft joists. Inside the screen walled area the beams support the mechanical equipment. A summary of the moments, left and right reactions, and required $I_x$ to keep the live load deflection to equal or less than the span divided by 240 (or 1.50 in.) is given below.

$$w_o = 1.35 \text{ kip/ft}$$
$$w_s = 1.27 \text{ kip/ft}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_u = 1.2 \times (1.35 \text{ kip/ft}) + 1.6 \times (1.27 \text{ kip/ft})$</td>
<td>$w_u = 1.35 \text{ kip/ft} + 1.27 \text{ kip/ft}^2$</td>
</tr>
<tr>
<td>$= 3.65 \text{ kip/ft}$</td>
<td>$= 2.62 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$M_u = \frac{3.65 \text{ kip/ft} \times (30.0 \text{ ft})^2}{8}$</td>
<td>$M_u = \frac{2.62 \text{ kip/ft} \times (30.0 \text{ ft})^2}{8}$</td>
</tr>
<tr>
<td>$= 411 \text{ kip-ft}$</td>
<td>$= 295 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$R_u = \frac{30.0 \text{ ft}}{2} \times (3.65 \text{ kip/ft})$</td>
<td>$R_u = \frac{30.0 \text{ ft}}{2} \times (2.62 \text{ kip/ft})$</td>
</tr>
<tr>
<td>$= 54.8 \text{ kips}$</td>
<td>$= 39.3 \text{ kips}$</td>
</tr>
<tr>
<td>$I_x \text{ req'd} = \frac{1.27 \text{ kip/ft} \times (30.0 \text{ ft})^4}{1,290(1.50 \text{ in.})}$</td>
<td>$I_x \text{ req'd} = \frac{1.27 \text{ kip/ft} \times (30.0 \text{ ft})^4}{1,290(1.50 \text{ in.})}$</td>
</tr>
<tr>
<td>$= 532 \text{ in.}^4$</td>
<td>$= 532 \text{ in.}^4$</td>
</tr>
</tbody>
</table>

From AISC Manual Table 3-2, for $L_b = 6 \text{ ft}$ and $C_b = 1.0$, select W21×55, which has a design flexural strength of 473 kip-ft, a design shear strength of 234 kips, and an $I_x$ of 1,140 in.$^4$.

From AISC Manual Table 3-2, for $L_b = 6 \text{ ft}$ and $C_b = 1.0$, select W21×55, which has an allowable flexural strength of 314 kip-ft, an allowable shear strength of 156 kips, and an $I_x$ of 1,140 in.$^4$. 

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
FLOOR MEMBER DESIGN AND SELECTION

Calculate dead load and live load.

Dead Load

<table>
<thead>
<tr>
<th>Component</th>
<th>Load (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab and Deck</td>
<td>57</td>
</tr>
<tr>
<td>Beams (est.)</td>
<td>8</td>
</tr>
<tr>
<td>Misc. (ceiling, mechanical, etc.)</td>
<td>10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>75</strong></td>
</tr>
</tbody>
</table>

Note: The weight of the floor slab and deck was obtained from the manufacturer’s literature.

Live Load

<table>
<thead>
<tr>
<th>Component</th>
<th>Load (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total (can be reduced for area per ASCE/SEI 7)</td>
<td>80</td>
</tr>
</tbody>
</table>

The floor and deck will be 3 in. of normal weight concrete, $f' = 4$ ksi, on 3-in. 20 gage, galvanized, composite deck, laid in a pattern of three or more continuous spans. The total depth of the slab is 6 in. The Steel Deck Institute maximum unshored span for construction with this deck and a three-span condition is 10 ft 11 in. The general layout for the floor beams is 10 ft on center; therefore, the deck does not need to be shored during construction. At 10 ft on center, this deck has an allowable superimposed live load capacity of 143 psf. In addition, it can be shown that this deck can carry a 2,000 pound load over an area of 2.5 ft by 2.5 ft as required by Section 4.4 of ASCE/SEI 7. The floor diaphragm and the floor loads extend 6 in. past the centerline of grid as shown on Sheet S4.1.
SELECT FLOOR BEAMS (composite and noncomposite)

Note: There are two early and important checks in the design of composite beams. First, select a beam that either does not require camber, or establish a target camber and moment of inertia at the start of the design process. A reasonable approximation of the camber is between $L/300$ minimum and $L/180$ maximum (or a maximum of 1½ to 2 in.).

Second, check that the beam is strong enough to safely carry the wet concrete and a 20 psf construction live load (per ASCE 37-05), when designed by the ASCE/SEI 7 load combinations and the provisions of Chapter F of the AISC Specification.

SELECT TYPICAL 45-FT INTERIOR COMPOSITE BEAM (10 FT ON CENTER)

Find a target moment of inertia for an unshored beam.

Hold deflection to around 2 in. maximum to facilitate concrete placement.

$$w_D = (0.057 \text{ kip/ft}^2 + 0.008 \text{ kip/ft}^2)(10.0 \text{ ft}) = 0.650 \text{ kip/ft}$$

$$I_{req} \approx \frac{0.650 \text{ kip/ft}(45.0 \text{ ft})^4}{1,290}(2.00 \text{ in.}) = 1,030 \text{ in.}^4$$

Determine the required strength to carry wet concrete and construction live load.

$$w_{DL} = 0.065 \text{ kip/ft}^2(10.0 \text{ ft}) = 0.650 \text{ kip/ft}$$

$$w_{LL} = 0.020 \text{ kip/ft}^2(10.0 \text{ ft}) = 0.200 \text{ kip/ft}$$

Determine the required flexural strength due to wet concrete only.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_U = 1.4(0.650 \text{ kip/ft})$</td>
<td>$w_U = 0.650 \text{ kip/ft}$</td>
</tr>
<tr>
<td>= 0.910 kip/ft</td>
<td></td>
</tr>
<tr>
<td>$M_U = \frac{0.910 \text{ kip/ft}(45.0 \text{ ft})^2}{8}$</td>
<td>$M_U = \frac{0.650 \text{ kip/ft}(45.0 \text{ ft})^2}{8}$ = 165 kip-ft</td>
</tr>
<tr>
<td>= 230 kip-ft</td>
<td></td>
</tr>
</tbody>
</table>

Determine the required flexural strength due to wet concrete and construction live load.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_U = 1.2(0.650 \text{ kip/ft}) + 1.6(0.200 \text{ kip/ft})$</td>
<td>$w_U = 0.650 \text{ kip/ft} + 0.200 \text{ kip/ft}$</td>
</tr>
<tr>
<td>= 1.10 kip/ft</td>
<td>= 0.850 kip/ft</td>
</tr>
<tr>
<td>$M_U = \frac{1.10 \text{ kip/ft}(45.0 \text{ ft})^2}{8}$</td>
<td>$M_U = \frac{0.850 \text{ kip/ft}(45.0 \text{ ft})^2}{8}$ controls</td>
</tr>
<tr>
<td>= 278 kip-ft</td>
<td>= 215 kip-ft controls</td>
</tr>
</tbody>
</table>

Use AISC Manual Table 3-2 to select a beam with $I_y \geq 1,030 \text{ in.}^4$. Select W21×50, which has $I_y = 984 \text{ in.}^4$, close to our target value, and has available flexural strengths of 413 kip-ft (LRFD) and 274 kip-ft (ASD).
Check for possible live load reduction due to area in accordance with Section 4.7.2 of ASCE/SEI 7.

For interior beams, $K_{LL} = 2$

The beams are at 10.0 ft on center, therefore the area $A_r = (45.0 \text{ ft})(10.0 \text{ ft}) = 450 \text{ ft}^2$.

Since $K_{LL} A_r = 2(450 \text{ ft}^2) = 900 \text{ ft}^2 > 400 \text{ ft}^2$, a reduced live load can be used.

From ASCE/SEI 7, Equation 4.7-1:

$$L = L_o \left(0.25 + \frac{15}{\sqrt{K_{LL} A_r}}\right)$$

$$= 80.0 \text{ psf} \left(0.25 + \frac{15}{\sqrt{900 \text{ ft}^2}}\right)$$

$$= 60.0 \text{ psf} \geq 0.50L_o = 40.0 \text{ psf}$$

Therefore, use 60.0 psf.

The beam is continuously braced by the deck.

The beams are at 10 ft on center, therefore the loading diagram is as shown below.

Calculate the required flexural strength from Chapter 2 of ASCE/SEI 7.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_0$ = 0.750 kip/ft</td>
<td>$w_o = 0.750 \text{ kip/ft} + 0.600 \text{ kip/ft}$</td>
<td></td>
</tr>
<tr>
<td>$w_1$ = 0.600 kip/ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_o = \frac{1.86 \text{ kip/ft}(45.0 \text{ ft})^2}{8}$</td>
<td>$M_o = \frac{1.35 \text{ kip/ft}(45.0 \text{ ft})^2}{8}$</td>
<td></td>
</tr>
<tr>
<td>= 471 kip-ft</td>
<td>= 342 kip-ft</td>
<td></td>
</tr>
</tbody>
</table>

Assume initially $a = 1.00$ in.

$Y_2 = Y_{con} - a / 2 = 6.00$ in. $- 1.00$ in. / 2 $= 5.50$ in.

Use AISC Manual Table 3-19 to check W21×50 selected above. Using required strengths of 471 kip-ft (LRFD) or 342 kip-ft (ASD) and a $Y_2$ value of 5.50 in.
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AMERICAN INSTITUTE OF STEEL CONSTRUCTION

Select W21×50 beam, where
PNA = Location 7 and ΣQn = 184 kips
ϕbMn = 598 kip-ft > 471 kip-ft o.k.

Select W21×50 beam, where
PNA = Location 7 and ΣQn = 184 kips
Mn/Ωn = 398 kip-ft > 342 kip-ft o.k.

Determine the effective width, beff.

Per Specification AISC Section I3.1a, the effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

1. One-eighth of the span of the beam, center-to-center of supports

\[
\frac{45.0 \text{ ft}}{8} \times 2 = 11.3 \text{ ft}
\]

2. One-half the distance to the centerline of the adjacent beam

\[
\frac{10.0 \text{ ft}}{2} \times 2 = 10.0 \text{ ft controls}
\]

3. The distance to the edge of the slab

Not applicable

Determine the height of the compression block, a.

\[
a = \frac{\sum Q_n}{0.85 f'_c b} = \frac{184 \text{ kips}}{0.85(4 \text{ ksi})(10.0 \text{ ft})(12 \text{ in./ft})} = 0.451 \text{ in.} < 1.00 \text{ in.} \quad \text{o.k.}
\]

Check the W21×50 end shear strength.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra = \frac{45.0 \text{ ft}}{2}(1.86 \text{ kip/ft})</td>
<td>Ra = \frac{45.0 \text{ ft}}{2}(1.35 \text{ kip/ft})</td>
</tr>
<tr>
<td>= 41.9 kips</td>
<td>= 30.4 kips</td>
</tr>
<tr>
<td>From AISC Manual Table 3-2,</td>
<td>From AISC Manual Table 3-2,</td>
</tr>
<tr>
<td>ϕaVa = 237 kips &gt; 41.9 kips o.k.</td>
<td>Va/Ωa = 158 kips &gt; 30.4 kips o.k.</td>
</tr>
</tbody>
</table>

Check live load deflection.

\[
\Delta_{LL} = \frac{l}{360} = \frac{45.0 \text{ ft}(12 \text{ in./ft})}{360} = 1.50 \text{ in.}
\]

For a W21×50, from AISC Manual Table 3-20,

\[\gamma_2 = 5.50 \text{ in.}\]
PNA Location 7  
$I_{LB} = 1,730$ in.$^4$

$$\Delta_{LL} = \frac{w_{LL}I^{4}}{1,290I_{LB}}$$

$$= \frac{0.600 \text{ kip/ft} (45.0 \text{ ft})^{4}}{1,290(1,730 \text{ in.}^4)}$$

$$= 1.10 \text{ in.} < 1.50 \text{ in.} \quad \text{o.k.}$$

Based on AISC Design Guide 3 (West, Fisher and Griffis, 2003) limit the live load deflection, using 50% of the (unreduced) design live load, to $L / 360$ with a maximum absolute value of 1.0 in. across the bay.

$$\Delta_{LL} = \frac{0.400 \text{ kip/ft} (45.0 \text{ ft})^{4}}{1,290(1,730 \text{ in.}^4)}$$

$$= 0.735 \text{ in.} < 1.00 \text{ in.} \quad \text{o.k.}$$

1.00 in. – 0.735 in. = 0.265 in.

Note: Limit the supporting girders to 0.265 in. deflection under the same load case at the connection point of the beam.

Determine the required number of shear stud connectors.

From AISC Manual Table 3-21, using perpendicular deck with one $\frac{3}{8}$-in.-diameter stud per rib in normal weight, 4 ksi concrete, in weak position; $Q_n = 17.2$ kips/stud.

$$\frac{\sum Q_s}{Q_n} = \frac{184 \text{ kips}}{17.2 \text{ kips/stud}} = 10.7 \text{ studs / side}$$

Therefore use 22 studs.

Based on AISC Design Guide 3, limit the wet concrete deflection in a bay to $L / 360$, not to exceed 1.00 in.

Camber the beam for 80% of the calculated wet deflection.

$$\Delta_{DL(\text{wet conc})} = \frac{0.650 \text{ kip/ft} (45.0 \text{ ft})^{4}}{1,290(984 \text{ in.}^4)}$$

$$= 2.10 \text{ in.}$$

Camber = 0.80(2.10 in.)

= 1.68 in.

Round the calculated value down to the nearest $\frac{1}{4}$ in; therefore, specify 1.50 in. of camber.

2.10 in. – 1.50 in. = 0.600 in.

1.00 in. – 0.600 in. = 0.400 in.

Note: Limit the supporting girders to 0.400 in. deflection under the same load combination at the connection point of the beam.
SELECT TYPICAL 30-FT INTERIOR COMPOSITE (OR NONCOMPOSITE) BEAM (10 FT ON CENTER)

Find a target moment of inertia for an unshored beam.

Hold deflection to around 1.50 in. maximum to facilitate concrete placement.

\[
I_{req} \approx \frac{0.650 \text{ kip/ft}(30.0 \text{ ft})^4}{1,290(1.50 \text{ in.})} = 272 \text{ in.}^4
\]

Determine the required strength to carry wet concrete and construction live load.

\[
w_{DL} = 0.065 \text{ kip/ft}^2(10.0 \text{ ft}) = 0.650 \text{ kip/ft}
\]

\[
w_{LL} = 0.020 \text{ kip/ft}^2(10.0 \text{ ft}) = 0.200 \text{ kip/ft}
\]

Determine the required flexural strength due to wet concrete only.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.4(0.650 \text{ kip/ft}) = 0.910 \text{ kip/ft} )</td>
<td>( w_u = 0.650 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( M_u = \frac{0.910 \text{ kip/ft}(30.0 \text{ ft})^2}{8} = 102 \text{ kip-ft} )</td>
<td>( M_u = \frac{0.650 \text{ kip/ft}(30.0 \text{ ft})^2}{8} = 73.1 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Determine the required flexural strength due to wet concrete and construction live load.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.2(0.650 \text{ kip/ft}) + 1.6(0.200 \text{ kip/ft}) = 1.10 \text{ kip/ft} )</td>
<td>( w_u = 0.650 \text{ kip/ft} + 0.200 \text{ kip/ft} = 0.850 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( M_u = \frac{1.10 \text{ kip/ft}(30.0 \text{ ft})^2}{8} = 124 \text{ kip-ft controls} )</td>
<td>( M_u = \frac{0.850 \text{ kip/ft}(30.0 \text{ ft})^2}{8} = 95.6 \text{ kip-ft controls} )</td>
</tr>
</tbody>
</table>

Use AISC Manual Table 3-2 to find a beam with an \( I_x \geq 272 \text{ in.}^4 \). Select W16×26, which has an \( I_x = 301 \text{ in.}^4 \) which exceeds our target value, and has available flexural strengths of 166 kip-ft (LRFD) and 110 kip-ft (ASD).

Check for possible live load reduction due to area in accordance with Section 4.7.2 of ASCE/SEI 7.

For interior beams, \( K_{LL} = 2 \).

The beams are at 10 ft on center, therefore the area \( A_T = 30.0 \text{ ft} \times 10.0 \text{ ft} = 300 \text{ ft}^2 \).

Since \( K_{LL}A_T = 2(300 \text{ ft}^2) = 600 \text{ ft}^2 > 400 \text{ ft}^2 \), a reduced live load can be used.

From ASCE/SEI 7, Equation 4.7-1:
\[ L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{lz} A_T}} \right) \]
\[ = 80.0 \text{ psf} \left( 0.25 + \frac{15}{\sqrt{600 \text{ ft}^2}} \right) \]
\[ = 69.0 \text{ psf} \geq 0.50 \, L_o = 40.0 \text{ psf} \]

Therefore, use 69.0 psf.

The beams are at 10 ft on center, therefore the loading diagram is as shown below.

\[ w_c = 0.750 \text{ kip/ft} \quad w_l = 0.690 \text{ kip/ft} \]

![Loading Diagram](image)

From Chapter 2 of ASCE/SEI 7, calculate the required strength.

\[
\begin{align*}
\text{LRFD} & \\
L_R & = 1.2(0.750 \text{ kip/ft}) + 1.6 (0.690 \text{ kip/ft}) \\
& = 2.00 \text{ kip/ft} \\
M_R & = \frac{2.00 \text{ kip/ft} \times (30.0 \text{ ft})^2}{8} \\
& = 225 \text{ kip-ft} \\
\text{ASD} & \\
L_A & = 0.750 \text{ kip/ft} + 0.690 \text{ kip/ft} \\
& = 1.44 \text{ kip/ft} \\
M_A & = \frac{1.44 \text{ kip/ft} \times (30.0 \text{ ft})^2}{8} \\
& = 162 \text{ kip-ft} \\
\end{align*}
\]

Assume initially \( a = 1.00 \)

\[ Y = 6.00 \text{ in.} - \frac{1.00 \text{ in.}}{2} \]
\[ = 5.50 \text{ in.} \]

Use AISC Manual Table 3-19 to check the W16\times26 selected above. Using required strengths of 225 kip-ft (LRFD) or 162 kip-ft (ASD) and a \( Y \) value of 5.50 in.

\[
\begin{align*}
\text{LRFD} & \\
\phi_b M_a = 248 \text{ kip-ft} > 225 \text{ kip-ft} & \text{o.k.} \\
\text{ASD} & \\
M_a / \Omega_a = 165 \text{ kip-ft} > 162 \text{ kip-ft} & \text{o.k.} \\
\end{align*}
\]

Determine the effective width, \( b_{eff} \).

From AISC Specification Section 13.1a, the effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

1. one-eighth of the span of the beam, center-to-center of supports
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

\[ \frac{30.0 \text{ ft}}{8} (2 \text{ sides}) = 7.50 \text{ ft} \quad \text{controls} \]

(2) one-half the distance to the centerline of the adjacent beam

\[ \frac{10.0 \text{ ft}}{2} (2 \text{ sides}) = 10.0 \text{ ft} \]

(3) the distance to the edge of the slab

Not applicable

Determine the height of the compression block, \( a \).

\[
\begin{align*}
\frac{96.0 \text{ kips}}{0.85 f_y} &= \frac{96.0 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})} \\
&= 0.314 \text{ in.} < 1.00 \text{ in.} \quad \text{o.k.}
\end{align*}
\]

Check the W16\( \times \)26 end shear strength.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = \frac{30.0 \text{ ft}}{2}(2.00 \text{ kip/ft}) )</td>
<td>( R_u = \frac{30.0 \text{ ft}}{2}(1.44 \text{ kip/ft}) )</td>
</tr>
<tr>
<td>= 30.0 kips</td>
<td>= 21.6 kips</td>
</tr>
</tbody>
</table>

From AISC Manual Table 3-2, \( \phi V_n = 106 \text{ kips} > 30.0 \text{ kips} \quad \text{o.k.} \)

From AISC Manual Table 3-2, \( V_n/\Omega = 70.5 \text{ kips} > 21.6 \text{ kips} \quad \text{o.k.} \)

Check live load deflection.

\[
\Delta_{LL} = \frac{l}{360} \\
= (30.0 \text{ ft})(12 \text{ in./ft})/360 \\
= 1.00 \text{ in.}
\]

For a W16\( \times \)26, from AISC Manual Table 3-20,

\( y_2 = 5.50 \text{ in.} \)

PNA Location 7

\( I_{LB} = 575 \text{ in.}^4 \)

\[
\Delta_{LL} = \frac{w_{LL}I^4}{1,290I_{LB}} \\
= \frac{0.690 \text{ kip/ft}(30.0 \text{ ft})^4}{1,290(575 \text{ in.}^4)} \\
= 0.753 \text{ in.} < 1.00 \text{ in.} \quad \text{o.k.}
\]
Based on AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to \( L/360 \) with a maximum absolute value of 1.0 in. across the bay.

\[
\Delta_{LL} = \frac{0.400 \text{ kip/ft} (30.0 \text{ ft})^4}{1,290 (575 \text{ in.}^4)}
\]

\[
= 0.437 \text{ in.} < 1.00 \text{ in.} \quad \text{o.k.}
\]

1.00 in. – 0.437 in. = 0.563 in.

Note: Limit the supporting girders to 0.563 in. deflection under the same load combination at the connection point of the beam.

Determine the required number of shear stud connectors.

From AISC Manual Table 3-21, using perpendicular deck with one \( \frac{3}{8} \)-in.-diameter stud per rib in normal weight, 4 ksi concrete, in the weak position; \( Q_o = 17.2 \) kips/stud

\[
\sum Q_o = \frac{96.0 \text{ kips}}{17.2 \text{ kips/stud}} = 5.58 \text{ studs/side}
\]

Use 12 studs

Note: Per AISC Specification Section I8.2d, there is a maximum spacing limit of 8(6 in.) = 4 ft not to exceed 36 in. between studs.

Therefore use 12 studs, uniformly spaced at no more than 36 in. on center.

Note: Although the studs may be placed up to 36 in. o.c. the steel deck must still be anchored to the supporting member at a spacing not to exceed 18 in. per AISC Specification Section I3.2c.

Based on AISC Design Guide 3, limit the wet concrete deflection in a bay to \( L/360 \), not to exceed 1.00 in.

Camber the beam for 80% of the calculated wet dead load deflection.

\[
\Delta_{DL(\text{wet conc})} = \frac{0.650 \text{ kip/ft} (30.0 \text{ ft})^4}{1,290 (301 \text{ in.}^4)}
\]

\[
= 1.36 \text{ in.}
\]

Camber = 0.800(1.36 in.)

\[
= 1.09 \text{ in.}
\]

Round the calculated value down to the nearest \( \frac{1}{4} \) in. Therefore, specify 1.00 in. of camber.

1.36 in. – 1.00 in. = 0.360 in.

1.00 in. – 0.360 in. = 0.640 in.

Note: Limit the supporting girders to 0.640 in. deflection under the same load combination at the connection point of the beam.

This beam could also be designed as a noncomposite beam. Use AISC Manual Table 3-2 with previous moments and shears.
Check beam deflections.

Check live load deflection of the W18×35 with an $I_x = 510 \text{ in.}^4$, from AISC Manual Table 3-2.

$$\Delta_{LL} = \frac{0.690 \text{ kip/ft} \ (30.0 \text{ ft})^4}{1,290 \ (510 \text{ in.}^4)}$$

$$= 0.850 \text{ in.} < 1.00 \text{ in.} \quad \text{o.k.}$$

Based on AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to $L/360$ with a maximum absolute value of 1.0 in. across the bay.

$$\Delta_{LL} = \frac{0.400 \text{ kip/ft} \ (30.0 \text{ ft})^4}{1,290 \ (510 \text{ in.}^4)}$$

$$= 0.492 \text{ in.} < 1.00 \text{ in.} \quad \text{o.k.}$$

1.00 in. – 0.492 in. = 0.508 in.

Note: Limit the supporting girders to 0.508 in. deflection under the same load combination at the connection point of the beam.

Note: Because this beam is stronger than the W16×26 composite beam, no wet concrete strength checks are required in this example.

Based on AISC Design Guide 3, limit the wet concrete deflection in a bay to $L/360$, not to exceed 1.00 in.

Camber the beam for 80% of the calculated wet deflection.

$$\Delta_{DE(wet\ conc)} = \frac{0.650 \text{ kip/ft} \ (30.0 \text{ ft})^4}{1,290 \ (510 \text{ in.}^4)}$$

$$= 0.800 \text{ in.} < 1.50 \text{ in.} \quad \text{o.k.}$$

Camber = $0.800(0.800 \text{ in.}) = 0.640 \text{ in.} < 0.750 \text{ in.}$

A good break point to eliminate camber is ¾ in.; therefore, do not specify a camber for this beam.

1.00 in. – 0.800 in. = 0.200 in.

Note: Limit the supporting girders to 0.200 in. deflection under the same load case at the connection point of the beam.
Therefore, selecting a W18×35 will eliminate both shear studs and cambering. The cost of the extra steel weight may be offset by the elimination of studs and cambering. Local labor and material costs should be checked to make this determination.
SELECT TYPICAL NORTH-SOUTH EDGE BEAM

The influence area \( (K_{LLA}) \) for these beams is less than 400 ft\(^2\), therefore no live load reduction can be taken.

These beams carry 5.50 ft of dead load and live load as well as a wall load.

The floor dead load is:

\[
w = 5.50 \text{ ft}(0.075 \text{ kips/ft}^2) = 0.413 \text{ kip/ft}
\]

Use 65 psf for the initial dead load.

\[
w_{D(initial)} = 5.50 \text{ ft}(0.065 \text{ kips/ft}^2) = 0.358 \text{ kips/ft}
\]

Use 10 psf for the superimposed dead load.

\[
w_{D(super)} = 5.50 \text{ ft}(0.010 \text{ kips/ft}^2) = 0.055 \text{ kips/ft}
\]

The dead load of the wall system at the floor is:

\[
w = 7.50 \text{ ft}(0.055 \text{ kip/ft}^2) + 6.00 \text{ ft}(0.015 \text{ kip/ft}^2)
\]
\[
= 0.413 \text{ kip/ft} + 0.090 \text{ kip/ft}
\]
\[
= 0.503 \text{ kip/ft}
\]

The total dead load is \( w_{DL} = 0.413 \text{ kip/ft} + 0.503 \text{ kip/ft} 
\]
\[
= 0.916 \text{ kip/ft}
\]

The live load is \( w_{LL} = 5.5 \text{ ft}(0.080 \text{ kip/ft}^2) 
\]
\[
= 0.440 \text{ kip/ft}
\]

The loading diagram is as follows.

Calculate the required strengths from Chapter 2 of ASCE/SEI 7.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.2(0.916 \text{ kip/ft}) + 1.6(0.440 \text{ kip/ft}) )</td>
<td>( w_u = 0.916 \text{ kip/ft} + 0.440 \text{ kip/ft} )</td>
</tr>
<tr>
<td>= 1.80 kip/ft</td>
<td>= 1.36 kip/ft</td>
</tr>
<tr>
<td>( M_u = \frac{1.80 \text{ kip/ft}(22.5 \text{ ft})^2}{8} )</td>
<td>( M_u = \frac{1.36 \text{ kip/ft}(22.5 \text{ ft})^2}{8} )</td>
</tr>
</tbody>
</table>
Because these beams are less than 25 ft long, they will be most efficient as noncomposite beams. The beams at the edges of the building carry a brick spandrel panel. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to \( L/600 \) or \( \frac{1}{2} \text{ in.} \) maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to \( L/600 \) or 0.3 in. max to accommodate the brick and \( L/360 \) or 0.25 in. max to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to \( L/600 \) or 0.25 in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. Note that it is typically not recommended to camber beams supporting spandrel panels.

Calculate minimum \( I_x \) to limit the superimposed dead and live load deflection to \( \frac{1}{4} \text{ in.} \)

\[
I_{req} = \frac{0.495 \text{ kip/ft}(22.5 \text{ ft})^4}{1,290(\frac{1}{4} \text{ in.})} = 393 \text{ in.}^4 \text{ controls}
\]

Calculate minimum \( I_x \) to limit the cladding and initial dead load deflection to \( \frac{3}{8} \text{ in.} \)

\[
I_{req} = \frac{0.861 \text{ kip/ft}(22.5 \text{ ft})^4}{1,290(\frac{3}{8} \text{ in.})} = 456 \text{ in.}^4
\]

Select beam from AISC Manual Table 3-2.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select W18×35 with ( I_x = 510 \text{ in.}^4 )</td>
<td>Select W18×35 with ( I_x = 510 \text{ in.}^4 )</td>
<td></td>
</tr>
<tr>
<td>( \phi \mu M_a = \phi \mu M_p )</td>
<td>( M_a / \Omega_a = M_p / \Omega_p )</td>
<td></td>
</tr>
<tr>
<td>= 249 kip-ft &gt; 114 kip-ft</td>
<td>= 166 kip-ft &gt; 86.1 kip-ft</td>
<td></td>
</tr>
<tr>
<td>( \phi V_a = 159 &gt; 20.3 \text{ kips} )</td>
<td>( V_a / \Omega_v = 106 \text{ kips} &gt; 15.3 \text{ kips} )</td>
<td></td>
</tr>
</tbody>
</table>
SELECT TYPICAL EAST-WEST SIDE GIRDER

The beams along the sides of the building carry the spandrel panel and glass, and dead load and live load from the intermediate floor beams. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to $L/600$ or $\frac{1}{6}$ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to $L/600$ or 0.3 in. max to accommodate the brick and $L/360$ or 0.25 in. max to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to $L/600$ or 0.25 in max. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. These beams will be part of the moment frames on the North and South sides of the building and therefore will be designed as fixed at both ends.

Establish the loading.

The dead load reaction from the floor beams is:

\[
P_D = 0.750 \text{ kip/ft}(45.0 \text{ ft} / 2) = 16.9 \text{ kips}
\]

\[
P_{D(initial)} = 0.650 \text{ kip/ft}(45.0 \text{ ft} / 2) = 14.6 \text{ kips}
\]

\[
P_{D(super)} = 0.100 \text{ kip/ft}(45.0 \text{ ft} / 2) = 2.25 \text{ kips}
\]

The uniform dead load along the beam is:

\[
w_D = 0.500 \text{ ft}(0.075 \text{ kip/ft}^2) + 0.503 \text{ kip/ft} = 0.541 \text{ kip/ft}
\]

\[
w_{D(initial)} = 0.500 \text{ ft}(0.065 \text{ kip/ft}^2) = 0.033 \text{ kip/ft}
\]

\[
w_{D(super)} = 0.500 \text{ ft}(0.010 \text{ kip/ft}^2) = 0.005 \text{ kip/ft}
\]

Select typical 30-ft composite (or noncomposite) girders.

Check for possible live load reduction in accordance with Section 4.7.2 of ASCE/SEI 7.

For edge beams with cantilevered slabs, $K_{LL} = 1$, per ASCE/SEI 7, Table 4-2. However, it is also permissible to calculate the value of $K_{LL}$ based upon influence area. Because the cantilever dimension is small, $K_{LL}$ will be closer to 2 than 1. The calculated value of $K_{LL}$ based upon the influence area is

\[
K_{LL} = \frac{(45.5 \text{ ft})(30.0 \text{ ft})}{45.0 \text{ ft}^2 + 0.500 \text{ ft}^2}(30.0 \text{ ft}) = 1.98
\]

The area $A_T = (30.0 \text{ ft})(22.5 \text{ ft} + 0.500 \text{ ft}) = 690 \text{ ft}^2$

Using Equation 4.7-1 of ASCE/SEI 7
\[
L = L_0 \left( 0.25 + \frac{15}{\sqrt{K_{LL} A_t}} \right)
\]
\[
= 80.0 \text{ psf} \left( 0.25 + \frac{15}{\sqrt{(1.98)(690 \text{ ft}^2)}} \right)
\]
\[
= 52.5 \text{ psf} \geq 0.50L_0 = 40.0 \text{ psf}
\]

Therefore, use 52.5 psf.

The live load from the floor beams is \( P_{LL} = 0.525 \text{ kip/ft}(45.0 \text{ ft} / 2) \)
\[
= 11.8 \text{ kips}
\]

The uniform live load along the beam is \( w_{LL} = 0.500 \text{ ft}(0.0525 \text{ kip/ft}^2) \)
\[
= 0.026 \text{ kip/ft}
\]

The loading diagram is shown below.

A summary of the moments, reactions and required moments of inertia, determined from a structural analysis of a fixed-end beam, is as follows:

Calculate the required strengths and select the beams for the floor side beams.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical side beam</td>
<td>Typical side beam</td>
</tr>
<tr>
<td>( R_u = 49.5 \text{ kips} )</td>
<td>( R_u = 37.2 \text{ kips} )</td>
</tr>
<tr>
<td>( M_u \text{ at ends} = 313 \text{ kip-ft} )</td>
<td>( M_u \text{ at ends} = 234 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( M_u \text{ at ctr.} = 156 \text{ kip-ft} )</td>
<td>( M_u \text{ at ctr.} = 117 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

The maximum moment occurs at the support with compression in the bottom flange. The bottom laterally braced at 10 ft o.c. by the intermediate beams.

Note: During concrete placement, because the deck is parallel to the beam, the beam will not have continuous lateral support. It will be braced at 10 ft o.c. by the intermediate beams. By inspection, this condition will not control because the maximum moment under full loading causes compression in the bottom flange, which is braced at 10 ft o.c.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate ( C_h ) = for compression in the bottom flange braced at 10 ft o.c.</td>
<td>Calculate ( C_h ) = for compression in the bottom flange braced at 10 ft o.c.</td>
</tr>
<tr>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>------</td>
<td>-----</td>
</tr>
<tr>
<td>( C_b = 2.21 ) (from computer output)</td>
<td>( C_b = 2.22 ) (from computer output)</td>
</tr>
<tr>
<td>Select ( W_{21} \times 44 )</td>
<td>Select ( W_{21} \times 44 )</td>
</tr>
<tr>
<td>With continuous bracing, from AISC Manual Table 3-2,</td>
<td>With continuous bracing, from AISC Manual Table 3-2,</td>
</tr>
<tr>
<td>( \phi_b M_a = \phi_b M_p )</td>
<td>( M_a / \Omega_b = M_p / \Omega_b )</td>
</tr>
<tr>
<td>( = 358 \text{ kip-ft} &gt; 156 \text{ kip-ft} ) \text{ o.k.}</td>
<td>( = 238 \text{ kip-ft} &gt; 117 \text{ kip-ft} ) \text{ o.k.}</td>
</tr>
<tr>
<td>For ( L_b = 10 \text{ ft} ) and ( C_b = 2.21 ), from AISC Manual Table 3-10,</td>
<td>For ( L_b = 10 \text{ ft} ) and ( C_b = 2.22 ), from AISC Manual Table 3-10,</td>
</tr>
<tr>
<td>( \phi M_a = (265 \text{ kip-ft})(2.21) )</td>
<td>( M_a / \Omega = (176 \text{ kip-ft})(2.22) )</td>
</tr>
<tr>
<td>( = 586 \text{ kip-ft} )</td>
<td>( = 391 \text{ kip-ft} )</td>
</tr>
<tr>
<td>According to AISC Specification Section F2.2, the nominal flexural strength is limited ( M_p ), therefore ( \phi_b M_a = \phi_b M_p \approx 358 \text{ kip-ft.} )</td>
<td>According to AISC Specification Section F2.2, the nominal flexural strength is limited ( M_p ), therefore ( M_a / \Omega_b = M_p / \Omega_b = 238 \text{ kip-ft.} )</td>
</tr>
<tr>
<td>( 358 \text{ kip-ft} &gt; 313 \text{ kip-ft} ) \text{ o.k.}</td>
<td>( 238 \text{ kip-ft} &gt; 234 \text{ kip-ft} ) \text{ o.k.}</td>
</tr>
<tr>
<td>From AISC Manual Table 3-2, a ( W_{21} \times 44 ) has a design shear strength of 217 kips. From Table 1-1, ( I_s = 843 \text{ in.}^4 )</td>
<td>From AISC Manual Table 3-2, a ( W_{21} \times 44 ) has an allowable shear strength of 145 kips. From Table 1-1, ( I_s = 843 \text{ in.}^4 )</td>
</tr>
<tr>
<td>Check deflection due to cladding and initial dead load.</td>
<td>Check deflection due to cladding and initial dead load.</td>
</tr>
<tr>
<td>( \Delta = 0.295 \text{ in.} &lt; \frac{3}{8} \text{ in.} ) \text{ o.k.}</td>
<td>( \Delta = 0.295 \text{ in.} &lt; \frac{3}{8} \text{ in.} ) \text{ o.k.}</td>
</tr>
<tr>
<td>Check deflection due to superimposed dead and live loads.</td>
<td>Check deflection due to superimposed dead and live loads.</td>
</tr>
<tr>
<td>( \Delta = 0.212 \text{ in.} &lt; 0.250 \text{ in.} ) \text{ o.k.}</td>
<td>( \Delta = 0.212 \text{ in.} &lt; 0.250 \text{ in.} ) \text{ o.k.}</td>
</tr>
</tbody>
</table>

Note that both of the deflection criteria stated previously for the girder and for the locations on the girder where the floor beams are supported have also been met.

Also noted previously, it is not typically recommended to camber beams supporting spandrel panels. The \( W_{21} \times 44 \) is adequate for strength and deflection, but may be increased in size to help with moment frame strength or drift control.
SELECT TYPICAL EAST-WEST INTERIOR GIRDER

Establish loads

The dead load reaction from the floor beams is

\[ P_{DL} = 0.750 \text{ kip/ft}(45.0 \text{ ft} + 30.0 \text{ ft})/2 \]
\[ = 28.1 \text{ kips} \]

Check for live load reduction due to area in accordance with Section 4.7.2 of ASCE/SEI 7.

For interior beams, \( K_{LL} = 2 \)

The area \( A_T = (30.0 \text{ ft})(37.5 \text{ ft}) = 1,130 \text{ ft}^2 \)

Using ASCE/SEI 7, Equation 4.7-1:

\[
L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL}A_T}} \right)
\]
\[
= 80.0 \text{ psf} \left( 0.25 + \frac{15}{\sqrt{2}(1,130 \text{ ft}^2)} \right)
\]
\[
= 45.2 \text{ psf} \geq 0.50 L_o = 40.0 \text{ psf}
\]

Therefore, use 45.2 psf.

The live load from the floor beams is \( P_{LL} = 0.0452 \text{ kip/ft}^2(10.0 \text{ ft})(37.5 \text{ ft}) \)
\[ = 17.0 \text{ kips} \]

\[ P_o = 28.1 \text{ kips} \]
\[ P_L = 17.0 \text{ kips} \]

Note: The dead load for this beam is included in the assumed overall dead load.

A summary of the simple moments and reactions is shown below:

A summary of the simple moments and reactions is shown below:

Calculate the required strengths and select the size for the interior beams.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical interior beam</td>
<td>Typical interior beam</td>
</tr>
<tr>
<td>( R_o = 60.9 \text{ kips} )</td>
<td>( R_o = 45.1 \text{ kips} )</td>
</tr>
<tr>
<td>( M_o = 609 \text{ kip-ft} )</td>
<td>( M_o = 451 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Check for beam requirements when carrying wet concrete.
Note: During concrete placement, because the deck is parallel to the beam, the beam will not have continuous lateral support. It will be braced at 10 ft on center by the intermediate beams. Also, during concrete placement, a construction live load of 20 psf will be present. This load pattern and a summary of the moments, reactions, and deflection requirements is shown below. Limit wet concrete deflection to 1.5 in.

![Beam Loading & Bracing Diagram](image)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical interior beam with wet concrete only</td>
<td>Typical interior beam with wet concrete only</td>
</tr>
<tr>
<td>$R_u = 34.2$ kips</td>
<td>$R_u = 24.4$ kips</td>
</tr>
<tr>
<td>$M_u = 342$ kip-ft</td>
<td>$M_u = 244$ kip-ft</td>
</tr>
</tbody>
</table>

Assume $I_x \geq 935 \text{ in.}^4$, where $935 \text{ in.}^4$ is determined based on a wet concrete deflection of 1.5 in.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical interior beam with wet concrete and construction load</td>
<td>Typical interior beam with wet concrete and construction load</td>
</tr>
<tr>
<td>$R_u = 41.3$ kips</td>
<td>$R_u = 31.9$ kips</td>
</tr>
<tr>
<td>$M_u$ (midspan) $= 413$ kip-ft</td>
<td>$M_u$ (midspan) $= 319$ kip-ft</td>
</tr>
</tbody>
</table>

Select a beam with an unbraced length of 10.0 ft and a conservative $C_b = 1.0$.

From AISC Manual Tables 3-2 and 3-10, select a W21×68, which has a design flexural strength of 532 kip-ft, a design shear strength of 272 kips, and from Table 1-1, an $I_x$ of 1,480 in.⁴

$\phi \cdot M_p = 532$ kip-ft $> 413$ kip-ft \hspace{1cm} \text{o.k.}$   

Check W21×68 as a composite beam.

From previous calculations:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical interior Beam</td>
<td>Typical interior beam</td>
</tr>
<tr>
<td>$R_u = 60.9$ kips</td>
<td>$R_u = 45.1$ kips</td>
</tr>
<tr>
<td>$M_u$ (midspan) $= 609$ kip-ft</td>
<td>$M_u$ (midspan) $= 451$ kip-ft</td>
</tr>
</tbody>
</table>

$Y_2$ (from previous calculations, assuming an initial $a = 1.00$ in.) $= 5.50$ in.

Using AISC Manual Table 3-19, check a W21×68, using required flexural strengths of 609 kip-ft (LRFD) and 451 kip-ft (ASD) and $Y_2$ value of 5.5 in.
Select a W21×68
At PNA Location 7, \( \sum Q_x = 250 \) kips
\( \phi_b M_a = 844 \text{ kip-ft} > 609 \text{ kip-ft} \quad \text{o.k.} \)
\( M_a / \Omega_b = 561 \text{ kip-ft} > 461 \text{ kip-ft} \quad \text{o.k.} \)

Based on AISC Design Guide 3, limit the wet concrete deflection in a bay to \( L/360 \), not to exceed 1.00 in.

Camber the beam for 80% of the calculated wet deflection.

\[
\Delta_{DL,\text{wet conc}} = \frac{24.4 \text{ kips} (30.0 \text{ ft})^{3/2} (12 \text{ in./ft})^{3/2}}{28 (29,000 \text{ ksi}) (1,480 \text{ in.}^{4})} = 0.947 \text{ in.}
\]

Camber = 0.80(0.947 in.) = 0.758 in.

Round the calculated value down to the nearest \( \frac{1}{4} \) in. Therefore, specify \( \frac{1}{4} \) in. of camber.

0.947 in. – \( \frac{1}{4} \) in. = 0.197 in. < 0.200 in.

Therefore, the total deflection limit of 1.00 in. for the bay has been met.

Determine the effective width, \( b_{\text{eff}} \).

Per AISC Specification Section I3.1a, the effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

(1) one-eighth of the span of the beam, center-to-center of supports

\[
\frac{30.0 \text{ ft}}{8} (2 \text{ sides}) = 7.50 \text{ ft} \quad \text{controls}
\]

(2) one-half the distance to the centerline of the adjacent beam

\[
\left( \frac{45.0 \text{ ft}}{2} + \frac{30.0 \text{ ft}}{2} \right) = 37.5 \text{ ft}
\]

(3) the distance to the edge of the slab

Not applicable.

Determine the height of the compression block.

\[
a = \frac{\sum Q_x}{0.85 f_c' b} = \frac{250 \text{ kips}}{0.85 (4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})} = 0.817 \text{ in.} < 1.00 \text{ in.} \quad \text{o.k.}
\]
Check end shear strength.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 60.9\text{ kips}$</td>
<td>$R_u = 45.1\text{ kips}$</td>
</tr>
<tr>
<td>From AISC <em>Manual</em> Table 3-2, $\phi V_u = 272\text{ kips} &gt; 60.9\text{ kips}$</td>
<td>From AISC <em>Manual</em> Table 3-2, $V_n / \Omega_f = 181\text{ kips} &gt; 45.1\text{ kips}$</td>
</tr>
</tbody>
</table>

Check live load deflection.

\[ \Delta_{LL} = \frac{l}{360} = \frac{(30.0\text{ ft})(12\text{ in.}/\text{ft})}{360} = 1.00\text{ in.} \]

From AISC *Manual* Table 3-20,

\[ W_{21\times68}: y_2 = 5.50\text{ in.}, \text{PNA Location 7} \]

\[ I_{LB} = 2,510\text{ in.}^4 \]

\[ \Delta_{LL} = \frac{P l^3}{28 E I_{LB}} = \frac{17.0\text{ kips}(30.0\text{ ft})^3(12\text{ in./ft})^3}{28(29,000\text{ ksi})(2,510\text{ in.}^4)} = 0.389\text{ in.} < 1.00\text{ in.} \quad \text{o.k.} \]

Based on AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to $L/360$ with a maximum absolute value of 1.00 in. across the bay.

The maximum deflection is,

\[ \Delta_{LL} = \frac{15.0\text{ kips}(30.0\text{ ft})^3(12\text{ in./ft})^3}{28(29,000\text{ ksi})(2,510\text{ in.}^4)} = 0.343\text{ in.} < 1.00\text{ in.} \quad \text{o.k.} \]

Check the deflection at the location where the floor beams are supported.

\[ \Delta_{LL} = \frac{15.0\text{ kips}(120\text{ in.})}{6(29,000\text{ ksi})(2,510\text{ in.}^4)} \left[ 3(360\text{ in.})(120\text{ in.}) - 4(120\text{ in.})^2 \right] = 0.297\text{ in.} > 0.265\text{ in.}$  \quad \text{o.k.} \]

Therefore, the total deflection in the bay is $0.297\text{ in.} + 0.735\text{ in.} = 1.03\text{ in.}$, which is acceptably close to the limit of 1.00 in, where $\Delta_{LL} = 0.735\text{ in.}$ is from the 45 ft interior composite beam running north-south.

Determine the required shear stud connectors.

Using *Manual* Table 3-21, for parallel deck with, $w_i / h_i > 1.5$, one $\frac{3}{8}\text{-in.}$-diameter stud in normal weight, 4-ksi concrete and $Q_n = 21.5\text{ kips/stud}$.

\[ \frac{\sum Q_n}{Q_n} = \frac{250\text{ kips}}{21.5\text{ kips/stud}} = 11.6\text{ studs/side} \]
Therefore, use a minimum 24 studs for horizontal shear.

Per AISC Specification Section 18.2d, the maximum stud spacing is 36 in.

Since the load is concentrated at \( \frac{1}{3} \) points, the studs are to be arranged as follows:

Use 12 studs between supports and supported beams at \( \frac{1}{3} \) points. Between supported beams (middle \( \frac{1}{3} \) of span), use 4 studs to satisfy minimum spacing requirements.

Thus, 28 studs are required in a 12:4:12 arrangement.

Notes: Although the studs may be placed up to 3'-0" o.c. the steel deck must still be anchored to be the supporting member at a spacing not to exceed 18 in. in accordance with AISC Specification Section 13.2c.

This W21×68 beam, with full lateral support, is very close to having sufficient available strength to support the imposed loads without composite action. A larger noncomposite beam might be a better solution.
COLUMN DESIGN AND SELECTION FOR GRAVITY COLUMNS

Estimate column loads

**Roof**  (from previous calculations)
- **Dead Load**: 20 psf
- **Live (Snow)**: 25 psf
- **Total**: 45 psf

Snow drift loads at the perimeter of the roof and at the mechanical screen wall from previous calculations

Reaction to column (side parapet):
\[
 w = \frac{3.73 \text{ kips}}{6.00 \text{ ft}} - (0.025 \text{ ksf})(23.0 \text{ ft}) = 0.0467 \text{ kip/ft}
\]

Reaction to column (end parapet):
\[
 w = \frac{16.0 \text{ kips}}{37.5 \text{ ft}} - (0.025 \text{ ksf})(15.5 \text{ ft}) = 0.0392 \text{ kip/ft}
\]

Reaction to column (screen wall along lines C & D):
\[
 w = \frac{4.02 \text{ kips}}{6.00 \text{ ft}} - (0.025 \text{ ksf})(22.5 \text{ ft}) = 0.108 \text{ kip/ft}
\]

Mechanical equipment and screen wall (average):
\[
 w = 40 \text{ psf}
\]
<table>
<thead>
<tr>
<th>Column</th>
<th>Loading Width</th>
<th>Loading Length</th>
<th>Area</th>
<th>DL</th>
<th>$P_D$</th>
<th>SL</th>
<th>$P_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A, 2F, 3A, 3F, 4A, 4F</td>
<td>23.0</td>
<td>30.0</td>
<td>690</td>
<td>0.020</td>
<td>13.8</td>
<td>0.025</td>
<td>17.3</td>
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<tr>
<td>5A, 5F, 6A, 6F, 7A, 7F</td>
<td></td>
<td></td>
<td></td>
<td>0.0467 klf</td>
<td>1.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>snow drifting side</td>
<td>30.0</td>
<td>0.413 klf</td>
<td></td>
<td>12.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exterior wall</td>
<td>30.0</td>
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<tr>
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<td>26.2</td>
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<td></td>
<td>18.7</td>
<td></td>
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</tr>
<tr>
<td>1B, 1E, 8B, 8E</td>
<td>3.50</td>
<td>22.5</td>
<td>78.8</td>
<td>0.020</td>
<td>1.58</td>
<td>0.025</td>
<td>1.97</td>
</tr>
<tr>
<td>snow drifting end</td>
<td>22.5</td>
<td></td>
<td></td>
<td>0.0418 klf</td>
<td>0.941</td>
<td></td>
<td></td>
</tr>
<tr>
<td>exterior wall</td>
<td>22.5</td>
<td>0.413 klf</td>
<td></td>
<td>9.29</td>
<td></td>
<td></td>
<td></td>
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<td>10.9</td>
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<td></td>
<td></td>
<td>2.91</td>
<td></td>
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<tr>
<td>1A, 1F, 8A, 8F</td>
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<td>357</td>
<td>0.020</td>
<td>6.36</td>
<td>0.025</td>
<td>7.95</td>
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<td>(78.8 ft³) / 2</td>
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<td>snow drifting end</td>
<td>11.8</td>
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<td></td>
<td>0.0418 klf</td>
<td>0.493</td>
<td></td>
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<tr>
<td>snow drifting side</td>
<td>15.5</td>
<td>0.0467 klf</td>
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<td>0.941</td>
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<tr>
<td>exterior wall</td>
<td>27.3</td>
<td>0.413 klf</td>
<td></td>
<td>11.3</td>
<td></td>
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<td></td>
<td>9.17</td>
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</tr>
<tr>
<td>1C, 1D, 8C, 8D</td>
<td>37.5</td>
<td>15.5</td>
<td>581</td>
<td>0.020</td>
<td>10.8</td>
<td>0.025</td>
<td>13.6</td>
</tr>
<tr>
<td>(78.8 ft³) / 2</td>
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<td></td>
<td>542</td>
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</tr>
<tr>
<td>snow-drifting end</td>
<td>26.3</td>
<td></td>
<td></td>
<td>0.0418 klf</td>
<td>1.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>exterior wall</td>
<td>26.3</td>
<td>0.413 klf</td>
<td></td>
<td>10.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td>21.7</td>
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<td>14.7</td>
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<td></td>
</tr>
<tr>
<td>2C, 2D, 7C, 7D</td>
<td>37.5</td>
<td>30.0</td>
<td>1,125</td>
<td>0.020</td>
<td>22.5</td>
<td>0.025</td>
<td>28.1</td>
</tr>
<tr>
<td>3C, 3D, 4C, 4D</td>
<td>22.5</td>
<td>30.0</td>
<td>675</td>
<td>0.020</td>
<td>13.5</td>
<td>0.025</td>
<td>16.9</td>
</tr>
<tr>
<td>5C, 5D, 6C, 6D</td>
<td></td>
<td></td>
<td></td>
<td>0.108 klf</td>
<td>3.24</td>
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</tr>
<tr>
<td>snow-drifting</td>
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</tr>
<tr>
<td>mechanical area</td>
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<td>0.108 klf</td>
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<tr>
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<td>0.040 klf</td>
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<td>18.0</td>
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<td></td>
<td>40.5</td>
<td>0.381</td>
<td></td>
<td>38.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Floor Loads (from previous calculations)
Dead load 75 psf
Live load 80 psf
Total load 155 psf

Calculate reduction in live loads, analyzed at the base of three floors using Section 4.7.2 of ASCE/SEI 7.

Note: The 6-in. cantilever of the floor slab has been ignored for the calculation of $K_{LL}$ for columns in this building because it has a negligible effect.

Columns: 2A, 2F, 3A, 3F, 4A, 4F, 5A, 5F, 6A, 6F, 7A, 7F
Exterior column without cantilever slabs
$K_{LL} = 4$
$L_o = 80.0$ psf
$n = 3$

$A_T = (23.0 \text{ ft})(30.0 \text{ ft})$
   $= 690 \text{ ft}^2$

Using ASCE/SEI 7 Equation 4.7-1

\[
L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right)
\]

\[
= 80.0 \text{ psf} \left( 0.25 + \frac{15}{\sqrt{(4)(3)(690 \text{ ft}^2)}} \right)
\]

\[
= 33.2 \text{ psf} \geq 0.4L_o = 32.0 \text{ psf}
\]

Use $L = 33.2$ psf.

Columns: 1B, 1E, 8B, 8E
Exterior column without cantilever slabs
$K_{LL} = 4$
$L_o = 80.0$ psf
$n = 3$

$A_T = (5.50 \text{ ft})(22.5 \text{ ft})$
   $= 124 \text{ ft}^2$

\[
L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right)
\]

\[
= 80.0 \text{ psf} \left( 0.25 + \frac{15}{\sqrt{(4)(3)(124 \text{ ft}^2)}} \right)
\]

\[
= 51.1 \text{ psf} \geq 0.4L_o = 32.0 \text{ psf}
\]

Use $L = 51.1$ psf.

Columns: 1A, 1F, 8A, 8F
Corner column without cantilever slabs
$K_{LL} = 4$
$L_o = 80.0$ psf
$n = 3$
\( A_T = (15.5 \text{ ft})(23.0 \text{ ft}) - (124 \text{ ft}^2 / 2) \)

\[ = 295 \text{ ft}^2 \]

\[ L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right) \geq 0.4 L_o \]

\[ = 80.0 \text{ psf} \left( 0.25 + \frac{15}{\sqrt{(4)(3)(295 \text{ ft}^2)}} \right) \geq 0.4(80.0 \text{ psf}) \]

\[ = 40.2 \text{ psf} \geq 32.0 \text{ psf} \]

Use \( L = 40.2 \text{ psf} \).

Columns: 1C, 1D, 8C, 8D
Exterior column without cantilever slabs
\( K_{LL} = 4 \)
\( L_o = 80.0 \text{ psf} \)
\( n = 3 \)

\( A_T = (15.5 \text{ ft})(37.5 \text{ ft}) - (124 \text{ ft}^2 / 2) \)

\[ = 519 \text{ ft}^2 \]

\[ L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right) \geq 0.4 L_o \]

\[ = 80.0 \text{ psf} \left( 0.25 + \frac{15}{\sqrt{(4)(3)(519 \text{ ft}^2)}} \right) \geq 0.4(80.0 \text{ psf}) \]

\[ = 35.2 \text{ psf} \geq 32.0 \text{ psf} \]

Use \( L = 35.2 \text{ psf} \).

Columns: 2C, 2D, 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D, 7C, 7D
Interior column
\( K_{LL} = 4 \)
\( L_o = 80.0 \text{ psf} \)
\( n = 3 \)

\( A_T = (37.5 \text{ ft})(30.0 \text{ ft}) \)

\[ = 1,125 \text{ ft}^2 \]

\[ L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right) \geq 0.4 L_o \]

\[ = 80.0 \text{ psf} \left( 0.25 + \frac{15}{\sqrt{(4)(3)(1,125 \text{ ft}^2)}} \right) \geq 0.4(80.0 \text{ psf}) \]

\[ = 30.3 \text{ psf} \leq 32.0 \text{ psf} \]

Use \( L = 32.0 \text{ psf} \).
<table>
<thead>
<tr>
<th>Column</th>
<th>Loading Width (ft)</th>
<th>Loading Length (ft)</th>
<th>Tributary Area (ft²)</th>
<th>DL (kip/ft²)</th>
<th>$P_D$ (kip)</th>
<th>LL (kip/ft²)</th>
<th>$P_L$ (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A, 2F, 3A, 3F, 4A, 4F (exterior wall)</td>
<td>23.0</td>
<td>30.0</td>
<td>690</td>
<td>0.075</td>
<td>51.8</td>
<td>0.0332</td>
<td>22.9</td>
</tr>
<tr>
<td></td>
<td>30.0</td>
<td></td>
<td></td>
<td>0.503 klf</td>
<td>15.1</td>
<td></td>
<td>66.9</td>
</tr>
<tr>
<td>5A, 5F, 6A, 6F, 7A, 7F (exterior wall)</td>
<td>30.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B, 1E, 8B, 8E (exterior wall)</td>
<td>5.50</td>
<td>22.5</td>
<td>124</td>
<td>0.075</td>
<td>9.30</td>
<td>0.0511</td>
<td>6.34</td>
</tr>
<tr>
<td></td>
<td>22.5</td>
<td></td>
<td></td>
<td>0.503 klf</td>
<td>11.3</td>
<td></td>
<td>20.6</td>
</tr>
<tr>
<td>1A, 1F, 8A, 8F (exterior wall)</td>
<td>23.0</td>
<td>15.5</td>
<td>357</td>
<td>0.075</td>
<td>22.1</td>
<td>0.0402</td>
<td>11.9</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1C, 1D, 8C, 8D (exterior wall)</td>
<td>37.5</td>
<td>15.5</td>
<td>581</td>
<td>0.075</td>
<td>38.9</td>
<td>0.0352</td>
<td>18.3</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2C, 2D, 3C, 3D, 4C, 4D (5C, 5D, 6C, 6D, 7C, 7D)</td>
<td>37.5</td>
<td>30.0</td>
<td>1,125</td>
<td>0.075</td>
<td>84.4</td>
<td>0.0320</td>
<td>36.0</td>
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</tbody>
</table>
## Column load summary

<table>
<thead>
<tr>
<th>Column</th>
<th>Floor</th>
<th>$P_D$</th>
<th>$P_L$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>kips</td>
<td>kips</td>
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<tr>
<td>2A, 2F, 3A, 3F, 4A, 4F</td>
<td>Roof</td>
<td>26.2</td>
<td>18.7</td>
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<tr>
<td>5A, 5F, 6A, 6F, 7A, 7F</td>
<td>4th</td>
<td>66</td>
<td>.9</td>
</tr>
<tr>
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<td>3rd</td>
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<tr>
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<td>66</td>
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<td>Total</td>
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<td>87.4</td>
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<td>1B, 1E, 8B, 8E</td>
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<td>.9</td>
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<td>4th</td>
<td>20</td>
<td>.6</td>
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<tr>
<td></td>
<td>3rd</td>
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</tr>
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<td>Total</td>
<td>72.7</td>
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<td>1A, 1F, 8A, 8F</td>
<td>Roof</td>
<td>17</td>
<td>.7</td>
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<td>4th</td>
<td>35</td>
<td>.8</td>
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<td>3rd</td>
<td>35.8</td>
<td>11.9</td>
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<td>Total</td>
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<td>1C, 1D, 8C, 8D</td>
<td>Roof</td>
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<td>3rd</td>
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<td>18.3</td>
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<td>Total</td>
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<td>2C, 2D, 7C, 7D</td>
<td>Roof</td>
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<td></td>
<td>4th</td>
<td>84</td>
<td>.4</td>
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<td>3rd</td>
<td>84.4</td>
<td>36.0</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>84</td>
<td>.4</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>276</td>
<td>136</td>
</tr>
<tr>
<td>3C, 3D, 4C, 4D</td>
<td>Roof</td>
<td>40.5</td>
<td>38.1</td>
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<tr>
<td></td>
<td>4th</td>
<td>84</td>
<td>.4</td>
</tr>
<tr>
<td></td>
<td>3rd</td>
<td>84.4</td>
<td>36.0</td>
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<tr>
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<td>2nd</td>
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<td>.4</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>294</td>
<td>146</td>
</tr>
</tbody>
</table>
SELECT TYPICAL INTERIOR LEANING COLUMNS

Columns: 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100 ft
Column unbraced length: $K_x L_x = K_y L_y = 13.5$ ft

From ASCE/SEI 7, determine the required strength,

$$P_u = 1.2(294 \text{ kips}) + 1.6(3)(36.0 \text{ kips}) + 0.5(38.1 \text{ kips}) = 545 \text{ kips}$$

$$P_a = 294 \text{ kips} + 0.75(3)(36.0 \text{ kips}) + 0.75(38.1 \text{ kips}) = 404 \text{ kips}$$

Using AISC Manual Table 4-1, enter with the effective length of 13.5 ft, and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{12\times65}$</td>
<td>$W_{12\times65}$</td>
</tr>
<tr>
<td>$\phi P_u = 696 \text{ kips} &gt; 545 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{P_u}{\Omega_c} = 463 \text{ kips} &gt; 404 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$W_{14\times68}$</td>
<td>$W_{14\times68}$</td>
</tr>
<tr>
<td>$\phi P_u = 656 \text{ kips} &gt; 545 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{P_u}{\Omega_c} = 436 \text{ kips} &gt; 404 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Columns: 2C, 2D, 7C, 7D

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100.0 ft
Column unbraced length: $K_x L_x = K_y L_y = 13.5$ ft

$$P_u = 1.2(276 \text{ kips}) + 1.6(3)(36.0 \text{ kips}) + 0.5(28.1 \text{ kips}) = 518 \text{ kips}$$

$$P_a = 276 \text{ kips} + 0.75(3)(36.0 \text{ kips}) + 0.75(28.1 \text{ kips}) = 378 \text{ kips}$$

Using AISC Manual Table 4-1, enter with the effective length of 13.5 ft, and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{12\times65}$</td>
<td>$W_{12\times65}$</td>
</tr>
<tr>
<td>$\phi P_u = 696 \text{ kips} &gt; 518 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{P_u}{\Omega_c} = 463 \text{ kips} &gt; 378 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$W_{14\times61}$</td>
<td>$W_{14\times61}$</td>
</tr>
<tr>
<td>$\phi P_u = 585 \text{ kips} &gt; 518 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{P_u}{\Omega_c} = 389 \text{ kips} &gt; 378 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
SELECT TYPICAL EXTERIOR LEANING COLUMNS

Columns: 1B, 1E, 8B, 8E

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100.0 ft
Column unbraced length: \( K_L_e = K_L_e = 13.5 \) ft

\[
P_u = 1.2(72.7 \text{ kips}) + 1.6(3)(6.34 \text{ kips}) + 0.5(2.91 \text{ kips}) = 119 \text{ kips}
\]

\[
Pa = 72.7 \text{ kips} + 0.75(3)(6.34 \text{ kips}) + 0.75(2.91 \text{ kips}) = 89.1 \text{ kips}
\]

Using AISC Manual Table 4-1, enter with the effective length of 13.5 ft, and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u )</td>
<td>( P_u )</td>
</tr>
<tr>
<td>119 kips</td>
<td>89.1 kips</td>
</tr>
</tbody>
</table>

\[
\phi P_u = 316 \text{ kips} > 119 \text{ kips} \quad \text{o.k.}
\]

\[
P_u / \Omega = 210 \text{ kips} > 89.1 \text{ kips} \quad \text{o.k.}
\]

Note: A 12 in. column was selected above for ease of erection of framing beams.
(Bolted double-angle connections can be used without bolt staggering.)
WIND LOAD DETERMINATION

Use the Envelope Procedure for simple diaphragm buildings from ASCE/SEI 7, Chapter 28, Part 2.

To qualify for the simplified wind load method for low-rise buildings, per ASCE/SEI 7, Section 26.2, the following must be true.

1. Simple diaphragm building  o.k.
2. Low-rise building <= 60 ft  o.k.
3. Enclosed  o.k.
4. Regular-shaped  o.k.
5. Not a flexible building  o.k.
6. Does not have response characteristics requiring special considerations  o.k.
7. Symmetrical shape  o.k.
8. Torsional load cases from ASCE/SEI 7, Figure 28.4-1 do not control design of MWFRS  o.k.

Define input parameters

1. Risk category:  II  from ASCE/SEI 7, Table 1.5-1
2. Basic wind speed  $V$:  115 mph (3-s) from ASCE/SEI 7, Figure 26.5-1A
3. Exposure category:  C  from ASCE/SEI 7, Section 26.7.3
4. Topographic factor, $K_{zt}$:  1.0 from ASCE/SEI 7, Section 26.8.2
5. Mean roof height:  55' - 0"
6. Height and exposure adjustment, $\lambda$:  1.59 from ASCE/SEI 7, Figure 28.6-1
7. Roof angle:  0°

$$p_r = \lambda K_{zt} p_{30}$$  
$$= (1.59)(1.0)(21.0 \text{ psf}) = 33.4 \text{ psf} \quad \text{Horizontal pressure zone A}$$  
$$= (1.59)(1.0)(13.9 \text{ psf}) = 22.1 \text{ psf} \quad \text{Horizontal pressure zone C}$$  
$$= (1.59)(1.0)(-25.2 \text{ psf}) = -40.1 \text{ psf} \quad \text{Vertical pressure zone E}$$  
$$= (1.59)(1.0)(-14.3 \text{ psf}) = -22.7 \text{ psf} \quad \text{Vertical pressure zone F}$$  
$$= (1.59)(1.0)(-17.5 \text{ psf}) = -27.8 \text{ psf} \quad \text{Vertical pressure zone G}$$  
$$= (1.59)(1.0)(-11.1 \text{ psf}) = -17.6 \text{ psf} \quad \text{Vertical pressure zone H}$$

$$a = 10\% \text{ of least horizontal dimension or } 0.4h, \text{ whichever is smaller, but not less than either } 4\% \text{ of least horizontal dimension or } 3 \text{ ft}$$

$$a = \text{the lesser of: }$$
$$10\% \text{ of least horizontal dimension} \quad 12.3 \text{ ft}$$
$$40\% \text{ of eave height} \quad 22 \text{ ft}$$

but not less than

 $$4\% \text{ of least horizontal dimension or } 3 \text{ ft} \quad 4.92 \text{ ft}$$

$$a = 12.3 \text{ ft}$$
$$2a = 24.6 \text{ ft}$$

Zone A – End zone of wall (width = 2a)
Zone C – Interior zone of wall
Zone E – End zone of windward roof (width = 2a)
Zone F – End zone of leeward roof (width = 2a)
Zone G – Interior zone of windward roof  
Zone H – Interior zone of leeward roof  

Calculate load to roof diaphragm

Mechanical screen wall height: 6 ft  
Wall height: $\frac{1}{2}[55.0 \text{ ft} - 3(13.5 \text{ ft})] = 7.25 \text{ ft}$  
Parapet wall height: 2 ft  
Total wall height at roof at screen wall: $6 \text{ ft} + 7.25 \text{ ft} = 13.3 \text{ ft}$  
Total wall height at roof at parapet: $2 \text{ ft} + 7.25 \text{ ft} = 9.25 \text{ ft}$  

Calculate load to fourth floor diaphragm

Wall height:  
$\frac{1}{2}(55.0 \text{ ft} - 40.5 \text{ ft}) = 7.25 \text{ ft}$  
$\frac{1}{2}(40.5 \text{ ft} - 27.0 \text{ ft}) = 6.75 \text{ ft}$  
Total wall height at floor: $6.75 \text{ ft} + 7.25 \text{ ft} = 14.0 \text{ ft}$  

Calculate load to third floor diaphragm

Wall height:  
$\frac{1}{2}(40.5 \text{ ft} - 27.0 \text{ ft}) = 6.75 \text{ ft}$  
$\frac{1}{2}(27.0 \text{ ft} - 13.5 \text{ ft}) = 6.75 \text{ ft}$  
Total wall height at floor: $6.75 \text{ ft} + 6.75 \text{ ft} = 13.5 \text{ ft}$  

Calculate load to second floor diaphragm

Wall height:  
$\frac{1}{2}(27.0 \text{ ft} - 13.5 \text{ ft}) = 6.75 \text{ ft}$  
$\frac{1}{2}(13.5 \text{ ft} - 0.0 \text{ ft}) = 6.75 \text{ ft}$  
Total wall height at floor: $6.75 \text{ ft} + 6.75 \text{ ft} = 13.5 \text{ ft}$  

Total load to diaphragm:

Load to diaphragm at roof:  
\[ w_{d,(A)} = (33.4 \text{ psf})(9.25 \text{ ft}) = 309 \text{ plf} \]  
\[ w_{d,(C)} = (22.1 \text{ psf})(9.25 \text{ ft}) = 204 \text{ plf} \text{ at parapet} \]  
\[ w_{d,(C)} = (22.1 \text{ psf})(13.3 \text{ ft}) = 294 \text{ plf} \text{ at screenwall} \]  

Load to diaphragm at fourth floor:  
\[ w_{d,(A)} = (33.4 \text{ psf})(14.0 \text{ ft}) = 468 \text{ plf} \]  
\[ w_{d,(C)} = (22.1 \text{ psf})(14.0 \text{ ft}) = 309 \text{ plf} \]  

Load to diaphragm at second and third floors:  
\[ w_{d,(A)} = (33.4 \text{ psf})(13.5 \text{ ft}) = 451 \text{ plf} \]  
\[ w_{d,(C)} = (22.1 \text{ psf})(13.5 \text{ ft}) = 298 \text{ plf} \]
Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

\[ l = \text{length of structure, ft} \]
\[ b = \text{width of structure, ft} \]
\[ h = \text{height of wall at building element, ft} \]

Determine the wind load to each frame at each level. Conservatively apply the end zone pressures on both ends of the building simultaneously.

Wind from a north or south direction:

Total load to each frame:
\[ P_{W(n-s)} = w_{d(A)} (2a) + w_{d(C)} \left( l/2 - 2a \right) \]

Shear in diaphragm:
\[ v_{(n-s)} = \frac{P_{W(n-s)}}{120 \text{ ft for roof}} \]
\[ v_{(n-s)} = \frac{P_{W(n-s)}}{90 \text{ ft for floors (deduction for stair openings)}} \]

Wind from an east or west direction:

Total load to each frame:
\[ P_{W(e-w)} = w_{d(A)} (2a) + w_{d(C)} \left( b/2 - 2a \right) \]

Shear in diaphragm:
\[ v_{(e-w)} = \frac{P_{W(e-w)}}{210 \text{ ft for roof and floors}} \]

<table>
<thead>
<tr>
<th>Level</th>
<th>( l ) (ft)</th>
<th>( b ) (ft)</th>
<th>( 2a )</th>
<th>( h ) (ft)</th>
<th>( P_{d(A)} ) (psf)</th>
<th>( P_{d(C)} ) (psf)</th>
<th>( w_{d(A)} ) (psf)</th>
<th>( w_{d(C)} ) (psf)</th>
<th>( P_{W(n-s)} ) (kips)</th>
<th>( P_{W(e-w)} ) (kips)</th>
<th>( v_{(n-s)} ) (plf)</th>
<th>( v_{(e-w)} ) (plf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screen</td>
<td>93.0</td>
<td>33.0</td>
<td>0</td>
<td>13.3</td>
<td>0</td>
<td>22.1</td>
<td>0</td>
<td>294</td>
<td>13.7</td>
<td>4.85</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Roof</td>
<td>120</td>
<td>90.0</td>
<td>24.6</td>
<td>9.25</td>
<td>33.4</td>
<td>22.1</td>
<td>309</td>
<td>204</td>
<td>14.8</td>
<td>11.8</td>
<td>238</td>
<td>79</td>
</tr>
<tr>
<td>4th</td>
<td>213</td>
<td>123</td>
<td>24.6</td>
<td>14.0</td>
<td>33.4</td>
<td>22.1</td>
<td>468</td>
<td>309</td>
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<td>109</td>
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<tr>
<td>3rd</td>
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<td>123</td>
<td>24.6</td>
<td>13.5</td>
<td>33.4</td>
<td>22.1</td>
<td>451</td>
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<td>35.5</td>
<td>22.1</td>
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</tr>
<tr>
<td>2nd</td>
<td>213</td>
<td>123</td>
<td>24.6</td>
<td>13.5</td>
<td>33.4</td>
<td>22.1</td>
<td>451</td>
<td>298</td>
<td>35.5</td>
<td>22.1</td>
<td>394</td>
<td>105</td>
</tr>
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<td>Base of Frame</td>
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<td></td>
<td></td>
<td>136</td>
<td>83.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The table above indicates the total wind load in each direction acting on a steel frame at each level. The wind load at the ground level has not been included in the chart because it does not affect the steel frame.
SEISMIC LOAD DETERMINATION

The floor plan area: 120 ft, column center line to column center line, by 210 ft, column center line to column center line, with the edge of floor slab or roof deck 6 in. beyond the column center line.

Area = (121 ft)(211 ft)
= 25,500 ft²

The perimeter cladding system length:

Length = (2)(123 ft) + (2)(213 ft)
= 672 ft

The perimeter cladding weight at floors:

Brick spandrel panel with metal stud backup (7.50 ft)(0.055 ksf) = 0.413 klf
Window wall system (6.00 ft)(0.015 ksf) = 0.090 klf
Total 0.503 klf

Typical roof dead load (from previous calculations):

Although 40 psf was used to account for the mechanical units and screen wall for the beam and column design, the entire mechanical area will not be uniformly loaded. Use 30% of the uniform 40 psf mechanical area load to determine the total weight of all of the mechanical equipment and screen wall for the seismic load determination.

Roof Area = (25,500 ft²)(0.020 ksf) = 510 kips
Wall perimeter = (672 ft)(0.413 klf) = 278 kips
Mechanical Area = (2,700 ft²)(0.300)(0.040 ksf) = 32.4 kips
Total 820 kips

Typical third and fourth floor dead load:

Note: An additional 10 psf has been added to the floor dead load to account for partitions per Section 12.7.2.2 of ASCE/SEI 7.

Floor Area = (25,500 ft²)(0.085 ksf) = 2,170 kips
Wall perimeter = (672 ft)(0.503 klf) = 338 kips
Total 2,510 kips

Second floor dead load: the floor area is reduced because of the open atrium

Floor Area = (24,700 ft²)(0.085 ksf) = 2,100 kips
Wall perimeter = (672 ft)(0.503 klf) = 338 kips
Total 2,440 kips

Total dead load of the building:

Roof 820 kips
Fourth floor 2,510 kips
Third floor 2,510 kips
Second floor 2,440 kips
Total 8,280 kips
Calculate the seismic forces.

Determine the seismic risk category and importance factors.

Office Building: Risk Category II from ASCE/SEI 7 Table 1.5-1

Seismic Importance Factor: $I_e = 1.00$ from ASCE/SEI 7 Table 1.5-2

The site coefficients are given in this example. $S_s$ and $S_i$ can also be determined from ASCE/SEI 7, Figures 22-1 and 22-2, respectively.

$$S_s = 0.121g$$

$$S_i = 0.060g$$

Soil, site class D (given)

$$F_a @ S_s \leq 0.25 = 1.6 \text{ from ASCE/SEI 7, Table 11.4-1}$$

$$F_v @ S_i \leq 0.1 = 2.4 \text{ from ASCE/SEI 7, Table 11.4-2}$$

Determine the maximum considered earthquake accelerations.

$$S_{MS} = F_a S_s = (1.6)(0.121g) = 0.194g \text{ from ASCE/SEI 7, Equation 11.4-1}$$

$$S_{MI} = F_v S_i = (2.4)(0.060g) = 0.144g \text{ from ASCE/SEI 7, Equation 11.4-2}$$

Determine the design earthquake accelerations.

$$S_{DS} = \frac{g}{2} S_{MS} = \frac{g}{2} (0.194g) = 0.129g \text{ from ASCE/SEI 7, Equation 11.4-3}$$

$$S_{DI} = \frac{g}{2} S_{MI} = \frac{g}{2} (0.144g) = 0.096g \text{ from ASCE/SEI 7, Equation 11.4-4}$$

Determine the seismic design category.

$$S_{DS} < 0.167g, \text{ Seismic Risk Category II: Seismic Design Category: A from ASCE/SEI 7, Table 11.6-1}$$

$$0.067g \leq S_{DI} < 0.133g, \text{ Seismic Risk Category II: Seismic Design Category: B from ASCE/SEI 7, Table 11.6-2}$$

Select the seismic force resisting system.

Seismic Design Category B may be used and it is therefore permissible to select a structural steel system not specifically detailed for seismic resistance, for which the seismic response modification coefficient, $R = 3$

Determine the approximate fundamental period.

Building Height, $h_a = 55.0$ ft

$$C_t = 0.02: \quad x = 0.75 \text{ from ASCE/SEI 7, Table 12.8-2}$$

$$T_a = C_t (h_a)^x = (0.02)(55.0 \text{ ft})^{0.75} = 0.404 \text{ sec from ASCE/SEI 7, Equation 12.8-7}$$

Determine the redundancy factor from ASCE/SEI 7, Section 12.3.4.1.

$$\rho = 1.0 \text{ because the Seismic Design Category = B}$$
Determine the vertical seismic effect term.

\[ E_v = 0.2S_{D_S}D \]
\[ = 0.2(0.129g)D \]
\[ = 0.0258D \]  

(ASCE 7 Eq. 12.4-4)

The following seismic load combinations are as specified in ASCE/SEI 7, Section 12.4.2.3.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1.2 + 0.2S_{D_S})D + pQ_E + L + 0.2S)</td>
<td>((1.0 + 0.14S_{D_S})D + H + F + 0.7pQ_E)</td>
</tr>
<tr>
<td>[=\left[1.2 + 0.2(0.129)\right]D + 1.0Q_E + L + 0.2S]</td>
<td>[=\left[1.0 + 0.14(0.129)\right]D + 0.0 + 0.0 + 0.7(1.0)Q_E]</td>
</tr>
<tr>
<td>[= 1.23D + 1.0Q_E + L + 0.2S]</td>
<td>[= 1.02D + 0.7Q_E]</td>
</tr>
<tr>
<td>((0.9 - 0.2S_{D_S})D + pQ_E + 1.6H)</td>
<td>((1.0 + 0.10S_{D_S})D + H + F + 0.525pQ_E + 0.75L + 0.75(L_e \text{ or } S \text{ or } R))</td>
</tr>
<tr>
<td>[= \left[0.9 - 0.2(0.129)\right]D + 1.0Q_E + 0.0]</td>
<td>[= \left[1.0 + 0.10(0.129)\right]D + 0.0 + 0.0 + 0.525(1.0)Q_E + 0.75L + 0.75S]</td>
</tr>
<tr>
<td>[= 0.874D + 1.0Q_E]</td>
<td>[= 1.01D + 0.525Q_E + 0.75L + 0.75S]</td>
</tr>
<tr>
<td>((0.6 - 0.14S_{D_S})D + 0.7pQ_E + H)</td>
<td>((0.6 - 0.14S_{D_S})D + 0.7Q_E + 0)</td>
</tr>
<tr>
<td>[= \left[0.6 - 0.14(0.129)\right]D + 0.7(1.0)Q_E + 0]</td>
<td>[= 0.582D + 0.7Q_E]</td>
</tr>
</tbody>
</table>

Note: \(pQ_E\) = effect of horizontal seismic (earthquake induced) forces

Overstrength Factor: \(\Omega_o = 3\) for steel systems not specifically detailed for seismic resistance, excluding cantilever column systems, per ASCE/SEI 7, Table 12.2-1.

Calculate the seismic base shear using ASCE/SEI 7, Section 12.8.1.

Determine the seismic response coefficient from ASCE/SEI 7, Equation 12.8-2

\[ C_e = \frac{S_{D_S}}{R} \left(\frac{T}{L_e}\right) \]
\[ = \frac{0.129}{\left(\frac{3}{1}\right)} \]
\[ = 0.0430 \quad \text{controls} \]

Let \(T_a = T\). From ASCE/SEI 7 Figure 22-12, \(T_L = 12 > T\) (midwestern city); therefore use ASCE/SEI 7, Equation 12.8-3 to determine the upper limit of \(C_e\).

\[ C_e = \frac{S_{D_S}}{T} \left(\frac{R}{L_e}\right) \]
\[ = \frac{0.096}{0.404\left(\frac{3}{1}\right)} \]
\[ = 0.0792 \]
From ASCE/SEI 7, Equation 12.8-5, $C_s$ shall not be taken less than:

$$C_s = 0.044 S_{Dd} L \geq 0.01$$

$$= 0.044(0.129)(1.0)$$

$$= 0.00568$$

Therefore, $C_s = 0.0430$.

Calculate the seismic base shear from ASCE/SEI 7 Equation 12.8-1

$$V = C_s W$$

$$= 0.0430(8,280 \text{ kips})$$

$$= 356 \text{ kips}$$

Calculate vertical distribution of seismic forces from ASCE/SEI 7, Section 12.8.3.

$$F_i = C_s V$$

$$= C_s (356 \text{ kips})$$

$$C_s = \frac{w_i h_i}{\sum w_i h_i} F_i = C_s V$$

(ASCE Eq. 12.8-12)

For structures having a period of 0.5 s or less, $k = 1$.

Calculate horizontal shear distribution at each level per ASCE/SEI 7, Section 12.8.4.

$$V_x = \sum_{i=1}^{n} F_i$$

(ASCE Eq. 12.8-13)

Calculate the overturning moment at each level per ASCE/SEI 7, Section 12.8.5.

$$M_x = \sum_{i=1}^{n} F_i (h_i - h_x)$$

<table>
<thead>
<tr>
<th>Level</th>
<th>$w_x$</th>
<th>$h_x$</th>
<th>$w_x h_x$</th>
<th>$C_{rx}$</th>
<th>$F_x$</th>
<th>$V_x$</th>
<th>$M_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>820</td>
<td>55.0</td>
<td>45,100</td>
<td>0.182</td>
<td>64.8</td>
<td>64.8</td>
<td></td>
</tr>
<tr>
<td>Fourth</td>
<td>2,510</td>
<td>40.5</td>
<td>102,000</td>
<td>0.411</td>
<td>146</td>
<td>211</td>
<td>940</td>
</tr>
<tr>
<td>Third</td>
<td>2,510</td>
<td>27.0</td>
<td>67,800</td>
<td>0.273</td>
<td>97.2</td>
<td>308</td>
<td>3,790</td>
</tr>
<tr>
<td>Second</td>
<td>2,440</td>
<td>13.5</td>
<td>32,900</td>
<td>0.133</td>
<td>47.3</td>
<td>355</td>
<td>7,940</td>
</tr>
<tr>
<td>Base</td>
<td>8,280</td>
<td>13.5</td>
<td>248,000</td>
<td>0.133</td>
<td>355</td>
<td></td>
<td>12,700</td>
</tr>
</tbody>
</table>

Calculate strength and determine rigidity of diaphragms.

Determine the diaphragm design forces from Section 12.10.1.1 of ASCE/SEI 7.

$F_{pm}$ is the largest of:

1. The force $F_i$ at each level determined by the vertical distribution above
2. \( F_{px} = \sum_{i=1}^{n} \frac{F_i}{w_{px}} w_{px} \leq 0.4S_{DS}I_x w_{px} \) from ASCE/SEI 7, Equation 12.10-1 and 12.10-3

\[
\leq 0.4(0.129)(1.0)w_{px} \\
\leq 0.0516w_{px}
\]

3. \( F_{px} = 0.2S_{DS}I_x w_{px} \) from ASCE/SEI 7, Equation 12.10-2

\[
= 0.2(0.129)(1.0)w_{px} \\
= 0.0258w_{px}
\]

<table>
<thead>
<tr>
<th></th>
<th>( w_{px} )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( F_{px} )</th>
<th>( V_{(n-e)} )</th>
<th>( V_{(c-e)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>820</td>
<td>64.8</td>
<td>42.3</td>
<td>21.2</td>
<td>64.8</td>
<td>297</td>
<td>170</td>
</tr>
<tr>
<td>Fourth</td>
<td>2,510</td>
<td>146</td>
<td>130</td>
<td>64.8</td>
<td>146</td>
<td>892</td>
<td>382</td>
</tr>
<tr>
<td>Third</td>
<td>2,510</td>
<td>97.2</td>
<td>130</td>
<td>64.8</td>
<td>130</td>
<td>791</td>
<td>339</td>
</tr>
<tr>
<td>Second</td>
<td>2,440</td>
<td>47.3</td>
<td>105</td>
<td>63.0</td>
<td>105</td>
<td>641</td>
<td>275</td>
</tr>
</tbody>
</table>

where

\( A = \text{force at a level based on the vertical distribution of seismic forces} \)

\[
B = F_{px} = \sum_{i=1}^{n} \frac{F_i}{w_{px}} w_{px} \leq 0.4S_{DS}I_x w_{px}
\]

\( C = 0.2S_{DS}I_x w_{px} \)

\( F_{px} = \max(A, B, C) \)

Note: The diaphragm shear loads include the effects of openings in the diaphragm and a 10% increase to account for accidental torsion.
Roof

Roof deck: 1½ in. deep, 22 gage, wide rib
Support fasteners: ½ in. puddle welds in 36 / 5 pattern
Sidelap fasteners: 3 #10 TEK screws
Joist spacing = s = 6.0 ft
Diaphragm length = 210 ft
Diaphragm width = l_v = 120 ft

By inspection, the critical condition for the diaphragm is loading from the north or south directions.

Calculate the required diaphragm strength, including a 10% increase for accidental torsion.

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\text{From the ASCE/SEI 7 load combinations for strength design, the earthquake load is,} & \text{From the ASCE/SEI 7 load combinations for allowable stress design, the earthquake load is,} \\
v_v = 1.0 \frac{Q_v}{l_v} & v_v = 0.7 \frac{Q_v}{l_v} \\
= 1.0(0.55) \frac{F_{mv}}{l_v} & = 0.7(0.55) \frac{F_{mv}}{l_v} \\
= 1.0(0.55) \left( \frac{64.8 \text{ kips}}{120 \text{ ft}} \right) & = 0.7(0.55) \left( \frac{64.8 \text{ kips}}{120 \text{ ft}} \right) \\
= 0.297 \text{ klf} & = 0.208 \text{ klf} \\
\text{The wind load is,} & \text{The wind load is,} \\
v_v = 1.0W & v_v = 0.6W \\
= 1.0(0.238 \text{ klf}) & = 0.6(0.238 \text{ klf}) \\
= 0.238 \text{ klf} & = 0.143 \text{ klf} \\
\hline
\end{array}
\]

Note: The 0.55 factor in the earthquake load accounts for half the shear to each braced frame plus the 10% increase for accidental torsion.

From the SDI Diaphragm Design Manual (SDI, 2004), the nominal shear strengths are:

1. For panel buckling strength, \(v_n = 1.425 \text{ klf}\)
2. For connection strength, \(v_n = 0.820 \text{ klf}\)

Calculate the available strengths.

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\text{Panel Buckling Strength (SDI, 2004)} & \text{Panel Buckling Strength (SDI, 2004)} \\
v_u = \phi v_n & v_u = \frac{v_n}{\Omega} \\
= 0.80(1.425 \text{ klf}) & = \frac{1.425 \text{ klf}}{2.00} \\
= 1.14 \text{ klf} > 0.297 \text{ klf} & = 0.713 \text{ klf} > 0.208 \text{ klf} \quad \text{o.k.} \\
\text{o.k.} & \text{o.k.} \\
\text{Connection Strength (SDI, 2004)} & \text{Connection Strength (SDI, 2004)} \\
\text{Earthquake} & \text{Earthquake} \\
\hline
\end{array}
\]
Check diaphragm flexibility.

From the Steel Deck Institute *Diaphragm Design Manual*,

\[
D_{xx} = 758 \text{ ft} \\
K_1 = 0.286 \text{ ft}^{-1} \\
K_2 = 870 \text{ kip/in.} \\
K_4 = 3.78
\]

\[
G' = \frac{K_2}{K_4 + 0.3D_{xx}} + 3K_1s
\]

\[
= \frac{870 \text{ kips/in.}}{3.78 + \frac{0.3(758 \text{ ft})}{6.00 \text{ ft}} + 3\left(\frac{0.286 \text{ ft}}{6.00 \text{ ft}}\right)(6.00 \text{ ft})}
\]

\[
= 18.6 \text{ kips/in.}
\]

Seismic loading to diaphragm.

\[
w = \left(\frac{64.8 \text{ kips}}{210 \text{ ft}}\right)
\]

\[
= 0.309 \text{ klf}
\]

Calculate the maximum diaphragm deflection.

\[
\Delta = \frac{wL^2}{8f'G'}
\]

\[
= \frac{(0.309 \text{ klf})(210 \text{ ft})^2}{8(120 \text{ ft})(18.6 \text{ kips/in.})}
\]

\[
= 0.763 \text{ in.}
\]

Story drift = 0.141 in. (from computer output)

The diaphragm deflection exceeds two times the story drift; therefore, the diaphragm may be considered to be flexible in accordance with ASCE/SEI 7, Section 12.3.1.3

The roof diaphragm is flexible in the N-S direction, but using a rigid diaphragm distribution is more conservative for the analysis of this building. By similar reasoning, the roof diaphragm will also be treated as a rigid diaphragm in the E-W direction.

*Third and Fourth floors*
Floor deck: 3 in. deep, 22 gage, composite deck with normal weight concrete, Support fasteners; ½ in. puddle welds in a 36 / 4 pattern
Sidetap fasteners: 1 button punched fastener
Beam spacing = \( s = 10.0 \) ft
Diaphragm length = 210 ft
Diaphragm width = 120 ft
\( l_v = 120 \text{ ft} - 30 \text{ ft} = 90 \text{ ft} \) to account for the stairwell

By inspection, the critical condition for the diaphragm is loading from the north or south directions

Calculate the required diaphragm strength, including a 10\% increase for accidental torsion.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the ASCE/SEI 7 load combinations for strength design, the earthquake load for the fourth floor is, ( v_e = 1.0 \frac{Q_e}{l_v} ) 1.0(0.55) ( \frac{F_{pw}}{l_v} ) 1.0(0.55) ( \frac{146 \text{ kips}}{90 \text{ ft}} ) 0.892 klf</td>
<td>From the ASCE/SEI 7 load combinations for strength design, the earthquake load is, ( v_e = 0.7 \frac{Q_e}{l_v} ) 0.7(0.55) ( \frac{F_{pw}}{l_v} ) 0.7(0.55) ( \frac{146 \text{ kips}}{90 \text{ ft}} ) 0.625 klf</td>
</tr>
<tr>
<td>For the fourth floor, the wind load is, ( v_r = 1.0W ) 1.0(0.409 klf) 0.409 klf</td>
<td>For the fourth floor, the wind load is, ( v_r = 0.6W ) 0.6(0.409 klf) 0.245 klf</td>
</tr>
<tr>
<td>From the ASCI/SEI 7 load combinations for strength design, the earthquake load for the third floor is, ( v_e = 1.0 \frac{Q_e}{l_v} ) 1.0(0.55) ( \frac{F_{pw}}{l_v} ) 1.0(0.55) ( \frac{130 \text{ kips}}{90 \text{ ft}} ) 0.794 klf</td>
<td>From the ASCI/SEI 7 load combinations for strength design, the earthquake load for the third floor is, ( v_e = 0.7 \frac{Q_e}{l_v} ) 0.7(0.55) ( \frac{F_{pw}}{l_v} ) 0.7(0.55) ( \frac{130 \text{ kips}}{90 \text{ ft}} ) 0.556 klf</td>
</tr>
<tr>
<td>For the third floor, the wind load is, ( v_r = 1.0W ) 1.0(0.394 klf) 0.394 klf</td>
<td>For the third floor, the wind load is, ( v_r = 0.6W ) 0.6(0.394 klf) 0.236 klf</td>
</tr>
</tbody>
</table>

From the SDI Diaphragm Design Manual, the nominal shear strengths are:

For connection strength, \( v_e = 5.16 \) klf

Calculate the available strengths.
### Connection Strength (same for earthquake or wind) (SDI, 2004)

\[
v_u = \phi v_u = 0.5 (5.16 \text{ klf}) = 2.58 \text{ klf} > 0.892 \text{ klf}
\]

**o.k.**

### Connection Strength (same for earthquake or wind) (SDI, 2004)

\[
v_u = \frac{v_u}{\Omega} = \frac{5.16 \text{ klf}}{3.25} = 1.59 \text{ klf} > 0.625 \text{ klf}
\]

**o.k.**

Check diaphragm flexibility.

From the Steel Deck Institute *Diaphragm Design Manual*,

\[
K_1 = 0.729 \text{ ft}^{-1} \quad K_2 = 870 \text{ kip/in.} \quad K_3 = 2,380 \text{ kip/in.} \quad K_4 = 3.78
\]

\[
G' = \left( \frac{K_2}{K_4 + 3K_3} \right) + K_3
\]

\[
= \left( \frac{870 \text{ kip/in.}}{3.78 + 3 \left( \frac{0.729}{\text{ft}} \right) (10.0 \ \text{ft})} \right) + 2,380 \text{ kip/in.}
\]

\[
= 2,410 \text{ kips/in.}
\]

**Fourth Floor**

Calculate seismic loading to diaphragm based on the fourth floor seismic load.

\[
w = \frac{(146 \text{ kips})}{(210 \ \text{ft})} = 0.695 \text{ klf}
\]

Calculate the maximum diaphragm deflection on the fourth floor.

\[
\Delta = \frac{wl^2}{8G'} = \frac{(0.695 \text{ klf})(210 \ \text{ft})^2}{8(90 \ \text{ft})(2,410 \text{ kips/in.})} = 0.0177 \text{ in.}
\]

**Third Floor**

Calculate seismic loading to diaphragm based on the third floor seismic load.

\[
w = \frac{(130 \text{ kips})}{(210 \ \text{ft})} = 0.619 \text{ klf}
\]

Calculate the maximum diaphragm deflection on the third floor.
\[ \Delta = \frac{wL^2}{8l/G'} \]
\[ = \frac{(0.619 \text{ klf})(210 \text{ ft})^2}{8(90 \text{ ft})(2,410 \text{ kips/in.})} \]
\[ = 0.0157 \text{ in.} \]

The diaphragm deflection at the third and fourth floors is less than two times the story drift (story drift = 0.245 in. from computer output); therefore, the diaphragm is considered rigid in accordance with ASCE/SEI 7, Section 12.3.1.3. By inspection, the floor diaphragm will also be rigid in the E-W direction.

**Second floor**

Floor deck: 3 in. deep, 22 gage, composite deck with normal weight concrete,
Support fasteners: 3/8 in. puddle welds in a 36 / 4 pattern
Sidetap fasteners: 1 button punched fasteners
Beam spacing = \( s \) = 10.0 ft
Diaphragm length = 210 ft
Diaphragm width = 120 ft
Because of the atrium opening in the floor diaphragm, an effective diaphragm depth of 75 ft will be used for the deflection calculations.

By inspection, the critical condition for the diaphragm is loading from the north or south directions.

Calculate the required diaphragm strength, including a 10% increase for accidental torsion.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the ASCE/SEI 7 load combinations for strength design, the earthquake load is, ( v_e ) = 1.0 ( \frac{Q_e}{l_v} )</td>
<td></td>
</tr>
<tr>
<td>[ = 1.0(0.55)\frac{F_{pu}}{l_v} ]</td>
<td></td>
</tr>
<tr>
<td>[ = 1.0(0.55)\frac{105 \text{ kips}}{90 \text{ ft}} ]</td>
<td></td>
</tr>
<tr>
<td>[ = 0.642 \text{ klf} ]</td>
<td></td>
</tr>
<tr>
<td>The wind load is, ( v_e ) = 1.0W</td>
<td></td>
</tr>
<tr>
<td>[ = 1.0(0.395 \text{ klf}) ]</td>
<td></td>
</tr>
<tr>
<td>[ = 0.395 \text{ klf} ]</td>
<td></td>
</tr>
<tr>
<td>From the ASCE/SEI 7 load combinations for strength design, the earthquake load is, ( v_e ) = 0.7 ( \frac{Q_e}{l_v} )</td>
<td></td>
</tr>
<tr>
<td>[ = 0.7(0.55)\frac{F_{pu}}{l_v} ]</td>
<td></td>
</tr>
<tr>
<td>[ = 0.7(0.55)\frac{105 \text{ kips}}{90 \text{ ft}} ]</td>
<td></td>
</tr>
<tr>
<td>[ = 0.449 \text{ klf} ]</td>
<td></td>
</tr>
<tr>
<td>The wind load is, ( v_e ) = 0.6W</td>
<td></td>
</tr>
<tr>
<td>[ = 0.6(0.395 \text{ klf}) ]</td>
<td></td>
</tr>
<tr>
<td>[ = 0.237 \text{ klf} ]</td>
<td></td>
</tr>
</tbody>
</table>

From the SDI *Diaphragm Design Manual*, the nominal shear strengths are:

For connection strength, \( v_n = 5.16 \text{ klf} \)

Calculate the available strengths.
Check diaphragm flexibility.

From the Steel Deck Institute *Diaphragm Design Manual*,

\[ K_1 = 0.729 \, \text{ft}^{-1} \quad K_2 = 870 \, \text{kip/in.} \quad K_3 = 2,380 \, \text{kip/in.} \quad K_4 = 3.78 \]

\[
G' = \left( \frac{K_2}{K_4 + 3K_3} \right) + K_3 \\
= \left( \frac{870 \, \text{kip/in.}}{3.78 + 3 \left( \frac{0.729}{\text{ft}} \right)(10 \, \text{ft})} \right) + 2,380 \, \text{kip/in.}
\]

= 2,410 kip/in.

Calculate seismic loading to diaphragm.

\[ w = \frac{(105 \, \text{kips})}{(210 \, \text{ft})} \]

= 0.500 klf

Calculate the maximum diaphragm deflection.

\[
\Delta = \frac{wL^2}{8bG'}
\]

\[
= \frac{(0.500 \, \text{klf})(210 \, \text{ft})^2}{8(75 \, \text{ft})(2,410 \, \text{kip/in.})}
\]

= 0.0152 in.

Story drift = 0.228 in. (from computer output)

The diaphragm deflection is less than two times the story drift; therefore, the diaphragm is considered rigid in accordance with ASCE/SEI 7, Section 12.3.1.3. By inspection, the floor diaphragm will also be rigid in the E-W direction.

*Horizontal shear distribution and torsion:*

Calculate the seismic forces to be applied in the frame analysis in each direction, including the effect of accidental torsion, in accordance with ASCE/SEI 7, Section 12.8.4.
<table>
<thead>
<tr>
<th></th>
<th>$F_y$</th>
<th>Load to Frame</th>
<th>Accidental Torsion</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kips</td>
<td>%</td>
<td>kips</td>
<td>%</td>
</tr>
<tr>
<td>Roof</td>
<td>64.8</td>
<td>50</td>
<td>32.4</td>
<td>5</td>
</tr>
<tr>
<td>Fourth</td>
<td>146</td>
<td>50</td>
<td>73.0</td>
<td>5</td>
</tr>
<tr>
<td>Third</td>
<td>97.2</td>
<td>50</td>
<td>48.6</td>
<td>5</td>
</tr>
<tr>
<td>Second</td>
<td>47.3</td>
<td>50</td>
<td>23.7</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$F_x$</th>
<th>Load to Frame</th>
<th>Accidental Torsion</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kips</td>
<td>%</td>
<td>kips</td>
<td>%</td>
</tr>
<tr>
<td>Roof</td>
<td>64.8</td>
<td>50</td>
<td>32.4</td>
<td>5</td>
</tr>
<tr>
<td>Fourth</td>
<td>146</td>
<td>50</td>
<td>73.0</td>
<td>5</td>
</tr>
<tr>
<td>Third</td>
<td>97.2</td>
<td>50</td>
<td>48.6</td>
<td>5</td>
</tr>
<tr>
<td>Second</td>
<td>47.3</td>
<td>50.8(1)</td>
<td>24.0</td>
<td>5</td>
</tr>
</tbody>
</table>

(1) Note: In this example, Grids A and F have both been conservatively designed for the slightly higher load on Grid A due to the atrium opening. The increase in load is calculated as follows:

\[
y = \frac{125,000 \text{ kip-ft}}{2,100 \text{ kips}} = 59.5 \text{ ft}
\]

\[
(100\%)(121 \text{ ft} - 59.5 \text{ ft})/121 \text{ ft} = 50.8\%
\]
MOMENT FRAME MODEL

Grids 1 and 8 were modeled in conventional structural analysis software as two-dimensional models. The second-order option in the structural analysis program was not used. Rather, for illustration purposes, second-order effects are calculated separately, using the “Approximate Second-Order Analysis” method described in AISc Specification Appendix 8.

The column and beam layouts for the moment frames follow. Although the frames on Grids A and F are the same, slightly heavier seismic loads accumulate on grid F after accounting for the atrium area and accidental torsion. The models are half-building models. The frame was originally modeled with W14×82 interior columns and W21×44 non-composite beams, but was revised because the beams and columns did not meet the strength requirements. The W14×82 column size was increased to a W14×90 and the W21×44 beams were upsized to W24×55 beams. Minimum composite studs are specified for the beams (corresponding to \( \sum Q_s = 0.25F/A_b \)), but the beams were modeled with a stiffness of \( I_{eq} = I_s \).

The frame was checked for both wind and seismic story drift limits. Based on the results on the computer analysis, the frame meets the \( L/400 \) drift criterion for a 10 year wind (0.7W) indicated in Commentary Section CC.1.2 of ASCE/SEI 7. In addition, the frame meets the 0.025\( h_{cx} \) allowable story drift limit given in ASCE/SEI 7 Table 12.12-1 for Seismic Risk Category II.

All of the vertical loads on the frame were modeled as point loads on the frame. The dead load and live load are shown in the load cases that follow. The wind, seismic, and notional loads from leaning columns are modeled and distributed 1/14 to exterior columns and 1/7 to the interior columns. This approach minimizes the tendency to accumulate too much load in the lateral system nearest an externally applied load.

Also shown in the models below are the remainder of the half-building model gravity loads from the interior leaning columns accumulated in a single leaning column which was connected to the frame portion of the model with pinned ended links. Because the second-order analyses that follow will use the “Approximate Second-Order Analysis (amplified first-order) approach given in the AISC Specification Appendix 8, the inclusion of the leaning column is unnecessary, but serves to summarize the leaning column loads and illustrate how these might be handled in a full second-order analysis. See Geschwindner (1994), “A Practical Approach to the ‘Leaning’ Column.”

There are five lateral load cases. Two are the wind load and seismic load, per the previous discussion. In addition, notional loads of \( N_t = 0.002Y_i \) were established. The model layout, nominal dead, live, and snow loads with associated notional loads, wind loads and seismic loads are shown in the figures below.

The same modeling procedures were used in the braced frame analysis. If column bases are not fixed in construction, they should not be fixed in the analysis.
FRAME LAYOUT (GRID A & F)
FRAME (1/2 BLDG)

NOMINAL DEAD LOADS

FRAME (1/2 BLDG)

NOTIONAL DEAD LOADS
Table II-64

### Frame (1/2 BLDG)

**Nominal Live Loads**

- Column 1: 11.9k
- Column 2: 22.9k
- Column 3: 22.9k
- Column 4: 22.9k
- Column 5: 22.9k
- Column 6: 22.9k
- Column 7: 11.9k
- Column 8: 20.5k

### Frame (1/2 BLDG)

**Notional Live Loads**

- Column 1: 0.0617k
- Column 2: 0.122k
- Column 3: 0.122k
- Column 4: 0.122k
- Column 5: 0.122k
- Column 6: 0.122k
- Column 7: 0.0617k
- Column 8: 0.122k

---

*Design Examples V14.1*
*American Institute of Steel Construction*
FRAME (1/2 BLDG)

NOMINAL SNOW LOADS

FRAME (1/2 BLDG)

NOTIONAL SNOW LOADS
FRAME (1/2 BLDG)

NOMINAL WIND LOADS

FRAME (1/2 BLDG)

SEISMIC LOADS (1.0Qe)
CALCULATION OF REQUIRED STRENGTH—THREE METHODS

Three methods for checking one of the typical interior column designs at the base of the building are presented below. All three of presented methods require a second-order analysis (either direct via computer analysis techniques or by amplifying a first-order analysis). A fourth method called the “First-Order Analysis Method” is also an option. This method does not require a second-order analysis; however, this method is not presented below. For additional guidance on applying any of these methods, see the discussion in AISC Manual Part 2 titled Required Strength, Stability, Effective Length, and Second-Order Effects.

GENERAL INFORMATION FOR ALL THREE METHODS

Seismic load combinations controlled over wind load combinations in the direction of the moment frames in the example building. The frame analysis was run for all LRFD and ASD load combinations; however, only the controlling combinations have been illustrated in the following examples. A lateral load of 0.2% of gravity load was included for all gravity-only load combinations.

The second-order analysis for all the examples below was carried out by doing a first-order analysis and then amplifying the results to achieve a set of second-order design forces using the approximate second-order analysis procedure from AISC Specification Appendix 8.

METHOD 1. DIRECT ANALYSIS METHOD

Design for stability by the direct analysis method is found in Chapter C of the AISC Specification. This method requires that both the flexural and axial stiffness are reduced and that 0.2% notional lateral loads are applied in the analysis to account for geometric imperfections and inelasticity. Any general second-order analysis method that considers both $P - \delta$ and $P - \Delta$ effects is permitted. The amplified first-order analysis method of AISC Specification Appendix 8 is also permitted provided that the $B_1$ and $B_2$ factors are based on the reduced flexural and axial stiffnesses. A summary of the axial loads, moments and 1st floor drifts from first-order analysis is shown below. The floor diaphragm deflection in the east-west direction was previously determined to be very small and will thus be neglected in these calculations. Second-order member forces are determined using the amplified first-order procedure of AISC Specification Appendix 8.

It was assumed, subject to verification, that $B_2$ is less than 1.7 for each load combination; therefore, per AISC Specification Section C2.2b(4), the notional loads were applied to the gravity-only load combinations. The required seismic load combinations are given in ASCE/SEI 7, Section 12.4.2.3.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams)</td>
<td>$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$ (Controls columns and beams)</td>
</tr>
<tr>
<td>From a first-order analysis with notional loads where appropriate and reduced stiffnesses:</td>
<td>From a first-order analysis with notional loads where appropriate and reduced stiffnesses:</td>
</tr>
<tr>
<td>$P_a = 317$ kips</td>
<td>$P_a = 295$ kips</td>
</tr>
<tr>
<td>$M_{1u} = 148$ kip-ft (from first-order analysis)</td>
<td>$M_{1a} = 77.9$ kip-ft</td>
</tr>
<tr>
<td>$M_{2u} = 233$ kip-ft (from first-order analysis)</td>
<td>$M_{2a} = 122$ kip-ft</td>
</tr>
<tr>
<td>First story drift with reduced stiffnesses = 0.718 in.</td>
<td>First story drift with reduced stiffnesses = 0.377 in.</td>
</tr>
</tbody>
</table>

Note: For ASD, ordinarily the second-order analysis must be carried out under 1.6 times the ASD load combinations and the results must be divided by 1.6 to obtain the required strengths. For this example, second-order analysis by the amplified first-order analysis method is used. The amplified first-order analysis method incorporates the 1.6 multiplier directly in the $B_1$ and $B_2$ amplifiers, such that no other modification is needed.

The required second-order flexural strength, $M_r$, and axial strength, $P_r$, are determined as follows. For typical
interior columns, the gravity-load moments are approximately balanced, therefore, \( M_{nt} = 0.0 \) kip-ft

Calculate the amplified forces and moments in accordance with AISC *Specification* Appendix 8.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_r = B_1 M_{nt} + B_2 M_{lt} ) (Spec. Eq. A-8-1)</td>
<td>( M_r = B_1 M_{nt} + B_2 M_{lt} ) (Spec. Eq. A-8-1)</td>
</tr>
<tr>
<td><strong>Determine</strong> ( B_1 )</td>
<td><strong>Determine</strong> ( B_1 )</td>
</tr>
<tr>
<td>( P_r = ) required second-order axial strength using LRFD or ASD load combinations, kips.</td>
<td>( P_r = ) required second-order axial strength using LRFD or ASD load combinations, kips.</td>
</tr>
<tr>
<td>Note that for members subject to axial compression, ( B_1 ) may be calculated based on the first-order estimate ( P_r = P_{nt} + P_{lt} ).</td>
<td>Note that for members subject to axial compression, ( B_1 ) may be calculated based on the first-order estimate ( P_r = P_{nt} + P_{lt} ).</td>
</tr>
<tr>
<td>Therefore, ( P_r = 317 ) kips (from the first-order computer analysis)</td>
<td>Therefore, ( P_r = 295 ) kips (from the first-order computer analysis)</td>
</tr>
<tr>
<td>( I_x = 999 ) in.(^4) (W14(\times)90)</td>
<td>( I_x = 999 ) in.(^4) (W14(\times)90)</td>
</tr>
<tr>
<td>( \tau_b = 1.0 )</td>
<td>( \tau_b = 1.0 )</td>
</tr>
<tr>
<td>( P_{ct} = \pi^2EI^* \left( KL \right)^2 )</td>
<td>( P_{ct} = \pi^2EI^* \left( KL \right)^2 )</td>
</tr>
<tr>
<td>( = \pi^2 \times (0.8)(29,000 \text{ ksi})(999 \text{ in.}^4) )</td>
<td>( = \pi^2 \times (0.8)(29,000 \text{ ksi})(999 \text{ in.}^4) )</td>
</tr>
<tr>
<td>( = 1\left(1.0\right)(13.5 \text{ ft})(12 \text{ in./ft}) ]^2</td>
<td>( = 1\left(1.0\right)(13.5 \text{ ft})(12 \text{ in./ft}) ]^2</td>
</tr>
<tr>
<td>( = 8,720 ) kips</td>
<td>( = 8,720 ) kips</td>
</tr>
<tr>
<td>( C_m = 0.6 - 0.4 \left( M_1 / M_2 \right) )</td>
<td>( C_m = 0.6 - 0.4 \left( M_1 / M_2 \right) )</td>
</tr>
<tr>
<td>( = 0.6 - 0.4 \left( \frac{148 \text{ kip-ft}}{233 \text{ kip-ft}} \right) )</td>
<td>( = 0.6 - 0.4 \left( \frac{77.9 \text{ kip-ft}}{122 \text{ kip-ft}} \right) )</td>
</tr>
<tr>
<td>( = 0.346 )</td>
<td>( = 0.345 )</td>
</tr>
<tr>
<td>( \alpha = 1.0 )</td>
<td>( \alpha = 1.6 )</td>
</tr>
<tr>
<td>( B_1 = \frac{C_m}{\alpha P_{ct}} \geq 1 )</td>
<td>( B_1 = \frac{C_m}{\alpha P_{ct}} \geq 1 )</td>
</tr>
<tr>
<td>( 1 - \frac{0.346}{P_{ct}} \geq 1 )</td>
<td>( 1 - \frac{0.345}{P_{ct}} \geq 1 )</td>
</tr>
<tr>
<td>( = \frac{0.359}{8,720 \text{ kips}} \geq 1 )</td>
<td>( = \frac{0.365}{8,720 \text{ kips}} \geq 1 )</td>
</tr>
<tr>
<td>( \geq 1; \text{ Use 1.0} )</td>
<td>( \geq 1; \text{ Use 1.0} )</td>
</tr>
<tr>
<td><strong>Determine</strong> ( B_2 )</td>
<td><strong>Determine</strong> ( B_2 )</td>
</tr>
<tr>
<td>( B_2 = \frac{1}{\alpha P_{sstory}} \geq 1 )</td>
<td>( B_2 = \frac{1}{\alpha P_{sstory}} \geq 1 )</td>
</tr>
<tr>
<td>( \text{where} ) ( \alpha = 1.0 )</td>
<td>( \text{where} ) ( \alpha = 1.6 )</td>
</tr>
<tr>
<td>( P_{sstory} = 5,440 ) kips (from computer output)</td>
<td>( P_{sstory} = 5,120 ) kips (from computer output)</td>
</tr>
<tr>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>------</td>
<td>-----</td>
</tr>
</tbody>
</table>
| \( P_{e\,\text{story}} \) may be taken as: \[
\frac{P_{e\,\text{story}}}{R_{sd}} = \frac{H L}{\Delta_H} \quad \text{(Spec. Eq. A-8-7)}
\]

where \[
R_{sd} = 1 - 0.15 \frac{P_{mf}}{P_{e\,\text{story}}} \quad \text{(Spec. Eq. A-8-8)}
\]

where \( P_{mf} = 2,250 \text{kips} \) (gravity load in moment frame)

\[
R_{sd} = 1 - 0.15 \frac{2,250 \text{kips}}{5,440 \text{kips}}
\]

\[
= 0.938
\]

\( H = 1.0Q_e \)

\[
= 1.0(196 \text{kips})
\]

\( = 196 \text{kips} \)

(Previous seismic force distribution calculations)

\( \Delta_H = 0.718 \text{in.} \) (from computer output)

\[
P_{e\,\text{story}} = 0.938 \frac{(196 \text{kips})(13.5 \text{ft})(12 \text{in./ft})}{0.718 \text{in.}}
\]

\[
= 41,500 \text{kips}
\]

\( B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,\text{story}}}} \geq 1 \quad \text{(Spec. Eq. A-8-6)}
\]

\[
= \frac{1}{1 - \frac{1.0(5,440 \text{kips})}{41,500 \text{kips}}}
\]

\[
= 1.15 \geq 1
\]

Because \( B_2 < 1.7 \), it is verified that it was unnecessary to add the notional loads to the lateral loads for this load combination.

**Calculate amplified moment**

From AISC *Specification* Equation A-8-1,

\[
M_e = (1.0)(0.0 \text{kip-ft}) + (1.15)(233 \text{kip-ft})
\]

\[
= 268 \text{kip-ft}
\]

**Calculate amplified axial load**

\[
P_{nt} = 317 \text{kips} \quad \text{(from computer analysis)}
\]

\( P_{e\,\text{story}} = 2,090 \text{kips} \) (gravity load in moment frame)

\[
R_{sd} = 1 - 0.15 \frac{2,090 \text{kips}}{5,120 \text{kips}}
\]

\[
= 0.939
\]

\( H = 0.75(0.7Q_e) \)

\[
= 0.75(0.7)(196 \text{kips})
\]

\( = 103 \text{kips} \)

(Previous seismic force distribution calculations)

\( \Delta_H = 0.377 \text{in.} \) (from computer output)

\[
P_{e\,\text{story}} = 0.939 \frac{(103 \text{kips})(13.5 \text{ft})(12 \text{in./ft})}{0.377 \text{in.}}
\]

\[
= 41,600 \text{kips}
\]

\( B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,\text{story}}}} \geq 1 \quad \text{(Spec. Eq. A-8-6)}
\]

\[
= \frac{1}{1 - \frac{1.6(5,120 \text{kips})}{41,600 \text{kips}}}
\]

\[
= 1.25 \geq 1
\]

Because \( B_2 < 1.7 \), it is verified that it was unnecessary to add the notional loads to the lateral loads for this load combination.

**Calculate amplified moment**

From AISC *Specification* Equation A-8-1,

\[
M_e = (1.0)(0.0 \text{kip-ft}) + (1.25)(122 \text{kip-ft})
\]

\[
= 153 \text{kip-ft}
\]

**Calculate amplified axial load**

\[
P_{nt} = 295 \text{kips} \quad \text{(from computer analysis)}
\]

For a long frame, such as this one, the change in load
For a long frame, such as this one, the change in load to the interior columns associated with lateral load is negligible.

\[ P_r = P_{rd} + B_2 P_{lt} \quad (\text{Spec. Eq. A-8-2}) \]

\[ = 317 \text{ kips} + (1.15)(0.0 \text{ kips}) \]

\[ = 317 \text{ kips} \]

The flexural and axial stiffness of all members in the moment frame were reduced using 0.8\(E\) in the computer analysis.

Check that the flexural stiffness was adequately reduced for the analysis per AISC Specification Section C2.3(2).

\[ \alpha = 1.0 \]

\[ P_r = 317 \text{ kips} \]

\[ P_y = AF_y = 26.5 \text{ in.}^2 (50.0 \text{ ksi}) = 1,330 \text{ kips} \]

(W14×90 column)

\[ \alpha P_y = \frac{1.0(317 \text{ kips})}{1,330 \text{ kips}} = 0.238 \leq 0.5 \]

Therefore, \( \tau_b = 1.0 \) \textbf{o.k.} \]

Note: By inspection \( \tau_b = 1.0 \) for all of the beams in the moment frame.

For the direct analysis method, \( K = 1.0 \).

From AISC Manual Table 4-1,

\[ P_c = 1,040 \text{ kips} \text{ (W14×90 @ } KL = 13.5 \text{ ft}) \]

From AISC Manual Table 3-2,

\[ M_{ct} = \phi_b M_{px} = 574 \text{ kip-ft (W14×90 with } L_b = 13.5 \text{ ft}) \]

\[ P_r = \frac{317 \text{ kips}}{1,040 \text{ kips}} = 0.305 \geq 0.2 \]

Because \( \frac{P_r}{P_c} \geq 0.2 \), use AISC Specification interaction Equation H1-1a.

\[ \frac{P_r}{P_c} + \left( \frac{8}{9} \right) \left( \frac{M_{cb}}{M_{ct}} + \frac{M_{cy}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1a}) \]

\[ P_r = 295 \text{ kips} \]

\[ P_y = AF_y = 26.5 \text{ in.}^2 (50.0 \text{ ksi}) = 1,330 \text{ kips} \]

(W14×90 column)

\[ \alpha P_y = \frac{1.6(295 \text{ kips})}{1,330 \text{ kips}} = 0.355 \leq 0.5 \]

Therefore, \( \tau_b = 1.0 \) \textbf{o.k.} \]

Note: By inspection \( \tau_b = 1.0 \) for all of the beams in the moment frame.

For the direct analysis method, \( K = 1.0 \).

From AISC Manual Table 4-1,

\[ P_c = 690 \text{ kips} \text{ (W14×90 @ } KL = 13.5 \text{ ft}) \]

From AISC Manual Table 3-2,

\[ M_{ct} = \frac{M_{pt}}{\Omega_b} = 382 \text{ kip-ft (W14×90 with } L_b = 13.5 \text{ ft}) \]

\[ P_r = \frac{295 \text{ kips}}{690 \text{ kips}} = 0.428 \geq 0.2 \]

Because \( \frac{P_r}{P_c} \geq 0.2 \), use AISC Specification interaction Equation H1-1a.

\[ \frac{P_r}{P_c} + \left( \frac{8}{9} \right) \left( \frac{M_{cb}}{M_{ct}} + \frac{M_{cy}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1a}) \]
METHOD 2. EFFECTIVE LENGTH METHOD

Required strengths of frame members must be determined from a second-order analysis. In this example the second-order analysis is performed by amplifying the axial forces and moments in members and connections from a first-order analysis using the provisions of AISC Specification Appendix 8. The available strengths of compression members are calculated using effective length factors computed from a sidesway stability analysis.

A first-order frame analysis is conducted using the load combinations for LRFD or ASD. A minimum lateral load (notional load) equal to 0.2% of the gravity loads is included for any gravity-only load combination. The required load combinations are given in ASCE/SEI 7 and are summarized in Part 2 of the AISC Manual.

A summary of the axial loads, moments and 1st floor drifts from the first-order computer analysis is shown below. The floor diaphragm deflection in the east-west direction was previously determined to be very small and will thus be neglected in these calculations.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.23D \pm 1.0QE + 0.5L + 0.2S) (Controls columns and beams)</td>
<td>(1.01D + 0.75L + 0.75(0.7QE) + 0.75S) (Controls columns and beams)</td>
</tr>
<tr>
<td>For Interior Column Design:</td>
<td>For Interior Column Design:</td>
</tr>
<tr>
<td>(P_u = 317) kips</td>
<td>(P_u = 295) kips</td>
</tr>
<tr>
<td>(M_{1u} = 148) kip-ft (from first-order analysis)</td>
<td>(M_{1u} = 77.9) kip-ft (from first-order analysis)</td>
</tr>
<tr>
<td>(M_{2u} = 233) kip-ft (from first-order analysis)</td>
<td>(M_{2u} = 122) kip-ft (from first-order analysis)</td>
</tr>
<tr>
<td>First-order first story drift = 0.575 in.</td>
<td>First-order first story drift = 0.302 in.</td>
</tr>
</tbody>
</table>

The required second-order flexural strength, \(M_r\), and axial strength, \(P_r\), are calculated as follows:

For typical interior columns, the gravity load moments are approximately balanced; therefore, \(M_{nt} = 0.0\) kips.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_r = B_1M_{nt} + B_2M_{lt}) ((\text{Spec. Eq. A-8-1}))</td>
<td>(M_r = B_1M_{nt} + B_2M_{lt}) ((\text{Spec. Eq. A-8-1}))</td>
</tr>
<tr>
<td>Determine (B_1).</td>
<td>Determine (B_1).</td>
</tr>
<tr>
<td>(P_r = ) required second-order axial strength using LRFD or ASD load combinations, kips</td>
<td>(P_r = ) required second-order axial strength using LRFD or ASD load combinations, kips</td>
</tr>
<tr>
<td>Note that for members subject to axial compression, (B_1) may be calculated based on the first-order estimate (P_r = P_{nt} + P_{lt}).</td>
<td>Note that for members subject to axial compression, (B_1) may be calculated based on the first-order estimate (P_r = P_{nt} + P_{lt}).</td>
</tr>
<tr>
<td>Therefore, (P_r = 317) kips (from first-order computer analysis)</td>
<td>Therefore, (P_r = 295) kips (from first-order computer analysis)</td>
</tr>
<tr>
<td>(I = 999) in.(^4) ((W14 \times 90))</td>
<td>(I = 999) in.(^4) ((W14 \times 90))</td>
</tr>
<tr>
<td>(P_{li} = \frac{\pi^2EI^*}{(KL)^2}) ((\text{Spec. Eq. A-8-5}))</td>
<td>(P_{li} = \frac{\pi^2EI^*}{(KL)^2}) ((\text{Spec. Eq. A-8-5}))</td>
</tr>
</tbody>
</table>
### LRFD

\[
\pi^2 \left( 29,000 \text{ ksi} \right) \left( 999 \text{ in.}^4 \right) \left(1.0 \right) \left(13.5 \text{ ft} \right) \left(12 \text{ in./ft} \right) \left[ \right]^2 = 10,900 \text{ kips}
\]

\[C_m = 0.6 - 0.4 \left( M_1 / M_2 \right) \text{ (Spec. Eq. A-8-4)} \]

\[= 0.6 - 0.4 \left( 148 \text{ kip-ft} / 233 \text{ kip-ft} \right) \]

\[= 0.346\]

\[\alpha = 1.0\]

\[B_1 = \frac{C_m}{1 - \frac{\alpha P_{story}}{P_{el}}} \geq 1 \text{ (Spec. Eq. A-8-3)}\]

\[= \frac{0.346}{1 - \frac{1.0 \left( 317 \text{ kips} \right)}{10,900 \text{ kips}}} \geq 1\]

\[= 0.356 \geq 1; \text{ Use } 1.0\]

Determine \( B_2 \).

\[B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{el}}} \geq 1 \text{ (Spec. Eq. A-8-6)}\]

where

\[\alpha = 1.0\]

\[P_{story} = 5,440 \text{ kips (from computer output)}\]

\[P_{el} \text{ may be taken as}\]

\[P_{el} = R_M \frac{HL}{\Delta_H} \text{ (Spec. Eq. A-8-7)}\]

where

\[R_M = 1 - 0.15 \frac{P_{el}}{P_{story}} \text{ (Spec. Eq. A-8-8)}\]

where

\[P_{el} = 2,250 \text{ kips} \text{ (gravity load in moment frame)}\]

\[R_M = 1 - 0.15 \frac{2,250 \text{ kips}}{5,440 \text{ kips}} = 0.938\]

\[H = 196 \text{ kips} \text{ (Lateral)}\]

(from previous seismic force distribution calculations)

\[\Delta_H = 0.575 \text{ in. (from computer output)}\]

### ASD

\[
\pi^2 \left( 29,000 \text{ ksi} \right) \left( 999 \text{ in.}^4 \right) \left(1.0 \right) \left(13.5 \text{ ft} \right) \left(12 \text{ in./ft} \right) \left[ \right]^2 = 10,900 \text{ kips}
\]

\[C_m = 0.6 - 0.4 \left( M_1 / M_2 \right) \text{ (Spec. Eq. A-8-4)} \]

\[= 0.6 - 0.4 \left( 148 \text{ kip-ft} / 233 \text{ kip-ft} \right) \]

\[= 0.345\]

\[\alpha = 1.6\]

\[B_1 = \frac{C_m}{1 - \frac{\alpha P_{story}}{P_{el}}} \geq 1 \text{ (Spec. Eq. A-8-3)}\]

\[= \frac{0.345}{1 - \frac{1.6 \left( 295 \text{ kips} \right)}{10,900 \text{ kips}}} \geq 1\]

\[= 0.361 \geq 1; \text{ Use } 1.0\]

Determine \( B_2 \).

\[B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{el}}} \geq 1 \text{ (Spec. Eq. A-8-6)}\]

where

\[\alpha= 1.6\]

\[P_{story} = 5,120 \text{ kips (from computer output)}\]

\[P_{el} \text{ may be taken as}\]

\[P_{el} = R_M \frac{HL}{\Delta_H} \text{ (Spec. Eq. A-8-7)}\]

where

\[R_M = 1 - 0.15 \frac{P_{el}}{P_{story}} \text{ (Spec. Eq. A-8-8)}\]

where

\[P_{el} = 2,090 \text{ kips} \text{ (gravity load in moment frame)}\]

\[R_M = 1 - 0.15 \frac{2,090 \text{ kips}}{5,120 \text{ kips}} = 0.939\]

\[H = 103 \text{ kips} \text{ (Lateral)}\]

(from previous seismic force distribution calculations)

\[\Delta_H = 0.302 \text{ in. (from computer output)}\]
LRFD

\[ P_{\text{story}} = 0.938 \left( \frac{196 \text{ kips} \times (13.5 \text{ ft})}{0.575 \text{ in.}} \right) \]
\[ = 51,800 \text{ kips} \]
\[ B_2 = \frac{1}{1 - \frac{\alpha P_{\text{story}}}{P_{\text{story}}}} \geq 1 \quad \text{(Spec. Eq. A-8-6)} \]
\[ = \frac{1}{1 - \frac{1.0(5,440 \text{ kips})}{51,800 \text{ kips}}} \geq 1 \]
\[ = 1.12 \geq 1 \]

Note: \( B_2 < 1.5 \), therefore use of the effective length method is acceptable.

**Calculate amplified moment**

From AISC Specification Equation A-8-1,

\[ M_r = (1.00)(0.0 \text{ kip-ft}) + (1.12)(233 \text{ kip-ft}) \]
\[ = 261 \text{ kip-ft} \]

**Calculate amplified axial load.**

\[ P_{\text{nt}} = 317 \text{ kips} \quad \text{(from computer analysis)} \]

For a long frame, such as this one, the change in load to the interior columns associated with lateral load is negligible.

Therefore, \( P_{\text{lt}} = 0 \)

\[ P_r = P_{\text{nt}} + B_2 P_{\text{lt}} \quad \text{(Spec. Eq. A-8-2)} \]
\[ = 317 \text{ kips} + (1.12)(0.0 \text{ kips}) \]
\[ = 317 \text{ kips} \]

**Determine the controlling effective length.**

For out-of-plane buckling in the moment frame

\[ K_y = 1.0 \]

\[ K_yL_y = 1.0(13.5 \text{ ft}) = 13.5 \text{ ft} \]

For in-plane buckling in the moment frame, use the story stiffness procedure from the AISC Specification Commentary for Appendix 7 to determine \( K_x \) with Specification Commentary Equation C-A-7-5.

ASD

\[ P_{\text{story}} = 0.939 \left( \frac{103 \text{ kips} \times (13.5 \text{ ft})}{0.302 \text{ in.}} \right) \]
\[ = 51,900 \text{ kips} \]
\[ B_2 = \frac{1}{1 - \frac{\alpha P_{\text{story}}}{P_{\text{story}}}} \geq 1 \quad \text{(Spec. Eq. A-8-6)} \]
\[ = \frac{1}{1 - \frac{1.6(5,120 \text{ kips})}{51,900 \text{ kips}}} \geq 1 \]
\[ = 1.19 \geq 1 \]

Note: \( B_2 < 1.5 \), therefore use of the effective length method is acceptable.

**Calculate amplified moment**

From AISC Specification Equation A-8-1,

\[ M_r = (1.00)(0.0 \text{ kip-ft}) + (1.19)(122 \text{ kip-ft}) \]
\[ = 145 \text{ kip-ft} \]

**Calculate amplified axial load.**

\[ P_{\text{nt}} = 295 \text{ kips} \quad \text{(from computer analysis)} \]

For a long frame, such as this one, the change in load to the interior columns associated with lateral load is negligible.

Therefore, \( P_{\text{lt}} = 0 \)

\[ P_r = P_{\text{nt}} + B_2 P_{\text{lt}} \quad \text{(Spec. Eq. A-8-2)} \]
\[ = 295 \text{ kips} + (1.19)(0.0 \text{ kips}) \]
\[ = 295 \text{ kips} \]

**Determine the controlling effective length.**

For out-of-plane buckling in the moment frame

\[ K_y = 1.0 \]

\[ K_yL_y = 1.0(13.5 \text{ ft}) = 13.5 \text{ ft} \]

For in-plane buckling in the moment frame, use the story stiffness procedure from the AISC Specification Commentary for Appendix 7 to determine \( K_x \) with Specification Commentary Equation C-A-7-5.
Simplifying and substituting terms previously calculated results in:

\[
K_s = \frac{P_{\text{s,story}}}{P_{\text{story}}} \geq \sqrt{\frac{P_s}{1.7HL}}
\]

where

\[
P_s = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29,000 \text{ksi}) (999 \text{ in}^2)}{[12 \text{ in./ft}(13.5 \text{ ft})]^2} = 10,900 \text{ kips}
\]

\[
K_s = \frac{5,440 \text{ kips}}{51,800 \text{ kips}} \geq \sqrt{\frac{10,900 \text{ kips}}{1.7(196 \text{ kips})(12 \text{ in./ft})(13.5 \text{ ft})}}
\]

\[
= 1.90 \geq 0.341
\]

Use \(K_s = 1.90\)

\[
\frac{K_sL_s}{r_s/r_y} = \frac{1.90(13.5 \text{ ft})}{1.66} = 15.5 \text{ ft}
\]

Because \(\frac{K_sL_s}{r_s/r_y} > K_rL_r\), use \(KL = 15.5 \text{ ft}\)

From AISC Manual Table 4-1,

\(P_s = 990 \text{ kips (W14\times90 @ KL = 15.5 ft)}\)

From AISC Manual Table 3-2,

\(M_{cs} = 574 \text{ kip-ft (W14\times90 with } L_b = 13.5 \text{ ft)}\)

\[
P_s = \frac{317 \text{ kips}}{990 \text{ kips}} = 0.320 \geq 0.2
\]
Because $\frac{P_e}{P_c} \geq 0.2$, use interaction Equation H1-1a.

$$\frac{P_e}{P_c} + \frac{8}{9} \left( \frac{M_{xx}}{M_{xx}^c} + \frac{M_{yy}}{M_{yy}^c} \right) \leq 1.0 \quad \text{(Spec. Eq. H1.1-a)}$$

$$0.320 + \frac{8}{9} \left( \frac{261 \text{ kip-ft}}{574 \text{ kip-ft}} \right) \leq 1.0$$

$$0.724 \leq 1.0 \quad \text{o.k.}$$

Because $\frac{P_e}{P_c} \geq 0.2$, use interaction Equation H1-1a.

$$\frac{P_e}{P_c} + \frac{8}{9} \left( \frac{M_{xx}}{M_{xx}^c} + \frac{M_{yy}}{M_{yy}^c} \right) \leq 1.0 \quad \text{(Spec. Eq. H1.1-a)}$$

$$0.447 + \frac{8}{9} \left( \frac{145 \text{ kip-ft}}{382 \text{ kip-ft}} \right) \leq 1.0$$

$$0.784 \leq 1.0 \quad \text{o.k.}$$

**METHOD 3. SIMPLIFIED EFFECTIVE LENGTH METHOD**

A simplification of the effective length method using a method of second-order analysis based upon drift limits and other assumptions is described in Chapter 2 of the AISC *Manual*. A first-order frame analysis is conducted using the load combinations for LRFD or ASD. A minimum lateral load (notional load) equal to 0.2% of the gravity loads is included for all gravity-only load combinations. The floor diaphragm deflection in the east-west direction was previously determined to be very small and will thus be neglected in these calculations.

### LRFD

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.23D \pm 1.0Q_E + 0.5L + 0.2S$</td>
<td>(Controls columns and beams)</td>
</tr>
<tr>
<td>From a first-order analysis</td>
<td></td>
</tr>
<tr>
<td>For interior column design:</td>
<td></td>
</tr>
<tr>
<td>$P_a = 317 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$M_{1a} = 148 \text{ kip-ft}$ (from first-order analysis)</td>
<td></td>
</tr>
<tr>
<td>$M_{2a} = 233 \text{ kip-ft}$ (from first-order analysis)</td>
<td></td>
</tr>
<tr>
<td>First story first-order drift = 0.575 in.</td>
<td></td>
</tr>
</tbody>
</table>

### ASD

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$</td>
<td>(Controls columns and beams)</td>
</tr>
<tr>
<td>From a first-order analysis</td>
<td></td>
</tr>
<tr>
<td>For interior column design:</td>
<td></td>
</tr>
<tr>
<td>$P_a = 295 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$M_{1a} = 77.9 \text{ kip-ft}$ (from first-order analysis)</td>
<td></td>
</tr>
<tr>
<td>$M_{2a} = 122 \text{ kip-ft}$ (from first-order analysis)</td>
<td></td>
</tr>
<tr>
<td>First story first-order drift = 0.302 in.</td>
<td></td>
</tr>
</tbody>
</table>

Then the following steps are executed.

### LRFD

**Step 1:**

Lateral load = 196 kips

Deflection due to first-order elastic analysis

$\Delta = 0.575 \text{ in. between first and second floor}$

Floor height = 13.5 ft

Drift ratio = (13.5 ft)(12 in./ft) / 0.575 in. = 282

**Step 2:**

Design story drift limit = $H/400$

Adjusted Lateral load = (282/400)(196 kips) = 138 kips

### ASD

**Step 1:**

Lateral load = 103 kips

Deflection due to first-order elastic analysis

$\Delta = 0.302 \text{ in. between first and second floor}$

Floor height = 13.5 ft

Drift ratio = (13.5 ft)(12 in./ft) / 0.302 in. = 536

**Step 2:**

Design story drift limit = $H/400$

Adjusted Lateral load = (536/400)(103 kips) = 138 kips
### LRFD

**Step 3:**

Load ratio $= (1.0) \frac{\text{total story load}}{\text{lateral load}}$

$$= (1.0) \frac{5,440 \text{ kips}}{138 \text{ kips}}$$

$$= 39.4$$

From AISC Manual Table 2-1:

$B_2 = 1.1$

Which matches the value obtained in Method 2 to the two significant figures of the table

Note: Intermediate values are not interpolated from the table because the precision of the table is two significant digits. Additionally, the design story drift limit used in Step 2 need not be the same as other strength or serviceability drift limits used during the analysis and design of the structure.

**Step 4.** Multiply all the forces and moment from the first-order analysis by the value of $B_2$ obtained from the table. This presumes that $B_1$ is less than or equal to $B_2$, which is usually the case for members without transverse loading between their ends.

$$M_r = B_2(M_{nt} + M_{lt})$$

$$= 1.1(0 \text{ kip-ft + 233 kip-ft}) = 256 \text{ kip-ft}$$

$$P_r = B_2(P_{nt} + P_{lt})$$

$$= 1.1(317 \text{ kips +0.0 kips}) = 349 \text{ kips}$$

From AISC Manual Table 4-1,

$P_c = 1,040 \text{ kips (W14×90 @ KL = 13.5 ft)}$

From AISC Manual Table 3-2,

$M_{cx} = \phi_r M_{px} = 574 \text{ kip-ft (W14×90 with } L_b = 13.5 \text{ ft)}$

$$\frac{P_r}{P_c} = \frac{349 \text{ kips}}{1,040 \text{ kips}} = 0.336 \geq 0.2$$

Because $\frac{P_r}{P_c} \geq 0.2$, use interaction Equation H1-1a.

### ASD

**Step 3:** (for an ASD design the ratio must be multiplied by 1.6)

Load ratio $= (1.6) \frac{\text{total story load}}{\text{lateral load}}$

$$= (1.6) \frac{5,120 \text{ kips}}{138 \text{ kips}}$$

$$= 59.4$$

From AISC Manual Table 2-1:

$B_2 = 1.2$

Which matches the value obtained in Method 2 to the two significant figures of the table

**Step 5.** Since the selection is in the shaded area of the chart, ($B_2 \leq 1.1$). For LRFD design, use $K = 1.0$.

Multiply both sway and non-sway moments by $B_2$.

$$M_r = B_2(M_{nt} + M_{lt})$$

$$= 1.2(0 \text{ kip-ft + 122 kip-ft}) = 146 \text{ kip-ft}$$

$$P_r = B_2(P_{nt} + P_{lt})$$

$$= 1.2(295 \text{ kips +0.0 kips}) = 354 \text{ kips}$$

From AISC Manual Table 4-1,

$P_c = 675 \text{ kips (W14×90 @ KL = 13.5 ft)}$

From AISC Manual Table 3-2,

$M_{cx} = M_{px} = 382 \text{ kip-ft (W14×90 with } L_b = 13.5 \text{ ft)}$

$$\frac{P_r}{P_c} = \frac{354 \text{ kips}}{675 \text{ kips}} = 0.524 \geq 0.2$$

Because $\frac{P_r}{P_c} \geq 0.2$, use interaction Equation H1-1a.
BEAM ANALYSIS IN THE MOMENT FRAME

The controlling load combinations for the beams in the moment frames are shown below and evaluated for the second floor beam. The dead load, live load and seismic moments were taken from a computer analysis. The table summarizes the calculation of $B_2$ for the stories above and below the second floor.

### 1st – 2nd

<table>
<thead>
<tr>
<th>LRFD Combination</th>
<th>ASD Combination 1</th>
<th>ASD Combination 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>196 kips</td>
<td>137 kips</td>
</tr>
<tr>
<td>$L$</td>
<td>13.5 ft</td>
<td>13.5 ft</td>
</tr>
<tr>
<td>$\Delta_{H}$</td>
<td>0.575 in.</td>
<td>0.402 in.</td>
</tr>
<tr>
<td>$P_{nf}$</td>
<td>2,250 kips</td>
<td>1,640 kips</td>
</tr>
<tr>
<td>$R_{d}$</td>
<td>0.938</td>
<td>0.937</td>
</tr>
<tr>
<td>$P_{story}$</td>
<td>51,800 kips</td>
<td>51,700 kips</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1.12</td>
<td>1.14</td>
</tr>
</tbody>
</table>

### 2nd – 3rd

<table>
<thead>
<tr>
<th>LRFD Combination</th>
<th>ASD Combination 1</th>
<th>ASD Combination 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>170 kips</td>
<td>119 kips</td>
</tr>
<tr>
<td>$L$</td>
<td>13.5 ft</td>
<td>13.5 ft</td>
</tr>
<tr>
<td>$\Delta_{H}$</td>
<td>0.728 in.</td>
<td>0.509 in.</td>
</tr>
<tr>
<td>$P_{nf}$</td>
<td>1,590</td>
<td>1,160</td>
</tr>
<tr>
<td>$R_{d}$</td>
<td>0.938</td>
<td>0.937</td>
</tr>
<tr>
<td>$P_{story}$</td>
<td>35,500 kips</td>
<td>35,500 kips</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1.12</td>
<td>1.14</td>
</tr>
</tbody>
</table>

For beam members, the larger of the $B_2$ values from the story above or below is used.

From computer output at the controlling beam:

$M_{dead} = 153$ kip-ft  
$M_{live} = 80.6$ kip-ft  
$M_{snow} = 0.0$ kip-ft  
$M_{earthquake} = 154$ kip-ft

<table>
<thead>
<tr>
<th>LRFD Combination</th>
<th>ASD Combination 1</th>
<th>ASD Combination 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_2M_B = 1.12(154 \text{ kip-ft})$</td>
<td>$B_2M_B = 1.14(154 \text{ kip-ft})$</td>
<td>$B_2M_B = 1.20(154 \text{ kip-ft})$</td>
</tr>
<tr>
<td>$= 172$ kip-ft</td>
<td>$= 176$ kip-ft</td>
<td>$= 185$ kip-ft</td>
</tr>
</tbody>
</table>
Calculate $C_b$ for compression in the bottom flange braced at 10.0 ft o.c.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| $C_b = 1.86$ (from computer output) | $1.02D + 0.7Q_E$
| $1.01D + 0.75(0.7Q_E) + 0.75L$
<p>| Check W24×55 | Check W24×55 |
| From AISC Manual Table 3-2, with continuous bracing | From AISC Manual Table 3-2, with continuous bracing |
| $\phi M_u = 503$ kip-ft | $\frac{M_u}{\Omega} = 334$ kip-ft |
| From AISC Manual Table 3-10, for $L_b = 10.0$ ft and $C_b = 1.86$ | From AISC Manual Table 3-10, for $L_b = 10.0$ ft and $C_b = 1.86$ |
| $\phi M_u = (386$ kip-ft)1.86 $\leq 503$ kip-ft | $\frac{M_u}{\Omega} = (256$ kip-ft)1.86 $\leq 334$ kip-ft |
| $= 718$ kip-ft $\leq 503$ kip-ft | $= 476$ kip-ft $\leq 334$ kip-ft |
| Use $\phi M_u = 503$ kip-ft $&gt; 400$ kip-ft o.k. | Use $\frac{M_u}{\Omega} = 334$ kip-ft $&gt; 279$ kip-ft o.k. |
| From AISC Manual Table 3-2, a W24×55 has a design shear strength of 252 kips and an $I_x$ of 1350 in.$^4$ | $1.01D + 0.75(0.7Q_E) + 0.75L$ |
| With continuous bracing | With continuous bracing |
| $\frac{M_u}{\Omega} = 334$ kip-ft | $\frac{M_u}{\Omega} = 334$ kip-ft |
| From AISC Manual Table 3-10, for $L_b = 10$ ft and $C_b = 2.01$ | From AISC Manual Table 3-10, for $L_b = 10$ ft and $C_b = 2.01$ |
| $\frac{M_u}{\Omega} = (256$ kip-ft)2.01 | $\frac{M_u}{\Omega} = (256$ kip-ft)2.01 |
| $= 515$ kip-ft $\leq 334$ kip-ft | $= 515$ kip-ft $\leq 334$ kip-ft |
| Use $\frac{M_u}{\Omega} = 334$ kip-ft $&gt; 312$ kip-ft o.k. | Use $\frac{M_u}{\Omega} = 334$ kip-ft $&gt; 312$ kip-ft o.k. |</p>
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From AISC Manual Table 3-2, a W24×55 has an allowable shear strength of 167 kips and an $I_x$ of 1,350 in.$^4$</td>
</tr>
</tbody>
</table>

The moments and shears on the roof beams due to the lateral loads were also checked but do not control the design.

The connections of these beams can be designed by one of the techniques illustrated in the Chapter IIB of the design examples.
BRACED FRAME ANALYSIS

The braced frames at Grids 1 and 8 were analyzed for the required load combinations. The stability design requirements from Chapter C were applied to this system.

The model layout, nominal dead, live, and snow loads with associated notional loads, wind loads and seismic loads are shown in the figures below:
Second-order analysis by amplified first-order analysis

In the following, the approximate second-order analysis method from AISC Specification Appendix 8 is used to account for second-order effects in the braced frames by amplifying the axial forces in members and connections from a first-order analysis.

A first-order frame analysis is conducted using the load combinations for LRFD and ASD. From this analysis the critical axial loads, moments and deflections are obtained.

A summary of the axial loads and 1st floor drifts from the first-order computer analysis is shown below. The floor diaphragm deflection in the north-south direction was previously determined to be very small and will thus be neglected in these calculations.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams)</td>
<td>$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$ (Controls columns and beams)</td>
</tr>
<tr>
<td>From a first-order analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>For interior column design:</td>
<td>$P_{nt} = 236$ kips</td>
<td>$P_{nt} = 219$ kips</td>
</tr>
<tr>
<td>$P_h = 146$ kips</td>
<td>$P_h = 76.6$ kips</td>
<td></td>
</tr>
<tr>
<td>The moments are negligible</td>
<td>The moments are negligible</td>
<td></td>
</tr>
<tr>
<td>First story first-order drift = 0.211 in.</td>
<td>First story first-order drift = 0.111 in.</td>
<td></td>
</tr>
</tbody>
</table>

The required second-order axial strength, $P_r$, is computed as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_r = P_{nt} + B_2P_h$</td>
<td>$P_r = P_{nt} + B_2P_h$</td>
</tr>
<tr>
<td>Determine $B_2$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{story}}} \geq 1$</td>
<td>$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{story}}} \geq 1$</td>
<td></td>
</tr>
<tr>
<td>$P_{story} = 5,440$ kips (previously calculated)</td>
<td>$P_{story} = 5,120$ kips (previously calculated)</td>
<td></td>
</tr>
<tr>
<td>$P_{story}$ may be calculated as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{story} = \frac{R_M H L}{\Delta H}$ (Spec. Eq. A-8-7)</td>
<td>$P_{story} = \frac{R_M H L}{\Delta H}$ (Spec. Eq. A-8-7)</td>
<td></td>
</tr>
<tr>
<td>where</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H = 196$ kips (from previous calculations)</td>
<td>$H = 103$ kips (from previous calculations)</td>
<td></td>
</tr>
<tr>
<td>$\Delta H = 0.211$ in. (from computer output)</td>
<td>$\Delta H = 0.111$ in. (from computer output)</td>
<td></td>
</tr>
<tr>
<td>$R_M = 1.0$ for braced frames</td>
<td>$R_M = 1.0$ for braced frames</td>
<td></td>
</tr>
<tr>
<td>$P_{story} = 1.0 \left( \frac{196 \text{ kips}}{0.211 \text{ in.}} \right) \left( 13.5 \text{ ft} \right) \left( 12 \text{ in./ft} \right)$ = 150,000 kips</td>
<td>$P_{story} = 1.0 \left( \frac{103 \text{ kips}}{0.111 \text{ in.}} \right) \left( 13.5 \text{ ft} \right) \left( 12 \text{ in./ft} \right)$ = 150,000 kips</td>
<td></td>
</tr>
</tbody>
</table>
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Note: Notice that the lower sidesway displacements of the braced frame produce much lower values of $B_2$ than those of the moment frame. Similar results could be expected for the other two methods of analysis.

Although not presented here, second-order effects should be accounted for in the design of the beams and diagonal braces in the braced frames at Grids 1 and 8.
ANALYSIS OF DRAG STRUTS

The fourth floor delivers the highest diaphragm force to the braced frames at the ends of the building: \( E = 80.3 \) kips (from previous calculations). This force is transferred to the braced frame through axial loading of the W18×35 beams at the end of the building.

The gravity dead loads for the edge beams are the floor loading of 75.0 psf (5.50 ft) plus the exterior wall loading of 0.503 kip/ft, giving a total dead load of 0.916 kip/ft. The gravity live load for these beams is the floor loading of 80.0 psf (5.50 ft) = 0.440 kip/ft. The resulting midspan moments are \( M_{\text{Dead}} = 58.0 \) kip-ft and \( M_{\text{Live}} = 27.8 \) kip-ft.

The controlling load combination for LRFD is \( 1.23D + 1.0Q_E + 0.50L \). The controlling load combinations for ASD are \( 1.01D + 0.75L + 0.75(0.7Q_E) \) or \( 1.02D + 0.7Q_E \).

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
M_a = 1.23(58.0 \text{ kip-ft}) + 0.50(27.8 \text{ kip-ft}) & M_a = 1.01(58.0 \text{ kip-ft}) + 0.75(27.8 \text{ kip-ft}) \\
= 85.2 \text{ kip-ft} & = 79.4 \text{ kip-ft} \\
\text{Load from the diaphragm shear due to earthquake loading} & \text{Load from the diaphragm shear due to earthquake loading} \\
F_p = 80.3 \text{ kips} & F_p = 0.75(0.70)(80.3 \text{ kips}) = 42.2 \text{ kips} \\
& \text{or} \\
& F_p = 0.70(80.3 \text{ kips}) = 56.2 \text{ kips} \\
\hline
\end{array}
\]

Only the two 45 ft long segments on either side of the brace can transfer load into the brace, because the stair opening is in front of the brace.

Use AISC Specification Section H2 to check the combined bending and axial stresses.

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
V = \frac{80.3 \text{ kips}}{2(45.0 \text{ ft})} = 0.892 \text{ kip/ft} & V = \frac{42.2 \text{ kips}}{2(45.0 \text{ ft})} = 0.469 \text{ kip/ft} \\
\text{The top flange bending stress is} & \text{The top flange stress due to bending} \\
f_b = \frac{M_a}{S_t} & f_b = \frac{M_a}{S_t} \\
\frac{85.2 \text{ kip-ft}(12 \text{ in./ft})}{57.6 \text{ in.}^3} & \frac{79.4 \text{ kip-ft}(12 \text{ in./ft})}{57.6 \text{ in.}^3} \\
= 17.8 \text{ ksi} & = 16.5 \text{ ksi} \\
\hline
\end{array}
\]
Note: It is often possible to resist the drag strut force using the slab directly. For illustration purposes, this solution will instead use the beam to resist the force independently of the slab. The full cross section can be used to resist the force if the member is designed as a column braced at one flange only (plus any other intermediate bracing present, such as from filler beams). Alternatively, a reduced cross section consisting of the top flange plus a portion of the web can be used. Arbitrarily use the top flange and 8 times an area of the web equal to its thickness times a depth equal to its thickness, as an area to carry the drag strut component.

Area = 6.00 in.(0.425 in.) + 8(0.300 in.)² = 2.55 in.² + 0.720 in.² = 3.27 in.²

Ignoring the small segment of the beam between Grid C and D, the axial stress due to the drag strut force is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_a = \frac{80.3 \text{kips}}{2(3.27 \text{ in.}^2)} )</td>
<td>( f_a = \frac{42.2 \text{kips}}{2(3.27 \text{ in.}^2)} )</td>
</tr>
<tr>
<td>= 12.3 ksi</td>
<td>= 6.45 ksi</td>
</tr>
</tbody>
</table>

Using AISC Specification Section H2, assuming the top flange is continuously braced:

\[ F_a = \phi_b F_y \]
\[ = 0.90(50 \text{ ksi}) \]
\[ = 45.0 \text{ ksi} \]

\[ F_{bw} = \phi_b F_y \]
\[ = 0.90(50 \text{ ksi}) \]
\[ = 45.0 \text{ ksi} \]

\[ \frac{f_a + f_{bw}}{F_a} \leq 1.0 \quad (\text{from Spec. Eq. H2-1}) \]
\[ \frac{12.3 \text{ksi} + 17.8 \text{ksi}}{45.0 \text{ksi} + 45.0 \text{ksi}} = 0.669 \quad \text{o.k.} \]

Load Combination 1:

\[ 6.45 \text{ksi} + 16.5 \text{ksi} = 0.768 \quad \text{o.k.} \]

Load Combination 2:
Note: Because the drag strut load is a horizontal load, the method of transfer into the strut, and the extra horizontal load which must be accommodated by the beam end connections should be indicated on the drawings.

\[
\frac{8.59 \text{ ksi}}{29.9 \text{ ksi}} + \frac{12.3 \text{ ksi}}{29.9 \text{ ksi}} = 0.699 \quad \text{o.k.}
\]
PART III EXAMPLE REFERENCES

ASCE (2010), Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, Reston, VA.


SJI (2005), Load Tables and Weight Tables for Steel Joists and Joist Girders, 42nd Ed., Steel Joist Institute, Forest, VA.

Part IV
Additional Resources

This part contains additional design aids that are not available in the AISC Steel Construction Manual.
DESIGN TABLE DISCUSSION

Table 4-1. W-Shapes in Axial Compression, 65 ksi steel
Available strengths in axial compression are given for W-shapes with $F_y = 65$ ksi (ASTM A913 Grade 65). The tabulated values are given for the effective length with respect to the y-axis ($KL_y$). However, the effective length with respect to the x-axis ($KL_x$), must also be investigated. To determine the available strength in axial compression, the table should be entered at the larger of ($KL_y$) and ($KL_y_{eq}$), where

$$ (KL)_{y_{eq}} = \frac{(KL)_x}{r_x} \frac{r_y}{(KL)_y} \quad (4-1) $$

The available strength is based on the limit states of flexural buckling, torsional buckling, and flexural-torsional buckling. The limit between elastic and inelastic buckling is $KL \leq 99.5$ with $F_y = 65$ ksi.

The slenderness limit between a nonslender web and a slender web is $\lambda_{yw} = 31.5$ with $F_y = 65$ ksi. All current ASTM A6 W-shapes have nonslender flanges with $F_y = 65$ ksi.

Values of the ratio $r_x/r_y$ and other properties useful in the design of W-shape compression members are listed at the bottom of Table 4-1.

Variables $P_{wo}$, $P_{wi}$, $P_{wb}$, and $P_{fb}$ shown in Table 4-1 can be used to determine the strength of W-shapes without stiffeners to resist concentrated forces applied normal to the face(s) of the flange(s). In these tables it is assumed that the concentrated forces act far enough away from the member ends that end effects are not considered (end effects are addressed in Chapter 9). When $P_r \leq \phi R_n$ or $R_n/\Omega$, column web stiffeners are not required. Figures 4-1, 4-2 and 4-3 illustrate the limit states and the applicable variables for each.

Web Local Yielding: The variables $P_{wo}$ and $P_{wi}$ can be used in the calculation of the available web local yielding strength for the column as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n$</td>
<td>$P_{wo} + P_{wb}l_b$</td>
<td>$R_n/\Omega = P_{wo} + P_{wb}l_b$</td>
</tr>
</tbody>
</table>

where

$$ R_n = F_{yw}l_w(5k + l_b) = 5F_{yw}t_wk + F_{yw}l_wl_b \text{, kips (AISC Specification Equation J10-2)} $$

$$ P_{wo} = \phi 5F_{yw}t_wk \text{ for LRFD and } 5F_{yw}t_wk/\Omega \text{ for ASD, kips} $$

$$ P_{wi} = \phi F_{yw}t_w \text{ for LRFD and } F_{yw}t_w/\Omega \text{ for ASD, kips/in.} $$

$k$ = distance from outer face of flange to the web toe of fillet, in.

$l_b$ = length of bearing, in.

$t_w$ = thickness of web, in.

$\phi$ = 1.00

$\Omega$ = 1.50

Web Compression Buckling: The variable $P_{wb}$ is the available web compression buckling strength for the column as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n$</td>
<td>$P_{wb}$</td>
<td>$R_n/\Omega = P_{wb}$</td>
</tr>
</tbody>
</table>

where

$$ R_n = 24t_w^3 \sqrt{EF_{yw}}/h \text{ (AISC Specification Equation J10-8)} $$


Flange Local Buckling: The variable $P_{fb}$ is the available flange local bending strength for the column as follows:

\[
P_{fb} = \phi 6.25 F_{yf} t_f^2 \text{ for LRFD and } 6.25 F_{yf} t_f^2 / \Omega \text{ for ASD, kips}
\]

where

- $R_n = 6.25 F_{yf} t_f^2$, kips (AISC Specification Equation J10-1)
- $P_{fb} = \phi 6.25 F_{yf} t_f^2$ for LRFD and $6.25 F_{yf} t_f^2 / \Omega$ for ASD, kips
- $\phi = 0.90$
- $\Omega = 1.67$

Fig. 4-2. Illustration of web compression buckling limit state *(AISC Specification Section J10.5).*
Fig. 4-3. Illustration of flange local bending limit state (AISC Specification Section J10.1).
### Table 4-1

Available Strength in Axial Compression, kips

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<tr>
<th>W-Shape</th>
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### Effective length, $K_L$ (ft), with respect to least radius of gyration, $r_g$

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### Properties

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### ASD LRFD

| $\Omega_c = 1.67$ | $\xi_c = 0.90$ |

<sup>Flange thickness is greater than 2 in. Special requirements may apply per AISCSpecification Section A3.1c.</sup>
**Table 4-1 (continued)**

**Available Strength in Axial Compression, kips**

*W-Shapes*

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*Flange thickness is greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.*

---

**Design Examples V14.1**

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
Table 4-1 (continued)

Available Strength in Axial Compression, kips

| Shape | W14 | Design | \( P_{wo} \), kips | \( P_{wi} \), kips/in. | \( P_{wb} \), kips | \( L_r \), ft | \( L_p \), ft | \( A_y \), in.² | \( I_{x} \), in.⁴ | \( I_{y} \), in.⁴ | \( r_y \), in. | \( f_{y}/r_{y} \) | \( P_{aw}(KL)^2/10^4 \), k-in.² | \( P_{aw}(KL)^2/10^4 \), k-in.² |
|-------|-----|--------|------------------|----------------------|----------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| ASD   | LRFD | ASD   | LRFD | ASD   | LRFD | ASD   | LRFD | ASD   | LRFD | ASD   | LRFD | ASD   | LRFD | ASD   | LRFD |
| 0     | 2940 | 4420  | 2670 | 4010  | 2410  | 3630  | 2210  | 3220  | 1960  | 2940  | 1760  | 2650  | 2020  | 3030  | 1820  |
| 6     | 2860 | 4300  | 2590 | 3890  | 2340  | 3520  | 2150  | 3220  | 1960  | 2940  | 1760  | 2650  | 2020  | 3030  | 1820  |

\( F_y = 65 \text{ ksi} \)

\( \phi_c = 0.90 \)
## Table 4-1 (continued)

### Available Strength in Axial Compression, kips

#### W-Shapes

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### Properties

- $P_{wo}$, kips: 249, 373, 228, 342, 197, 295, 166, 249, 145, 218, 125, 187
- $P_{wb}$, kips/in.: 29.5, 44.2, 28.0, 41.9, 25.6, 38.4, 22.8, 34.1, 21.0, 31.5, 19.1, 28.6
- $P_{wo}$, kips: 543, 816, 464, 697, 356, 535, 251, 377, 197, 297, 147, 222
- $P_{wb}$, kips: 289, 434, 258, 388, 215, 323, 180, 270, 148, 222, 123, 184

### Effective length, KL (ft), with respect to least radius of gyration, $r_y$

- $L_p$, ft: 12.3, 11.6, 11.6, 11.6, 11.5, 11.5
- $L_r$, ft: 48.7, 44.3, 41.5, 39.1, 36.8, 34.9

### Axial Compression, kips

- $A_p$, in.$^2$: 42.7, 38.8, 35.3, 32.0, 29.1, 26.5
- $l_p$, in.$^4$: 1710, 1530, 1380, 1240, 1110, 999
- $I_p$, in.$^4$: 677, 548, 495, 447, 402, 362
- $r_y$, in.: 3.98, 3.76, 3.74, 3.73, 3.71, 3.70
- $f_p/r_y$: 1.59, 1.67, 1.67, 1.67, 1.66, 1.66

### $P_{wo}(KL)^2/10^4$, k-in.$^2$

- 48900, 43800, 39500, 35500, 31800, 28600

### $P_{wb}(KL)^2/10^4$, k-in.$^2$

- 19400, 15700, 14200, 12800, 11500, 10400

### ASD LRFD

- $\Omega_c = 1.67$, $\phi_c = 0.90$

---

**Design Examples V14.1**

**AMERICAN INSTITUTE OF STEEL CONSTRUCTION**
### Table 4-1 (continued)

**Available Strength in Axial Compression, kips**

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#### Properties

| $P_{aw}$, kips | 160 | 240 | 135 | 202 | 116 | 177 | 101 | 151 | 100 | 150 | 67.7 | 131 | 74.0 | 111 |
| $P_{aw}$, kips/ln | 22.1 | 33.2 | 19.5 | 29.3 | 18.0 | 27.0 | 16.3 | 24.4 | 16.0 | 24.1 | 14.7 | 22.1 | 13.2 | 19.8 |
| $P_{aw}$, kips | 229 | 344 | 157 | 236 | 124 | 186 | 91.3 | 137 | 87.4 | 131 | 67.9 | 102 | 49.1 | 73.8 |
| $P_m$, kips | 178 | 267 | 150 | 225 | 126 | 190 | 101 | 152 | 106 | 159 | 86.1 | 129 | 68.3 | 103 |

| $L_p$, ft | 7.68 | 7.68 | 7.62 | 7.59 | 5.95 | 5.92 | 5.86 |
| $L_f$, ft | 26.7 | 25.2 | 24.0 | 22.7 | 18.4 | 17.6 | 16.8 |

| $A_p$, in.² | 24.0 | 21.8 | 20.0 | 17.9 | 15.6 | 14.1 | 12.6 |
| $I_p$, in.⁴ | 881 | 795 | 722 | 640 | 541 | 484 | 428 |
| $I_f$, in.⁴ | 148 | 134 | 121 | 107 | 57.7 | 51.4 | 45.2 |
| $r_y$, in | 2.48 | 2.48 | 2.46 | 2.46 | 1.92 | 1.91 | 1.89 |
| $r_y/r_f$ | 2.44 | 2.44 | 2.44 | 2.44 | 3.07 | 3.06 | 3.08 |

| $P_{aw}(KL)^2/10^4$, k-in.² | 25200 | 22800 | 20700 | 18300 | 15500 | 13900 | 12300 |
| $P_{aw}(KL)^2/10^4$, k-in.² | 4240 | 3840 | 3460 | 3060 | 1650 | 1470 | 1290 |

#### ASD

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| $\phi_c = 0.90$ |

### Note:
- Heavy line indicates $KL/r_f$, with respect to least radius of gyration, $r_f$.
- Shape is slender for compression with $F_y = 65$ ksi.

### Design Examples V14.1

*AMERICAN INSTITUTE OF STEEL CONSTRUCTION*
### Table 4-1 (continued)

#### Available Strength in Axial Compression, kips

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#### Properties

- Flange thickness is greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.

| P<sub>mo</sub>, kips | 1370 | 2050 | 1170 | 1750 | 1020 | 1530 | 865 | 1300 | 746 | 1120 | 639 | 959 |
| P<sub>mo</sub>, kips/in. | 77.1 | 116 | 70.6 | 106 | 66.3 | 99.5 | 60.7 | 91.0 | 55.9 | 83.9 | 51.1 | 76.7 |
| P<sub>mo</sub>, kips | 11400 | 17200 | 8770 | 13200 | 7270 | 10900 | 5560 | 8350 | 4340 | 6530 | 3340 | 5020 |
| P<sub>n</sub>, kips | 2130 | 3200 | 1790 | 2690 | 1480 | 2230 | 1230 | 1850 | 1040 | 1570 | 878 | 1320 |
| L<sub>p</sub>, ft | 10.7 | 10.6 | 10.5 | 10.3 | 10.3 | 10.2 |
| L<sub>r</sub>, ft | 116 | 105 | 96.8 | 88.0 | 80.7 | 73.9 |
| A<sub>v</sub>, in. | 98.9 | 89.5 | 81.9 | 74.1 | 67.7 | 61.8 |
| I<sub>v</sub>, in. | 4060 | 3550 | 3110 | 2720 | 2420 | 2140 |
| I<sub>r</sub>, in. | 1190 | 1050 | 937 | 828 | 742 | 664 |
| r<sub>y</sub>, in. | 3.47 | 3.42 | 3.38 | 3.34 | 3.31 | 3.28 |
| r<sub>y</sub>/r<sub>y</sub> | 1.85 | 1.84 | 1.82 | 1.81 | 1.80 | 1.80 |
| P<sub>av</sub>(KL)<sup>2</sup>/10<sup>4</sup>, k-in.<sup>2</sup> | 116000 | 102000 | 89000 | 77900 | 69300 | 61300 |
| P<sub>av</sub>(KL)<sup>2</sup>/10<sup>5</sup>, k-in.<sup>2</sup> | 34100 | 30100 | 26800 | 23700 | 21200 | 19000 |
| ASD | LRFD |
| ω<sub>r</sub> = 1.67 | ϕ<sub>r</sub> = 0.90 |

Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 4-1 (continued)

**Available Strength in Axial Compression, kips**

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**Effective length, $KL$, (ft), with respect to least radius of gyration, $r_y$**

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**Properties**

| $P_{aw}$, kips | 535 | 803 | 449 | 674 | 377 | 566 | 317 | 475 | 262 | 392 | 210 | 315 |
| $P_{aw}$, kips/in. | 45.9 | 68.9 | 41.6 | 62.4 | 37.7 | 56.6 | 34.2 | 51.4 | 30.8 | 46.2 | 26.4 | 39.7 |
| $P_{aw}$, kips | 2420 | 3640 | 1800 | 2710 | 1330 | 2000 | 1000 | 1500 | 726 | 1090 | 613 | 921 |
| $P_m$, kips | 737 | 1110 | 592 | 890 | 477 | 717 | 380 | 571 | 300 | 450 | 238 | 358 |
| $L_r$, ft | 67.4 | 60.7 | 54.8 | 49.1 | 44.2 | 39.9 |
| $A_g$, in.$^2$ | 56.0 | 50.0 | 44.7 | 39.9 | 35.2 | 31.2 |
| $l_x$, in. | 1890 | 1650 | 1430 | 1240 | 1070 | 933 |
| $l_y$, in. | 589 | 517 | 454 | 398 | 345 | 301 |
| $r_y$, in. | 3.25 | 3.22 | 3.19 | 3.16 | 3.13 | 3.11 |
| $r_x/r_y$ | 1.79 | 1.78 | 1.77 | 1.77 | 1.76 | 1.76 |
| $P_{aw}(KL)^2/10^4$, k-in.$^2$ | 54100 | 47200 | 40900 | 35500 | 30600 | 26700 |
| $P_{aw}(KL)^2/10^4$, k-in.$^2$ | 16900 | 14800 | 13000 | 11400 | 9870 | 8620 |

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### Table 4-1 (continued)

#### Available Strength in Axial Compression, kips

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#### Design Examples V14.1

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**Properties**

- L_p, ft
- Ax, in.²
- I_x, in.⁴
- I_y, in.⁴
- r_x
- r_y
- P = (KL)²/10⁴, k-in.²
- P_x = (KL)²/10⁴, k-in.²

**American Institute of Steel Construction**
### Table 4-1 (continued)

**Available Strength in Axial Compression, kips**

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*Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.*

- $F_y = 65$ ksi
- $\phi_c = 0.90$

**Effective length, $KL$, (ft), with respect to least radius of gyration, $r_y$**

**Properties**

- $P_{wo}$, kips
- $P_{wi}$, kips
- $P_{wb}$, kips
- $L_p$, ft
- $L_r$, ft
- $A_g$, in.²
- $I_x$, in.⁴
- $I_y$, in.⁴
- $r_y$, in.
- $r_x/r_y$
- $P_{ey} (KL)^2/104$, k-in.²
- $P_{ex} (KL)^2/104$, k-in.²

- **ASD LRFD**

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<th>$P_{n}/\Omega_c$</th>
<th>$\phi_c P_n$</th>
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**Available Strength in Axial Compression, kips**

- **W-Shapes**

**Design Examples V14.1**

**American Institute of Steel Construction**
### Table 4-1 (continued)
**Available Strength in Axial Compression, kips**

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| Effective length, \( KL \) (ft), with respect to least radius of gyration, \( r_y \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( P_{aw} \), kips | 286 | 429 | 239 | 358 | 195 | 293 | 157 | 236 | 129 | 194 | 107 | 161 |
| \( P_{aw}/\text{Kips} \) | 32.7 | 49.1 | 29.5 | 44.2 | 26.2 | 39.3 | 23.0 | 34.5 | 20.4 | 30.6 | 18.2 | 27.3 |
| \( P_{aw} \), kips | 1080 | 1630 | 786 | 1180 | 556 | 835 | 374 | 563 | 261 | 392 | 186 | 280 |
| \( L_{p} \), ft | 380 | 571 | 305 | 459 | 238 | 358 | 184 | 277 | 144 | 217 | 112 | 169 |
| \( A_{p} \), in.² | 8.30 | 8.21 | 8.15 | 8.05 | 8.02 | 7.96 |
| \( I_{p} \), in.⁴ | 49.6 | 44.8 | 39.9 | 35.5 | 32.1 | 29.2 |
| \( A_{y} \), in.² | 32.9 | 29.3 | 26.0 | 22.7 | 19.9 | 17.7 |
| \( I_{y} \), in.⁴ | 716 | 623 | 534 | 455 | 394 | 341 |
| \( r_{y} \), in. | 236 | 207 | 179 | 154 | 134 | 116 |
| \( r_{y}/r_{p} \) | 1.74 | 1.74 | 1.73 | 1.73 | 1.71 | 1.71 |
| \( P_{aw}(KL)^{2}/10^{4} \), in.² | 20500 | 17800 | 15300 | 13000 | 11300 | 9760 |
| \( P_{aw}(KL)^{2}/10^{4} \), in.² | 6750 | 5920 | 5120 | 4410 | 3840 | 3320 |

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#### Available Strength in Axial Compression, kips

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#### Properties

- \( P_{wo}, \) kips
- \( P_{wi}, \) kips/ln
- \( P_{wb}, \) kips
- \( P_{ey}, \) \((KL)^2/10^4\), k-in.²
- \( P_{ex}, \) \((KL)^2/10^4\), k-in.²
- \( A_g, \) in.²
- \( I_x, \) in.⁴
- \( I_y, \) in.⁴
- \( r_x/r_y, \)
- \( L_p, \) ft
- \( L_r, \) ft

#### Notes
- Effective length, \( KL \), with respect to least radius of gyration, \( r_y \)

### Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 4-1 (continued)
**Available Strength in Axial Compression, kips**

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**Effective length, \( KL (ft) \), with respect to least radius of gyration, \( r_y \)**

<table>
<thead>
<tr>
<th>Design</th>
<th>( KL / r_y ) with respect to least radius of gyration, ( r_y )</th>
<th>ASD</th>
<th>LRFD</th>
</tr>
</thead>
</table>
0       | 0                                                             | 767 | 666  |
6       | 6                                                             | 660 | 572  |
7       | 7                                                             | 631 | 546  |
9       | 9                                                             | 599 | 518  |
10      | 10                                                            | 565 | 488  |
11      | 11                                                            | 530 | 457  |
12      | 12                                                            | 495 | 426  |
13      | 13                                                            | 458 | 394  |
14      | 14                                                            | 422 | 362  |
15      | 15                                                            | 386 | 331  |
16      | 16                                                            | 352 | 301  |
17      | 17                                                            | 318 | 271  |
18      | 18                                                            | 285 | 243  |
19      | 19                                                            | 256 | 218  |
20      | 20                                                            | 231 | 197  |
21      | 21                                                            | 191 | 163  |
22      | 22                                                            | 160 | 137  |
23      | 23                                                            | 137 | 116  |
24      | 24                                                            | 118 | 100  |
25      | 25                                                            | 103 | 87.5 |
26      | 26                                                            | 90.3| 76.9 |
27      | 27                                                            | 79.9| 68.1 |

**Properties**

- \( P_{w0}, \) kips
- \( P_{w1}, \) kips/in.
- \( P_{wb}, \) kips
- \( P_{bo}, \) kips
- \( L_{p}, \) ft
- \( L_{fr}, \) ft
- \( A_{y}, \) in.²
- \( I_{y}, \) in.⁴
- \( I_{fr}, \) in.⁴
- \( r_{y}, \) in.
- \( r_{fr}, \) in.
- \( P_{w0} (KL)^{1/2}, \) k-in.²
- \( P_{w1} (KL)^{1/2}, \) k-in.²

**ASD LRFD**

- Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.

- \( \Omega_c = 1.67 \)  
- \( \eta_c = 0.90 \)
DESIGN TABLE DISCUSSION

**Table 6-1. W-Shapes in Combined Flexure and Axial Force, 65 ksi**

Steel W-shapes with $F_y = 65$ ksi (ASTM A913 Grade 65) and subject to combined axial force (tension or compression) and flexure may be checked for compliance with the provisions of Section H1.1 and H1.2 of the AISC *Specification* using values listed in Table 6-1 and the appropriate interaction equations provided in the following sections.

Values $p$, $b_x$, $b_y$, $t_y$, and $t_r$ presented in Table 6-1 are defined as follows.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Axial Compression</strong></td>
<td>$p = \frac{1}{\phi_k P_n}$, (kips)$^{-1}$</td>
<td>$p = \frac{\Omega_p}{P_n}$, (kips)$^{-1}$</td>
</tr>
<tr>
<td><strong>Strong Axis Bending</strong></td>
<td>$b_x = \frac{8}{9\phi_b M_{nx}}$, (kip-ft)$^{-1}$</td>
<td>$b_x = \frac{8\Omega_b}{9M_{nx}}$, (kip-ft)$^{-1}$</td>
</tr>
<tr>
<td><strong>Weak Axis Bending</strong></td>
<td>$b_y = \frac{8}{9\phi_b M_{ny}}$, (kip-ft)$^{-1}$</td>
<td>$b_y = \frac{8\Omega_b}{9M_{ny}}$, (kip-ft)$^{-1}$</td>
</tr>
<tr>
<td><strong>Tension Yielding</strong></td>
<td>$t_y = \frac{1}{\phi_k F_{yA}}$, (kips)$^{-1}$</td>
<td>$t_y = \frac{\Omega_t}{F_{yA}}$, (kips)$^{-1}$</td>
</tr>
<tr>
<td><strong>Tension Rupture</strong></td>
<td>$t_r = \frac{1}{\phi_k F_{u}(0.75A_g)}$, (kips)$^{-1}$</td>
<td>$t_r = \frac{\Omega_t}{F_{u}(0.75A_g)}$, (kips)$^{-1}$</td>
</tr>
</tbody>
</table>

**Combined Flexure and Compression**

Equations H1-1a and H1-1b of the AISC *Specification* may be written as follows using the coefficients listed in Table 6-1 and defined previously.

When $pP_r \geq 0.2$:

$$pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0 \quad (6-1)$$

When $pP_r < 0.2$:

$$\frac{1}{2} pP_r + \frac{\phi}{2}\left(b_x M_{rx} + b_y M_{ry}\right) \leq 1.0 \quad (6-2)$$

The designer may check acceptability of a given shape using the appropriate interaction equation from the preceding list. See Ammansour (2000) for more information on this method, including an alternative approach for selection of a trial shape.

**Combined Flexure and Tension**

Equations H1-1a and H1-1b of the AISC *Specification* may be written as follows using the coefficients listed in Table 6-1 and defined previously.

When $pP_r \geq 0.2$:

$$\left(t_y \text{ or } t_r\right)P_r + b_x M_{rx} + b_y M_{ry} \leq 1.0 \quad (6-3)$$

When $pP_r < 0.2$:

$$\frac{1}{2}\left(t_y \text{ or } t_r\right)P_r + \frac{\phi}{2}\left(b_x M_{rx} + b_y M_{ry}\right) \leq 1.0 \quad (6-4)$$

The larger value of $t_y$ and $t_r$ should be used in the preceding equations.
The designer may check acceptability of a given shape using the appropriate interaction equation from above along with variables $t_x$, $t_y$, $b_x$ and $b_y$. See Aminmansour (2006) for more information on this method.

It is noted that the values for $t_x$ listed in Table 6-1 are based on the assumption that $A_e = 0.75A_g$. See Part 5 for more information on this assumption. When $A_e > 0.75A_g$, the tabulated values for $t_x$ are conservative. When $A_e < 0.75A_g$, $t_x$ must be calculated based upon the actual value of $A_e$.

**General Considerations for Use of Values Listed in Table 6-1**

The following remarks are offered for consideration in use of the values listed in Table 6-1.

1. Values of $p$, $b_x$ and $b_y$ already account for section compactness and can be used directly.
2. Tabulated values of $b_x$ assumed $C_b = 1.0$. A procedure for determining $b_x$ when $C_b > 1.0$ follows.
3. Given that the limit state of lateral-torsional buckling does not apply to W-shapes bent about their
   weak axis, values of $b_x$ are independent of unbraced length and $C_b$.
4. Values of $b_y$ equally apply to combined flexure and compression as well as combined flexure and
   tension.
5. Smaller values of variable $p$ for a given $KL$ and smaller values of $b_x$ for a given $L_b$ indicate higher
   strength for the type of load in question. For example, a section with a smaller $p$ at a certain $KL$ is
   more effective in carrying axial compression than another section with a larger value of $p$ at the same
   $KL$. Similarly, a section with a smaller $b_x$ is more effective for flexure at a given $L_b$ than another
   section with a larger $b_x$ for the same $L_b$. This information may be used to select more efficient shapes
   when relatively large amounts of axial load or bending are present.

**Determination of $b_x$ when $C_b > 1.0$**

The tabulated values of $b_x$ assume that $C_b = 1.0$. These values may be modified in accordance with AISC Specification Sections F1 and H1.2. The following procedure may be used to account for $C_b > 1.0$.

$$b_x(C_b > 1.0) = \frac{b_x(C_b = 1.0)}{C_b} \geq b_{x_{min}}$$

(6-5)

Values of $b_{x_{min}}$ are listed in Table 6-1 at $L_b = 0$ ft. See Aminmansour (2009) for more information on this method. Values for $p$, $b_x$, $t_x$, $t_y$ and $t_r$ presented in Table 6-1 have been multiplied by $10^3$. Thus, when used in the appropriate interaction equation they must be multiplied by $10^{-3} (0.001)$. 
### Table 6-1

**Combined Flexure and Axial Force**

<table>
<thead>
<tr>
<th>Shape</th>
<th>W44×</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>335°</td>
</tr>
<tr>
<td></td>
<td>p × 10³</td>
</tr>
<tr>
<td>Design</td>
<td>ASD</td>
</tr>
<tr>
<td></td>
<td>(kip)⁻¹</td>
</tr>
<tr>
<td>0</td>
<td>0.276</td>
</tr>
<tr>
<td>11</td>
<td>0.309</td>
</tr>
<tr>
<td>12</td>
<td>0.315</td>
</tr>
<tr>
<td>13</td>
<td>0.323</td>
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<td>14</td>
<td>0.331</td>
</tr>
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<td>15</td>
<td>0.341</td>
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<tr>
<td>16</td>
<td>0.352</td>
</tr>
<tr>
<td>17</td>
<td>0.363</td>
</tr>
<tr>
<td>18</td>
<td>0.376</td>
</tr>
<tr>
<td>19</td>
<td>0.391</td>
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<tr>
<td>20</td>
<td>0.409</td>
</tr>
<tr>
<td>22</td>
<td>0.449</td>
</tr>
<tr>
<td>24</td>
<td>0.498</td>
</tr>
<tr>
<td>26</td>
<td>0.558</td>
</tr>
<tr>
<td>28</td>
<td>0.629</td>
</tr>
<tr>
<td>30</td>
<td>0.719</td>
</tr>
<tr>
<td>32</td>
<td>0.818</td>
</tr>
<tr>
<td>34</td>
<td>0.923</td>
</tr>
<tr>
<td>36</td>
<td>1.03</td>
</tr>
<tr>
<td>38</td>
<td>1.15</td>
</tr>
<tr>
<td>40</td>
<td>1.28</td>
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<tr>
<td>42</td>
<td>1.41</td>
</tr>
<tr>
<td>44</td>
<td>1.55</td>
</tr>
<tr>
<td>46</td>
<td>1.69</td>
</tr>
<tr>
<td>48</td>
<td>1.84</td>
</tr>
<tr>
<td>50</td>
<td>2.00</td>
</tr>
</tbody>
</table>

| Effective length, KL (ft), with respect to least radius of gyration, rₓ, or Unbraced Length, Lb (ft), for X-X axis bending |
|----------------|------|------|------|------|------|------|------|------|------|------|------|------|
|                | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD |
|                | (kip)⁻¹ | (kip-ft)⁻¹ | (kip)⁻¹ | (kip-ft)⁻¹ | (kip)⁻¹ | (kip-ft)⁻¹ |
| 0              | 0.261 | 0.174 | 0.301 | 0.200 | 0.333 | 0.221 |
| 11             | 0.338 | 0.226 | 0.390 | 0.260 | 0.432 | 0.288 |

| Other Constants and Properties |
|-------------------------------|------|------|------|------|------|------|
| bₓ × 10³ (kip-ft)⁻¹ | 1.16 | 0.773 | 1.34 | 0.889 | 1.51 | 1.00 |
| tₓ × 10³ (kip)⁻¹     | 0.261 | 0.174 | 0.301 | 0.200 | 0.333 | 0.221 |
| tₓ × 10³ (kip)⁻¹     | 0.338 | 0.226 | 0.390 | 0.260 | 0.432 | 0.288 |
| rₓ, in.            | 5.10 | 5.10 | 5.10 | 5.10 | 5.10 | 5.10 |
| rᵧ, in.            | 3.49 | 3.49 | 3.47 | 3.47 | 3.47 | 3.47 |

*Shape is slender for compression with $F_y = 65$ ksi.

*Shape does not meet the $h/t_w$ limit for shear in AISC Specification Section G2.1(a) with $F_y = 65$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$. 
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

<table>
<thead>
<tr>
<th>Shape</th>
<th>W44×</th>
<th>W40×</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2303</td>
<td>5933</td>
</tr>
<tr>
<td></td>
<td>5933</td>
<td></td>
</tr>
<tr>
<td>Design</td>
<td>ASD LRFD</td>
<td>ASD LRFD</td>
</tr>
<tr>
<td>-------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td>p x 10^3</td>
<td>b x 10^3</td>
<td>p x 10^3</td>
</tr>
<tr>
<td>(kips)^1</td>
<td>(kip-ft)^1</td>
<td>(kips)^1</td>
</tr>
<tr>
<td>-------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td>0</td>
<td>0.443</td>
<td>0.295</td>
</tr>
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<td>11</td>
<td>0.492</td>
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<td>12</td>
<td>0.502</td>
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</tr>
<tr>
<td>13</td>
<td>0.513</td>
<td>0.342</td>
</tr>
<tr>
<td>14</td>
<td>0.526</td>
<td>0.350</td>
</tr>
<tr>
<td>15</td>
<td>0.540</td>
<td>0.359</td>
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<tr>
<td>16</td>
<td>0.555</td>
<td>0.368</td>
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<td>17</td>
<td>0.573</td>
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<td>0.459</td>
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<td>0.560</td>
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<td>28</td>
<td>0.946</td>
<td>0.630</td>
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<td>30</td>
<td>1.08</td>
<td>0.719</td>
</tr>
<tr>
<td>32</td>
<td>1.23</td>
<td>0.818</td>
</tr>
<tr>
<td>34</td>
<td>1.39</td>
<td>0.924</td>
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<td>36</td>
<td>1.56</td>
<td>1.04</td>
</tr>
<tr>
<td>38</td>
<td>1.73</td>
<td>1.15</td>
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<td>40</td>
<td>1.92</td>
<td>1.28</td>
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<td>2.12</td>
<td>1.41</td>
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<tr>
<td>44</td>
<td>2.33</td>
<td>1.55</td>
</tr>
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<td>46</td>
<td>2.54</td>
<td>1.69</td>
</tr>
<tr>
<td>48</td>
<td>2.77</td>
<td>1.84</td>
</tr>
<tr>
<td>50</td>
<td>3.00</td>
<td>2.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending</th>
</tr>
</thead>
<tbody>
<tr>
<td>b_x x 10^3</td>
</tr>
<tr>
<td>t_y x 10^3</td>
</tr>
<tr>
<td>Combined Flexure and Axial Force</td>
</tr>
<tr>
<td>b_y x 10^3</td>
</tr>
<tr>
<td>t_y x 10^3</td>
</tr>
<tr>
<td>r_y, in.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Constants and Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>b_y x 10^3</td>
</tr>
<tr>
<td>t_y x 10^3</td>
</tr>
<tr>
<td>r_y, in.</td>
</tr>
</tbody>
</table>

*Shape is slender for compression with F_y = 65 ksi.
*Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
*Shape does not meet the h/t_w limit for shear in AISC Specification Section G2.1(a) with F_y = 65 ksi; therefore, \( \phi_v = 0.90 \) and \( \Omega_c = 1.67 \).
### Combined Flexure and Axial Force

<table>
<thead>
<tr>
<th>Shape</th>
<th>W40×</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Table 6-1 (continued)

<table>
<thead>
<tr>
<th>Design</th>
<th>W40×</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Other Constants and Properties

<table>
<thead>
<tr>
<th></th>
<th>431^h</th>
<th>397^h</th>
<th>652^h</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.

^h Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
Table 6-1 (continued)  
Combined Flexure  
and Axial Force  
W-Shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>W40×</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p \times 10^3$</td>
</tr>
<tr>
<td></td>
<td>ASD</td>
</tr>
<tr>
<td>Design</td>
<td></td>
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<tr>
<td>0</td>
<td>0.234</td>
</tr>
<tr>
<td>11</td>
<td>0.265</td>
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<td>12</td>
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<tr>
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<td>0.279</td>
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<td>0.287</td>
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<td>16</td>
<td>0.306</td>
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<tr>
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<td>0.317</td>
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<td>0.389</td>
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<td>0.429</td>
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<td>0.477</td>
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<td>0.535</td>
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<td>0.605</td>
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<td>1.08</td>
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<td>42</td>
<td>1.19</td>
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<tr>
<td>44</td>
<td>1.30</td>
</tr>
<tr>
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</tr>
<tr>
<td>48</td>
<td>1.55</td>
</tr>
<tr>
<td>50</td>
<td>1.68</td>
</tr>
</tbody>
</table>

Effective length, $KL_{(ft)}$, with respect to least radius of gyration, $r_y$,  
or Unbraced Length, $L_b_{(ft)}$, for X-X axis bending

<table>
<thead>
<tr>
<th>b x × 10^3</th>
<th>t x × 10^3 (kip-ft)^{-1}</th>
<th>t y × 10^3 (kip-ft)^{-1}</th>
<th>p × 10^3</th>
<th>b x × 10^3</th>
<th>t x × 10^3 (kip-ft)^{-1}</th>
<th>t y × 10^3 (kip-ft)^{-1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.989</td>
<td>0.658</td>
<td>1.02</td>
<td>0.675</td>
<td>1.62</td>
<td>1.08</td>
<td></td>
</tr>
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<td>0.234</td>
<td>0.155</td>
<td>0.242</td>
<td>0.161</td>
<td>0.263</td>
<td>0.175</td>
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<td>0.303</td>
<td>0.202</td>
<td>0.314</td>
<td>0.210</td>
<td>0.341</td>
<td>0.227</td>
<td></td>
</tr>
</tbody>
</table>

$r_y/r_{fy}$  
4.58                  4.58                  6.19

$r_{fy}$, in.  
3.60                  3.60                  2.57

Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.  
Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

*W-Shapes*

<table>
<thead>
<tr>
<th>Shape</th>
<th>$W40\times$</th>
<th>$W24\times$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p \times 10^3$</td>
<td>$b_x \times 10^3$</td>
</tr>
<tr>
<td>Design</td>
<td>ASD LRFD ASD LRFD ASD LRFD ASD LRFD ASD LRFD ASD LRFD</td>
<td>(kip)$^{-1}$ (kip-ft)$^{-1}$ (kip)$^{-1}$ (kip-ft)$^{-1}$ (kip)$^{-1}$ (kip-ft)$^{-1}$</td>
</tr>
<tr>
<td>0</td>
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<td>0.276 0.184 0.168</td>
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#### Other Constants and Properties

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$F_y = 65$ ksi

- Shape is slender for compression with $F_y = 65$ ksi.
- Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

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#### Design

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<th>p × 10^3</th>
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#### Effective length, KL (ft), with respect to least radius of gyration, r_y

#### Other Constants and Properties

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<th>t_y × 10^3 (kip-ft)</th>
<th>b_x × 10^3 (kip-ft)</th>
<th>t_x × 10^3 (kip-ft)</th>
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**Note:** Heavy line indicates KL/r_y equal to or greater than 200.

---

**Design Examples V14.1**

**AMERICAN INSTITUTE OF STEEL CONSTRUCTION**
### Combined Flexure and Axial Force

#### W-Shapes

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<td>2.59 1.72 0.790 0.526</td>
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#### Other Constants and Properties

- $b_y \times 10^3$ (kip-ft)$^{-1}$: 2.08 1.38 1.51 1.00 2.32 1.55
- $t_y \times 10^2$ (kips)$^{-1}$: 0.332 0.221 0.350 0.233 0.372 0.247
- $t_r \times 10^3$ (kips)$^{-1}$: 0.431 0.287 0.454 0.302 0.482 0.322
- $r_{xy}/r_y$ (in.): 6.27 4.59 6.26
- $r_y$, in.: 2.52 3.55 2.54

Shape is slender for compression with $F_y = 65$ ksi.

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
Table 6-1 (continued)  
Combined Flexure and Axial Force  
W-Shapes  

<table>
<thead>
<tr>
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<th>211°</th>
<th>199°</th>
</tr>
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<tr>
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<td>1×10³</td>
<td>5×10²</td>
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<tr>
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<td>bₓ</td>
<td>p</td>
<td>bₓ</td>
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<td>(kip)</td>
<td>(kip-ft)</td>
<td>(kip)</td>
<td>(kip-ft)</td>
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<td>ASD LRFD</td>
<td>ASD LRFD</td>
<td>ASD LRFD</td>
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<td>0.401</td>
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Effective length, KL (ft), with respect to least radius of gyration, ry, or Unbraced Length, Lb (ft), for X-X axis bending:

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<th>Effective length</th>
<th>KL (ft)</th>
<th>Lb (ft)</th>
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<td>215°</td>
<td>1.76</td>
<td>2.61</td>
</tr>
<tr>
<td>211°</td>
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<td>3.14</td>
</tr>
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<td>199°</td>
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Other Constants and Properties:

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<tr>
<th>bᵧ × 10³ (kip-ft)⁻¹</th>
<th>1.76</th>
<th>1.74</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>tᵧ × 10³ (kips)⁻¹</td>
<td>0.405</td>
<td>0.275</td>
<td>0.378</td>
</tr>
<tr>
<td>tᵧ × 10³ (kips)⁻¹</td>
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<td>0.437</td>
<td>0.567</td>
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<tr>
<td>rₓ, in.</td>
<td>3.54</td>
<td>2.51</td>
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</tr>
<tr>
<td>rᵧ, in.</td>
<td>4.58</td>
<td>6.29</td>
<td>4.64</td>
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</tbody>
</table>

Note: Heavy line indicates KL/rᵧ equal to or greater than 200.

v Shape does not meet the h/tw limit for shear in AISC Specification Section G2.1(a) with Fᵧ = 65 ksi; therefore, \( \phi_v = 0.90 \) and \( \Omega_v = 1.67 \).

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

**W-Shapes**

<table>
<thead>
<tr>
<th>Design</th>
<th>Shape</th>
<th>$p \times 10^3$ (kip)</th>
<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$ (kip)</th>
<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$ (kip)</th>
<th>$b_x \times 10^3$</th>
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</thead>
<tbody>
<tr>
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<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
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<td>0.526</td>
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<td>6.62</td>
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</table>

<table>
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<tr>
<th>Effective length, $K_L$ (ft), with respect to least radius of gyration, $r_y$, or Unbraced Length, $L_b$ (ft)</th>
<th>$p \times 10^3$ (kip)</th>
<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$ (kip)</th>
<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$ (kip)</th>
<th>$b_x \times 10^3$</th>
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<tbody>
<tr>
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<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
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</table>

Other Constants and Properties

- $b_y \times 10^3$ (kip-ft)$^{-1}$: 3.10, 2.06, 3.61, 2.40, 4.41, 2.94
- $t_y \times 10^3$ (kip-ft)$^{-1}$: 0.482, 0.321, 0.521, 0.347, 0.587, 0.390
- $t_r \times 10^3$ (kip-ft)$^{-1}$: 0.625, 0.417, 0.676, 0.451, 0.761, 0.507
- $r_x/r_y, \text{ in.}$: 6.31, 6.38, 6.55
- $r_y, \text{ in.}$: 2.49, 2.40, 2.29

$^a$ Shape is slender for compression with $F_y = 65$ ksi.

$^b$ Shape does not meet the $h/t_w$ limit for shear in AISC Specification Section G2.1(a) with $F_y = 65$ ksi; therefore, $\phi = 0.90$ and $\Omega_v = 1.67$.

Note: Heavy line indicates $K_L/r_y$ equal to or greater than 200.
### Combined Flexure and Axial Force

#### W-Shapes

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#### Other Constants and Properties

| bᵧ x 10³ (kip-ft)⁻¹ | 0.472 | 0.314 | 0.604 | 0.402 | 0.665 | 0.443 |
| tᵧ x 10³ (kip)⁻¹ | 0.134 | 0.0890 | 0.165 | 0.110 | 0.180 | 0.120 |
| tᵧ x 10³ (kip)⁻¹ | 0.174 | 0.116 | 0.214 | 0.142 | 0.233 | 0.155 |
| rₛ/rᵧ | 3.95 | 4.00 | 3.99 |
| rᵧ, in. | 4.10 | 4.00 | 3.96 |

* Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
## Combined Flexure and Axial Force

### W-Shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>Design</th>
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<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$</th>
<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$</th>
<th>$b_x \times 10^3$</th>
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<td>ASD LRFD</td>
<td>ASD LRFD</td>
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### Other Constants and Properties

| $b_y \times 10^3$ (kip-ft)$^{-1}$ | 0.745 | 0.495 | 0.843 | 0.561 | 0.935 | 0.622 |
| $t_y \times 10^3$ (kips)$^{-1}$ | 0.198 | 0.131 | 0.221 | 0.147 | 0.242 | 0.161 |
| $t_z \times 10^3$ (kips)$^{-1}$ | 0.256 | 0.171 | 0.287 | 0.192 | 0.314 | 0.210 |
| $r_{y/f_y}$ | 4.01 | 4.05 | 4.05 |
| $r_y$, in. | 3.92 | 3.88 | 3.85 |

$^a$ Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
Table 6-1 (continued)

Combined Flexure and Axial Force

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<td>Design</td>
<td>ASD</td>
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</tbody>
</table>

<table>
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<tr>
<th>p × 10³</th>
<th>bₓ × 10³</th>
<th>p × 10³</th>
<th>bₓ × 10³</th>
<th>p × 10³</th>
<th>bₓ × 10³</th>
</tr>
</thead>
<tbody>
<tr>
<td>(kips)⁻¹</td>
<td>(kip-ft)⁻¹</td>
<td>(kips)⁻¹</td>
<td>(kip-ft)⁻¹</td>
<td>(kips)⁻¹</td>
<td>(kip-ft)⁻¹</td>
</tr>
</tbody>
</table>

| 0       | 0.265  | 0.176  | 0.129  | 0.294  | 0.196  | 0.214  | 0.142  | 0.321  | 0.214  | 0.230  | 0.153  |
| 11      | 0.297  | 0.197  | 0.129  | 0.325  | 0.216  | 0.214  | 0.142  | 0.354  | 0.236  | 0.230  | 0.153  |
| 12      | 0.303  | 0.202  | 0.130  | 0.331  | 0.221  | 0.215  | 0.143  | 0.361  | 0.240  | 0.231  | 0.154  |
| 13      | 0.310  | 0.207  | 0.131  | 0.338  | 0.225  | 0.218  | 0.145  | 0.369  | 0.245  | 0.235  | 0.156  |
| 14      | 0.318  | 0.212  | 0.134  | 0.347  | 0.231  | 0.222  | 0.148  | 0.377  | 0.251  | 0.239  | 0.159  |
| 15      | 0.327  | 0.218  | 0.136  | 0.357  | 0.237  | 0.225  | 0.150  | 0.387  | 0.257  | 0.243  | 0.162  |
| 16      | 0.337  | 0.224  | 0.138  | 0.367  | 0.244  | 0.229  | 0.152  | 0.397  | 0.264  | 0.247  | 0.164  |
| 17      | 0.347  | 0.231  | 0.140  | 0.379  | 0.252  | 0.233  | 0.155  | 0.408  | 0.272  | 0.252  | 0.167  |
| 18      | 0.359  | 0.239  | 0.142  | 0.391  | 0.260  | 0.237  | 0.158  | 0.419  | 0.280  | 0.256  | 0.170  |
| 19      | 0.371  | 0.247  | 0.145  | 0.405  | 0.269  | 0.241  | 0.160  | 0.436  | 0.290  | 0.261  | 0.174  |
| 20      | 0.385  | 0.256  | 0.147  | 0.420  | 0.280  | 0.245  | 0.163  | 0.453  | 0.301  | 0.266  | 0.177  |
| 22      | 0.417  | 0.277  | 0.229  | 0.152  | 0.455  | 0.302  | 0.255  | 0.169  | 0.490  | 0.326  | 0.276  | 0.184  |
| 24      | 0.454  | 0.302  | 0.238  | 0.158  | 0.496  | 0.330  | 0.264  | 0.176  | 0.535  | 0.356  | 0.287  | 0.191  |
| 26      | 0.498  | 0.331  | 0.247  | 0.164  | 0.544  | 0.362  | 0.275  | 0.183  | 0.588  | 0.391  | 0.299  | 0.199  |
| 28      | 0.551  | 0.367  | 0.256  | 0.171  | 0.602  | 0.401  | 0.286  | 0.191  | 0.652  | 0.434  | 0.313  | 0.208  |
| 30      | 0.614  | 0.409  | 0.267  | 0.178  | 0.671  | 0.447  | 0.299  | 0.199  | 0.727  | 0.484  | 0.327  | 0.218  |
| 32      | 0.690  | 0.459  | 0.278  | 0.185  | 0.755  | 0.503  | 0.313  | 0.208  | 0.820  | 0.545  | 0.343  | 0.228  |
| 34      | 0.779  | 0.518  | 0.291  | 0.194  | 0.853  | 0.567  | 0.327  | 0.218  | 0.925  | 0.616  | 0.361  | 0.240  |
| 36      | 0.874  | 0.581  | 0.305  | 0.203  | 0.956  | 0.636  | 0.344  | 0.229  | 1.049  | 0.690  | 0.385  | 0.256  |
| 38      | 0.973  | 0.648  | 0.322  | 0.214  | 1.070  | 0.709  | 0.371  | 0.247  | 1.168  | 0.769  | 0.417  | 0.277  |
| 40      | 1.08   | 0.717  | 0.346  | 0.230  | 1.185  | 0.785  | 0.399  | 0.266  | 1.287  | 0.852  | 0.449  | 0.299  |

<table>
<thead>
<tr>
<th>Effective length, K₁ (ft), with respect to least radius of gyration, rᵧ, or Unbraced Length, Lb (ft), for X-X axis bending</th>
</tr>
</thead>
<tbody>
<tr>
<td>bₓ × 10³ (kip-ft)⁻¹</td>
</tr>
<tr>
<td>103</td>
</tr>
<tr>
<td>1.03</td>
</tr>
</tbody>
</table>

Other Constants and Properties

<table>
<thead>
<tr>
<th>bᵧ × 10³ (kip-ft)⁻¹</th>
<th>tᵧ × 10³ (kip)⁻¹</th>
<th>rᵧ, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>1.41</td>
<td>3.83</td>
</tr>
<tr>
<td>0.757</td>
<td>1.23</td>
<td>3.82</td>
</tr>
<tr>
<td>0.818</td>
<td>4.05</td>
<td>3.80</td>
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</tbody>
</table>

Shape is slender for compression with Fᵧ = 65 ksi.

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
Table 6-1 (continued)  
Combined Flexure  
and Axial Force  
W-Shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>262(^2)</th>
<th>256(^2)</th>
<th>247(^2)</th>
</tr>
</thead>
<tbody>
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<td>(p \times 10^3)</td>
<td>(b_x \times 10^3)</td>
<td>(p \times 10^3)</td>
</tr>
<tr>
<td>(p \times 10^3)</td>
<td>(b_x \times 10^3)</td>
<td>(p \times 10^3)</td>
<td>(b_x \times 10^3)</td>
</tr>
<tr>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
</tr>
<tr>
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<td>0.350</td>
<td>0.233</td>
<td>0.166</td>
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<tr>
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<td>0.386</td>
<td>0.257</td>
<td>0.233</td>
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<tr>
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<td>0.393</td>
<td>0.262</td>
<td>0.251</td>
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<tr>
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<td>0.402</td>
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<td>0.255</td>
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<td>0.445</td>
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</table>

Other Constants and Properties

\(b_y \times 10^3\) (kip-ft)\(^{-1}\) | 1.34 | 0.894 | 2.00 | 1.33 | 1.44 | 0.960
| \(t_y \times 10^3\) (kips)\(^{-1}\) | 0.333 | 0.221 | 0.341 | 0.227 | 0.354 | 0.236
| \(t_r \times 10^3\) (kips)\(^{-1}\) | 0.432 | 0.288 | 0.443 | 0.295 | 0.460 | 0.307
| \(r_x/r_y\) | 4.07 | 5.62 | 4.06 |
| \(r_y\), in. | 3.76 | 2.65 | 3.74 |

\(^a\) Shape is slender for compression with \(F_y = 65\) ksi.

Note: Heavy line indicates \(KL/r_y\) equal to or greater than 200.
<table>
<thead>
<tr>
<th>Shape</th>
<th>Design</th>
<th>232&lt;sup&gt;2&lt;/sup&gt;</th>
<th>231&lt;sup&gt;2&lt;/sup&gt;</th>
<th>210&lt;sup&gt;2&lt;/sup&gt;</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(p \times 10^3)</td>
<td>(b_x \times 10^3)</td>
<td>(p \times 10^3)</td>
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<tr>
<td></td>
<td></td>
<td>(kips)&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>(kip-ft)&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>(kips)&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
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<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
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</table>

**Other Constants and Properties**

\[
b_y \times 10^4 (\text{kip-ft})^{-1} = 2.25 \\
t_y \times 10^3 (\text{kip})^{-1} = 0.378 \\
t \times 10^3 (\text{kip})^{-1} = 0.490 \\
r_{x,y}, \text{in.} = 2.62 \\
r_{x,y} = 5.65 \\
r_{y} = 4.07 \\
r_{y}, \text{in.} = 2.58
\]

Notes:
- **Shape is slender for compression with** \(F_y = 65\) ksi.
- **Note:** Heavy line indicates \(KL/r_y\) equal to or greater than 200.
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

**W-Shapes**

<table>
<thead>
<tr>
<th>Design</th>
<th>( p \times 10^3 )</th>
<th>( b_x \times 10^3 )</th>
<th>( p \times 10^3 )</th>
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#### Other Constants and Properties

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\textsuperscript{\textdegree} Shape is slender for compression with \( F_y = 65 \) ksi.

\textsuperscript{\textast} Shape does not meet the \( h/t_w \) limit for shear in AISC Specification Section G2.1(a) with \( F_y = 65 \) ksi; therefore, \( \phi_y = 0.90 \) and \( \Omega_y = 1.67 \).

**Note:** Heavy line indicates \( KL/\gamma_y \) equal to or greater than 200.
Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

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Other Constants and Properties

- \( b_y \times 10^3 \) (kip-ft)\(^t\) = 3.55, 2.36, 3.87, 2.57, 4.59, 3.05
- \( t_y \times 10^3 \) (kips)\(^t\) = 0.547, 0.364, 0.580, 0.386, 0.644, 0.428
- \( t_r \times 10^3 \) (kips)\(^t\) = 0.709, 0.473, 0.752, 0.502, 0.835, 0.557
- \( r_y/r_y \) = 5.76, 5.79, 5.88
- \( r_y \) in. = 2.50, 2.47, 2.38

\(^t\) Shape is slender for compression with \( F_y = 65 \) ksi.
\(^v\) Shape does not meet the \( h/t_w \) limit for shear in AISC Specification Section G2.1(a) with \( F_y = 65 \) ksi; therefore, \( \phi_v = 0.90 \) and \( \Omega_v = 1.67 \).

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.
### Combined Flexure and Axial Force

*W-Shapes*

**$F_y = 65$ ksi**

| Shape | W33× | **| **| **| **| **| **| **| **| **| **| **| **| **|
|-------|------|---|---|---|---|---|---|---|---|---|---|---|---|
| Design | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD |
| $p \times 10^3$ (kip)$^{-1}$ | $b_x \times 10^3$ (kip-ft)$^{-1}$ | $p \times 10^3$ (kip)$^{-1}$ | $b_x \times 10^3$ (kip-ft)$^{-1}$ | $p \times 10^3$ (kip)$^{-1}$ | $b_x \times 10^3$ (kip-ft)$^{-1}$ |
| 0 | 0.225 | 0.150 | 0.117 | 0.247 | 0.164 | 0.193 | 0.128 | 0.274 | 0.182 | 0.216 | 0.144 |
| 11 | 0.253 | 0.168 | 0.117 | 0.278 | 0.211 | 0.197 | 0.131 | 0.324 | 0.216 | 0.221 | 0.147 |
| 12 | 0.259 | 0.172 | 0.117 | 0.284 | 0.211 | 0.194 | 0.129 | 0.333 | 0.222 | 0.224 | 0.148 |
| 13 | 0.265 | 0.176 | 0.119 | 0.291 | 0.211 | 0.199 | 0.133 | 0.333 | 0.222 | 0.224 | 0.148 |
| 14 | 0.272 | 0.181 | 0.120 | 0.298 | 0.211 | 0.199 | 0.133 | 0.333 | 0.222 | 0.224 | 0.148 |
| 15 | 0.280 | 0.186 | 0.122 | 0.308 | 0.205 | 0.202 | 0.135 | 0.343 | 0.227 | 0.227 | 0.151 |
| 16 | 0.288 | 0.192 | 0.124 | 0.317 | 0.211 | 0.205 | 0.137 | 0.354 | 0.235 | 0.231 | 0.154 |
| 17 | 0.298 | 0.198 | 0.125 | 0.326 | 0.218 | 0.208 | 0.139 | 0.365 | 0.243 | 0.235 | 0.156 |
| 18 | 0.308 | 0.205 | 0.127 | 0.339 | 0.226 | 0.211 | 0.141 | 0.378 | 0.252 | 0.238 | 0.159 |
| 19 | 0.319 | 0.212 | 0.129 | 0.352 | 0.234 | 0.214 | 0.143 | 0.393 | 0.261 | 0.242 | 0.161 |
| 20 | 0.331 | 0.220 | 0.131 | 0.365 | 0.243 | 0.218 | 0.145 | 0.408 | 0.272 | 0.246 | 0.164 |
| 22 | 0.359 | 0.239 | 0.202 | 0.397 | 0.264 | 0.225 | 0.149 | 0.444 | 0.295 | 0.265 | 0.170 |
| 24 | 0.392 | 0.261 | 0.208 | 0.434 | 0.289 | 0.232 | 0.154 | 0.486 | 0.323 | 0.264 | 0.176 |
| 26 | 0.432 | 0.288 | 0.215 | 0.479 | 0.318 | 0.240 | 0.160 | 0.537 | 0.357 | 0.274 | 0.182 |
| 28 | 0.480 | 0.319 | 0.221 | 0.532 | 0.354 | 0.248 | 0.165 | 0.598 | 0.398 | 0.285 | 0.189 |
| 30 | 0.536 | 0.357 | 0.229 | 0.596 | 0.397 | 0.257 | 0.171 | 0.671 | 0.446 | 0.296 | 0.197 |
| 32 | 0.605 | 0.403 | 0.237 | 0.674 | 0.449 | 0.267 | 0.177 | 0.761 | 0.506 | 0.308 | 0.205 |
| 34 | 0.684 | 0.455 | 0.245 | 0.761 | 0.507 | 0.277 | 0.184 | 0.859 | 0.571 | 0.322 | 0.214 |
| 36 | 0.766 | 0.510 | 0.254 | 0.854 | 0.568 | 0.288 | 0.192 | 0.963 | 0.641 | 0.337 | 0.224 |
| 38 | 0.854 | 0.568 | 0.264 | 0.951 | 0.633 | 0.300 | 0.200 | 1.07 | 0.714 | 0.353 | 0.235 |
| 40 | 0.946 | 0.629 | 0.274 | 1.05 | 0.701 | 0.314 | 0.209 | 1.19 | 0.791 | 0.378 | 0.251 |

| Effective length, $KL$, (ft), with respect to least radius of gyration, $r_y$, or Unbraced Length, $L_b$ (ft), for X-X axis bending |
|-------|------|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.647 | 0.584 | 0.972 | 0.647 | 1.10 | 0.729 |
| 11 | 0.225 | 0.150 | 0.247 | 0.164 | 0.274 | 0.182 |
| 12 | 0.292 | 0.195 | 0.321 | 0.214 | 0.356 | 0.237 |

**Other Constants and Properties**

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*Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.*
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

**W-Shapes**

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<td>0.592</td>
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</table>

**Effective length, \( KL \) (ft), with respect to least radius of gyration, \( r_y \), and Unbraced Length, \( L_b \) (ft), for X-X axis bending**

<table>
<thead>
<tr>
<th>Design</th>
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<tr>
<td>ASD</td>
<td>ASD</td>
<td>ASD</td>
<td>ASD</td>
</tr>
<tr>
<td>LRFD</td>
<td>LRFD</td>
<td>LRFD</td>
<td>LRFD</td>
</tr>
</tbody>
</table>

\( K \) = 1.36 \times 10^3 \text{(kip-ft)}^{-1}, \( \beta \times 10^3 \) \( L_b \) (ft)\text{, for X-X axis bending}

**Other Constants and Properties**

<table>
<thead>
<tr>
<th>( b_y \times 10^3 ) (kip-ft)(^{-1} )</th>
<th>( t_y \times 10^3 ) (kips)(^{-1} )</th>
<th>( t_r \times 10^3 ) (kips)(^{-1} )</th>
<th>( r_x/r_y )</th>
<th>( r_y ), in.</th>
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<td>0.287</td>
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<td>0.469</td>
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<td>1.00</td>
<td>0.240</td>
<td>0.313</td>
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</table>

\( F_y = 65 \text{ ksi} \)

\( F_y \text{ is slender for compression with } F_y = 65 \text{ ksi.} \)
### Combined Flexure and Axial Force

#### W-Shapes

<table>
<thead>
<tr>
<th>Design</th>
<th>Shape</th>
<th>b ( \times 10^3 )</th>
<th>( p \times 10^3 )</th>
<th>( b_x \times 10^3 )</th>
<th>( p \times 10^3 )</th>
<th>( b_x \times 10^3 )</th>
<th>( p \times 10^3 )</th>
<th>( b_x \times 10^3 )</th>
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<td>ASDF</td>
<td>ASD</td>
<td>ASD</td>
<td>ASD</td>
<td>ASD</td>
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<td>0.381</td>
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<td>0.485</td>
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<td>0.766</td>
<td>0.551</td>
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<td>0.374</td>
<td>1.480</td>
<td>0.984</td>
<td>0.657</td>
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<td>0.614</td>
<td>0.408</td>
<td>1.660</td>
<td>1.100</td>
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<td>0.846</td>
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</table>

#### Effective length, \( KL \) (ft), with respect to least radius of gyration, \( r_y \), or Unbraced Length, \( L_b \) (ft), for X-X axis bending

\[ b_y \times 10^3 (\text{kip-ft})^{-1} = 1.67 \quad 1.11 \quad 1.86 \quad 1.24 \quad 3.25 \quad 2.16 \]
\[ t_y \times 10^3 (\text{kips})^{-1} = 0.393 \quad 0.262 \quad 0.435 \quad 0.289 \quad 0.519 \quad 0.345 \]
\[ t_r \times 10^3 (\text{kips})^{-1} = 0.510 \quad 0.340 \quad 0.564 \quad 0.376 \quad 0.673 \quad 0.449 \]

#### Other Constants and Properties

\[ r_y, r_x \]
\[ r_y, \text{in.} \]
\[ r_y, \text{ft} \]

\[ \frac{r_y}{r_x} = 3.93 \quad 3.93 \quad 5.48 \]
\[ r_y, \text{in.} = 3.59 \quad 3.56 \quad 2.50 \]

\[ \text{Shape is slender for compression with } F_y = 50 \text{ ksi.} \]

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Design</th>
<th>Effective length, KL (ft), with respect to least radius of gyration, $r_y$</th>
<th>Other Constants and Properties</th>
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<tbody>
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<td>$p \times 10^3$</td>
<td>$b_x \times 10^3$</td>
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<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
</tr>
<tr>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
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<tr>
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<td>LRFD</td>
<td>ASD</td>
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<tr>
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<td>LRFD</td>
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<td>LRFD</td>
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</table>

- **W33**

<table>
<thead>
<tr>
<th>$b_x \times 10^3$ (kip-ft)$^{-1}$</th>
<th>$t_y \times 10^3$ (kips)$^{-1}$</th>
<th>$t_r \times 10^3$ (kips)$^{-1}$</th>
<th>$r_y/\bar{r}_y$</th>
<th>$r_y$, in.</th>
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<tr>
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<td>2.37</td>
<td>0.619</td>
<td>0.803</td>
<td>5.52</td>
<td>2.39</td>
</tr>
</tbody>
</table>

- **Notes:**
  - Shape is slender for compression with $F_y = 65$ ksi.
  - Shape does not meet the $h/t_w$ limit for shear in AISC Specification Section G2.1(a) with $F_y = 65$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_r = 1.67$.
  - Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
### Table 6-1 (continued) Combined Flexure and Axial Force

#### W-Shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>Design</th>
<th>$p \times 10^3$</th>
<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$</th>
<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$</th>
<th>$b_x \times 10^3$</th>
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<td>0.223</td>
<td>0.189</td>
<td>0.245</td>
<td>0.183</td>
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<td></td>
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<td>0.592</td>
<td>0.439</td>
<td>0.149</td>
<td>0.126</td>
<td>0.208</td>
<td>0.138</td>
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<td>0.277</td>
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<tr>
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<td>0.101</td>
<td>0.245</td>
<td>0.138</td>
</tr>
<tr>
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<td>0.192</td>
<td>0.287</td>
<td>0.185</td>
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<td>0.497</td>
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<td>0.147</td>
<td>0.113</td>
<td>0.301</td>
<td>0.186</td>
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</table>

#### Other Constants and Properties

- $b_y \times 10^3$ (kip-ft)$^{-1}$: 3.54, 3.55, 0.884, 0.588, 0.982, 0.654
- $t_y \times 10^3$ (kip)$^{-1}$: 0.740, 0.493, 0.223, 0.149, 0.245, 0.163
- $t_r \times 10^3$ (kip)$^{-1}$: 0.961, 0.640, 0.290, 0.193, 0.317, 0.212

**Note:** Heavy line indicates $KL/r_y$ equal to or greater than 200.

---

1. Shape is slender for compression with $F_y = 65$ ksi.
2. Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
3. Shape does not meet the $h/t_w$ limit for shear in AISC Specification Section G2.1(a) with $F_y = 65$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

**Note:**
- $F_y = 65$ ksi
- $r_y$, in., 3.65
- $r_y$, ft, 0.222
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

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<tr>
<td></td>
<td>(p \times 10^3)</td>
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<td>-----</td>
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<td>0.266</td>
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<td>0.304</td>
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</table>

#### Other Constants and Properties

- \(b_y \times 10^3\) (kip-ft)
- \(t_y \times 10^3\) (kip)
- \(t_x \times 10^3\) (kip)

| \(r_y/r_f\) | 3.67 | 3.69 | 3.71 |
| \(r_y\), in. | 3.60 | 3.58 | 3.53 |

\(a\): Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

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**Effective length, \( KL \) (ft), with respect to least radius of gyration, \( r_y \), or Unbraced Length, \( L_b \) (ft), for X-X axis bending**

**Other Constants and Properties**

| \( b_y \times 10^3 \) (kip-ft)\(^{-1} \) | 1.57 | 1.04 | 1.77 |
| \( t_y \times 10^3 \) (kips)\(^{-1} \) | 0.371 | 0.247 | 0.412 |
| \( t_r \times 10^3 \) (kips)\(^{-1} \) | 0.481 | 0.321 | 0.535 |

**Notes:**
- Shape is slender for compression with \( F_y = 65 \) ksi.
- Other notes and properties.
### Table 6-1 (continued)  
Combined Flexure and Axial Force  
W-Shapes

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Other Constants and Properties

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Shape is slender for compression with $F_y = 65$ ksi.  
Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
\[ F_y = 65 \text{ ksi} \]

### Combined Flexure and Axial Force

#### W-Shapes

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<th>( b_x \times 10^3 )</th>
<th>( p \times 10^3 )</th>
<th>( b_x \times 10^3 )</th>
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Other Constants and Properties

\[ b_y \times 10^3, t_y \times 10^2, t_x \times 10^2 \]

\[ r_y, r_y', \Phi_v, \Omega_v \]

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.

Note: \( \Phi_v = 0.90 \) and \( \Omega_v = 1.67 \).
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

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<td>(kip-ft)$^{-1}$</td>
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<td>1.04 1.12 0.746</td>
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<table>
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<tr>
<th>Effective length, $KL$ (ft), with respect to least radius of gyration, $r_y$, or Unbraced Length, $L_b$ (ft), for X-X axis bending</th>
<th>b$_y$ $\times 10^3$ (kip-ft)$^{-1}$</th>
<th>t$_y$ $\times 10^3$ (kips)$^{-1}$</th>
<th>t$_r$ $\times 10^3$ (kips)$^{-1}$</th>
<th>r$_y$/r$_x$</th>
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<td>0.210</td>
<td>0.140</td>
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1 Shape is slender for compression with $F_y = 65$ ksi.
2 Shape does not meet compact limit for flexure with $F_y = 65$ ksi.
3 Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
4 Shape does not meet the $h/t_w$ limit for shear in AISC Specification Section G2.1(a) with $F_y = 65$ ksi; therefore, $\phi = 0.90$ and $\Omega_x = 1.67$.

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

#### W-Shapes

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**Other Constants and Properties**

- \( b_y \times 10^3 \) (kip-ft): 0.982 0.654 1.09 0.724 1.21 0.803
- \( t_y \times 10^3 \) (kips): 0.236 0.157 0.259 0.172 0.285 0.190
- \( t_r \times 10^3 \) (kips): 0.306 0.204 0.336 0.224 0.370 0.246

\( r_x / r_y \) 3.51 3.51 3.52

\( r_y \), in. 3.48 3.45 3.41

\( F_y = 65 \text{ ksi} \)

**Notes:**

- Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.

Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
## Table 6-1 (continued)

### Combined Flexure and Axial Force

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### Other Constants and Properties

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<th>( t_y \times 10^3 ) (kip)&lt;sup&gt;-1&lt;/sup&gt;</th>
<th>( t_r \times 10^3 ) (kip-ft)&lt;sup&gt;-1&lt;/sup&gt;</th>
<th>( r_y / r_y )</th>
<th>( r_y ), in.</th>
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\( F_y = 65 \text{ ksi} \)

Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

W-Shapes

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Other Constants and Properties

| | **b_y × 10^3** | **t_y × 10^3** | **t_r × 10^3** |
| | (kip-ft)^{-1} | (kips)^{-1} | (kip-ft)^{-1} |
| 1.78 | 1.18 | 2.02 | 1.34 | 2.25 | 1.49 |
| 0.402 | 0.268 | 0.450 | 0.299 | 0.489 | 0.326 |
| 0.522 | 0.348 | 0.584 | 0.389 | 0.635 | 0.423 |

- **r_y, in.**
  - 3.32
  - 3.29
  - 3.25

- **r_y/f_y**
  - 3.55
  - 3.56
  - 3.57

- Shape is slender for compression with $F_y = 65$ ksi.
Table 6-1 (continued)  
Combined Flexure and Axial Force  
W-Shapes

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<td>p x 10³</td>
<td>b_x x 10³</td>
<td>p x 10³</td>
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Other Constants and Properties

| b_y x 10³ (kip-ft)⁻¹ | 2.51 | 1.67 | 2.81 | 1.87 | 4.76 | 3.17 |
| t_y x 10³ (kips)⁻¹ | 0.540 | 0.359 | 0.595 | 0.396 | 0.680 | 0.452 |
| t_r x 10³ (kips)⁻¹ | 0.700 | 0.467 | 0.772 | 0.514 | 0.882 | 0.588 |
| r_s/r_y | 3.56 | 3.59 | 5.07 |
| r_y, in. | 3.23 | 3.20 | 2.21 |

Shape is slender for compression with F_y = 65 ksi.  
Note: Heavy line indicates KL/r_y equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

*W*-Shapes

### Design Examples V14.1

**AMERICAN INSTITUTE OF STEEL CONSTRUCTION**
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

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**Other Constants and Properties**

- b_y × 10^3 (kip-ft)^{-1} = 8.25, 5.49
- t_y × 10^3 (kips) = 1.04, 0.692
- t_r × 10^3 (kips)^{-1} = 1.35, 0.900

- r_y, in. = 5.17, 3.39
- r_y, ft = 2.07, 3.27

**Notes:**
- Shape is slender for compression with \( F_y = 65 \) ksi.
- Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
- Shape does not meet the \( h/t_w \) limit for shear in AISC Specification Section G2.1(a) with \( F_y = 65 \) ksi; therefore, \( \phi_v = 0.90 \) and \( \Omega_v = 1.67 \).

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.
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<th>( p \times 10^3 )</th>
<th>( b_y \times 10^3 )</th>
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### Effective length, \( L_{KL} \) (ft), with respect to least radius of gyration, \( r_y \), or Unbraced Length, \( L_b \) (ft), for X-X axis bending:

### Other Constants and Properties:

<table>
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<tr>
<th>( b_y \times 10^3 ) (kip-ft)(^{-1} )</th>
<th>( t_y \times 10^3 ) (kips)(^{-1} )</th>
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\( r_y, \) in. 3.41 3.41 3.41

\( L_b \) (ft) 3.20 3.17 3.14

### Notes:

- \( F_y = 65 \text{ ksi} \)

- The table contains combined flexure and axial force data for W-shapes.

- For flange thickness greater than 2 in., special requirements may apply per AISC Specification Section A3.1c.

- All values are rounded.

- For X-X axis bending, flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
## Combined Flexure and Axial Force

### W-Shapes

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<td>p (\times 10^3)</td>
<td>b_x (\times 10^3)</td>
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### Effective length, KL (ft), with respect to least radius of gyration, r_y,
or Unbraced Length, L_b (ft), for X-X axis bending

<table>
<thead>
<tr>
<th>b_y (\times 10^3) (kip-ft) (^{-1})</th>
<th>t_y (\times 10^3) (kips) (^{-1})</th>
<th>t_y (\times 10^3) (kips) (^{-1})</th>
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### Other Constants and Properties

- W24 \(= 65 \text{ ksi}\)
- Combined Flexure and Axial Force
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

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</table>

Other Constants and Properties

| $b_y \times 10^3$ (kip-ft)$^{-1}$ | 2.38 | 1.59 | 2.61 | 1.74 | 2.94 | 1.96 |
| $t_y \times 10^3$ (kip) | 0.497 | 0.331 | 0.537 | 0.358 | 0.597 | 0.398 |
| $t_r \times 10^3$ (kip) | 0.645 | 0.430 | 0.697 | 0.465 | 0.775 | 0.517 |

Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION

---

$F_y = 65$ ksi

$\text{Shape is slender for compression with } F_y = 65 \text{ ksi.}$
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

<table>
<thead>
<tr>
<th>Shape</th>
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<tbody>
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<td>131c</td>
</tr>
<tr>
<td></td>
<td>ASD</td>
</tr>
<tr>
<td></td>
<td>( p \times 10^3 )</td>
</tr>
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<td>0.687</td>
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<td>48</td>
<td>6.48</td>
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</table>

#### Other Constants and Properties

- \( b_y \times 10^3 \) (kip-ft)<sup>-1</sup> = 3.36, 2.24, 3.84, 2.55, 4.48, 2.98
- \( t_y \times 10^3 \) (kip-ft)<sup>-1</sup> = 0.666, 0.443, 0.747, 0.497, 0.837, 0.557
- \( t_x \times 10^3 \) (kip-ft)<sup>-1</sup> = 0.864, 0.576, 0.969, 0.646, 1.09, 0.724

| \( r_s/r_y \) | 3.43 | 3.44 | 3.47 |
| \( r_y \), in. | 2.97 | 2.94 | 2.91 |

<sup>5</sup> Shape is slender for compression with \( F_y = 65 \text{ ksi} \).

<sup>6</sup> Shape does not meet compact limit for flexure with \( F_y = 65 \text{ ksi} \).

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.
### Table 6-1 (continued)
#### Combined Flexure and Axial Force

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<tr>
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<tr>
<td>b_x × 10^3 (kip-ft)^{-1}</td>
<td>p × 10^3 (kips)^{-1}</td>
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#### Effective length, KL (ft), with respect to least radius of gyration, ry, or Unbraced Length, Lb (ft), for X-X axis bending

<table>
<thead>
<tr>
<th>r_y, in.</th>
<th>5.03</th>
<th>4.98</th>
<th>5.02</th>
</tr>
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<tbody>
<tr>
<td>b_y × 10^3 (kip-ft)^{-1}</td>
<td>6.60</td>
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<tr>
<td>t_y × 10^3 (kips)^{-1}</td>
<td>0.848</td>
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<tr>
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<td>r_y, in.</td>
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<td>4.98</td>
<td>5.02</td>
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</table>

Note: Heavy line indicates KL/r_y equal to or greater than 200.

---

IV-55

Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

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<th>$b_x \times 10^3$</th>
<th>$p \times 10^3$</th>
<th>$b_x \times 10^3$</th>
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<th>$b_x \times 10^3$</th>
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</table>

**Other Constants and Properties**

<table>
<thead>
<tr>
<th>$b_y \times 10^3$ (kip-ft)$^{-1}$</th>
<th>9.58</th>
<th>6.38</th>
<th>11.20</th>
<th>7.44</th>
<th>17.50</th>
<th>11.60</th>
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</table>

---

*$^*$ Shape is slender for compression with $F_y = 65$ ksi.

$^\dagger$ Shape does not meet the $h/t_w$ limit for shear in AISC Specification Section G2.1(a) with $F_y = 65$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

Note: Heavy line indicates $KL/ir_y$ equal to or greater than 200.
### Table 6-1 (continued)  
Combined Flexure and Axial Force  
W-Shapes

<table>
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#### Other Constants and Properties

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<th>(t_y \times 10^3) (kips)(^{-1})</th>
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<th>(r_y, r_y,) in.</th>
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\(F_y = 65\) ksi  

\(b_y\) is slender for compression with \(F_y = 65\) ksi.  

\(t_y\) is slender for shear in AISC Specification Section G2.1(a) with \(F_y = 65\) ksi; therefore, \(\phi = 0.90\) and \(\Omega_x = 1.67\).  

Note: Heavy line indicates \(KL/r_y\) equal to or greater than 200.
### Combined Flexure and Axial Force

#### W-Shapes

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<td>$b_x \times 10^{3}$</td>
<td>$p \times 10^{3}$</td>
<td>$b_x \times 10^{3}$</td>
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#### Effective length, KL (ft), with respect to least radius of gyration, $r_y$,
or Unbraced Length, $L_b$ (ft), for X-X axis bending

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<th>$t_y \times 10^{3}$</th>
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#### Other Constants and Properties

| $b_y \times 10^{3}$ (kip-ft)$^{-1}$ | 2.54 | 1.69 | 2.96 | 1.97 | 3.33 | 2.22 |
| $t_y \times 10^{3}$ (kips)$^{-1}$ | 0.526 | 0.350 | 0.595 | 0.396 | 0.662 | 0.441 |
| $t_x \times 10^{3}$ (kips)$^{-1}$ | 0.683 | 0.455 | 0.772 | 0.514 | 0.859 | 0.573 |

| $r_y$, in. | 2.99 | 2.95 | 2.93 |

| $r_y/r_f$ | 3.13 | 3.11 | 3.11 |

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

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<th>$p \times 10^3$</th>
<th>$b_x \times 10^3$</th>
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### Other Constants and Properties

| $b_y \times 10^3$ (kip-ft)$^{-1}$ | 3.63 | 2.41 | 4.02 | 2.67 | 4.44 | 2.96 |
| $t_y \times 10^3$ (kips)$^{-1}$ | 0.716 | 0.476 | 0.788 | 0.524 | 0.862 | 0.574 |
| $t_r \times 10^3$ (kips)$^{-1}$ | 0.929 | 0.619 | 1.02 | 0.682 | 1.12 | 0.746 |

$r_y$, in.

| W21 | 3.11 | 3.12 | 3.12 |
| W21 | 2.92 | 2.90 | 2.89 |

$^c$ Shape is slender for compression with $F_y = 65$ ksi.
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<td><strong>ASD LRFD</strong></td>
<td><strong>ASD LRFD</strong></td>
<td><strong>ASD LRFD</strong></td>
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| 0 | 0.950 | 0.632 | 1.24 | 0.825 | 1.10 | 0.731 | 1.40 | 0.930 | 1.29 | 1.06 |
| 6 | 1.09 | 0.724 | 1.25 | 0.833 | 1.24 | 0.827 | 1.41 | 0.941 | 1.29 | 1.06 |
| 7 | 1.15 | 0.763 | 1.29 | 0.861 | 1.30 | 0.866 | 1.47 | 0.975 | 1.32 | 1.12 |
| 8 | 1.22 | 0.811 | 1.34 | 0.891 | 1.37 | 0.914 | 1.52 | 1.01 | 1.38 | 1.16 |
| 9 | 1.31 | 0.869 | 1.39 | 0.924 | 1.47 | 0.976 | 1.58 | 1.05 | 1.45 | 1.21 |
| 10 | 1.41 | 0.938 | 1.44 | 0.966 | 1.58 | 1.05 | 1.64 | 1.09 | 1.52 | 1.26 |
| 11 | 1.53 | 1.02 | 1.50 | 0.996 | 1.73 | 1.15 | 1.71 | 1.14 | 1.60 | 1.32 |
| 12 | 1.68 | 1.12 | 1.56 | 1.04 | 1.90 | 1.26 | 1.79 | 1.19 | 1.67 | 1.38 |
| 13 | 1.86 | 1.24 | 1.62 | 1.08 | 2.10 | 1.40 | 1.87 | 1.24 | 1.74 | 1.45 |
| 14 | 2.08 | 1.38 | 1.70 | 1.13 | 2.35 | 1.56 | 1.96 | 1.30 | 1.81 | 1.52 |
| 15 | 2.34 | 1.56 | 1.77 | 1.18 | 2.64 | 1.76 | 2.06 | 1.37 | 1.88 | 1.60 |
| 16 | 2.65 | 1.77 | 1.86 | 1.24 | 3.00 | 2.00 | 2.17 | 1.44 | 2.04 | 2.25 |
| 17 | 3.00 | 1.99 | 1.96 | 1.30 | 3.39 | 2.25 | 2.29 | 1.52 | 2.39 | 2.55 |
| 18 | 3.36 | 2.23 | 2.08 | 1.38 | 3.80 | 2.53 | 2.51 | 1.67 | 2.49 | 2.78 |
| 19 | 3.74 | 2.49 | 2.25 | 1.50 | 4.23 | 2.82 | 2.72 | 1.81 | 2.57 | 3.04 |
| 20 | 4.15 | 2.76 | 2.42 | 1.61 | 4.69 | 3.12 | 2.94 | 1.96 | 2.82 | 3.20 |
| 22 | 5.02 | 3.34 | 2.77 | 1.84 | 5.67 | 3.78 | 3.37 | 2.24 | 3.48 | 3.78 |
| 24 | 5.97 | 3.97 | 3.12 | 2.07 | 6.75 | 4.49 | 3.81 | 2.53 | 4.38 | 4.70 |
| 26 | 7.01 | 4.66 | 3.46 | 2.30 | 7.93 | 5.27 | 4.25 | 2.83 | 5.12 | 5.44 |
| 30 | 9.33 | 6.21 | 4.16 | 2.77 | 10.6 | 7.02 | 5.13 | 3.41 | 7.14 | 7.51 |

**Other Constants and Properties**

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<th>t_y × 10^3</th>
<th>t_x × 10^3</th>
<th>r_y, in.</th>
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*Shape is slender for compression with F_y = 65 ksi.*

Note: Heavy line indicates KL/r_y equal to or greater than 200.
### Table 6-1 (continued)

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<tr>
<th>Shape</th>
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#### Other Constants and Properties

| $b_y \times 10^3$ (kip-ft)$^1$ | 11.2 | 7.47 | 12.6 | 8.40 | 18.5 | 12.3 |
| $t_y \times 10^3$ (kips)$^1$ | 1.28 | 0.855 | 1.40 | 0.934 | 1.54 | 1.02 |
| $t_x \times 10^3$ (kips)$^1$ | 1.67 | 1.11 | 1.82 | 1.21 | 2.00 | 1.33 |
| $r_{x/y}$ | 4.78 | 4.82 | 6.19 |
| $y$ | 1.80 | 1.77 | 1.35 |

*Shape is slender for compression with $F_y = 65$ ksi.

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**  
**W-Shapes**

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### Other Constants and Properties

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<th>f_y×10^3</th>
<th>(kips)^1</th>
<th>t_y×10^3</th>
<th>(kips)^1</th>
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Notes:
- Shape is slender for compression with \( F_y = 65 \) ksi.
- Shape does not meet compact limit for flexure with \( F_y = 65 \) ksi.
- Shape does not meet the \( h/t_w \) limit for shear in AISC Specification Section G2.1(a) with \( F_y = 65 \) ksi; therefore, \( \phi_v = 0.90 \) and \( \Omega_v = 1.67 \).

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

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<th>Shape</th>
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<td>b x 10^3</td>
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<td>(kip-ft)^1</td>
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<td>ASD</td>
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### Other Constants and Properties

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<th>b x 10^3 (kip-ft)^1</th>
<th>t x 10^3 (kips)^1</th>
<th>t x 10^3 (kips)^1</th>
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<td>1.71</td>
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**Notes:**
- Shape is slender for compression with \( F_y = 65 \text{ ksi} \).
- Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
- Shape does not meet the \( h/t_w \) limit for shear in AISC Specification Section G2.1(a) with \( F_y = 65 \text{ ksi}; \) therefore, \( \phi_v = 0.90 \) and \( \Omega_v = 1.67 \).
- Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.

---

**Design Examples V14.1**

*American Institute of Steel Construction*
### Table 6-1 (continued)

Combined Flexure and Axial Force

**W-Shapes**

- **Design**
  - **Shape**
    - **W18**
  - **Effective length, KL (ft), with respect to least radius of gyration, \( r_y \)**
    - 0
    - 6
    - 7
    - 8
    - 9
    - 10
    - 11
    - 12
    - 13
    - 14
    - 15
    - 16
    - 17
    - 18
    - 19
    - 20
    - 21
    - 22
    - 23
    - 24
    - 25
    - 26
    - 27
    - 28
    - 29
    - 30
    - 31
    - 32
    - 33
    - 34
    - 35
    - 36
    - 37
    - 38
    - 39
  - **Other Constants and Properties**
    - **Properties**
      - \( b_y \times 10^3 \) (kip-ft)\(^{-1} \)
      - \( t_y \times 10^3 \) (kip)\(^{-1} \)
      - \( t_r \times 10^3 \) (kip-ft)\(^{-1} \)
      - \( r_y/r_{f_y} \)
      - \( r_{f_y} \), in.

**Design Examples V14.1**

**AMERICAN INSTITUTE OF STEEL CONSTRUCTION**
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

<table>
<thead>
<tr>
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<th>158</th>
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<td>b_x x 10^3 (kip-ft)</td>
<td>p x 10^3 (kips)</td>
</tr>
<tr>
<td>ASD</td>
<td></td>
<td></td>
<td>ASD</td>
</tr>
<tr>
<td>LRFD</td>
<td></td>
<td></td>
<td>LRFD</td>
</tr>
<tr>
<td>LRFD</td>
<td></td>
<td></td>
<td>LRFD</td>
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<td>0.620</td>
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<td>0.620</td>
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<td>0.852</td>
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</table>

#### Other Constants and Properties

- **b_y x 10^3 (kip-ft)^-1**: 2.30, 1.53, 2.59, 1.72, 2.89, 1.92
- **t_x x 10^3 (kips)^-1**: 0.457, 0.304, 0.500, 0.333, 0.555, 0.369
- **t_y x 10^3 (kips)^-1**: 0.593, 0.395, 0.649, 0.432, 0.720, 0.460
- **r_x, in.**: 2.97, 2.97, 2.96
- **t_y, in.**: 2.79, 2.76, 2.74
### Combined Flexure and Axial Force

#### W-Shapes

**Table 6-1 (continued)**

- **Design Examples V14.1**
- **American Institute of Steel Construction**

<table>
<thead>
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<td>W18x</td>
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<td>**Effective length, <em>KL</em> (ft), with respect to least radius of gyration, <em>ry</em></td>
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</tr>
<tr>
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<td>0.612</td>
<td>0.407</td>
<td>0.851</td>
</tr>
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<td>0.435</td>
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<td><strong>Other Constants and Properties</strong></td>
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<td>2.14</td>
<td>3.57</td>
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<tr>
<td>( t_y \times 10^3 ) (kips(^{-1})</td>
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<td>0.407</td>
<td>0.671</td>
</tr>
<tr>
<td>( t_r \times 10^3 ) (kips(^{-1})</td>
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<td>2.97</td>
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<td>( r_y ), in.</td>
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### Combined Flexure and Axial Force

**W-Shapes**

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<td>b_x \times 10^3</td>
<td>p \times 10^3</td>
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<td>ASD</td>
<td>LRFD</td>
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<td>0.604</td>
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### Effective Length, KL (ft), with respect to least radius of gyration, r_y

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<td>0.826</td>
<td>0.550</td>
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<tr>
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<td>0.886</td>
<td>0.589</td>
</tr>
<tr>
<td>7</td>
<td>0.908</td>
<td>0.604</td>
</tr>
<tr>
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<td>0.622</td>
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<td>11</td>
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<td>0.762</td>
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<td>14</td>
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<td>0.803</td>
</tr>
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<td>15</td>
<td>1.28</td>
<td>0.849</td>
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</tbody>
</table>

### Other Constants and Properties

<table>
<thead>
<tr>
<th>b_y \times 10^3 (kip-ft)^{-1}</th>
<th>t_y \times 10^3 (kip)</th>
<th>t_x \times 10^3 (kip)</th>
<th>r_y, in.</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2.66</td>
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\(^c\) Shape is slender for compression with \(F_y = 65\) ksi.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

<table>
<thead>
<tr>
<th>Design</th>
<th>( p \times 10^3 )</th>
<th>( b_y \times 10^3 )</th>
<th>( F_y = 65 \text{ ksi} )</th>
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</thead>
<tbody>
<tr>
<td>( W_{18} \times )</td>
<td>( p \times 10^3 )</td>
<td>( b_y \times 10^3 )</td>
<td>( p \times 10^3 )</td>
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#### Effective length, \( KL \), \( L_b \), or \( r_y \) with respect to least radius of gyration, \( r_y \), \( t_y \) x 10³ (kip-ft)⁻¹, or \( t_x \) x 10³ (kip-ft)⁻¹

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<th>( r_y ) x 10³ (kip-ft)⁻¹</th>
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#### Other Constants and Properties

- **Shape is slender for compression with** \( F_y = 65 \text{ ksi} \).
- **Shape does not meet compact limit for flexure with** \( F_y = 65 \text{ ksi} \).
- **Note:** Heavy line indicates \( KL/r_y \) equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

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### Other Constants and Properties

| $b_y \times 10^3$ (kip-ft)$^{-1}$ | 13.3     | 8.85     | 14.8     | 9.86     | 16.5     | 11.0     |
| $t_y \times 10^3$ (kip)$^{-1}$   | 1.46     | 0.971    | 1.59     | 1.06     | 1.75     | 1.16     |
| $t_r \times 10^3$ (kip)$^{-1}$   | 1.89     | 1.26     | 2.06     | 1.37     | 2.27     | 1.51     |
| $r_y \times r_y$                | 4.45     | 4.44     | 4.47     |          |          |          |
| $t_y \times r_y$                | 1.68     | 1.67     | 1.65     |          |          |          |

$^a$ Shape is slender for compression with $F_y = 65$ ksi.

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
### Combined Flexure and Axial Force

**W-Shapes**

**Table 6-1 (continued)**

- **Shape**: W18
- **$F_y = 65$ ksi**

#### Effective Length, KL (ft), with respect to least radius of gyration, $r_y$, or unbraced length, $L_b$ (ft), for X-X axis bending

$\phi_v = 0.90$ and $\Omega_v = 1.67$.

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<th>Design</th>
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#### Other Constants and Properties

- $b_y \times 10^3$ (kip-ft)$^{-1}$: 23.4, 15.6, 27.4, 18.2, 34.0, 22.6
- $t_y \times 10^3$ (kip): 1.90, 1.27, 2.18, 1.45, 2.49, 1.66
- $t_y \times 10^3$ (kips): 2.47, 1.65, 2.82, 1.88, 3.24, 2.16
- $r_y/r_y$: 5.62, 5.68, 5.77
- $r_y$, in.: 1.29, 1.27, 1.22

---

*Shape is slender for compression with $F_y = 65$ ksi.

*Shape does not meet the $h/t_w$ limit for shear in AISC Specification Section G2.1(a) with $F_y = 65$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

**W-Shapes**

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Other Constants and Properties

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### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

\[
F_y = 65 \text{ ksi}
\]

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<th>Effective length, (KL) (ft), with respect to least radius of gyration, (r_y), or Unbraced Length, (L_b) (ft), for X-X axis bending</th>
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<tbody>
<tr>
<td>(b_y \times 10^3) (kip-ft)(^{-1})</td>
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<td>(t_y \times 10^3) (kips)(^{-1})</td>
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<tr>
<td>(t_r \times 10^3) (kips)(^{-1})</td>
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<td>(r_y/r_f)</td>
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<td>(r_y), in.</td>
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\(^{2}\) Shape is slender for compression with \(F_y = 65\) ksi.

Note: Heavy line indicates \(KL/r_y\) equal to or greater than 200.
Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

\[ F_y = 65 \text{ ksi} \]

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<th>( 30^\circ )</th>
<th>( 36^\circ )</th>
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<td>( b_x \times 10^3 )</td>
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</table>

Effective length, \( KL \), with respect to least radius of gyration, \( r_y \), or Unbraced Length, \( L_b \), for X-X axis bending

\[ p \times 10^3 \times 10^3 \times 10^3 \times p \times 10^3 \times 10^3 \times 10^3 \times 10^3 \]

Other Constants and Properties

\[ b_f \times 10^3 \times (\text{kip-ft})^{-1} \]

\[ f_y \times 10^3 \times (\text{kip})^{-1} \]

\[ t_x \times 10^3 \times (\text{kip-ft})^{-1} \]

\[ r_x, r_y \]

\[ \phi = 0.90 \text{ and } \Omega = 1.67. \]

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

*W-Shapes*

\[
F_y = 65 \text{ ksi}
\]

<table>
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<th>Shape</th>
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<th>( F_y = 65 \text{ ksi} )</th>
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**Notes:**

1. Shape is slender for compression with \( F_y = 65 \text{ ksi} \).
2. Shape does not meet the \( h / t_w \) limit for shear in AISC Specification Section G2.1(a) with \( F_y = 65 \text{ ksi} \); therefore, \( \phi_y = 0.90 \) and \( \Omega_y = 1.67 \).

Note: Heavy line indicates \( KL / r_y \) equal to or greater than 200.
Table 6-1 (continued)

Combined Flexure and Axial Force
W-Shapes

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Other Constants and Properties

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^h Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

$F_y = 65$ ksi

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<td>$p \times 10^3$ (kip)(^-1)</td>
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Other Constants and Properties

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<th>$t_y \times 10^3$ (kip)(^-1)</th>
<th>$t_x \times 10^3$ (kip-ft)(^-1)</th>
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$\frac{P_y}{P_x} = 1.70$ | $\frac{P_y}{P_x} = 1.69$ | $\frac{P_y}{P_x} = 1.67$

$r_y$, in. | 4.49 | 4.43 | 4.38

$^a$ Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
### Table 6-1 (continued) Combined Flexure and Axial Force W-Shapes

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<th>$p \times 10^3$</th>
<th>$b_x \times 10^3$</th>
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<td>0.266</td>
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</table>

**Other Constants and Properties**

| $b_{y} \times 10^{3}$ (kip-ft)$^{-1}$ | 0.631  | 0.420  | 0.682  | 0.454  | 0.741  | 0.493  |
| $t_{y} \times 10^{3}$ (kips)$^{-1}$  | 0.206  | 0.137  | 0.220  | 0.146  | 0.236  | 0.157  |
| $t_{r} \times 10^{3}$ (kips)$^{-1}$  | 0.267  | 0.178  | 0.285  | 0.190  | 0.306  | 0.204  |

| $r_{y}/t_{y}$ | 1.67  | 1.66  | 1.66  |
| $r_{y}$, in. | 4.34  | 4.31  | 4.27  |

$^{a}$ Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

**W-Shapes**

**Shape**

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<th>311&lt;sup&gt;b&lt;/sup&gt;</th>
<th>283&lt;sup&gt;h&lt;/sup&gt;</th>
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#### Effective length, KL (ft), with respect to least radius of gyration, r<sub>y</sub>, or Unbraced Length, Lb (ft), for X-X axis bending

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<th>b&lt;sub&gt;x&lt;/sub&gt; x 10&lt;sup&gt;3&lt;/sup&gt;</th>
<th>p x 10&lt;sup&gt;3&lt;/sup&gt;</th>
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#### Other Constants and Properties

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<th>b&lt;sub&gt;y&lt;/sub&gt; x 10&lt;sup&gt;3&lt;/sup&gt; (kip-ft)&lt;sup&gt;-1&lt;/sup&gt;</th>
<th>t&lt;sub&gt;y&lt;/sub&gt; x 10&lt;sup&gt;3&lt;/sup&gt; (kip)&lt;sup&gt;-1&lt;/sup&gt;</th>
<th>t&lt;sub&gt;x&lt;/sub&gt; x 10&lt;sup&gt;3&lt;/sup&gt; (kip)&lt;sup&gt;-1&lt;/sup&gt;</th>
<th>r&lt;sub&gt;y&lt;/sub&gt;, in.</th>
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<sup>a</sup> Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

$F_y = 65$ ksi

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<td>$p \times 10^3$</td>
<td>$b_x \times 10^3$</td>
<td>$p \times 10^3$</td>
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Other Constants and Properties

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$r_s/r_y$ | 1.62 | 1.62 | 1.61
$r_y$, in. | 4.13 | 4.10 | 4.07

Design Examples V14.1
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

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### Other Constants and Properties

| \( b_y \times 10^3 \) (kip-ft)\(^{-1} \) | 1.52 | 1.01 | 1.68 | 1.12 | 1.88 | 1.25 |
| \( t_y \times 10^3 \) (kips)\(^{-1} \) | 0.452 | 0.301 | 0.496 | 0.330 | 0.550 | 0.366 |
| \( t_r \times 10^3 \) (kips)\(^{-1} \) | 0.587 | 0.391 | 0.644 | 0.429 | 0.714 | 0.476 |

| \( r_s/r_f \) | 1.60 | 1.60 | 1.60 |
| \( r_y \), in. | 4.05 | 4.02 | 4.00 |

---

Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Combined Flexure and Axial Force

**W-Shapes**

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<th>( b_x \times 10^3 )</th>
<th>( p \times 10^3 )</th>
<th>( b_x \times 10^3 )</th>
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**Other Constants and Properties**

- \( b_y \times 10^3 \) (kip-ft)\(^{-1}\) = \( 2.06 \times 10^{-3} \) when \( K \le 1.4 \), and \( 1.37 \times 10^{-3} \) when \( K > 1.4 \)
- \( t_y \times 10^3 \) (kip)\(^{-1}\) = \( 0.602 \times 10^{-3} \) when \( K \le 1.4 \), and \( 0.400 \times 10^{-3} \) when \( K > 1.4 \)
- \( t_r \times 10^3 \) (kip)\(^{-1}\) = \( 0.781 \times 10^{-3} \) when \( K \le 1.4 \), and \( 0.520 \times 10^{-3} \) when \( K > 1.4 \)

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<th>( r_y/r_y )</th>
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<td>1.67</td>
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Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

| W-Shapes |
|------------------|------------------|------------------|------------------|------------------|
| **Shape** | **Design** | ***P* × 10^3** | ***b_x* × 10^3** | ***P* × 10^3** | ***b_x* × 10^3** | ***P* × 10^3** | ***b_x* × 10^3** |
| | (kip) | (kip-ft)^3 | | (kip) | (kip-ft)^3 | (kip) | (kip-ft)^3 |
| | **ASD** | **LRFD** | **ASD** | **LRFD** | **ASD** | **LRFD** | **ASD** | **LRFD** |
| 0 | 0.803 | 0.534 | 1.45 | 0.962 | 0.883 | 0.587 | 1.64 | 1.09 |
| 11 | 0.904 | 0.602 | 1.45 | 0.962 | 0.996 | 0.663 | 1.64 | 1.09 |
| 12 | 0.925 | 0.615 | 1.45 | 0.962 | 1.02 | 0.678 | 1.64 | 1.09 |
| 13 | 0.948 | 0.631 | 1.45 | 0.968 | 1.04 | 0.695 | 1.64 | 1.09 |
| 14 | 0.974 | 0.648 | 1.47 | 0.971 | 1.07 | 0.714 | 1.64 | 1.09 |
| 15 | 1.00 | 0.667 | 1.50 | 0.995 | 1.10 | 0.735 | 1.67 | 1.11 |
| 16 | 1.03 | 0.687 | 1.52 | 1.01 | 1.14 | 0.758 | 1.69 | 1.13 |
| 17 | 1.07 | 0.710 | 1.54 | 1.02 | 1.18 | 0.783 | 1.72 | 1.14 |
| 18 | 1.10 | 0.735 | 1.56 | 1.04 | 1.22 | 0.811 | 1.75 | 1.16 |
| 19 | 1.15 | 0.762 | 1.58 | 1.05 | 1.26 | 0.841 | 1.78 | 1.18 |
| 20 | 1.19 | 0.792 | 1.61 | 1.07 | 1.31 | 0.874 | 1.81 | 1.20 |
| 21 | 1.22 | 0.820 | 1.66 | 1.10 | 1.37 | 0.909 | 1.84 | 1.23 |
| 22 | 1.29 | 0.860 | 1.66 | 1.10 | 1.43 | 0.951 | 1.87 | 1.24 |
| 23 | 1.41 | 0.941 | 1.71 | 1.14 | 1.57 | 1.04 | 1.93 | 1.29 |
| 24 | 1.56 | 1.041 | 1.77 | 1.18 | 1.73 | 1.15 | 2.00 | 1.33 |
| 25 | 1.74 | 1.161 | 1.83 | 1.22 | 1.93 | 1.28 | 2.08 | 1.38 |
| 26 | 1.95 | 1.291 | 1.90 | 1.26 | 2.16 | 1.44 | 2.16 | 1.44 |
| 27 | 2.20 | 1.471 | 1.97 | 1.31 | 2.45 | 1.63 | 2.25 | 1.50 |
| 28 | 2.49 | 1.661 | 2.04 | 1.36 | 2.77 | 1.84 | 2.34 | 1.56 |
| 29 | 2.79 | 1.861 | 2.12 | 1.41 | 3.10 | 2.06 | 2.45 | 1.63 |
| 30 | 3.11 | 2.071 | 2.21 | 1.47 | 3.45 | 2.30 | 2.60 | 1.73 |
| 31 | 3.44 | 2.291 | 2.33 | 1.55 | 3.83 | 2.55 | 2.78 | 1.85 |
| 32 | 3.80 | 2.531 | 2.48 | 1.65 | 4.22 | 2.81 | 2.96 | 1.97 |
| 33 | 4.17 | 2.771 | 2.62 | 1.74 | 4.63 | 3.08 | 3.13 | 2.08 |
| 34 | 4.55 | 3.031 | 2.76 | 1.84 | 5.06 | 3.37 | 3.31 | 2.20 |
| 35 | 4.96 | 3.301 | 2.91 | 1.93 | 5.51 | 3.67 | 3.46 | 2.32 |
| 36 | 5.38 | 3.581 | 3.05 | 2.03 | 5.98 | 3.96 | 3.66 | 2.43 |

**Other Constants and Properties**

- **b_y × 10^3** (kip-ft)^3
- **t_y × 10^3** (kip)^3
- **t_r × 10^3** (kip-ft)^3
- **r_x / r_y**
- **r_y, in.**

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<th><strong>t_r × 10^3</strong></th>
<th><strong>r_x / r_y</strong></th>
<th><strong>r_y, in.</strong></th>
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*Shape does not meet compact limit for flexure with *F_y* = 65 ksi.*
### Table 6-1 (continued)

Combined Flexure and Axial Force

**W-Shapes**

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**Effective length, KL (ft), with respect to least radius of gyration, \(r_y\), or Unbraced Length, \(L_b\) (ft), for X-X axis bending**

| b_y \times 10^3 (kip-ft)⁻¹ | 6.12 | 4.07 | 6.77 | 4.50 | 7.43 | 4.94 |
| t_y \times 10^3 (kips)⁻¹ | 1.07 | 0.712 | 1.18 | 0.784 | 1.28 | 0.855 |
| t_r \times 10^3 (kips)⁻¹ | 1.39 | 0.926 | 1.53 | 1.02 | 1.67 | 1.11 |

| r_x/r_y | 2.44 | 2.44 | 2.44 |
| r_y, in. | 2.48 | 2.48 | 2.46 |

**Other Constants and Properties**

---

**F_y = 65 ksi**

Design Examples V14.1

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
Table 6-1 (continued)

Combined Flexure and Axial Force

W-Shapes

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<td>p × 10^3 (kips)^{-1}</td>
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Other Constants and Properties

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<th>t_z × 10^5 (kip)</th>
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r_y, in.

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Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.
### Combined Flexure and Axial Force

#### W-Shapes

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Other Constants and Properties

- \( b_y \times 10^3 \) (kip-ft)^{-1}: 15.8, 22.6, 25.9, 17.2
- \( t_y \times 10^2 \) (kip): 2.04, 2.29, 2.57, 1.71
- \( t_x \times 10^2 \) (kip): 2.65, 2.98, 3.33, 2.22
- \( r_{x,y} \): 3.08, 3.79, 3.81
- \( r_y \), in.: 1.89, 1.55, 1.53

### Notes
- Shape is slender for compression with \( F_y = 65 \text{ ksi} \).
- Other Constants and Properties.

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.
Table 6-1 (continued)  
Combined Flexure and Axial Force  
W-Shapes

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**Effective length, K; (ft)**, with respect to least radius of gyration, rₓ, or Unbraced Length, Lb (ft), for X-X axis bending

- 0  | 62.4 |
- 6  | 65.8 |
- 7  | 7.57 |
- 8  | 8.97 |
- 9  | 11.1 |
- 10 | 13.6 |
- 11 | 16.5 |
- 12 | 19.5 |
- 13 | 23.1 |
- 14 | 26.8 |
- 15 | 30.7 |
- 16 | 34.9 |
- 17 | 39.4 |
- 18 | 45.0 |
- 19 | 51.7 |
- 20 | 58.4 |

**Other Constants and Properties**

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° Shape is slender for compression with Fᵧ = 65 ksi.

° Shape does not meet compact limit for flexure with Fᵧ = 65 ksi.

° Shape does not meet the h/tₓ limit for shear in AISC Specification Section G2.1(a) with Fᵧ = 65 ksi; therefore, φᵧ = 0.90 and Ωv = 1.67.

Note: Heavy line indicates K; rᵧ equal to or greater than 200.
### Table 6-1 (continued)
**Combined Flexure and Axial Force**

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\(^{h}\text{Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.}\)
Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

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Other Constants and Properties

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\(b_y\) Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
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### Combined Flexure and Axial Force

#### W-Shapes

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### Combined Flexure and Axial Force

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$^*$ Shape does not meet compact limit for flexure with $F_y = 65$ ksi.
### Combined Flexure and Axial Force

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### Other Constants and Properties

- **Effective length, \( K_L \) (ft), with respect to least radius of gyration, \( r_y \), or Unbraced Length, \( L_b \) (ft), for X-X axis bending**
- **Shape does not meet compact limit for flexure with \( F_y = 65 \text{ ksi} \).**

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Other Constants and Properties

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Shape does not meet compact limit for flexure with $F_y = 65$ ksi.

Note: Heavy line indicates $KL/\gamma_y$ equal to or greater than 200.
Table 6-1 (continued)  
Combined Flexure and Axial Force  
W-Shapes

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Other Constants and Properties

| bₓ × 10³ (kip-ft)⁻¹ | 16.3 | 10.9 | 23.8 | 15.9 | 28.7 | 19.1 |
| tₓ × 10³ (kips)⁻¹   | 2.20 | 1.46 | 2.49 | 1.66 | 2.92 | 1.94 |
| tᵧ × 10³ (kips)⁻¹   | 2.85 | 1.90 | 3.24 | 2.16 | 3.79 | 2.53 |
| rₓ, rᵧ               | 2.64 | 3.41 | 3.43 | 3.43 |
| rᵧ, in.              | 1.94 | 1.54 | 1.52 | 1.52 |

\* Shape is slender for compression with Fᵧ = 65 ksi.

Note: Heavy line indicates KL/rᵧ equal to or greater than 200.
### Combined Flexure and Axial Force

**W-Shapes**

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### Other Constants and Properties

- $b_y \times 10^3$ (kip-ft)$^{-1}$: 34.3, 22.8, 74.9, 49.8, 92.0, 61.2
- $t_y \times 10^3$ (kips)$^{-1}$: 3.36, 2.23, 3.96, 2.64, 4.61, 3.07
- $t_r \times 10^3$ (kips)$^{-1}$: 4.36, 2.90, 5.14, 3.43, 5.98, 3.99
- $r_s/r_y$, in.: 3.42, 5.79, 5.86
- $r_y$, in.: 1.51, 0.848, 0.822

$c$ Shape is slender for compression with $F_y = 65$ ksi.
$f$ Shape does not meet compact limit for flexure with $F_y = 65$ ksi.

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
Table 6-1 (continued)

Combined Flexure and Axial Force

W-Shapes

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Other Constants and Properties

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6 $r_y / r_y$ 6.04  6.14
7 $r_y$, in.  0.773  0.753

6 Shape is slender for compression with $F_y = 65$ ksi.
7 Shape does not meet compact limit for flexure with $F_y = 65$ ksi.
9 Shape does not meet the $h/t_w$ limit for shear in AISC Specification Section G2.1(a) with $F_y = 65$ ksi; therefore, $\psi = 0.90$ and $\Omega_c = 1.67$.

Note: Heavy line indicates $KL / r_y$ equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

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**Other Constants and Properties**

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F_y = 65 ksi

**Effective length, KL (ft), with respect to least radius of gyration, r_y**

**or Unbraced Length, L_b (ft), for X-X axis bending**

**Other Constants and Properties**

| r_y, in. | 2.68 | 2.65 | 2.63 |

**Design Examples V14.1**

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Combined Flexure and Axial Force

#### W-Shapes

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<th>$p \times 10^3$ (kip)</th>
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**Effective length, $KL$ (ft), with respect to least radius of gyration, $r_y$, or Unbraced Length, $L_b$ (ft), for X-X axis bending**

### Other Constants and Properties

- $b_y \times 10^3$ (kip-ft)$^{-1}$
- $t_y \times 10^3$ (kip)$^{-1}$
- $t_y \times 10^3$ (kip)$^{-1}$

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Other Constants and Properties

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<th>t_r × 10^3 (kip-ft)^{-1}</th>
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Note: Heavy line indicates KL/r_y equal to or greater than 200.
## Table 6-1 (continued)

**Combined Flexure and Axial Force**  
W-Shapes

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| Effective length, \( KL \) (ft), with respect to least radius of gyration, \( r_y \), or Unbraced Length, \( L_b \) (ft), for X-X axis bending |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 39                | 33                | 30                | 39                | 33                | 30                |

### Other Constants and Properties

- \( b_y \times 10^3 \) (kip-ft): 15.9 10.8 20.5 13.7 31.0 20.6
- \( t_y \times 10^5 \) (kip): 2.23 1.49 2.65 1.76 2.91 1.93
- \( t_x \times 10^5 \) (kip): 2.90 1.93 3.43 2.29 3.77 2.51
- \( r_{xy} \): 2.17 2.16 3.20
- \( r_{yx} \) in.: 1.98 1.94 1.37

Note: Shape does not meet compact limit for flexure with \( F_y = 65 \text{ ksi} \).

---

Design Examples V14.1  
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

| Shape | Design
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#### Effective Length, \( L_b \) (ft), for X-X axis bending

\[ p \times 10^3 \] (kip-ft) \[ b_x \times 10^3 \] (kip-ft)

\[ 36.5 \] \[ 24.3 \] \[ 44.9 \] \[ 29.9 \]

\[ 81.8 \] \[ 54.4 \]

\[ 3.38 \] \[ 2.25 \] \[ 3.96 \] \[ 2.63 \]

\[ 4.57 \] \[ 3.04 \]

\[ 4.38 \] \[ 2.92 \] \[ 5.14 \] \[ 3.42 \]

\[ 5.93 \] \[ 3.95 \]

\[ r_y/\ell_y \] \[ 3.20 \] \[ 3.21 \] \[ 4.74 \]

\[ r_y, \text{ in.} \] \[ 1.36 \] \[ 1.33 \] \[ 0.874 \]

\[ ^\text{Note: Heavy line indicates } KL/\ell_y \text{ equal to or greater than } 200. \]

\[ ^\text{Shape is slender for compression with } F_y = 65 \text{ ksi.} \]
## Table 6-1 (continued)

### Combined Flexure and Axial Force

*W-Shapes*

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**Other Constants and Properties**

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<th>ASD LRFD</th>
<th>ASD LRFD</th>
<th>ASD LRFD</th>
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<td>0.785</td>
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*Shape is slender for compression with $F_y = 65$ ksi.*

*Shape does not meet compact limit for flexure with $F_y = 65$ ksi.*

Note: Heavy line indicates $KL/r_y$ equal to or greater than 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

**W-Shapes**

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**Other Constants and Properties**

- $b_y \times 10^3$ (kip-ft)$^{-1}$
  - 8.38
- $t_y \times 10^3$ (kips)$^{-1}$
  - 1.30
- $t_x \times 10^3$ (kips)$^{-1}$
  - 1.69
- $r_x / r_y$
  - 1.75
- $r_y$, in.
  - 2.12

Note: Heavy line indicates $KL / r_y$ greater than or equal to 200.
### Table 6-1 (continued)

**Combined Flexure and Axial Force**

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### Other Constants and Properties

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<th>tᵧ × 10² (kips)</th>
<th>tₓ × 10² (kips)</th>
<th>rₓ/rᵧ</th>
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Note: Heavy line indicates KL/rᵧ equal to or greater than 200.

Shape does not meet compact limit for flexure with Fᵧ = 65 ksi.
### Combined Flexure and Axial Force

#### W-Shapes

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<th>$b_x \times 10^3$ (kip-ft)$^{-1}$</th>
<th>$p \times 10^3$ (kips)$^{-1}$</th>
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#### Effective length, $KL$ (ft), with respect to least radius of gyration, $r_y$, or Unbraced Length, $L_b$ (ft), for X-X axis bending:

- $b_y \times 10^3$ (kip-ft)$^{-1}$: 20.4, 13.6, 27.1, 18.1
- $t_y \times 10^3$ (kip)$^{-1}$: 2.81, 1.87, 3.11, 2.07
- $t_r \times 10^3$ (kip)$^{-1}$: 3.65, 2.43, 4.04, 2.89

#### Other Constants and Properties:

- $r_x/r_y$: 1.72, 2.13
- $r_y$, in.: 2.02, 1.62

**Note:** Heavy line indicates $KL/r_y$ greater than or equal to 200.

---

*Shape does not meet compact limit for flexure with $F_y = 65$ ksi.*
### Table 6-1 (continued)

#### Combined Flexure and Axial Force

**W-Shapes**

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<th>Effective length, KL (ft), with respect to least radius of gyration, ( r_y ), or Unbraced Length, ( L_b ) (ft), for X-X axis bending</th>
<th>( b_y \times 10^3 ) (kip-ft)(^{-1})</th>
<th>( t_y \times 10^3 ) (kips)(^{-1})</th>
<th>( t_r \times 10^3 ) (kips)(^{-1})</th>
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\(^1\) Shape does not meet compact limit for flexure with \( F_y = 65 \text{ ksi} \).

Note: Heavy line indicates \( KL/r_y \) equal to or greater than 200.

---

Other Constants and Properties

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Other Constants and Properties

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\(^\dagger\) Shape is slender for compression with $F_y = 65$ ksi.

\(^\ddagger\) Shape does not meet compact limit for flexure with $F_y = 65$ ksi.

Note: Heavy line indicates $KL/r_y$ greater than or equal to 200.